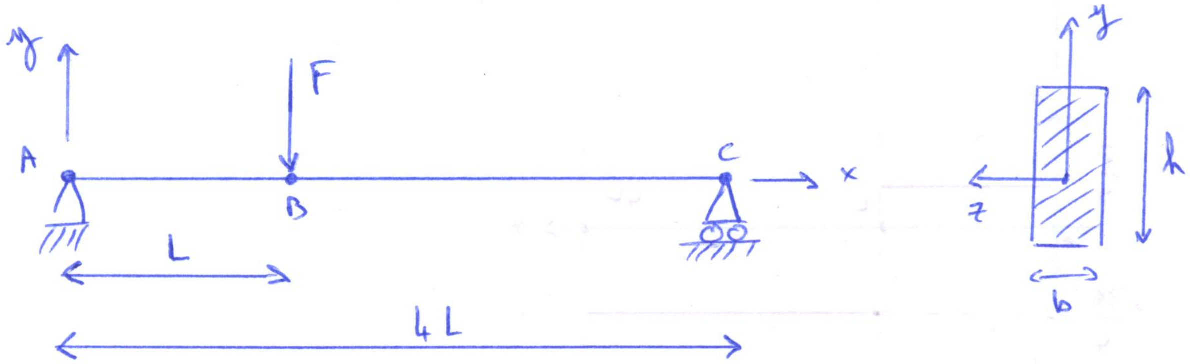
[sdm - 0015]



① Inconnues de liaison

$$\begin{cases} X_A = 0 \\ Y_A + Y_C - F = 0 \\ (\text{en } A) : -FL + Y_C \cdot 4L = 0 \end{cases}$$

Donc $\begin{cases} X_A = 0 \\ Y_C = \dfrac{F}{4} \\ Y_A = \dfrac{3F}{4} \end{cases}$

② 1ère coupe : $0 \leq x \leq L$

$$\begin{cases} N + X_A = 0 \\ T + Y_A = 0 \\ (\text{à la coupe}) \quad M - Y_A x = 0 \end{cases}$$

Donc $\begin{cases} N = 0 \\ T = -\dfrac{3F}{4} \\ M = \dfrac{3F}{4} x \end{cases}$

2ème coupe : $L \leq x \leq 4L$

$$\begin{cases} N + X_A = 0 \\ T - F + Y_A = 0 \\ (\text{à la coupe}) \quad M + F(x-L) - Y_A x = 0 \end{cases}$$

Donc $\begin{cases} N = 0 \\ T = \dfrac{F}{4} \\ M = F\left(L - \dfrac{x}{4}\right) \end{cases}$

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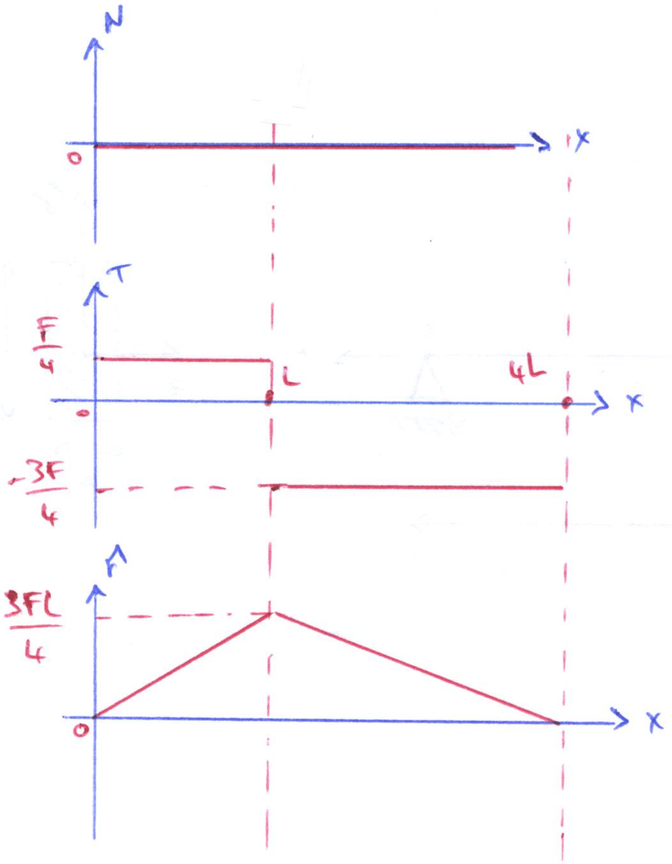

4 On a $\frac{d^2v}{dx^2} = \frac{Mf_z}{EI_z}$

et $\frac{d\theta}{dx} = \frac{Mf_z}{EI_z}$ avec $I_z = \frac{bh^3}{12}$

cas $x \in [0;L]$

$$\theta(x) = \int \frac{Mf_z}{EI_z} dx = \frac{1}{EI_z} \int \frac{3F}{4} x \, dx = \frac{3F}{4EI_z} \cdot \frac{x^2}{2} + C_1$$

$$\theta(x) = \frac{3F}{8E} \cdot \frac{12}{bh^3} \cdot x^2 + C_1$$

$$V(x) = \iint \frac{Mf_z}{EI_z} dx\, dx = \frac{1}{EI_z} \iint \frac{3F}{4} x \, dx dx$$

$$= \frac{3F}{4EI_z} \frac{x^3}{6} + C_1 x + C_2$$

$$V(x) = \frac{F x^3}{\cancel{12} E} \cdot \frac{\cancel{12}}{bh^3} + C_1 x + C_2$$

Cas $x \in [L ; 4L]$

$$\theta(x) = \int \frac{M_{fz}}{EI_z}\,dx = \frac{1}{EI_z}\int F\left(L - \frac{x}{4}\right)dx$$

$$= \frac{1}{EI_z}\left(FLx - \frac{Fx^2}{8}\right) + C_3$$

$$\theta(x) = \frac{12}{Ebh^3} F\left(Lx - \frac{x^2}{8}\right) + C_3$$

$$V(x) = \iint \frac{M_{fz}}{EI_z}\,dx\,dx = \frac{1}{EI_z}\iint F\left(L - \frac{x}{4}\right)dx\,dx$$

$$= \frac{12}{Ebh^3}\int\left(\left(FLx - F\frac{x^2}{8}\right) + C_3\right)dx$$

$$V(x) = \frac{12}{Ebh^3}\left(FL\frac{x^2}{2} - F\frac{x^3}{24}\right) + C_3x + C_4$$

conditions limites : $V(0) = 0$ ① et $V(4L) = 0$ ②

conditions de continuité : $V(L^-) = V(L^+)$ ③

$\theta(L^-) = \theta(L^+)$ ④

① $\Rightarrow V(0) = \boxed{C_2 = 0}$

④ $\Rightarrow \frac{3F}{8E}\cdot\frac{12}{bh^3}L^2 + C_1 = \frac{12}{Ebh^3}FL^2\cdot\frac{7}{8} + C_3$

On pose $\alpha = \frac{12FL^2}{Ebh^3}$.

donc $\frac{3}{8}\alpha + C_1 = \frac{7}{8}\alpha + C_3$

donc $\boxed{C_1 - C_3 = \frac{\alpha}{2}}$ (a)

② $\Rightarrow$ $\alpha\left(8L - \frac{8}{3}L\right) + 4L\,C_3 + C_4 = 0$

$\boxed{\alpha L \cdot \frac{16}{3} + 4L\,C_3 + C_4 = 0}$ (b)

③ $\Rightarrow$ $\frac{F l^3}{E b h^3} + C_1 l + \underbrace{C_2}_{=0} = \frac{12}{E b h^3}\left(\frac{FL^3}{2} - \frac{FL^3}{24}\right) + C_3 l + C_4$

$\frac{\alpha L}{12} + C_1 L = \alpha\left(\frac{L}{2} - \frac{L}{24}\right) + C_3 L + C_4$

$\boxed{\frac{\alpha L}{12} + C_1 L = \frac{11L}{24}\alpha + C_3 L + C_4}$ (c)

avec (a) et (c) $\Rightarrow$ $\frac{\alpha L}{12} + \frac{\alpha L}{2} + \cancel{C_3 L} = \frac{11L}{24}\alpha + \cancel{C_3 L} + C_4$

donc $C_4 = \alpha L\left(\frac{1}{12} + \frac{1}{2} - \frac{11}{24}\right)$

$\boxed{C_4 = \frac{\alpha L}{8}}$ (d)

avec (b) et (d) $\Rightarrow$ $\alpha L\,\frac{16}{3} + 4L\,C_3 + \frac{\alpha L}{8} = 0$

donc $C_3 = -\alpha\left(\frac{16}{4 \cdot 3} + \frac{1}{4 \cdot 8}\right)$

$\boxed{C_3 = \frac{-131}{96}\alpha}$ (e) avec (a) et (e) $\Rightarrow$ $c_1 = \frac{\alpha}{2} + c_3$

$$\boxed{c_1 = -\frac{7}{8}\alpha}$$