$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad A = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$Sp(A) = \{2, 1, 0\}$$

$$X_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad X = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$X_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad P = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

on $\quad X' = AX \iff X' = PDP^{-1}X$

$$\iff (P^{-1}X)' = D(P^{-1}X)$$

on pose $\quad Y = P^{-1}X$

d'où $\quad Y' = DY \iff \begin{cases} y_1' = 0 \\ y_2' = y_2 \\ y_3' = 2y_3 \end{cases} \iff \begin{cases} y_1 = cte \\ y_2 = \lambda e^t \\ y_3 = \beta e^{2t} \end{cases}$

donc $\quad X = PY \iff \begin{pmatrix} x \\ y \\ z \end{pmatrix} = P \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$