

On pose  $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$  et  $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 1 \end{pmatrix}$

$$Sp(A) = \{0, 6\}$$

$$X_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$X_3 = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

$$X_2 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

Donc  $A$  est diagonalisable

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

On a  $X' = AX \Leftrightarrow X' = PD P^{-1}X$

$$\Leftrightarrow P^{-1}X' = D P^{-1}X$$

on pose  $Y = P^{-1}X$  d'où  $Y' = DY$

donc  $y_1 = c_1 e_1$

$y_2 = c_2 e_2$

$y_3 = \lambda e^{6t}$   $\lambda \in \mathbb{R}$

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = P \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$