Regression Notes

Useful Link

- http://web.pdx.edu/~newsomj/mvclass/ho_link.pdf explanation of link functions

TABLE 1 Transformation model. Interpretation of linear predictors $\boldsymbol{x}^\top\boldsymbol{\beta}$ under different link functions $g = F^{-1}$

Link F^{-1}	Interpretation of $\mathbf{x}^\top\boldsymbol{\beta}$
probit	$\mathbb{E}(\alpha(Y)\|\mathbf{x}) = \mathbf{x}^\top\boldsymbol{\beta}$
logit	$\frac{F_{Y\|\mathbf{X}=\mathbf{x}}(y\|\mathbf{x})}{1-F_{Y\|\mathbf{X}=\mathbf{x}}(y\|\mathbf{x})} = \exp\left(-\mathbf{x}^\top\boldsymbol{\beta}\right)\frac{F_Y(y)}{1-F_Y(y)}$
cloglog	$1 - F_{Y\|\mathbf{X}=\mathbf{x}}(y\|\mathbf{x}) = (1 - F_Y(y))^{\exp(-\mathbf{x}^\top\boldsymbol{\beta})}$
loglog	$F_{Y\|\mathbf{X}=\mathbf{x}}(y\|\mathbf{x}) = F_Y(y)^{\exp(\mathbf{x}^\top\boldsymbol{\beta})}$

The parameters $\boldsymbol{\beta}$ describe a deviation from the baseline distribution F_Y in terms of the linear predictor $\boldsymbol{x}^\top\boldsymbol{\beta}$. For a probit link, the linear predictor is the conditional mean of the transformed counts $\alpha(Y)$. This interpretation, except for the fact that the intercept is now understood as being part of the transformation function α, is the same as in the traditional approach of first transforming the counts and only then estimating the mean using least-squares. However, the transformation α is not heuristically chosen or defined a priori but estimated from data through parameters $\boldsymbol{\vartheta}$, as explained below. For a logit link, $\exp\left(-\boldsymbol{x}^\top\boldsymbol{\beta}\right)$ is the odds ratio comparing the conditional odds $F_{Y|\mathbf{X}=\mathbf{x}}/1 - F_{Y|\mathbf{X}=\mathbf{x}}$ with the baseline odds $F_Y/1 - F_Y$. The complementary log-log (cloglog) link leads to a discrete version of the Cox proportional hazards model, such that $\exp\left(-\boldsymbol{x}^\top\boldsymbol{\beta}\right)$ is the hazard ratio comparing the conditional cumulative hazard function $\log(1 - F_{Y|\mathbf{X}=\mathbf{x}})$ with the baseline cumulative hazard function $\log(1 - F_Y)$. The log-log link leads to the reverse time hazard ratio with multiplicative changes in $\log(F_Y)$. All models in Table 1 are parameterized to relate positive values of $\boldsymbol{x}^\top\boldsymbol{\beta}$ to larger means independent of the specified link $g = F^{-1}$.