Proyecciones Matemáticas Del Caribe
Universidad del Atlántico - Colombia
Facultad de Ciencias Básicas
©Programa de Matemáticas

Vol. I, Nº 1, Julio (2014)
Matemáticas: 1–**??**

Separation axioms on enlargements of generalized topologies

C. Carpintero, N. Rajesh E. Rosas

Recibido Junio 20, 2013 Aceptado Julio 2, 2013

Abstract

The concept of enlargement of a generalized topology was introduced by Császár [3]. In this paper, we introduce the notion of κ-T_i (i = 0, 1/2, 1, 2) and study some properties of them.

1. Introduction

Let X be a nonempty set and μ be a collection of subsets of X. Then μ is called a generalized topology on X if and only if $\emptyset \in g$ and $G_i \in g$ for $i \in I \neq \emptyset$ implies $\underset{i\in I}{\cup} G_i \in g$. We call the pair (X, g) a generalized topological spaces on X. The members of μ are called μ-open sets [1] and the complement of a μ-open set is called a μ-closed set. The generalized-closure of a set A of X, denoted by $c_\mu(A)$, is the intersection of all μ-closed sets containing A and the generalized-interior of A, denoted by $i_\mu(A)$, is the union of μ-open sets included in A. Let μ be a generalized topology on X. A mapping $\kappa : \mu \to P(X)$ is called an enlargement [3] on X if $M \subseteq \kappa M$ ($= \kappa(M)$) whenever $M \in \mu$. Let μ be a generalized topology on X and $\kappa : \mu \to P(X)$ an enlargement of μ. Let us say that a subset $A \subseteq X$ is κ_μ-open [3] if and only if $x \in A$ implies the existence of a μ-open set M such that $x \in M$ and $\kappa M \subseteq A$. The collection of all κ_μ-open sets is a generalized topology on X [3]. A subset $A \subseteq X$ is said to be κ_μ-closed if and only if $X \backslash A$ is κ_μ-open [3]. The set c_κ (briefly $c_\kappa A$) is defined in [3] as the following:

$c_\kappa(A) = \{x \in X : \kappa(M) \cap A \neq \emptyset$ for every μ-open set M containing x$\}$.

2. Preliminaries

Definition 2.1. *[3] Let (X, μ) and (Y, ν) be a generalized topological spaces. A function $f : (X, \mu) \to (Y, \nu)$ is said to be (κ, λ)-continuous if $x \in X$ and $N \in \upsilon$, $f(x) \in N$ imply the existence of $M \in \mu$ such that $x \in M$ and $f(\kappa M) \subset \lambda N$.*

Keywords: Generalized Topology, enlargements.

AMSC(2013): 54A05, 54A10, 54D10

Theorem 2.2. *[3] Let (X,μ) and (Y,ν) be a generalized topological spaces and $f:(X,\mu)\to(Y,\nu)$ a (κ,λ)-continuous function. Then the following hold:*

1. *$f(c_\kappa(A))\subset c_\lambda(f(A))$ holds for every subset A of (X,μ).*
2. *for every λ_ν-open set B of (Y,ν), $f^{-1}(B)$ is κ_μ-open in (X,μ).*

3. Enlargement-separation axioms

Definition 3.1. *Let $\kappa:\mu\to P(X)$ be an enlargement and A a subset of X. Then the κ_μ-closure of A is denoted by $c_{\kappa_\mu}(A)$ and is defined as the intersection of all κ_μ-closed sets containing A.*

Remark 3.2. *Since The collection of all κ_μ-open sets is a generalized topology on X, then for any $A\in X$, $c_{\kappa_\mu}(A)$ is a κ_μ-closed set*

Proposition 3.3. *Let $\kappa:\mu\to P(X)$ be an enlargement and A a subset of X. Then $c_{\kappa_\mu}(A)=\{y\in X : V\cap A\neq\emptyset$ for every $V\in\kappa_\mu$ such that $y\in V\}$.*

Demostración. Denote $E=\{y\in X : V\cap A\neq\emptyset$ for every $V\in\kappa_\mu$ such that $y\in V\}$. We shall prove that $c_{\kappa_\mu}(A)=E$. Let $x\notin E$. Then there exists a κ_μ-open set V containing x such that $V\cap A=\emptyset$. This implies that $X\backslash V$ is κ_μ-closed and $A\subset X\backslash V$. Hence $c_{\kappa_\mu}(A)\subset X\backslash V$. It follows that $x\notin c_{\kappa_\mu}(A)$. Thus we have that $c_{\kappa_\mu}(A)\subset E$. Conversely, let $x\notin c_{\kappa_\mu}(A)$. Then there exists a κ_μ-closed set F such that $A\subset F$ and $x\notin F$. Then we have that $x\in X\backslash F$, $X\backslash F\in\kappa_\mu$ and $(X\backslash F)\cap A=\emptyset$. This implies that $x\notin E$. Hence $E\subset c_{\kappa_\mu}(A)$. Therefore $c_{\kappa_\mu}(A)=E$. □

Example 3.4. *Let $X=\{a,b,c,d\}$ and $\mu=P(X)\backslash\{$all proper subsets of X which contains $d\}$.*
The enlargement κ add's the element d to each nonempty μ-open set. Then $\kappa_\mu=\{\emptyset,X\}$. Now put $A=\{a\}$. Obviously $c_{\kappa_\mu}(A)=X$ and $c_\kappa(A)=\{a,d\}$. This example shows that $c_\kappa\subsetneq c_{\kappa_\mu}$.

Example 3.5. *Let $X=\Re$ be the real line and $\mu=\emptyset\cup\{\Re\backslash\{x\},x\neq 0\}$. The enlargement κ is defined as $\kappa(A)=c_\mu(A)$. Then $\kappa_\mu=\{\emptyset,X\}$.*

Example 3.6. *Let $X=\Re$ and $\mu=\{\emptyset,\Re\}\cup\{A_a=(a,+\infty)$ for all $a\in\Re\}$. The enlargement map κ is defined as follows:*

$$\kappa(A)=\begin{cases} A & \text{if } A=(0,+\infty),\\ \Re & \text{if } A\neq(0,+\infty),\\ \emptyset & \text{if } A=\emptyset.\end{cases}$$

The generalized κ_μ topology on X is $\{\emptyset,\Re,(0,+\infty)\}$.

Definition 3.7. *An enlargement κ on μ is said to be open, if for every μ-neighborhood U of $x \in X$, there exists a κ_μ-open set B such that $x \in B$ and $\kappa(U) \supset B$.*

Example 3.8. *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Define $\kappa : \mu \to P(X)$ as follows:*

$$\kappa(A) = \begin{cases} A & \text{if } b \in A \\ c_\mu(A) & \text{if } b \notin A. \end{cases}$$

The enlargement κ on μ is open.

Proposition 3.9. *If $\kappa : \mu \to P(X)$ is an open enlargement and A a subset of X, then $c_\kappa(A) = c_{\kappa_\mu}(A)$ and $c_\kappa(c_\kappa(A)) = c_\kappa(A)$ hold and $c_\kappa(A)$ is κ_μ-closed in (X, μ).*

Demostración. Suppose that $x \notin c_\kappa(A)$. Then there exists a μ-open set U containing x such that $\kappa(U) \cap A = \emptyset$. Since κ is an open enlargement, by Definition 3.7, there exists a κ_μ-open set V such that $x \in V \subset \kappa(U)$ and so $V \cap A = \emptyset$. By Proposition 3.3, $x \notin c_{\kappa_\mu}(A)$, it follows that, $c_{\kappa_\mu}(A) \subset c_\kappa(A)$. By Corollary 1.7 of [3], we have $c_\kappa(A) \subset c_{\kappa_\mu}(A)$. In consequence, we obtain that $c_\kappa(c_\kappa(A)) = c_\kappa(A)$. By Proposition 1.3 of [3], we obtain that $c_\kappa(A)$ is a κ_μ-closed in (X, μ). □

Definition 3.10. *Let μ be a generalized topology on X and $\kappa : \mu \to P(X)$ an enlargement of μ. Then a subset A of a generalized topological space (X, μ) is said to be a generalized κ_μ-closed (abbreviated by g.κ_μ-closed) set in (X, μ), if $c_\kappa(A) \subset U$ whenever $A \subset U$ and $U \in \kappa_\mu$.*

Proposition 3.11. *Every κ_μ-closed set is g.κ_μ-closed.*

Demostración. Straightforward. □

Remark 3.12. *A subset A is g.id_μ-closed if and only if A is g_μ-closed in the sense of Maragathavalli et. al. [4].*

Theorem 3.13. *Let κ be an enlargement of a generalized topological space (X, μ). If A is g.κ_μ-closed in (X, μ), then $c_\kappa(\{x\}) \cap A \neq \emptyset$ for every $x \in c_\kappa(A)$.*

Demostración. Let A be a g.κ_μ-closed set of (X, μ). Suppose that there exists a point $x \in c_\kappa(A)$ such that $c_\kappa(\{x\}) \cap A = \emptyset$. By Proposition 1.3 of [3], $c_\kappa(\{x\})$ is a μ-closed. Put $U = X \backslash c_\kappa(\{x\})$. Then, we have that $A \subset U$, $x \in U$ and U is a μ-open set of (X, μ). Since A is a g.κ_μ-closed set, $c_\kappa(A) \subset U$. Thus, we have $x \notin c_\kappa(A)$. This is a contradiction. □

The converse of the above Theorem not necessarily is true, as we can see.

Example 3.14. *Let N be the set of all natural numbers and μ the discrete topology on N. Let i_0 be a fixed odd number. Define $\kappa : \mu \to P(N)$ as follows:*

$$\kappa(\{n\}) = \begin{cases} \{2i : i \in N\} & \text{if } n \text{ is an even number} \\ \{2i+1 : i \in N\} & \text{if } n = i_0 \\ \{n\} & \text{if } n \text{ is an odd number } \neq i_0 \end{cases}$$

and $\kappa(A) = N$ for the rest. Clearly, κ is an enlargement on μ. Take $A = \{2, 4\}$. It easy to see that $c_\kappa(A) = \{2i : i \in N\}$ and $c_\kappa(\{x\}) \cap A \neq \emptyset$ for every $x \in c_\kappa(A)$ but A not is a g.κ_μ-closed set.

Theorem 3.15. *Let μ be a generalized topology on X and $\kappa : \mu \to P(X)$ an enlargement on μ.*

1. *If a subset A is g.κ_μ-closed in (X, μ), then $c_\kappa(A)\backslash A$ does not contain any nonempty κ_μ-closed set.*
2. *If $\kappa : \mu \to P(X)$ be an open enlargement on (X, μ), then the converse of (1) is true.*

Demostración. (1). Suppose that there exists a κ_μ-closed set F such that $F \subset c_\kappa(A)\backslash A$. Then, we have that $A \subset X\backslash F$ and $X\backslash F$ is κ_μ-open. It follows from assumption that $c_\kappa(A) \subset X\backslash F$ and so $F \subset (c_\kappa(A)\backslash A) \cap (X\backslash c_\kappa(A))$. Therefore, we have that $F = \emptyset$. (2). Let U be a κ_μ-open set such that $A \subset U$. Since κ is an open enlargement, it follows from Proposition 3.9 that $c_\kappa(A)$ is κ_μ-closed in (X, μ). Thus using Proposition 1.1 of [3], we have that $c_{\kappa_\mu}(A) \cap X\backslash U$, say F, is a κ_μ-closed set in (X, μ). Since $X\backslash U \subset X\backslash A$, $F \subset c_{\kappa_\mu}(A)\backslash A$. Using the assumption of the converse of (1) above, $F = \emptyset$ and hence $c_{\kappa_\mu}(A) \subset U$. □

Lemma 3.16. *Let A be a subset of a generalized topological space (X, μ) and $\kappa : \mu \to P(X)$ an enlargement on (X, μ). Then for each $x \in X$, $\{x\}$ is κ_μ-closed or $(X\backslash\{x\})$ is g.κ_μ-closed set of (X, μ).*

Demostración. Suppose that $\{x\}$ is not κ_μ-closed. Then $X\backslash\{x\}$ is not κ_μ-open. Let U be any κ_μ-open set such that $X\backslash\{x\} \subset U$. Then since $U = X$, $c_\kappa(X\backslash\{x\}) \subset U$. Therefore, $X\backslash\{x\}$ is g.κ_μ-closed. □

Definition 3.17. *A generalized topological space (X, μ) is said to be a κ-$T_{1/2}$ space, if every g.κ_μ-closed set of (X, μ) is κ_μ-closed.*

Theorem 3.18. *A generalized topological space (X, μ) is κ-$T_{1/2}$ if and only if for each $x \in X$, $\{x\}$ is κ_μ-closed or κ_μ-open in (X, μ).*

Demostración. Necessity: It is obtained by Lemma 3.16 and Definition 3.17. Sufficiency: Let F be g.κ_μ-closed in (X, μ). We shall prove that $c_{\kappa_\mu}(F) = F$. It is sufficient to show that $c_{\kappa_\mu}(F) \subset F$. Assume that there exists a point x

such that $x \in c_{\kappa_\mu}(F)\backslash F$. Then by assumption, $\{x\}$ is κ_μ-closed or κ_μ-open. Case(i): $\{x\}$ is κ_μ-closed set. For this case, we have a κ_μ-closed set $\{x\}$ such that $\{x\} \subset c_{\kappa_\mu}(F)\backslash F$. This is a contradiction to Theorem 3.15 (1). Case(ii): $\{x\}$ is κ_μ-open set. Using Corollary 1.7 of [3], we have $x \in c_{\kappa_\mu}(F)$. Since $\{x\}$ is κ_μ-open, it implies that $\{x\} \cap F \neq \emptyset$. This is a contradiction. Thus, we have that, $c_\kappa(F) = F$ and so, by Proposition 1.4 of [3] F is κ_μ-closed. □

Definition 3.19. *Jet $\kappa : \mu \to P(X)$ be an enlargement. A generalized topological space (X, μ) is said to be:*

1. *κ-T_0 if for any two distinct points $x, y \in X$ there exists a μ-open set U such that either $x \in U$ and $y \notin \kappa(U)$ or $y \in U$ and $x \notin \kappa(U)$.*

2. *κ-T_1 if for any two distinct points $x, y \in X$ there exist two μ-open sets U and V containing x and y, respectively such that $y \notin \kappa(U)$ and $x \notin \kappa(V)$.*

3. *κ-T_2 if for any two distinct points $x, y \in X$ there exist two μ-open sets U and V containing x and y, respectively such that $\kappa(U) \cap \kappa(V) = \emptyset$.*

Theorem 3.20. *Let A be a subset of a generalized topological space (X, μ) and $\kappa : \mu \to P(X)$ an open enlargement on (X, μ). Then (X, μ) is a κ-T_0 space if and only if for each pair $x, y \in X$ with $x \neq y$, $c_\kappa(\{x\}) = c_\kappa(\{y\})$ holds.*

Demostración. Let x and y be any two distinct points of a κ-T_0 space. Then by Definition3.19, there exists a μ-open set U such that $x \in U$ and $y \notin \kappa(U)$. It follows that there exists a μ-open set S such that $x \in S$ and $S \subset \kappa(U)$. Hence, $y \in X\backslash\kappa(U) \subset X\backslash S$. Because $X\backslash S$ is a μ-closed set, we obtain that $c_\kappa(\{y\}) \subset X\backslash S$ and so $c_\kappa(\{x\}) \neq c_\kappa(\{y\})$. Conversely, suppose that $x \neq y$ for any $x, y \in X$. Then we have that, $c_\kappa(\{x\}) \neq c_\kappa(\{y\})$. Thus, we assume that there exists $z \in c_\kappa(\{x\})$ but $z \notin c_\kappa(\{y\})$. If $x \in c_\kappa(\{y\})$, then we obtain $c_\kappa(\{x\}) \subset c_\kappa(\{y\})$. This implies that $z \in c_\kappa(\{y\})$. This is a contradiction, in consequence, $x \in c_\kappa(\{y\})$. Therefore, there exists a μ-open set W such that $x \in W$ and $\kappa(W) \cap \{y\} = \emptyset$. Thus, we have that $x \in W$ and $y \notin \kappa(W)$. Hence, (X, μ) is a κ-T_0 space. □

Example 3.21. *In the Example 3.14. Take $A = \{2, 4\}$. $c_\kappa(A) - A = \{2i : i \in N - \{1, 2\}\}$ not contain nonempty κ_μ-open set and A is not $g.\kappa_\mu$-closed set.*

Theorem 3.22. *A generalized topological space (X, μ) is κ-T_1 if and only if every singleton set of X is κ_μ-closed.*

Demostración. The proof follows from the respective definitions. □

From Theorems 3.18, 3.22 and Definition 3.19 we obtain the following:

$$\kappa\text{-}T_2 \to \kappa\text{-}T_1 \to \kappa\text{-}T_{1/2} \to \kappa\text{-}T_0.$$

Definition 3.23. *Let (X, μ) be a generalized topological space. Then the sequence $\{x_k\}$ is said to κ-converge to a point $x_0 \in X$, denoted $x_k \underrightarrow{\kappa} x_0$, if for every μ-open set U containing x_0 there exists a positive integer n such that $x_k \in \kappa(U)$ for all $k \geq n$.*

Theorem 3.24. *Let (X, μ) be a κ-T_2 space. If $\{x_k\}$ is said to κ-converge sequence, Then it κ-converge to at most one point.*

Demostración. Let $\{x_k\}$ be a sequence in X κ-converging to x and y. Then by definition of κ-T_2 space, there exist $U, V \in \mu$ such that $x \in U, y \in V$ and $\kappa(U) \cap \kappa(V) = \emptyset$. Since $x_k \underrightarrow{\kappa} x$, there exists a positive integer n_1 such that $x_k \in \kappa(U)$ for all $k \geq n_1$. Also $x_k \underrightarrow{\kappa} y$, therefore there exists a positive integer n_2 such that $x_k \in \kappa(V)$, for all $k \geq n_2$. Let $n_0 = \text{máx}(n_1, n_2)$. Then $x_k \in \kappa(U)$ and $x_k \in \kappa(V)$, for all $k \geq n_0$ or $x_k \in \kappa(U) \cap \kappa(V)$, for all $k \geq n_0$. This contradiction proves that $\{x_k\}$ κ-converges to at most one point. □

Remark 3.25. *Note that the above results generalize the well known separation axioms in general topology in an structure more weaker than a topology.*

4. Additional Properties

Proposition 4.1. *Let $f : (X, \mu) \to (Y, \nu)$ be a (κ, λ)-continuous injection. If (Y, ν) is λ-T_1 (resp. λ-T_2), then (X, μ) is κ-T_1 (resp. κ-T_2).*

Demostración. Suppose that (Y, ν) is λ-T_2. Let x and x' be distinct points of X. Then there exist two open sets V and W of Y such that $f(x) \in V, f(x') \in W$ and $\lambda(V) \cap \lambda(W) = \emptyset$. Since f is (κ, λ)-continuous, for V and W there exist two open sets U, X such that $x \in U, x' \in S, f(\kappa(U)) \subset \lambda(V)$ and $f(\kappa(S)) \subset \lambda(W)$. Therefore, we have $\kappa(U) \cap \kappa(S) = \emptyset$ and hence (X, μ) is κ-T_2. The proof of the case of λ-T_1 is proved similarly. □

Definition 4.2. *An enlargement $\kappa : \mu \times \nu \to P(X \times Y)$ is said to be associated with κ_1 and κ_2, if $\kappa(U \times V) = \kappa_1(U) \times \kappa_2(V)$ holds for each $(\neq \emptyset)U \in \mu$, $(\neq \emptyset)V \in \nu$.*

Definition 4.3. *An enlargement $\kappa : \mu \times \nu \to P(X \times Y)$ is said to be regular with respect to κ_1 and κ_2, if for each point $(x, y) \in X \times Y$ and each $\mu \times \nu$-open set W containing (x, y), there exists $U \in \mu$ and $V \in \nu$ such that $x \in U$, $y \in V$ and $\kappa_1(U) \times \kappa_2(V) \subset \kappa(W)$.*

Proposition 4.4. *Let $\kappa : \mu \times \mu \to P(X \times X)$ be an enlargement associated with κ_1 and κ_1. If $f : (X, \mu) \to (Y, \nu)$ is (κ_1, κ_2)-continuous and (Y, ν) is a κ_2-T_2 space, then the set $A = \{(x, y) \in X \times X : f(x) = f(y)\}$ is a κ-closed set of $(X \times X, \mu \times \mu)$.*

Demostración. We show that $c_\kappa(A) \subset A$. Let $(x, y) \in X \times X \backslash A$. Then, there exist $U, V \in \nu$ such that $f(x) \in U, f(y) \in V$ and $\kappa_2(U) \cap \kappa_2(V) = \emptyset$. Moreover, for U and V there exist $W, S \in \mu$ such that $x \in W, y \in S$ and $f(\kappa_1(W)) \subset \kappa_2(U)$ and $f(\kappa_1(S)) \subset \kappa_2(V)$. Therefore, we have $\kappa(W \times S) \cap A = \emptyset$. This shows that $(x, y) \notin c_\kappa(A)$. $\square$

Corollary 4.5. *If $\kappa : \mu \times \mu \to P(X \times X)$ is an enlargement associated with κ_1 and κ_1 and it is regular with respect to κ_1 and κ_1. A generalized topological space (X, μ) is κ_1-T_2 if and only if the diagonal set $\Delta = \{(x, x) : x \in X\}$ is κ-closed in $(X \times X, \mu \times \mu)$.*

Proposition 4.6. *Let $\kappa : \mu \times \nu \to P(X \times Y)$ be an enlargement associated with κ_1 and κ_1. If $f : (X, \mu) \to (Y, \nu)$ is (κ_1, κ_2)-continuous and (Y, ν) is a κ_2-T_2 space, then the graph of f, $G(f) = \{(x, f(x)) \in X \times Y\}$ is a κ-closed set of $(X \times Y, \mu \times \nu)$.*

Demostración. The proof is similar to that of Proposition 4.4. $\square$

Definition 4.7. *An enlargement κ on μ is said to be regular, if for any μ-open neighborhoods U, V of $x \in X$, there exists a μ-open neighborhood W of x such that $\kappa(U) \cap \kappa(V) \supset \kappa(W)$.*

Theorem 4.8. *Suppose that κ_1 is a regular enlargement and $\kappa : \mu \times \nu \to P(X \times Y)$ is regular with respect to κ_1 and κ_2. Let $f : (X, \mu) \to (Y, \nu)$ be a function whose graph $G(f)$ is κ-closed in $(X \times Y, \mu \times \nu)$. If a subset B is κ_2-compact in (Y, ν), then $f^{-1}(B)$ is κ_1-closed in (X, μ).*

Demostración. Suppose that $f^{-1}(B)$ is not κ_1-closed. Then, there exists a point x such that $x \in c_{\kappa_1}(f^{-1}(B))$ and $x \notin f^{-1}(B)$. Since $(x, b) \notin G(f)$ for each $b \in B$ and $G(f) \supset c_\kappa(G(f))$, there exists a $\mu \times \nu$-open set W such that $(x, b) \in W$ and $\kappa(W) \cap G(f) = \emptyset$. By the regularity of κ, for each $b \in B$ we can take two sets $U(b) \in \mu$ and $V(b) \in \nu$ such that $x \in U(b), b \in V(b)$ and $\kappa_1(U(b)) \times \kappa_2(V(b)) \subset \kappa(W)$. Then we have $f(\kappa_1(U(b))) \cap \kappa_2(V(b)) = \emptyset$. Since $\{V(b) : b \in B\}$ is a ν-open cover of B, there exists a finite number of points $b_1, ..., b_n \in B$ such that $B \subset \bigcup\limits_{i=1}^{n} \kappa_2(V(b_i))$, by the κ_2-compactness of B. By the regularity of κ_1, there exists $U \in \mu$ such that $x \in U$, $\kappa_1(U) \subset \bigcap\limits_{i=1}^{n} \kappa_1(U(b_i))$. Therefore, we have $\kappa_1(U) \cap f^{-1}(B) \subset \bigcup\limits_{i=1}^{n} \kappa_1(U(b_i)) \cap f^{-1}(\kappa_2(V(b_i))) = \emptyset$. This shows that $x \notin c_{\kappa_1}(f^{-1}(B))$, thus we have a contradiction. $\square$

Theorem 4.9. *Let $f : (X, \mu) \to (Y, \nu)$ be a function whose graph $G(f)$ is κ-closed in $(X \times Y, \mu \times \nu)$ and suppose that the following conditions hold:*

1. *$\kappa_1 : \mu \to P(X)$ is open,*
2. *$\kappa_2 : \nu \to P(Y)$ is regular, and*

3. *$\kappa : \mu \times \nu \to P(X \times Y)$ is an enlargement associated with κ_1 and κ_2 and κ is regular with respect to κ_1 and κ_2.*

If every cover of A by κ_1-open sets of (X, μ) has a finite subcover, then $f(A)$ is κ_2-closed in (Y, ν).

Demostración. The proof is similar to that of Theorem 4.8 □

Proposition 4.10. *Let $\kappa : \mu \times \nu \to P(X \times Y)$ be an enlargement associated with κ_1 and κ_2. If $f : (X, \mu) \to (Y, \nu)$ is (κ_1, κ_2)-continuous and (Y, ν) is a κ_2-T_2, then the graph of f, $G(f) = \{(x, f(x)) \in X \times Y\}$ is a κ-closed set of $(X \times Y, \mu \times \nu)$.*

Demostración. The proof is similar to that of Proposition 4.4. □

Definition 4.11. *A function $f : (X, \mu) \to (Y, \nu)$ is said to be (κ, λ)-closed, if for any κ_μ-closed set A of (X, μ), $f(A)$ is λ_ν-closed in (Y, ν).*

Theorem 4.12. *Suppose that f is (κ, λ)-continuous and (id, λ)-closed. If for every g.κ_μ-closed set A of (X, μ), then the image $f(A)$ is g.λ_ν-closed.*

Demostración. Let V be any λ_ν-open set of (Y, ν) such that $f(A) \subset V$. By the Theorem 2.2 (2), $f^{-1}(V)$ is κ_μ-open. Since A is g.κ_μ-closed and $A \subset f^{-1}(V)$, we have $c_\kappa(A) \subset f^{-1}(V)$, and hence $f(c_\kappa(A)) \subset V$. It follows from Proposition 1.3 of [3] and assumption that $f(c_\kappa(A))$ is λ_ν-closed. Therefore we have $c_\lambda(f(A)) \subset c_\lambda(f(c_\kappa(A))) = f(c_\kappa(A)) \subset V$. This implies $f(A)$ is g.λ_ν-closed. □

Theorem 4.13. *If $f : (X, \mu) \to (Y, \nu)$ is (κ, λ)-continuous and (id, λ)-closed. If f is injective and (Y, ν) is λ-$T_{1/2}$, then (X, μ) is κ-$T_{1/2}$.*

Demostración. Let A be a g.κ_μ-closed set of (X, μ). We shows that A is κ_μ-closed. By Theorem 4.12 and assumptions it is obtained that $f(A)$ is g.λ_ν-closed and hence $f(A)$ is λ_μ-closed. Since f is (κ, λ)-continuous, $f^{-1}(f(A))$ is κ_μ-closed by using Theorem 2.2(2). □

Referencias

[1] A. Császár, Generalized topology, generalized continuity, *Acta Math. Hungar.*, **96 (2002)**, 351-357.

[2] A. Császár, Generalized open sets in generalized topology, *Acta Math. Hungar.*, **106 (2005)**, 53-66.

[3] A. Császár, Enlargements and generalized topologies *Acta Math. Hungar.*, **120 (2008)**, 351-354.

[4] S. Maragathavalli, M. Sheik John and D. Sivaraj, On g-closed sets in generalized topological spaces, *J. Adv. Res. Pure Maths.* **2(1) (2010)**, 57-64.