Avanzamento tesi

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General picture

Perturbative Quantum Field Theory.

Framework with which appear **powers of distributions**,

in general **ill defined**.

To look at this isue we chose to work with the **analytic regularisation**,

used with **success on Minkowski spacetime** (see [?] & [?]).

We will look at this issue on **curved space time** .

- Classical field theory.
- Quantum level, then pertubative theory.
- Overview of the existing tools.

Scalar field theory [?] [?]

Classical scalar field theory

(M, g): Minkowski spacetime.

Lagrangian: $L = g^{ab}\partial_a\phi\partial_b\phi + m^2\phi^2$.

Eq. of motion: $P\phi \doteq (\Box + m^2)\phi = 0$.

$\Delta = (\Delta_{adv} - \Delta_{ret}) \in D'(M^2)$.

Δ : causal propagator.

$\forall\phi \in \mathcal{S}(M),\ \phi = \Delta f,\ f \in D(M, \mathbb{R})$.

$(\mathcal{S}(M), \sigma)$: Symplectic space.

$\sigma(\phi_f, \phi_g) = \langle f, \Delta g\rangle \doteq \Delta(f, g)$.

To implemente dynamic: $\phi(Pf) = 0$.

Quantum scalar field theory

To promote $\phi(f)$ to a proper quantum field: $[\phi(f_1), \phi(f_2)] = i\Delta(f_1, f_2)\mathbb{I}$.

We have a $*$-algebra over the fields.

Perturbative theory $\rightarrow$ powers of distributions ill defined at the origin.

e.g. $L(x) = L_{free} + L_{int}$, with $L_{int} = - :\phi^4(x):$

$$:\phi(x)^4: \star :\phi(y)^4: = :\phi(x)^4\phi(y)^4: + 16 :\phi(x)^3\phi(y)^3: \Delta^+(x,y) + 72 :\phi(x)^2\phi(y)^2: \left(\Delta^+(x,y)\right)^2 + \dots$$

$$:\phi(x)^4: \star_T :\phi(y)^4: = :\phi(x)^4\phi(y)^4: + 16 :\phi(x)^3\phi(y)^3: \Delta_F(x,y) + 72 :\phi(x)^2\phi(y)^2: (\Delta_F(x,y))^2 + \dots$$

[?]

$u : C_0^\infty(\mathbb{R}^n) \to \mathbb{R}$ distribution : $\mathcal{D}'(\mathbb{R}^n)$

Singular support

$$singsupp(u) \doteq \{x \in \mathbb{R}^n \mid \exists U_x \ni x \ : \ u|_{U_x} \notin C^\infty(U_x)\}$$

Wave Front set

$$WF(u) = \left\{(x,k) \in T^*\mathbb{R}^n \setminus \{0\} \mid x \in singsupp(u), \ \forall f \ : \ \hat{fu} \ \begin{matrix}\text{does not decay} \\ \text{rapidly in direction k}\end{matrix} \right\}$$

Microlocal Analysis

if $u, v \in \mathcal{D}'(\mathbb{R}^n)$ and $WF(u) \oplus WF(v) \neq 0$,
then $\exists!\ u.v \in \mathcal{D}'(\mathbb{R}^n)$.

with $WF(u) \oplus WF(v) = \{(x, k_1 + k_2) \mid (x, k_1) \in WF(u)\ ;\ (x, k_2) \in WF(v)\}$.

[?]

$u : C_0^\infty(\mathbb{R}^n) \to \mathbb{R}$ distribution : $\mathcal{D}'(\mathbb{R}^n)$

Scaling degree

$u \in \mathcal{D}'(\mathbb{R}^n)$

$$sd(u) \doteq inf \left\{ \omega \in \mathbb{R}^n \mid \lim_{\rho \to 0} \rho^\omega u_\rho = 0 \right\}, \quad u_\rho(f) = \int_{\mathbb{R}^d} dx \; u(\rho x) f(x).$$

Theorem

$v \in D'(\mathbb{R}^d \backslash \{0\})$ *distribution not define in* 0.

- $sd(v) < d \;\Rightarrow\; \exists!$ *extension* $\tilde{v} \in \mathcal{D}'(\mathbb{R}^d)$ *of* v*:* $sd(\tilde{v}) = sd(v)$.
- $sd(v) \geq d \;\Rightarrow\; \exists$ *extensions* $\tilde{v} \in \mathcal{D}'(\mathbb{R}^d)$ *of* v*:* $sd(\tilde{v}) = sd(v)$, *uniquely defined on a finite set of test functions.*

Goal

Using the **analytic regularisation** , develop **a way to compute** these extended distributions in **curved spacetime** .

Analytic regularisation , e.g. regularise $\Delta_F(x,y)^{n+\lambda}$, $n \in \mathbb{N}, \lambda \in \mathbb{C}$

$$\tilde{\Delta}_F(x,y)^n = \lim_{\lambda \to 0} \Delta_F(x,y)^{n+\lambda} \; - \; (poles).$$

The noncommutative algebra $\mathbb{R}^3_\lambda$, [?]

$\mathbb{R}^3_\lambda$ is a Poisson algebra of function on $\mathbb{R}^3$ endowed with the Wick-Voros product,

$$\phi \star \psi(x) = exp\left[\frac{\lambda}{2}(\delta_{\mu\nu}x_0 + i\epsilon_{\mu\nu\rho}x_\rho)\frac{\partial}{\partial y_\mu}\frac{\partial}{\partial z_\nu}\right]\phi(y)\psi(z)\Big|_{y=z=x},$$

with the coordinate functions x_μ, $[x_\mu, x_\nu]_\star = i\lambda\epsilon_{\mu\nu\rho}x_\rho$, $(\mu = 1, 2, 3)$.

Gauge action, [?]

The vector fields are the derivations of the algebra $\mathbb{R}^3_\lambda$,
$Der(\mathbb{R}^3_\lambda) = \left\{D_\mu = \frac{i}{\kappa^2}[x_\mu, .],\ \mu = 1, 2, 3\right\}$.
Connection , linear map $\nabla : \mathbb{R}^3_\lambda \times Der(\mathbb{R}^3_\lambda) \to \mathbb{R}^3_\lambda$, $\nabla_{D_\mu} = D_\mu + A_\mu$, with
$A_\mu = \nabla_{D_\mu}(\mathbb{I})$, and $A^{cov}_\mu = A_\mu + \frac{i}{\kappa^2}x_\mu$.
Curvature (of ∇), linear map $F : \mathbb{R}^3_\lambda \to \mathbb{R}^3_\lambda$,
$F_{\mu\nu} = (D_\mu A_\nu - D_\nu A_\mu) + [A_\mu, A_\nu] + \frac{\lambda}{\kappa^2}\epsilon_{\mu\nu\rho}A_\rho$.
Action (massless),
$S_{cl} = Tr\left[F^\dagger_{\mu\nu}F_{\mu\nu} + \gamma\epsilon_{\mu\nu\rho}\left(A^{cov}_\mu F_{\nu\rho} + \zeta A^{cov}_\mu A^{cov}_\nu A^{cov}_\rho\right) + \mu A^{cov}_\mu A^{cov}_\mu\right]$
(vacuum $A_\mu = 0$).

A. Géré, P. Vitale, and J.-C. Wallet. *Quantum gauge models on noncommutative 3-d space.* **(2013)** [?]

Gauge fixed action

$$
\begin{aligned}
S &= S_{free} + S_{int} \\
S_{free} &= Tr\left(A_\mu\left[-2\delta_{\mu\nu}D^2\right]A_\nu - \bar{c}D^2c\right) \\
S^3_{int} &= Tr\left(4iA_\mu A_\nu(D_\mu A_\nu) - 4iA_\mu A_\nu(D_\nu A_\mu) - \frac{8i\lambda}{3\kappa^2}\epsilon_{\mu\nu\rho}A_\mu A_\nu A_\rho - i(D_\mu\bar{c})[A_\mu, c]\right)
\end{aligned}
$$

One point function

Conclusion

Classical vacuum not stable against the quantum fluctuation.
We cannot keep the masslessness imposed at classical level.

No infrared divergence.

Seminar on the Weyl quantization

Prof. Claudio Bartocci and Prof. Nicola Pinamonti.
[?]

Seminar on representation theory

Workshop on Applied Harmonic Analysis - Genova, sept. 2013.
Prof. Filippo De Mari and Prof. Ernesto De Vito.
[?]

Scientific activities

Workshop - Foundations and Constructive Aspects of QFT
University of Wuppertal,
Germany, May 31 - June 1, 2013
Financial support from the University of Wuppertal.

Summer Graduate School - Mathematical General Relativity
Cortona, Italy, July 29 - August 9, 2013
Granted full support for accomodation and board.

Workshop - Noncommutative Field Theory and Gravity
Corfu, Greece, September 8 - 15, 2013
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References