[rdm - 0017]

① $h = 2 + 1 - 3 \times 1 = 0 \Rightarrow$ Isostatique.

② PFS
$$\begin{cases} X_A = 0 \\ Y_A - F + Y_B - F = 0 \\ (\text{en } A) \quad -FL + Y_B 2L - F3L = 0 \end{cases}$$

donc
$$\begin{cases} X_A = 0 \\ Y_B = 2F \\ Y_A = 0 \end{cases}$$

③ 1ère coupe : $x \in [0; L]$

$$\begin{cases} N + X_A = 0 \\ T + Y_A = 0 \\ (\text{à la coupe}) \quad M - Y_A \cdot x = 0 \end{cases}$$

Donc $|N = T = M = 0$

2ème coupe : $x \in [L; 2L]$

$$\begin{cases} N + X_A = 0 \\ T + Y_A - F = 0 \\ (\text{à la coupe}) \quad M - Y_A \cdot x + F(x - L) = 0 \end{cases}$$

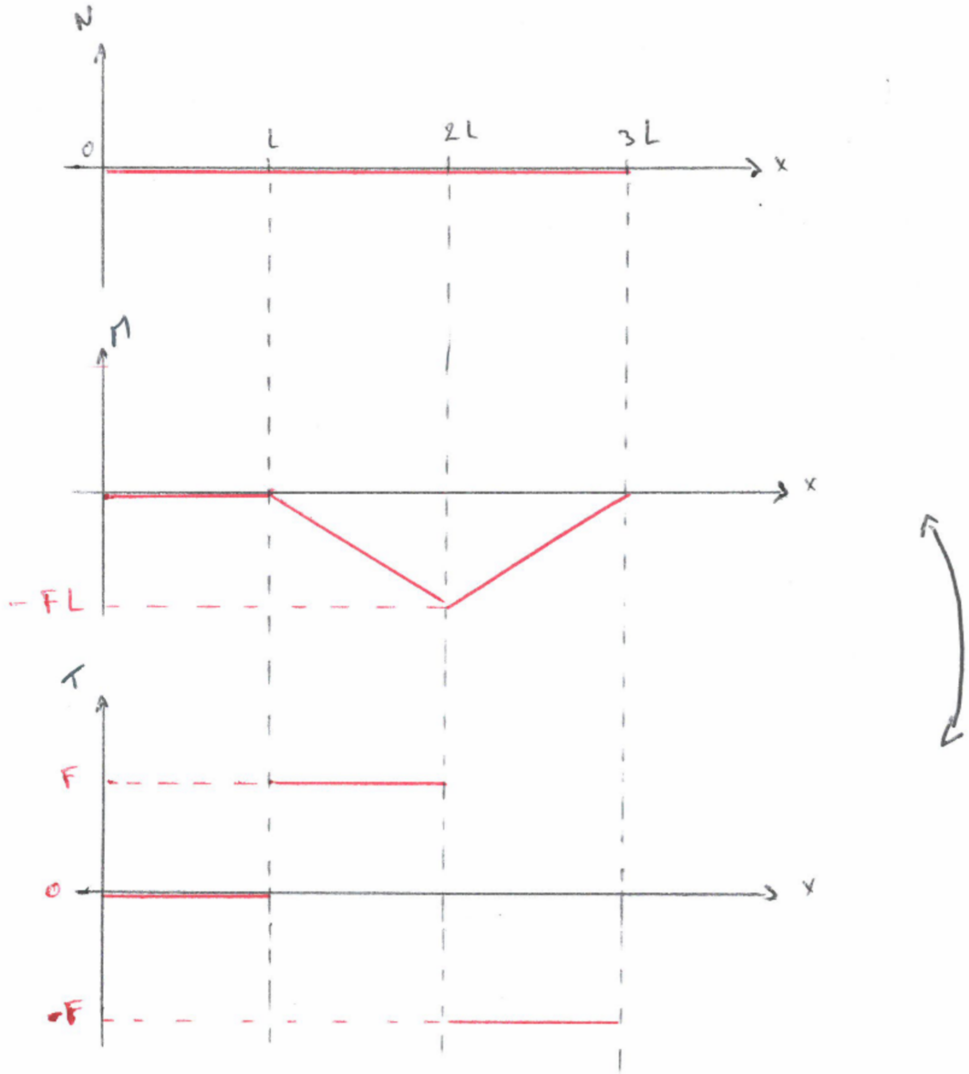
donc
$$\begin{cases} N = 0 \\ T = F \\ M = F(L - x) \end{cases}$$

3ème coupe : $x \in [2L; 3L]$

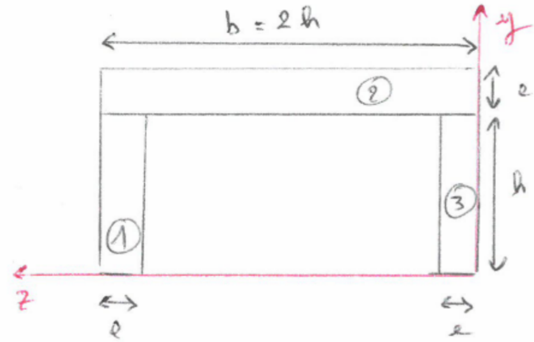
$$\begin{cases} N + X_A = 0 \\ T + Y_A - F + Y_B = 0 \\ (\text{à la coupe}) \quad M - Y_A \cdot x + F(x - L) - Y_B(x - 2L) = 0 \end{cases}$$

Donc $\begin{cases} N = 0 \\ T = -F \\ M = F(L-x) + 2F\,(x-2L) = F(x-3L) \end{cases}$

On peut alors tracer les diagrammes des efforts internes



4.a



Dans le repère $(y O z)$,

$$G_1 \left(2h - \frac{e}{2} \, , \, \frac{h}{2} \right)$$

$$G_2 \left(h \, , \, h + \frac{e}{2} \right)$$

$$G_3 \left(\frac{e}{2} \, , \, \frac{h}{2} \right)$$

$z \qquad y$

surface de 1 : $S_1 = e h$

" " 2 : $S_2 = 2 h e$

" " 3 : $S_3 = e h$

Donc,

$$z_G = \frac{S_1 \, z_{G_1} + S_2 \, z_{G_2} + S_3 \, z_{G_3}}{S_1 + S_2 + S_3}$$

$$= \frac{eh\left(2h - \frac{e}{2}\right) + 2he\,h + eh\,\frac{e}{2}}{4he}$$

$$= \frac{1}{4} \left(2h - \frac{e}{2} + 2h + \frac{e}{2} \right)$$

$$z_G = h$$

et,

$$y_G = \frac{1}{4} \left(\frac{h}{2} + 2\left(h + \frac{e}{2}\right) + \frac{h}{2} \right)$$

$$= \frac{3h + e}{4}$$

3.b

$$I_{G_1 z}^{(1)} = \frac{e h^3}{12} = I_{G_3 z}^{(3)}$$

$$I_{G_2 z}^{(2)} = \frac{2 h e^3}{12} = \frac{h e^3}{6}$$

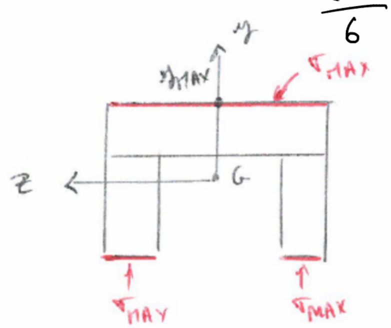
donc,

$$I_{Gz} = \frac{e h^3}{12} \times 2 + \frac{h e^3}{6} + S_1 \times \left(\frac{3h+e}{4} - \frac{h}{2}\right)^2 + S_2 \times \left(\frac{3h+e}{4} - h - \frac{e}{2}\right)^2 + S_3 \times \left(\frac{3h+e}{4} - \frac{h}{2}\right)^2$$

$$= \frac{e h^3}{6} + \frac{e^3 h}{6} + S_1 \times \left(\frac{h+e}{4}\right)^2 + S_2 \times \left(\frac{h+e}{4}\right)^2 + S_3 \left(\frac{h+e}{4}\right)^2$$

$$I_{Gz} = \frac{e h^3}{6} + \frac{e^3 h}{6} + 4 e h \left(\frac{h+e}{4}\right)^2$$

3.c

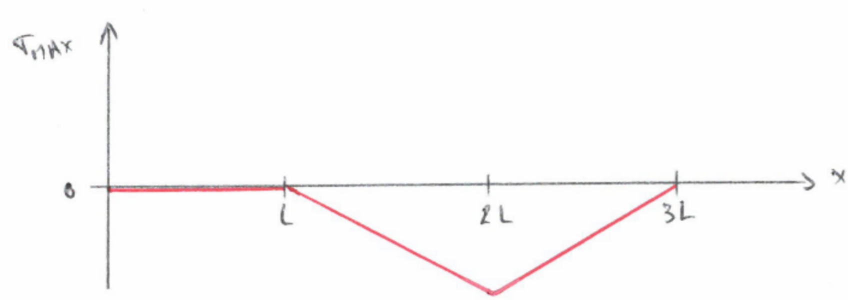


$$\sigma_{MAX} = \frac{|Mf_z|}{I_{Gz}} \, y_{MAX}$$

avec $y_{MAX} = (h+e) - \left(\frac{3h+e}{4}\right) = \frac{h+3e}{4}$

- $\underline{x \in [0; L]}$ $\quad \sigma_{MAX} = 0$
- $\underline{x \in [L; 2L]}$ $\quad \sigma_{MAX} = \frac{F}{I_{Gz}} (L - x) \cdot y_{MAX}$
- $\underline{x \in [2L; 3L]}$ $\quad \sigma_{MAX} = \frac{F}{I_{Gz}} (x - 3L) \cdot y_{MAX}$

3.d



(3.e) $\sigma_{xx} = -\dfrac{M_{fz}}{I_{Gz}}\, y$

- $x \in [0; L]$ $\quad \sigma_{xx} = 0$

- $x \in [L; 2L]$ $\quad \sigma_{xx} = -\dfrac{F}{I_{Gz}}(L-x)\,y$

- $x \in [2L; 3L]$ $\quad \sigma_{xx} = -\dfrac{F}{I_{Gz}}(x-3L)\,y$

(3.f) la contrainte de l'ensemble de la structure est localisée en $x = 2L$ et y_{MAX}.