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Simulating Multi-tone Free-running oscillators with Optimal Sweep Following

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With ever increasing operating frequencies of electric circuits, it becomes more and more common that very different time scales occur in circuit simulation. The fast frequency f_f can be up to 10^3 times the slow frequency f_s. This leads to transient behaviour as shown in Figure 1.

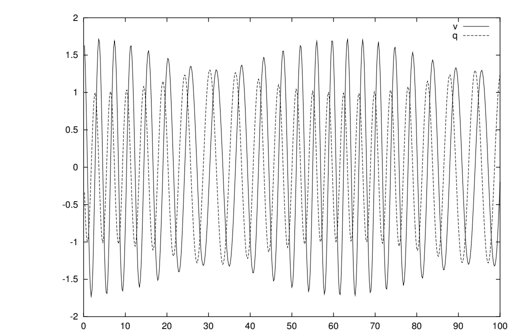

Figure 1: Transient behaviour of multi-tone circuit

Traditional simulation techniques have problems with such widely separate time scales, since they require an amount of computation time $O(f_f/f_s)$. Therefore, there is much interest in methods that attempt to split the fast time scale from the slow time scale. Our new method, named Optimal Sweep Following, handles autonomous circuits in which Frequency Modulation (FM) takes place. Competing methods only handle Amplitude Modulated (AM) circuits. The method makes an optimal splitting of the time into low- and high-frequency components.

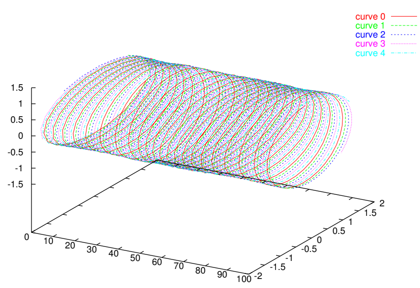

Figure 2: Surface traced out by solutions

Suppose we have a set $\mathcal{C}_0$ of initial values, parametrised by a parameter $s \in [0, 2\pi]$, i.e. $\mathcal{C}_0 = \{\hat{\mathbf{x}}_0(s) \mid 0 \leq s \leq 2\pi\}$. If we compute the solution, starting in each initial value in turn, we obtain a 2D surface in the "solution space". For a given $s \in [0, 2\pi]$, consider the solution $\hat{\mathbf{x}}(s,t)$ with initial condition $\hat{\mathbf{x}}_0(s)$, i.e. the solution to the problem

$$\frac{d}{dt}\mathbf{q}(\hat{\mathbf{x}}(s,t)) + \mathbf{j}(\hat{\mathbf{x}}(s,t)) = \mathbf{s}(s,t),$$
$$\hat{\mathbf{x}}(s,0) = \hat{\mathbf{x}}_0(s).$$

The parametrisation $\hat{\mathbf{x}}$ of the sweep induces a coordinate system (s,t) on the sweep. This coordinate system may be very skew. which means that information is not efficiently represented. Therefore, we will investigate coordinate transforms of the form

$$u = s + \alpha(t), \quad v = t,$$

for some differentiable function α. The function α can be selected in an optimal way, so as to "straighten" the curves in Figure 2 as much as possible.

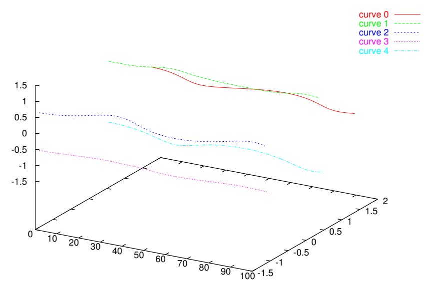

Figure 3: Surface in (u,v)-coordinates

The resulting curves are much smoother, which means that we can take larger steps in the v ($= t$) coordinate. This in turn means faster computations.