04.0012.C

$$P_A(\lambda) = \begin{vmatrix} 1-\lambda & 0 & -1 \\ -1 & 1-\lambda & 0 \\ 0 & -1 & 1-\lambda \end{vmatrix} \qquad C_1 \leftarrow C_1 + (1-\lambda)C_3$$

$$= \begin{vmatrix} 0 & 0 & \boxed{-1} \\ -1 & 1-\lambda & 0 \\ (1-\lambda)^2 & -1 & 1-\lambda \end{vmatrix}$$

$$= (-1)^{1+3} \cdot (-1) \cdot \begin{vmatrix} -1 & 1-\lambda \\ (1-\lambda)^2 & -1 \end{vmatrix}$$

$$= (-1) \cdot \left(1 - (1-\lambda)^3 \right)$$

$$P_A(\lambda) = (1-\lambda)^3 - 1$$

$$P_A(\lambda) = 0 \iff (1-\lambda)^3 - 1 = 0$$

$$\iff (1-\lambda)^3 = 1$$

$$\iff \lambda = 0$$

Donc $\text{Spec}(A) = \{0\}$ \qquad car $\lambda \in \mathbb{R}$.