## Algebraic and Noncommutative approaches to Quantum Field Theory

Antoine Géré

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# Part I (blablabla)

#### Chapter 1

### (blablabla)

- 1.1 (blablabla)
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