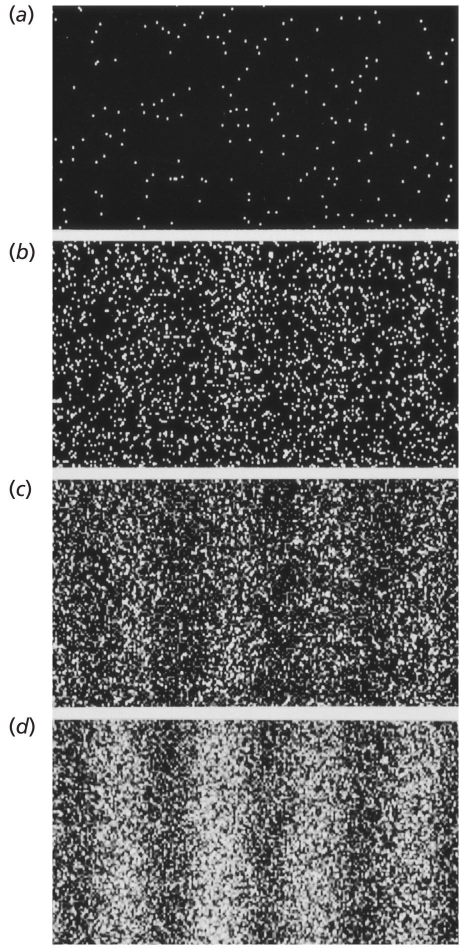
CHAPTER 17

Wave–Particle Duality and Quantum Physics



Electron interference pattern produced by electrons incident on a barrier containing two slits: (*a*) 10 electrons; (*b*) 100 electrons; (*c*) 3,000 electrons; (*d*) 70,000 electrons. The maxima and minima demonstrate the wave nature of the electron as it traverses the slits. Individual dots on the screen indicate the particle nature of the electron as it exchanges energy with the detector. The pattern is the same whether electrons or photons (particles of light) are used.

We have seen that the propagation of waves through space is quite different from the propagation of particles. Waves bend around corners (diffraction) and interfere with one another, producing interference patterns. When a wave encounters a small aperture, the wave spreads out on the other side as if the aperture were a point source (Figure 15-23). When two waves of equal intensity I_0 and originating from coherent sources meet in space, the result can be a wave of intensity $4I_0$ (constructive interference), an intensity of zero (destructive interference), or a wave of intensity between zero and $4I_0$, depending on the phase difference between the waves at their meeting point.

The propagation of particles is quite different. Particles travel along well-defined paths. When two particles meet in space, they never produce an interference pattern.

Particles and waves also exchange energy differently. Particles exchange energy in collisions that occur at specific points in space and time. The energy of waves, on the other hand, is spread out in space and deposited continuously as the wave front interacts with matter.

Sometimes the propagation of a wave cannot be distinguished from that of a beam of particles. When the wavelength λ is very small compared with the sizes of apertures and obstacles, diffraction effects are negligible and the wave appears to travel along a well-defined path. Also, interference maxima and minima are so close together in space as to be unobservable. Similarly, when there are very many small particles each exchanging a small amount of energy, the exchange cannot be distinguished from that of a wave. For example, you do not observe the individual air molecules bouncing off your face when the wind blows on it. Instead, the interaction with billions of particles is perceived to be continuous, as if the particles were a wave.

At the beginning of the twentieth century, it was thought that sound, light, and other electromagnetic radiation such as radio were waves, whereas electrons, protons, atoms, and similar constituents of nature were understood to be particles. The first 30 years of the new century revealed such startling developments in theoretical and experimental physics as the finding that light, thought to be a wave, actually exchanges energy in discrete lumps, or quanta, just like particles, and that an electron, thought to be a particle, exhibits diffraction and interference as it propagates through space, just like a wave.

The fact that light exchanges energy like a particle implies that light energy is not continuous but is *quantized*. Similarly, the wave nature of the electron, along with the fact that the standing-wave condition requires a discrete set of frequencies, implies that the energy of an electron in a confined region of space is not continuous, but is quantized to a discrete set of values.

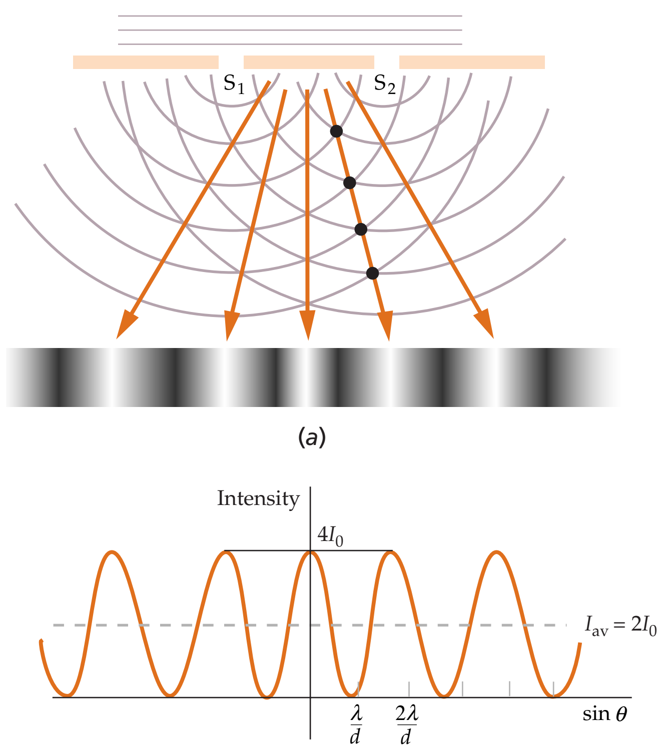
In this chapter we begin by discussing some basic properties of light and electrons, examining both their wave and particle characteristics. We then consider some of the detailed properties of matter waves, showing, in particular, how standing waves imply the quantization of energy. We will then discuss some of the important features of the theory of quantum physics, which was developed in the 1920s and which has been extremely successful in describing nature. Quantum physics is now the basis of our understanding of the microscopic world.

17-1 Light

The question of whether light consists of a beam of particles or waves in motion is one of the most interesting in the history of science. Newton used a particle theory of light to explain the laws of reflection and refraction, but for refraction he needed to assume that light travels faster in water or glass than in air, an assumption later shown to be false. The chief early proponents of the wave theory were Robert Hooke and Christian Huygens, who explained refraction by assuming that light travels more slowly in glass or water than in air. Newton rejected the wave theory because in his time light was believed to travel only in straight lines—diffraction had not yet been observed.

Because of Newton's great reputation and authority, his particle theory of light was accepted for more than a century. Then in 1801 Thomas Young demonstrated the wave nature of light in a famous experiment in which two coherent light sources are produced by illuminating a pair of narrow, parallel slits with a single source (Figure 17-1). We saw in Chapter 16 that when light encounters a small opening, the opening acts as a point source of waves (Figure 16-10). In Young's experiment, each slit acts as a line source, which is equivalent to a point source in two dimensions. The interference pattern is observed on a screen placed behind the slits. Interference maxima occur at angles such that the path difference is an integral number of wavelengths. Similarly, interference minima occur when the path difference is one half-wavelength or any odd number of

Figure 17-1 (*a*) Two slits act as coherent sources of light for the observation of interference in Young's experiment. Waves from the slits overlap and produce an interference pattern on a screen far away. (*b*) Graph of the intensity pattern produced in (*a*). The intensity is maximum at points where the path difference is an integral number of wavelengths and zero where the path difference is an odd number of half-wavelengths.



half-wavelengths. Figure 17-1*b* shows a graph of the intensity pattern seen on the screen. This and many other experiments demonstrate that light propagates like a wave.

In the early nineteenth century, the French physicist Augustin Fresnel (1788–1827) performed extensive experiments on interference and diffraction and put the wave theory on a mathematical basis. Among his results, he showed that the observed straight-line propagation of light is a result of the very short wavelengths of visible light.

The classical wave theory of light culminated in 1860 when James Clerk Maxwell published his mathematical theory of electromagnetism. This theory yielded a wave equation that predicted the existence of electromagnetic waves that propagate with a speed that can be calculated from the laws of electricity and magnetism. The fact that the result of this calculation was $c \approx 3 \times 10^8$ m/s, the same as the measured value for the speed of light, suggested to Maxwell that light is an electromagnetic wave. The eye is sensitive to electromagnetic waves with wavelengths in the range from about 400 nm (1 nm $= 10^{-9}$ m) to about 700 nm. This range is called visible light. Other electromagnetic waves such as microwaves, radio, television, and X rays differ from light only in wavelength and frequency.

17-2 The Particle Nature of Light: Photons

The diffraction of light and the existence of an interference pattern in the two-slit experiment give clear evidence that light has wave properties. However, early in the twentieth century it was found that light energy is exchanged in discrete amounts—an unwavelike property.

The Photoelectric Effect

The quantum nature of light and the quantization of energy were suggested by Einstein in 1905 in his explanation of the photoelectric effect. Einstein's work marked the beginning of quantum theory, and for it he received the Nobel prize for physics. Figure 17-2 shows a schematic diagram of the basic apparatus for studying the photoelectric effect. When light is incident on a clean metal surface C, electrons are emitted. Some of these electrons strike the second metal plate A, constituting an electric current between the plates. The maximum energy of the emitted electrons can be measured. Experiments give the surprising result that the maximum kinetic energy of the ejected electrons is *independent of the intensity* of the incident light. From the wave theory of light, we would expect that increasing the rate at which light energy falls on the metal surface would increase the amount of energy absorbed by individual electrons and therefore would increase the maximum kinetic energy of the electrons emitted. This is not what happens. The maximum kinetic energy of the ejected electrons is the same for a given wavelength of incident light, no matter how intense the light. Einstein suggested that this experimental result can be explained if light energy is quantized in small bundles called **photons**. The energy of each photon is given by

$$E = hf = \frac{hc}{\lambda} \qquad 17\text{-}1$$

Einstein equation for photon energy

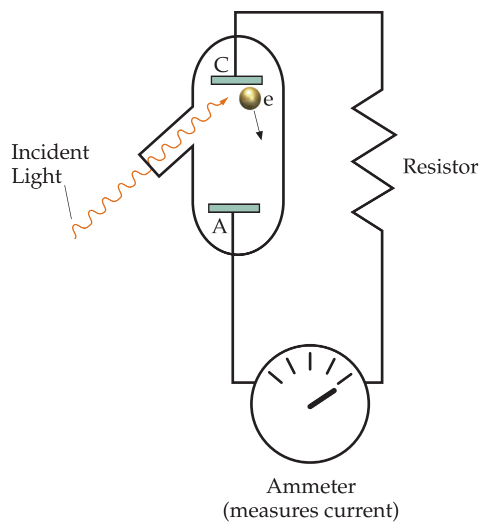


Figure 17-2 Schematic drawing of the apparatus for studying the photoelectric effect. Light strikes the cathode C and ejects electrons. The number of electrons that reach the anode A is measured by the current in an ammeter placed in a circuit between A and C.

optional

where f is the frequency, and h is a constant now known as **Planck's constant**.* The accepted value of this constant is now

$$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s} = 4.136 \times 10^{-15}\ \mathrm{eV \cdot s} \tag{17-2}$$

Equation 17-1 is sometimes called the **Einstein equation**.

In this picture, a light beam consists of a beam of particles—photons, each having energy hf. The intensity of a light beam (energy per unit area per unit time) is the number of photons per unit area per unit of time times the energy of each photon. The interaction of the light beam with the metal surface consists of collisions between photons and electrons. In these collisions, the photon disappears, giving all of its energy to the electron. An electron emitted from a metal surface exposed to light thus receives its energy from a single photon. When the intensity of light is increased, more photons fall on the surface per unit time, and more electrons are ejected. However, each photon still has the same energy hf, so the energy absorbed by each electron is the same.

If ϕ is the minimum energy necessary to remove an electron from a metal surface, the maximum kinetic energy of the electrons emitted is given by

$$K_{\mathrm{max}} = \left(\tfrac{1}{2}mv^2\right)_{\mathrm{max}} = hf - \phi \tag{17-3}$$

Einstein's photoelectric equation

The quantity ϕ, called the **work function,** is a characteristic of the particular metal. (Some electrons will have kinetic energies less than $hf - \phi$ because of the loss of energy from traveling through the metal.)

According to Einstein's photoelectric equation, a plot of K_{max} versus frequency f should be a straight line with the slope h. This was a bold prediction, for at the time there was no evidence that Planck's constant had any application outside of the then-mysterious phenomenon of blackbody radiation. In addition, there were no experimental data on K_{max} versus frequency because no one before that time had even suspected that the frequency of the light was related to K_{max}. This prediction was difficult to verify experimentally, but eventually careful experiments by R. A. Millikan about ten years later showed that Einstein's equation is correct. Figure 17-3 shows a plot of Millikan's data.

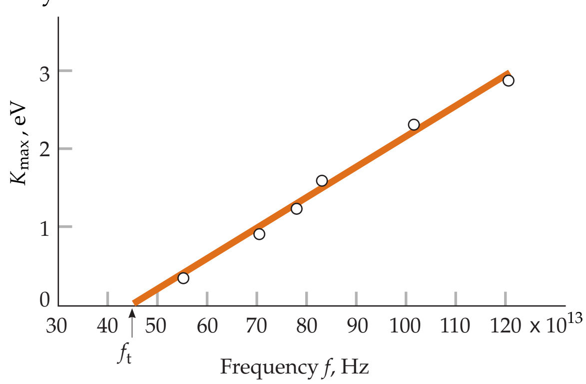


Figure 17-3 Millikan's data for the maximum kinetic energy K_{max} versus frequency f for the photoelectric effect. The data fall on a straight line that has a slope h, as predicted by Einstein a decade before the experiment was performed.

Photons with frequencies less than a **threshold frequency** f_t, and therefore with wavelengths greater than a **threshold wavelength** $\lambda_t = c/f_t$, do not have enough energy to eject an electron from a particular metal. The threshold frequency and the corresponding threshold wavelength can be related to the work function ϕ by setting the maximum kinetic energy of the electrons equal to zero in Equation 17-3. Then

$$\phi = hf_t = \frac{hc}{\lambda_t} \tag{17-4}$$

Work functions for metals are typically a few electron volts. Since light wavelengths are usually given in nanometers and energies in electron volts, it is useful to have the value of hc in electron volt-nanometers:

$$hc = (4.136 \times 10^{-15}\ \mathrm{eV \cdot s})(2.997 \times 10^{8}\ \mathrm{m/s}) = 1.240 \times 10^{-6}\ \mathrm{eV \cdot m}$$

or

$$hc = 1240\ \mathrm{eV \cdot nm} \tag{17-5}$$

* In 1900, the German physicist Max Planck had introduced this constant to explain discrepancies between theoretical curves and experimental data related to the spectrum of blackbody radiation. Planck also assumed that the radiation was emitted and absorbed by a blackbody in quanta of energy hf, but he considered his assumption to be just a calculational device rather than a fundamental property of electromagnetic radiation. We discuss blackbody radiation in Chapter 21.

Example 17-1

Calculate the photon energies for light of wavelengths 400 nm (violet) and 700 nm (red). (These are the approximate wavelengths at the two extremes of the visible spectrum.)

1. The energy is related to the wavelength by Equation 17-1: $E = hf = \dfrac{hc}{\lambda}$
2. For $\lambda = 400$ nm, the energy is: $E = \dfrac{hc}{\lambda} = \dfrac{1240\text{ eV}\cdot\text{nm}}{400\text{ nm}} = 3.10\text{ eV}$
3. For $\lambda = 700$ nm, the energy is: $E = \dfrac{hc}{\lambda} = \dfrac{1240\text{ eV}\cdot\text{nm}}{700\text{ nm}} = 1.77\text{ eV}$

Remark We can see from these calculations that visible light contains photons with energies that range from about 1.8 to 3.1 eV. X rays, which have much shorter wavelengths, contain photons with energies of the order of keV. Gamma rays emitted by nuclei have even shorter wavelengths and photons with energies of the order of MeV.

Example 17-2 *try it yourself*

The intensity of sunlight at the earth's surface is approximately 1400 W/m^2. Assuming that the average photon energy is 2 eV (corresponding to a wavelength of about 600 nm), calculate the number of photons that strike an area of 1 cm^2 in 1 s.

Cover the column to the right and try these on your own before looking at the answers.

Steps	*Answers*
1. The number N of photons is related to the total energy.	$E = Nhf = N(2\text{ eV})$
2. Use 1 W = 1 J/s to find the energy in joules striking an area of 1 cm^2 in 1 s.	$E = 0.14$ J
3. Use the conversion factor 1 eV = 1.6×10^{-19} J to find the energy in eV striking an area of 1 cm^2 in 1 s.	$E = 8.75 \times 10^{17}$ eV
4. Use this value of E to solve for N.	$N = 4.38 \times 10^{17}$ photons

Remark This is an enormous number of photons. In most everyday situations, the number of photons is so great that the quantization of light is not noticeable.

Exercise Find the energy of a photon corresponding to electromagnetic radiation in the FM radio band of wavelength 3 m. (*Answer* 4.13×10^{-7} eV)

Exercise Find the wavelength of a photon whose energy is (*a*) 0.1 eV, (*b*) 1 keV, and (*c*) 1 MeV. (*Answers* (*a*) 12.4 μm, (*b*) 1.24 nm, (*c*) 1.24 pm)

optional

Compton Scattering

Further evidence of the correctness of the photon concept was furnished by Arthur H. Compton, who measured the scattering of X rays by electrons in 1923. According to classical theory, when an electromagnetic wave of frequency f_1 is incident on material containing charges, the charges will oscillate with this frequency and reradiate electromagnetic waves of the same frequency. Compton pointed out that if the scattering process were considered to be a collision between a photon and an electron, the electron would recoil and thus absorb energy. The scattered photon would then have less energy and therefore a lower frequency and larger wavelength than the incident photon.

According to classical wave theory, the energy and momentum of an electromagnetic wave are related by

$$E = pc \tag{17-6}$$

If a photon has energy $E = hf = hc/\lambda$, its momentum should then be $p = E/c = hf/c = h/\lambda$:

$$p = \frac{h}{\lambda} \tag{17-7}$$

Momentum of a photon

Compton applied the laws of conservation of momentum and energy to the collision of a photon and an electron to calculate the momentum p_2 and thus the wavelength $\lambda_2 = h/p_2$ of a scattered photon (Figure 17-4). Because the calculation requires Einstein's theory of special relativity, we present only the result here. The wavelengths λ_1, associated with the incoming photon, and λ_2, associated with the scattered photon, are related to each other and to the scattering angle θ by

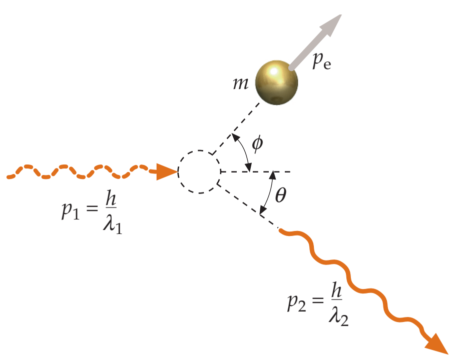


Figure 17-4 The scattering of light by an electron is considered as a collision of a photon of momentum h/λ_1 and a stationary electron. The scattered photon has less energy and therefore a greater wavelength.

$$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - \cos\theta) \tag{17-8}$$

where m_e is the mass of the electron. The change in wavelengths is independent of the original wavelength. The quantity $h/m_e c$ depends only on the mass of the electron. It has dimensions of length and is called the Compton wavelength. Its value is

$$\lambda_C = \frac{h}{m_e c} = \frac{hc}{m_e c^2} = \frac{1240 \text{ eV·nm}}{5.11 \times 10^5 \text{ eV}} = 2.43 \times 10^{-12} \text{ m} \tag{17-9}$$
$$= 2.43 \text{ pm}$$

where 1 pm $= 10^{-12}$ m $= 10^{-3}$ nm. Because $\lambda_2 - \lambda_1$ is small, it is difficult to observe unless λ_1 is so small that the fractional change $(\lambda_2 - \lambda_1)/\lambda_1$ is appreciable. Compton used X rays of wavelength 71.1 pm. The energy of a photon of this wavelength is $E = hc/\lambda = (1240 \text{ eV·nm})/(0.0711 \text{ nm}) = 17.4$ keV. Compton's experimental results for $\lambda_2 - \lambda_1$ as a function of scattering angle θ agreed with Equation 17-8, thereby confirming the correctness of the photon concept, that is, of the particle nature of light.

optional

Example 17-3

An X-ray photon of wavelength 6 pm makes a head-on collision with an electron so that it is scattered by an angle of 180° (Figure 17-5). (*a*) What is the change in wavelength of the photon? (*b*) What is the kinetic energy of the recoiling electron?

Picture the Problem We can calculate the change in wavelength and the new wavelength from Equation 17-8. We then use the new wavelength to find the energy of the scattered photon, and we use conservation of energy to find the energy of the recoiling electron.

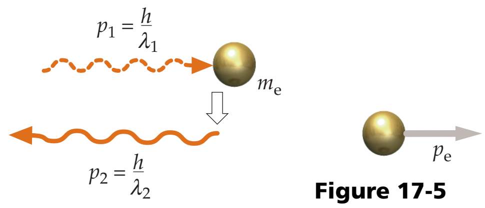


Figure 17-5

(*a*) Use Equation 17-8 to calculate the change in wavelength:

$$\Delta\lambda = \lambda_2 - \lambda_1$$
$$= \frac{h}{m_e c}(1 - \cos\theta) = 2.43 \text{ pm } (1 - \cos 180°)$$
$$= 2.43 \text{ pm } (1 - (-1)) = 4.86 \text{ pm}$$

(*b*)1. The energy of the recoiling electron equals the energy of the incident photon E_1 minus the energy of the scattered photon E_2:

$$K_e = E_1 - E_2 = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

2. Calculate the energy of the incident photon:

$$E_1 = \frac{hc}{\lambda_1} = \frac{1240 \text{ eV}\cdot\text{nm}}{6.0 \text{ pm}} = \frac{1.24 \text{ keV}\cdot\text{nm}}{6.0 \times 10^{-3} \text{ nm}} = 207 \text{ keV}$$

3. Calculate λ_2 from the given wavelength of the incident photon and the change found in step 1:

$$\lambda_2 = \lambda_1 + \Delta\lambda = 6 \text{ pm} + 4.86 \text{ pm} = 10.86 \text{ pm}$$

4. Use this result to find E_2:

$$E_2 = \frac{hc}{\lambda_2} = \frac{1240 \text{ eV}\cdot\text{nm}}{10.86 \text{ pm}} = \frac{1.24 \text{ keV}\cdot\text{nm}}{10.86 \times 10^{-3} \text{ nm}} = 114 \text{ keV}$$

5. Substitute the calculated values of E_1 and E_2 to find the energy of the recoiling electron:

$$K_e = E_1 - E_2 = 207 \text{ keV} - 114 \text{ keV} = 93 \text{ keV}$$

17-3 Energy Quantization in Atoms

Ordinary white light has a continuous spectrum, that is, it contains all the wavelengths in the visible spectrum. But when atoms in a gas at low pressure are excited by an electric discharge, they emit light of specific wavelengths that are characteristic of the type of atom. Since the energy of a photon is related to its wavelength by $E = hf = hc/\lambda$, a discrete set of wavelengths implies a discrete set of energies. Conservation of energy then implies that when an atom radiates, its internal energy changes by a discrete amount. This led Niels Bohr in 1913 to postulate that the internal energy of an atom can have only a discrete set of values. That is, the internal energy of an atom is **quantized.** When an atom radiates light of frequency f, the atom makes a transition from one allowed level to another level that is lower in energy by $\Delta E = hf$. Bohr was able to construct a model of the hydrogen atom that had a discrete set of energy levels consistent with the observed spectrum of emitted light.* However, the *reason* for the quantization of energy levels in atoms and other systems remained a mystery until the wave nature of electrons was discovered a decade later.

*We study the Bohr model in Chapter 37.

17-4 Electrons and Matter Waves

In 1897, J. J. Thomson showed that the rays of a cathode-ray tube (Figure 17-6) consist of electrically charged particles, and he showed that all the particles have the same charge-to-mass ratio q/m. He also showed that particles with this charge-to-mass ratio can be obtained using any material for the cathode, which means that these particles, now called **electrons,** are a fundamental constituent of all matter.

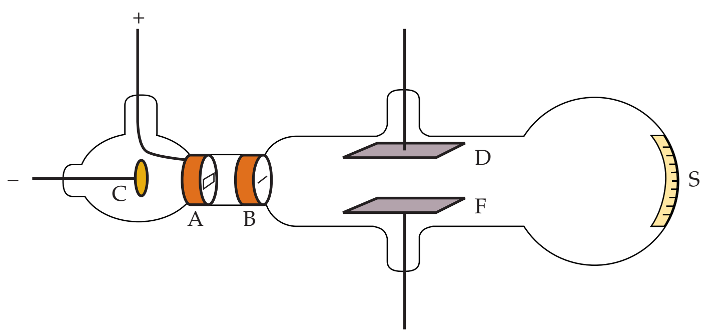


Figure 17-6 Schematic diagram of the cathode-ray tube Thomson used to measure q/m for the particles that comprise cathode rays (electrons). Electrons from the cathode C pass through the slits at A and B and strike a phosphorescent screen S. The beam can be deflected by an electric field between plates D and F or by a magnetic field (not shown).

The de Broglie Hypothesis

Since light seems to have both wave and particle properties, it is natural to ask whether matter—electrons, protons, etc.—might also have both wave and particle characteristics. In 1924, a French physics student, Louis de Broglie, suggested this idea in his doctoral dissertation. de Broglie's work was highly speculative since there was no evidence at that time of any wave aspects of matter.

For the wavelength of electron waves, de Broglie chose

$$\lambda = \frac{h}{p} \qquad \text{17-10}$$

de Broglie relation for the wavelength of electron waves

where p is the momentum of the electron. Note that this is the same as Equation 17-7 for a photon. For the frequency of electron waves de Broglie chose the Einstein equation relating the frequency and energy of a photon:

$$f = \frac{E}{h} \qquad \text{17-11}$$

de Broglie relation for the frequency of electron waves

These equations are thought to apply to all matter. However, for macroscopic objects, the wavelengths calculated from Equation 17-10 are so small that it is impossible to observe the usual wave properties of interference or diffraction. Even a dust particle as small as 1 μg is much too massive for any wave characteristics to be noticed, as we see in the following example.

Example 17-4 *try it yourself*

Find the de Broglie wavelength of a particle of mass 10^{-6} g moving with a speed of 10^{-6} m/s.

Cover the column to the right and try this on your own before looking at the answer.

Step	*Answer*
Write down the definition of the de Broglie wavelength and substitute the given data.	$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(10^{-9}\ \text{kg})(10^{-6}\ \text{m/s})}$ $= 6.63 \times 10^{-19}\ \text{m}$

Remark This wavelength is much smaller than the diameter of the atomic nucleus, which is about 10^{-15} m.

Since the wavelength found in Example 17-4 is much smaller than any possible apertures or obstacles, diffraction or interference of such waves cannot be observed. In fact, the propagation of waves of very small wavelengths is indistinguishable from the propagation of particles. The momentum of the particle in Example 17-4 was only 10^{-15} kg·m/s. A macroscopic particle with a greater momentum would have an even smaller de Broglie wavelength. We therefore do not observe the wave properties of such macroscopic objects as baseballs and billiard balls.

Exercise Find the de Broglie wavelength of a baseball of mass 0.17 kg moving at 100 km/h. (*Answer* 1.4×10^{-34} m)

The situation is different for low-energy electrons and other microscopic particles. Consider a particle with kinetic energy K. Its momentum is found from

$$K = \frac{p^2}{2m}$$

or

$$p = \sqrt{2mK}$$

Its wavelength is then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

If we multiply both numerator and denominator by c we obtain*

$$\lambda = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{2mc^2K}} \qquad \text{17-12}$$

Wavelength associated with a particle of mass m

where we have used $hc = 1240$ eV·nm. For electrons, $mc^2 = 0.511$ MeV. Then

$$\lambda = \frac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{2mc^2K}} = \frac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{2(0.511 \times 10^6\ \text{eV})K}}$$

* Equations 17-12 and 17-13 do not hold for relativistic particles whose kinetic energies are comparable to their rest energies mc^2.

optional

or

$$\lambda = \frac{1.23}{\sqrt{K}}\ \text{nm} \qquad (K \text{ in electron volts}) \qquad 17\text{-}13$$

Electron wavelength

Exercise Find the wavelength of an electron whose kinetic energy is 10 eV. (*Answer* 0.388 nm. This is on the same order of magnitude as the size of the atom and the spacing of atoms in a crystal.)

Electron Interference and Diffraction

The observation of diffraction and interference of electron waves would provide the crucial evidence that electrons have wave properties. This evidence was obtained accidentally in 1927 by C. J. Davisson and L. H. Germer as they were studying electron scattering from a nickel target at the Bell Telephone Laboratories. After an accidental break in the vacuum system they were using, they were obliged to heat the target to remove an oxide coating that had accumulated. Afterward, they found that the scattered-electron intensity as a function of the scattering angle showed maxima and minima. By chance they had observed electron diffraction. After realizing that the scattering pattern had changed because the heating procedure had caused the target to crystallize, they prepared a target consisting of a single crystal of nickel and investigated this phenomenon extensively. Figure 17-7*a* illustrates their experiment. Electrons from an electron gun are directed at a crystal and detected at some angle ϕ that can be varied. Figure 17-7*b* shows a typical pattern observed. There is a strong scattering maximum at an angle of 50°. The angle for maximum scattering of waves from a crystal depends on the wavelength of the waves and the spacing of the atoms in the crystal. Using the known spacing of the atoms in their crystal, Davisson and Germer calculated the wavelength that could produce such a maximum and found that it agreed with the de Broglie equation (Equation 17-10) for the electron energy they were using. By varying the energy of the incident electrons, they could vary the electron wavelengths and produce maxima and minima at different locations in the diffraction patterns. In all cases, the measured wavelengths agreed with de Broglie's hypothesis.

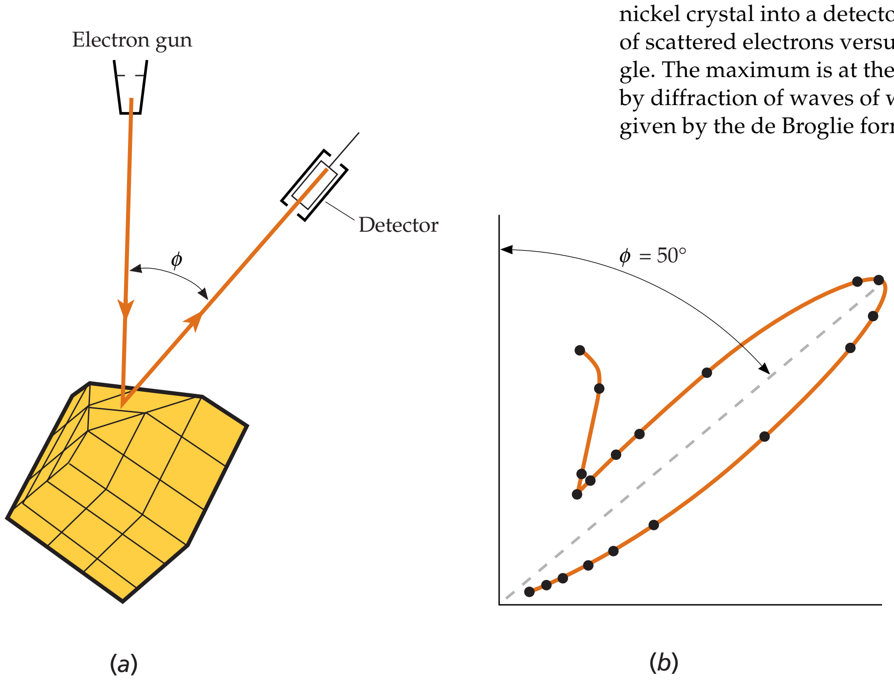
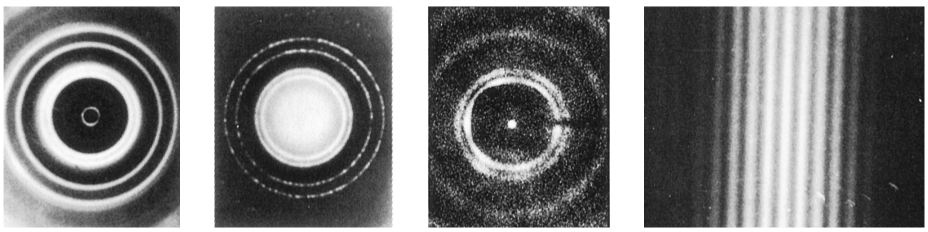


Figure 17-7 The Davisson–Germer experiment. (*a*) Electrons are scattered from a nickel crystal into a detector. (*b*) Intensity of scattered electrons versus scattering angle. The maximum is at the angle predicted by diffraction of waves of wavelength λ given by the de Broglie formula.

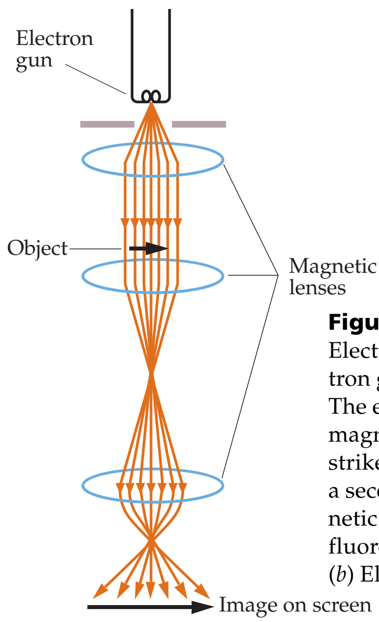
Another demonstration of the wave nature of electrons was provided in the same year by G. P. Thomson (son of J. J. Thomson), who observed electron diffraction in the transmission of electrons through thin metal foils. A metal foil consists of tiny, randomly oriented crystals. The diffraction pattern resulting from such a foil is a set of concentric circles. Figure 17-8*a* and *b* shows the diffraction pattern observed using X rays and electrons on an aluminum-foil target. Figure 17-8*c* shows the diffraction patterns of neutrons on a copper-foil target. Note the similarity of the patterns. The diffraction of hydrogen and helium atoms was observed in 1930. In all cases, the measured wavelengths agree with the de Broglie predictions. Figure 17-8*d* shows a diffraction pattern produced by electrons incident on two narrow slits. This experiment is equivalent to Young's famous double-slit experiment with light; the pattern is identical to that observed with photons of the same wavelength (compare with Figure 17-1).



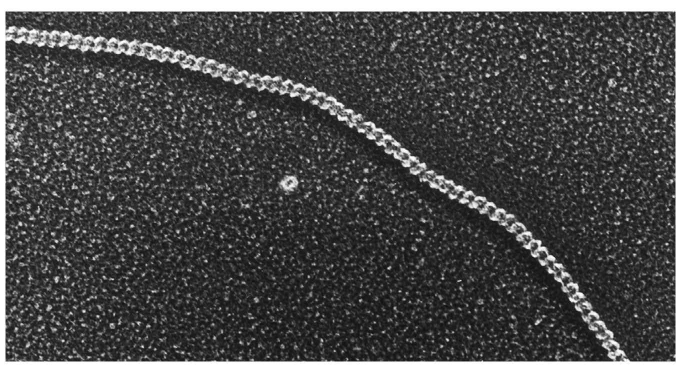
(*a*) (*b*) (*c*) (*d*)

Figure 17-8 (*a*) Diffraction pattern produced by X rays of wavelength 0.071 nm on an aluminum-foil target; (*b*) diffraction pattern produced by 600-eV electrons (λ = 0.050 nm) on an aluminum-foil target; and (*c*) diffraction of 0.0568 eV neutrons (λ = 0.12 nm) incident on a copper foil. (*d*) A two-slit electron diffraction–interference pattern.

Shortly after the wave properties of the electron were demonstrated, it was suggested that electrons rather than light might be used to "see" small objects. As was mentioned in Chapter 15, reflected waves can resolve details of objects only when the details are larger than the wavelength of the reflected wave. Beams of electrons, which can be focused electrically, can have very small wavelengths—much shorter than visible light. Today, the electron microscope is an important research tool used to visualize specimens at scales far smaller than those possible with a light microscope (Figure 17-9).



(*a*)



(*b*)

Figure 17-9 (*a*) Electron microscope. Electrons from a heated filament (the electron gun) are accelerated to a high energy. The electron beam is made parallel by a magnetic focusing lens. The electrons strike a thin target and are then focused by a second magnetic lens. The third magnetic lens projects the electron beam onto a fluorescent screen to produce the image. (*b*) Electron micrograph of DNA.

Standing Waves and Energy Quantization

Given that electrons have wavelike properties, it should be possible to produce standing electron waves. We saw in Chapter 16 that standing waves on a string or standing sound waves occur only for a discrete set of wavelengths and frequencies. If energy is associated with the frequency of a standing wave, as in $E = hf$ (Equation 17-11), then standing waves imply a discrete set of energies. In other words, standing waves imply that energy is quantized.

The idea that the discrete energy states in atoms could be explained by standing waves led to the development by Erwin Schrödinger and others in 1928 of a detailed mathematical theory known as quantum theory, quantum mechanics, or wave mechanics. In this theory, the electron is described by a wave function that obeys a wave equation called the Schrödinger equation. The form of the Schrödinger equation for a particular situation depends on the forces acting on the particle, which are described by the potential energy functions associated with those forces. In Chapter 36 we discuss this equation, which is somewhat similar to the classical wave equations for sound or light. Schrödinger solved the standing-wave problem for the hydrogen atom, the simple harmonic oscillator, and other systems of interest. He found that the allowed frequencies, combined with $E = hf$, resulted in the set of energy levels found experimentally for the hydrogen atom, thereby demonstrating that quantum theory provides a general method of finding the quantized energy levels for a given system. Quantum theory is the basis for our understanding of the modern world, from the inner workings of the atomic nucleus to the radiation spectra of distant galaxies.

17-5 The Interpretation of the Wave Function

The wave function for waves on a string is the string displacement y. The wave function for sound waves can be either the displacement s of the air molecules or the density ρ. The wave function for light and other electromagnetic waves is the electric field $\vec{E}$. What is the wave function for electron waves? The symbol we use for this wave function is ψ (the Greek letter psi). When Schrödinger published his wave equation, neither he nor anyone else knew just how to interpret the wave function ψ. We can get a hint about how to interpret ψ by considering the quantization of light waves. For sound or light waves, the energy per unit volume in the wave is proportional to the square of the wave function. Since the energy of a light wave is quantized, the energy per unit volume is proportional to the number of photons per unit volume. We might therefore expect the square of the photon's wave function to be proportional to the number of photons per unit volume in a light wave. But suppose we have a very low-energy source of light that emits just one photon at a time. In any unit volume, there is either one photon or none. The square of the wave function must then describe the *probability* of finding a photon in some unit volume.

The Schrödinger equation describes a single particle. The square of the wave function for a particle must then describe the *probability* of finding the particle in some unit volume. The probability of finding the particle in some volume element must also be proportional to the size of the volume element dV. Thus, in one dimension, the probability of finding a particle in a region dx at the position x is $\psi^2(x)\,dx$. If we call this probability $P(x)\,dx$, where $P(x)$ is the **probability density,** we have

$$P(x) = \psi^2(x) \tag{17-14}$$

Probability density

Generally, the wave function depends on time as well as position and is written $\Psi(x,t)$, with an uppercase psi. However, for standing waves, the probability density is independent of time. Since we will be concerned mostly with standing waves in this chapter, we omit the time dependence of the wave function, and write it $\psi(x)$ or just ψ.

The probability of finding the particle in dx at point x_1 *or* at point x_2 is the sum of the separate probabilities $P(x_1)\,dx + P(x_2)\,dx$. If we have a particle at all, the probability of finding the particle somewhere must be 1. Then the sum of the probabilities over all the possible values of x must equal 1. That is,

$$\int_{-\infty}^{\infty} \psi^2\, dx = 1 \qquad \text{17-15}$$

Normalization condition

Equation 17-15 is called the **normalization condition.** If ψ is to satisfy the normalization condition, it must approach zero as x approaches infinity. This places a restriction on the possible solutions of the Schrödinger equation.

Example 17-5

A classical point particle moves back and forth with constant speed between two walls at $x = 0$ and $x = 8$ cm (Figure 17-10). (*a*) What is the probability density $P(x)$? (*b*) What is the probability of finding the particle at $x = 2$ cm? (*c*) What is the probability of finding the particle between $x = 3.0$ cm and $x = 3.4$ cm?

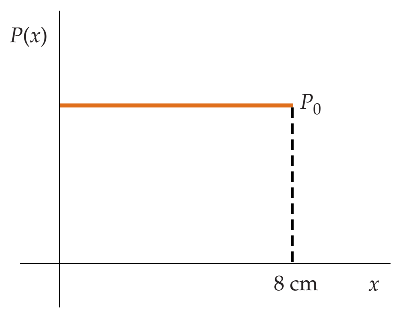


Figure 17-10 Probability function $P(x)$.

Picture the Problem The probability of finding a classical particle in some region dx is proportional to the time spent in that region, dx/v, where v is the speed. Since the speed is constant, the probability density $P(x)$ is constant, independent of x, for $0 < x < 8$ cm. Outside of this range, $P(x)$ is zero. We can find the constant by normalization, that is, by requiring that the probability of finding the particle somewhere between $x = 0$ and $x = 8$ cm is 1.

(*a*) 1. The probability density $P(x)$ is constant between the walls:

$$P(x) = P_0, \quad 0 < x < 8\text{ cm}$$
$$P(x) = 0, \quad x < 0 \text{ or } x > 8\text{ cm}$$

2. Apply the normalization condition:

$$\int_{-\infty}^{+\infty} P(x)\,dx = \int_0^{8\text{ cm}} P_0\,dx = P_0(8\text{ cm}) = 1$$

3. Solve for P_0:

$$P(x) = P_0 = \frac{1}{8\text{ cm}}$$

(*b*) The probability of finding the particle in some range dx is proportional to dx. Since $dx = 0$, the probability of finding the particle at the point $x = 2$ cm is 0.

(*c*) Since the probability density is constant, the probability of a particle being in some range Δx in the region $0 < x < 8$ cm is $P_0\,\Delta x$. The probability of the particle being in the region 3.0 cm $< x <$ 3.4 cm is thus:

$$P_0\,\Delta x = \frac{1}{8\text{ cm}}\,0.4\text{ cm} = 0.05$$

Remark Note in step 2 of part (*a*) that we need only integrate from 0 to 8 cm because $P(x)$ is zero outside this range.

17-6 Wave–Particle Duality

We have seen that light, which we ordinarily think of as a wave, exhibits particle properties when it interacts with matter, as in the photoelectric effect or in Compton scattering. Electrons, which we usually think of as particles, exhibit the wave properties of interference and diffraction. All carriers of momentum and energy, such as electrons, atoms, light, or sound, have both particle and wave characteristics. It might be tempting to say that an electron, for example, is both a wave and a particle, but what does this mean? In classical physics, the concepts of waves and particles are mutually exclusive. A **classical particle** behaves like a piece of shot; it can be localized and scattered, it exchanges energy suddenly at a point in space, and it obeys the laws of conservation of energy and momentum in collisions. It does *not* exhibit interference or diffraction. A **classical wave,** on the other hand, behaves like a water wave; it exhibits diffraction and interference, and its energy is spread out continuously in space and time. Nothing can be both a classical particle and a classical wave at the same time.

After Thomas Young observed the two-slit interference pattern with light in 1801, light was thought to be a classical wave. On the other hand, the electrons discovered by J. J. Thomson were thought to be classical particles. We now know that these classical concepts of waves and particles do not adequately describe the complete behavior of any phenomenon.

> All carriers of energy and momentum, such as light and electrons, propagate like a wave and exchange energy like a particle.

Often the concepts of the classical particle and the classical wave give the same results. When the wavelength is very small, diffraction effects are negligible, so the waves travel in straight lines like classical particles. Also, interference is not seen for waves of very small wavelength because the interference maxima and minima are too closely spaced to be observed. It then makes no difference which concept we use. When diffraction is negligible, we can think of light as a wave propagating along rays or as a beam of photon particles. Similarly, we can think of an electron as a wave propagating in straight lines along rays or, more commonly, as a particle.

We can also use either the wave or particle concept to describe exchanges of energy if we have a large number of particles and we are interested only in the average values of energy and momentum exchanges.

The Two-Slit Experiment Revisited The wave–particle duality of nature is illustrated by the analysis of the experiment in which an electron is incident on a barrier with two slits. The analysis is the same whether we use electrons or photons (light). To describe the propagation of the electron, we must use wave theory. Consider an electron wave that traverses both slits of the two-slit barrier. The two slits act as point sources of spherical electron waves. The wave function at a point on a screen or film far from the slits depends on the path difference from the two slits. At points for which the path difference is 0 or an integral number of wavelengths, the wave function ψ is maximum. Since the probability of detecting the electron is proportional to ψ^2, the electron is most likely to arrive at these points. At points for which the path difference is a half-wavelength or an odd number of half-wavelengths, the wave function ψ is zero, implying that there is zero probability of the electron arriving at such a point. The chapter opening photo on page 509 shows the interference pattern produced by 10 electrons, 100 electrons, 3,000 electrons, and 70,000 electrons. Note that, although the electron propagates through the slits like a wave, it interacts with the screen at a single point like a particle.

The Uncertainty Principle An important consequence of the wave–particle duality of nature is the uncertainty principle, which states that it is impossible in principle to simultaneously measure both the position and momentum of a particle with unlimited precision. A common way to measure the position of an object is to look at it with light. When we do this, we scatter light from the object and determine the position by the direction of the scattered light. If we use light of wavelength λ, we can measure the position only to an uncertainty of the order of λ because of diffraction effects:

$$\Delta x \sim \lambda$$

To reduce the uncertainty in position we therefore use light of very short wavelength, perhaps even X rays. In principle, there is no limit to the accuracy of such a position measurement because there is no limit on how small the wavelength λ can be.

If we know the mass of a particle, we can determine its momentum by measuring its position at two nearby times and computing its velocity. If we use light of wavelength λ, the photons carry momentum h/λ. When these photons are scattered by the particle under scrutiny, the momentum of the particle is changed by the scattering in an uncontrollable way. Each photon carries momentum h/λ, so the uncertainty in the momentum of the particle, introduced by looking at it, is of the order of h/λ:

$$\Delta p \sim \frac{h}{\lambda}$$

When the wavelength of the radiation is small, the momentum of each photon will be large and the momentum measurement will have a large uncertainty. This uncertainty cannot be eliminated by reducing the intensity of light; such a reduction merely reduces the number of photons in the beam. To "see" the particle we must scatter at least one photon. Therefore, the uncertainty in the momentum measurement of the particle will be large if λ is small, and the uncertainty in the position measurement of the particle will be large if λ is large.

Of course we could always "look at" the particles by scattering electrons instead of photons, but the same difficulty remains. If we use low-momentum electrons to reduce the uncertainty in the momentum measurement, we have a large uncertainty in the position measurement because of diffraction of the electrons. The relation between the wavelength and momentum $\lambda = h/p$ is the same for electrons as for photons.

The product of the intrinsic uncertainties in position and momentum is

$$\Delta x\, \Delta p \sim \lambda \times \frac{h}{\lambda} = h$$

If we define precisely what we mean by uncertainties in measurement, we can give a precise statement of the uncertainty principle. If Δx and Δp are defined to be the standard deviations in the measurements of position and momentum, it can be shown that their product must be greater than or equal to $\hbar/2$:

$$\Delta x\, \Delta p \geq \tfrac{1}{2}\hbar \qquad \text{17-16}$$

where $\hbar$ (read h bar) $= h/2\pi$.*

Equation 17-16 provides a statement of the uncertainty principle first enunciated by Werner Heisenberg in 1927. In practice, the experimental uncertainties are usually much greater than the intrinsic lower limit that results from wave–particle duality.

*The combination $h/2\pi$ occurs so often that it is given a special symbol, somewhat analogous to giving the special symbol ω for $2\pi f$, which occurs often in oscillations.

optional

17-7 A Particle in a Box

We can illustrate many of the important features of quantum physics by considering the simple problem of a particle of mass m confined to a one-dimensional box of length L, like the particle in Example 17-5. This situation is analogous to an electron confined within an atom, or a proton confined within a nucleus. When a classical particle bounces back and forth between the walls of the box, its energy and momentum can have any values. However, according to quantum theory, the particle is described by a wave function ψ, whose square describes the probability of finding the particle in some region. Since we are assuming that the particle is indeed inside the box, the wave function must be zero everywhere outside the box. If the box is between $x = 0$ and $x = L$, we have

$$\psi = 0 \qquad \text{for } x \leq 0 \quad \text{and} \quad \text{for } x \geq L$$

In particular, since the wave function is continuous, it must be zero at the end points of the box $x = 0$ and $x = L$. This is the same condition as that for standing waves on a string fixed at $x = 0$ and $x = L$, and the results are the same. The allowed wavelengths for a particle in the box are those such that the length L equals an integral number of half-wavelengths (Figure 17-11).

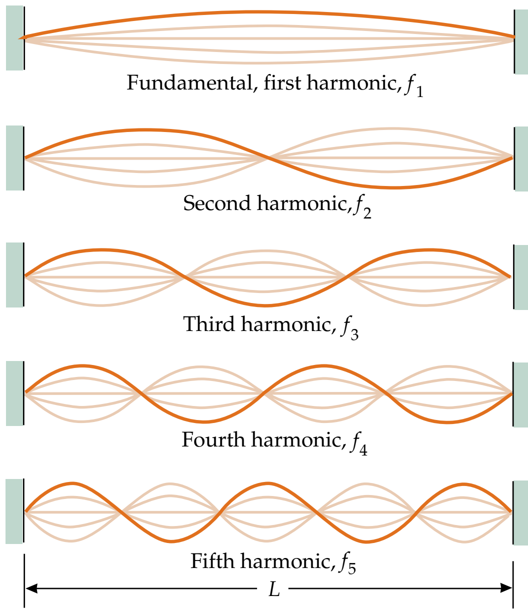


Figure 17-11 Standing waves on a string fixed at both ends. The standing-wave condition is the same as for standing electron waves in a box.

$$L = n\frac{\lambda_n}{2}, \qquad n = 1, 2, 3, \ldots \tag{17-17}$$

Standing-wave condition, particle in a box of length L

The total energy of a particle in a box is its kinetic energy:

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Substituting the de Broglie relation $p_n = h/\lambda_n$, we get

$$E_n = \frac{p_n^2}{2m} = \frac{(h/\lambda_n)^2}{2m}$$

Then the standing-wave condition $\lambda_n = 2L/n$ gives the allowed energies:

$$E_n = \frac{h^2}{2m\lambda_n^2} = n^2\frac{h^2}{8mL^2} = n^2E_1 \tag{17-18}$$

Allowed energies for a particle in a box

where

$$E_1 = \frac{h^2}{8mL^2} \tag{17-19}$$

Ground-state energy for a particle in a box

is the energy of the lowest state, the ground state.

The condition $\psi = 0$ at $x = 0$ and $x = L$ is called a **boundary condition.** Boundary conditions in quantum theory lead to energy quantization. Figure 17-12 shows the energy-level diagram for a particle in a box. Note that the lowest energy is not zero. This result is a general feature of quantum theory.

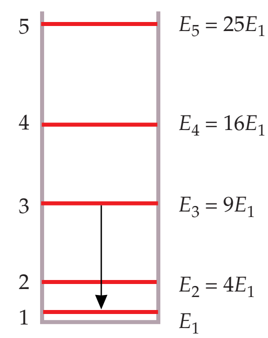


Figure 17-12 Energy-level diagram for a particle in a box. Classically, a particle can have any energy value. Quantum mechanically, only those energy values given by Equation 17-18 are allowed. A transition between the state $n = 3$ and the ground state $n = 1$ is indicated by the vertical arrow.

When a particle is confined to some region of space, it has a minimum energy, which is called the **zero-point energy.** The smaller the region of space, the greater the zero-point energy, as indicated by the fact that E_1 varies as $1/L^2$ in Equation 17-19.

If an electron is in some energy state E_i, it can make a transition to another energy state E_f with the emission of a photon (if $E_f < E_i$) or the absorption of a photon (if $E_f > E_i$). The transition from state 3 to the ground state is indicated in Figure 17-12 by the vertical arrow. The frequency of the emitted photon is found from conservation of energy*

$$hf = E_i - E_f \tag{17-20}$$

The wavelength of the photon is then

$$\lambda = \frac{c}{f} = \frac{hc}{E_i - E_f} \tag{17-21}$$

Wave Functions for Standing Waves

The instantaneous shape of a vibrating string fixed at $x = 0$ and $x = L$ is given by Equation 16-15:

$$y_n = A_n \sin k_n x$$

where A_n is a constant, and $k_n = 2\pi/\lambda_n$ is the wave number. The wave functions for a particle in a box (which can be obtained by solving the Schrödinger equation, as we will see in Chapter 36) are the same:

$$\psi_n(x) = A_n \sin k_n x$$

where $k_n = 2\pi/\lambda_n$. Using $\lambda_n = 2L/n$, we have

$$k_n = \frac{2\pi}{\lambda_n} = \frac{2\pi}{2L/n} = \frac{n\pi}{L}$$

The wave functions can thus be written

$$\psi_n(x) = A_n \sin \frac{n\pi x}{L}$$

The constant A_n is determined by the normalization condition (Equation 17-15):

$$\int_{-\infty}^{\infty} \psi^2\, dx = \int_0^L A_n^2 \sin^2 \frac{n\pi x}{L}\, dx = 1$$

Note that we need integrate only from $x = 0$ to $x = L$ because $\psi(x)$ is zero everywhere else. The integration can be done using tables. The result is

$$A_n = \sqrt{\frac{2}{L}}$$

independent of n. The normalized wave functions for a particle in a box are thus

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \tag{17-22}$$

Wave functions for a particle in a box

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* This equation was first proposed by Niels Bohr in his model of the hydrogen atom in 1913, about 10 years before de Broglie's suggestion that electrons have wave properties. We study the Bohr model in Chapter 37.

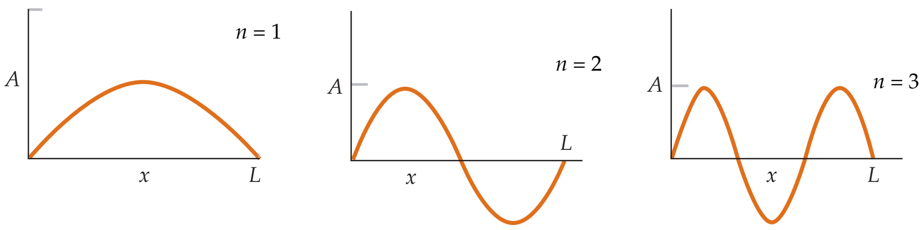
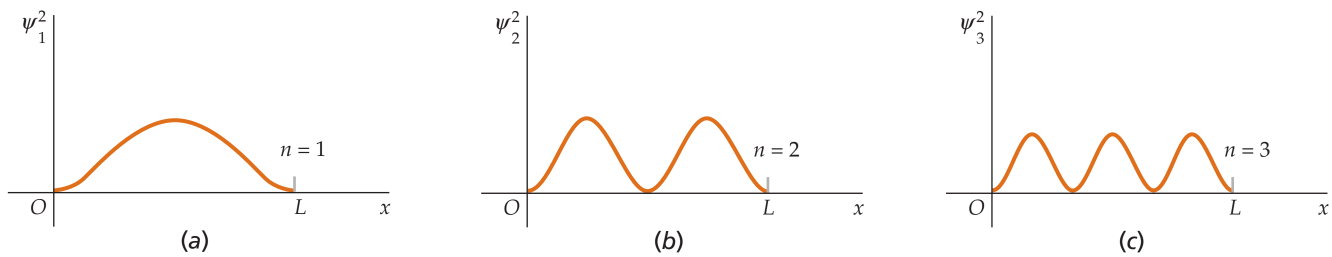


Figure 17-13 Standing-wave functions for $n = 1, 2,$ and 3.

These functions for $n = 1, 2,$ and 3 are shown in Figure 17-13.

The number n is called a **quantum number.** It characterizes the wave function for a particular state and the energy of that state. In our one-dimensional problem, it arises from the boundary condition on the wave function that it must be zero at $x = 0$ and $x = L$. In three-dimensional problems, three quantum numbers arise, one associated with a boundary condition in each dimension.

Figure 17-14 shows plots of ψ^2 for the ground state $n = 1$, the first excited state $n = 2$, the second excited state $n = 3$, and the state $n = 10$. In the ground state, the particle is most likely to be found near the center of the box, as indicated by the maximum value of ψ^2 at $x = L/2$. In the first excited state, the particle is never found exactly in the center of the box because ψ^2 is zero at $x = L/2$. For very large values of n, the maxima and minima of ψ^2 are very close together, as illustrated for $n = 10$. The average value of ψ^2 is indicated in this figure by the dashed line. For very large values of n, the maxima are so closely spaced that ψ^2 cannot be distinguished from its average value. The fact that $(\psi^2)_{av}$ is constant across the whole box means that the particle is equally likely to be found anywhere in the box—the same as the classical result. This is an example of **Bohr's correspondence principle**:



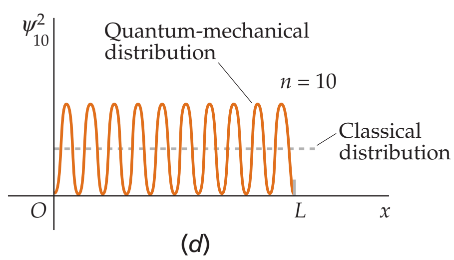


Figure 17-14 ψ^2 versus x for a particle in a box of length L for (*a*) the ground state, $n = 1$; (*b*) the first excited state, $n = 2$; (*c*) the second excited state, $n = 3$; and (*d*) the state $n = 10$. For large n, the maxima and minima of ψ^2 are so close together that individual maxima may be hard to distinguish. The average value of ψ^2 is indicated in (*d*) by the dashed line. It gives the classical prediction that the particle is equally likely to be found at any point in the box.

> In the limit of very large quantum numbers, the classical calculation and the quantum calculation must yield the same results.

Bohr's correspondence principle

When the quantum numbers are very large, the energy is very large. For large energies, the percentage change in energy between adjacent quantum states is very small, so energy quantization is not important (see Problem 83).

We are so accustomed to thinking of the electron as a classical particle that we tend to think of an electron in a box as a particle bouncing back and forth between the walls. But the probability distributions shown in Figure 17-14 are stationary; that is, they do not depend on time. A better picture for an electron in a bound state is a cloud of charge with the charge density proportional to ψ^2. The graphs in Figure 17-14 can then be thought of as plots of the charge density versus x for the various states. In the ground state, $n = 1$, the electron cloud is centered in the middle of the box and is spread out over most of the box, as indicated in Figure 17-14*a*. In the first excited state, $n = 2$, the charge density of the electron cloud has two maxima, as indicated in Figure 17-14*b*. For very large values of n, there are many closely spaced maxima and minima in the charge density resulting in an average charge density that is approximately uniform throughout the box. This probability-cloud picture of an electron is very useful in understanding the structure of atoms and molecules. However, it should be noted that whenever an electron is observed to interact with matter or radiation, it is always observed as a whole unit charge.

Example 17-6

An electron is in a one-dimensional box of length 0.1 nm. (*a*) Find the ground-state energy. (*b*) Find the energy in electron volts of the five lowest states and make an energy-level diagram. (*c*) Find the wavelength of the photon emitted for each transition from the state $n = 3$ to a lower-energy state.

Picture the Problem For (*a*) and (*b*), the energies are given by $E_n = n^2 E_1$, where $E_1 = h^2/8mL^2 = (hc)^2/8(mc^2)L^2$. For (*c*), the photon wavelengths are given by $\lambda = hc/(E_\mathrm{i} - E_\mathrm{f})$.

(*a*) Use $hc = 1240$ eV·nm, and $mc^2 = 5.11 \times 10^5$ eV to calculate E_1:

$$E_1 = \frac{(hc)^2}{8(mc^2)L^2} = \frac{(1240\ \mathrm{eV\cdot nm})^2}{8(5.11 \times 10^5\ \mathrm{eV})(0.1\ \mathrm{nm})^2} = 37.6\ \mathrm{eV}$$

(*b*) Calculate $E_n = n^2 E_1$ for $n = 2, 3, 4$, and 5:

$$E_2 = (2)^2(37.6\ \mathrm{eV}) = 150\ \mathrm{eV}$$

$$E_3 = (3)^2(37.6\ \mathrm{eV}) = 338\ \mathrm{eV}$$

$$E_4 = (4)^2(37.6\ \mathrm{eV}) = 602\ \mathrm{eV}$$

$$E_5 = (5)^2(37.6\ \mathrm{eV}) = 940\ \mathrm{eV}$$

(*c*) 1. Use the energies in (*b*) to calculate the wavelength for a transition from state 3 to state 2:

$$\lambda = \frac{hc}{E_3 - E_2} = \frac{1240\ \mathrm{eV\cdot nm}}{338\ \mathrm{eV} - 150\ \mathrm{eV}} = 6.60\ \mathrm{nm}$$

2. Then use the energies in (*a*) and (*b*) to calculate the wavelength for a transition from state 3 to state 1:

$$\lambda = \frac{hc}{E_3 - E_1} = \frac{1240\ \mathrm{eV\cdot nm}}{338\ \mathrm{eV} - 37.6\ \mathrm{eV}} = 4.13\ \mathrm{nm}$$

Remarks The energy-level diagram is shown in Figure 17-15. The transitions from $n = 3$ to $n = 2$ and from $n = 3$ to $n = 1$ are indicated by the vertical arrows. The ground-state energy of 37.6 eV is of the same order of magnitude as the kinetic energy of the electron in the ground state of the hydrogen atom, which is 13.6 eV. In the hydrogen atom, the electron also has potential energy of -27.2 eV in the ground state, giving a total ground-state energy of -13.6 eV.

Exercise Calculate the wavelength of the photon emitted if the electron makes a transition from $n = 4$ to $n = 3$. (*Answer* 4.70 nm)

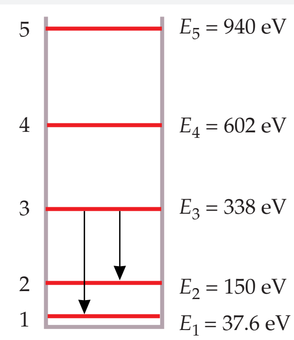


Figure 17-15

optional

17-8 Expectation Values

The solution of a classical mechanics problem is typically specified by giving the position of a particle as a function of time. But the wave nature of matter prevents us from doing this for microscopic systems. The most that we can know is the probability of measuring a certain value of position x. If we measure the position for a large number of identical systems, we get a range of values corresponding to the probability distribution. The average value of x obtained from such measurements is called the **expectation value** and is written $\langle x \rangle$. The expectation value of x is the same as the average value of x that we would expect to obtain from a measurement of the positions of a large number of particles with the same wave function $\psi(x)$.

Since $\psi^2(x)\,dx$ is the probability of finding a particle in the region dx, the expectation value of x is

$$\langle x \rangle = \int x\psi^2(x)\,dx \qquad \text{17-23}$$

Expectation value of x defined

The expectation value of any function $f(x)$ is given by

$$\langle f(x) \rangle = \int f(x)\psi^2(x)\,dx \qquad \text{17-24}$$

Expectation value of f(x) defined

Calculating Probabilities and Expectation Values*

The problem of a particle in a box allows us to illustrate the calculation of the probability of finding the particle in various regions of the box, and the expectation values for various energy states. We give two examples, using the wave functions given by Equation 17-22.

*These calculations are somewhat complicated and may be skipped over on a first reading. Students required to perform similar calculations in problems will find these examples helpful.

Example 17-7

A particle in a one-dimensional box of length L is in the ground state. Find the probability of finding the particle (a) in the region $\Delta x = 0.01L$ at $x = \frac{1}{2}L$, and (b) in the region $0 < x < \frac{1}{4}L$.

Picture the Problem The probability of finding the particle in some range dx is $\psi^2\,dx$. For (a) (Figure 17-16a), the region $\Delta x = 0.01L$ is so small that we can neglect the variation in $\psi(x)$ and just compute $\psi^2\,\Delta x$. For (b) (Figure 17-16b), we must take into account the variation of $\psi(x)$ and integrate from 0 to $L/4$. These probabilities are indicated by the shaded regions in the figures.

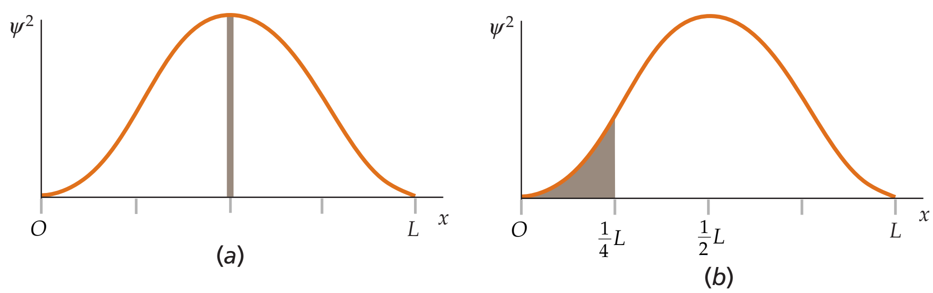


Figure 17-16

(a)1. The probability of finding the particle in some range dx is $\psi^2\,dx$:

$$P(x)\,dx = \psi^2(x)\,dx = \frac{2}{L}\sin^2\frac{\pi x}{L}\,dx$$

2. Since ψ^2 does not vary rapidly near $x = L/2$, and the region $\Delta x = 0.01L$ is very small compared with L, we do not need to integrate. The approximate probability is $\psi^2(x)\,\Delta x$. Substitute $x = \frac{1}{2}L$ and $\Delta x = 0.01L$:

$$P = \frac{2}{L}\left(\sin^2\frac{\pi}{2}\right)(0.01L) = \frac{2}{L}(1.0)(0.01L) = 0.02$$

(b)1. For the region $0 < x < L/4$, integrate from $x = 0$ to $x = L/4$:

$$P = \int_0^{L/4}\frac{2}{L}\sin^2\frac{\pi x}{L}\,dx$$

2. Change the integration variables to $\theta = \pi x/L$:

$$P = \frac{2}{L}\frac{L}{\pi}\int_0^{\pi/4}\sin^2\theta\,d\theta$$

3. The integral can be found in tables:

$$\int_0^{\pi/4}\sin^2\theta\,d\theta = \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\Bigg|_0^{\pi/4} = \frac{\pi}{8} - \frac{1}{4}$$

4. Use this result to calculate the probability:

$$P = \frac{2}{\pi}\int_0^{\pi/4}\sin^2\theta\,d\theta = \frac{2}{\pi}\left(\frac{\pi}{8} - \frac{1}{4}\right) = 0.091$$

Remarks: The chance of finding the particle in the region $\Delta x = 0.01L$ at $x = \frac{1}{2}L$ is approximately 2%. The chance of finding the particle in the region $0 < x < L/4$ is about 9.1%.

Example 17-8

(*a*) Find $\langle x\rangle$ for a particle in its ground state in a box of length L, and (*b*) find $\langle x^2\rangle$.

Picture the Problem We use $\langle f(x)\rangle = \int f(x)\psi^2(x)\,dx$ with

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$$

(a)1. Write $\langle x\rangle$ using the ground-state wave function given by Equation 17-22 with $n = 1$:

$$\langle x\rangle = \int_{-\infty}^{+\infty} x\psi^2(x)\,dx = \int_0^L x\,\frac{2}{L}\sin^2\frac{\pi x}{L}\,dx$$

2. Substitute $\theta = \pi x/L$:

$$\langle x\rangle = \frac{2}{L}\left(\frac{L}{\pi}\right)^2\int_0^{\pi}\theta\sin^2\theta\,d\theta = \frac{2L}{\pi^2}\int_0^{\pi}\theta\sin^2\theta\,d\theta$$

3. Evaluate the integral by looking it up in tables:

$$\int_0^{\pi}\theta\sin^2\theta\,d\theta = \left[\frac{\theta^2}{4} - \frac{\theta\sin 2\theta}{4} - \frac{\cos 2\theta}{8}\right]_0^{\pi} = \frac{\pi^2}{4}$$

4. Substitute this value into the expression in step 2:

$$\langle x\rangle = \frac{2L}{\pi^2}\int_0^{\pi}\theta\sin^2\theta\,d\theta = \frac{2L}{\pi^2}\frac{\pi^2}{4} = \frac{L}{2}$$

(b)1. Repeat step 1 for $\langle x^2\rangle$:

$$\langle x^2\rangle = \int_{-\infty}^{+\infty} x^2\psi^2(x)\,dx = \int_0^L x^2\,\frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx$$

2. Again, substitute $\theta = \pi x/L$:

$$\langle x^2\rangle = \frac{2}{L}\left(\frac{L}{\pi}\right)^3\int_0^{\pi}\theta^2\sin^2\theta\,d\theta = \frac{2L^2}{\pi^3}\int_0^{\pi}\theta^2\sin^2\theta\,d\theta$$

optional

3. Evaluate the integral by looking it up in tables:

$$\int_0^\pi \theta^2 \sin^2 \theta \, d\theta = \left[\frac{\theta^3}{6} - \left(\frac{\theta^2}{4} - \frac{1}{8}\right)\sin 2\theta - \frac{\theta \cos 2\theta}{4}\right]_0^\pi = \frac{\pi^3}{6} - \frac{\pi}{4}$$

4. Substitute this value into the expression in step 2 of part (*b*):

$$\langle x^2 \rangle = \frac{2L^2}{\pi^3}\int_0^\pi \theta^2 \sin^2 \theta \, d\theta = \frac{2L^2}{\pi^3}\left(\frac{\pi^3}{6} - \frac{\pi}{4}\right) = L^2\left(\frac{1}{3} - \frac{1}{2\pi^2}\right) = 0.283L^2$$

Remarks The expectation value of x is $L/2$, as we would expect, because the probability distribution is symmetric about the midpoint of the box. Note that $\langle x^2 \rangle$ is not equal to $\langle x \rangle^2$.

17-9 Energy Quantization in Other Systems

The quantized energies of a system are generally determined by solving the Schrödinger equation for that system. The form of the Schrödinger equation depends on the potential energy of the particle. The potential energy for a one-dimensional box from $x = 0$ to $x = L$ is shown in Figure 17-17. This potential-energy function is called an **infinite square-well potential** and is described mathematically by

$$U(x) = 0, \qquad 0 < x < L$$

$$U(x) = \infty, \qquad x < 0 \quad \text{or} \quad x > L \qquad \text{17-25}$$

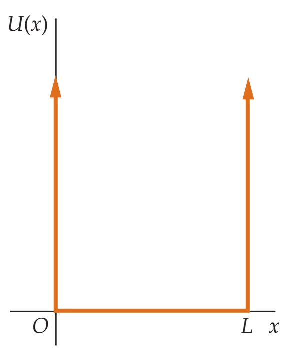


Figure 17-17 The infinite square-well potential energy. For $x < 0$ and $x > L$, the potential energy $U(x)$ is infinite. The particle is confined to the region in the well $0 < x < L$.

Inside the box, the particle moves freely so the potential energy is zero. Outside the box, the potential energy is infinite, so the particle cannot exist outside the box no matter what its energy. We did not need to solve the Schrödinger equation for this potential because the wave functions and quantized frequencies are the same as for a string fixed at both ends, which we studied in Chapter 16. Although this problem seems artificial, it is actually useful in dealing with some physical problems, such as a neutron inside a nucleus.

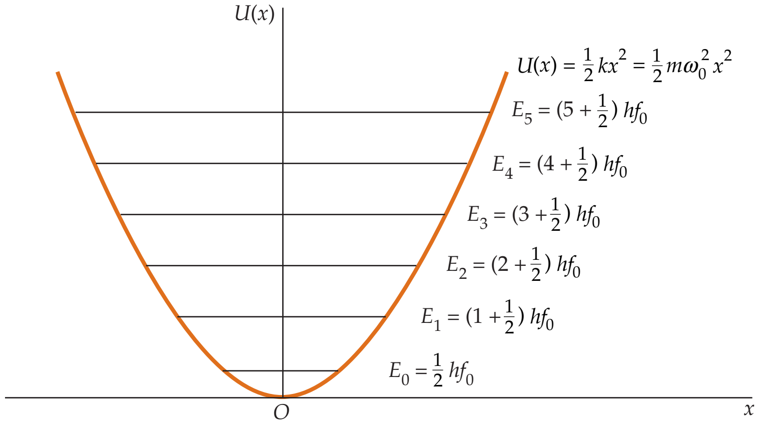


Figure 17-18 Harmonic oscillator potential-energy function. The allowed energy levels are indicated by the equally spaced horizontal lines.

The Harmonic Oscillator

More realistic than the particle in a box is the harmonic oscillator, which applies to an object of mass m on a spring of force constant k or to any system undergoing small oscillations about a stable equilibrium. Figure 17-18 shows the potential-energy function

$$U(x) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega_0^2 x^2$$

where $\omega_0 = \sqrt{k/m}$ is the natural frequency of the oscillator. Classically, the object oscillates between $x = +A$ and $x = -A$. Its total energy is $E = \frac{1}{2}m\omega_0^2 A^2$, which can have any nonnegative value, including zero.

In quantum theory, the particle is represented by the wave function $\psi(x)$, which is determined by solving the Schrödinger equation for this potential. Normalizable wave functions $\psi_n(x)$ occur only for discrete values of the energy E_n given by

$$E_n = (n + \tfrac{1}{2})hf_0, \qquad n = 0, 1, 2, 3, \ldots \qquad \text{17-26}$$

where $f_0 = \omega_0/2\pi$ is the frequency of the oscillator. Note that the energy levels of a harmonic oscillator are evenly spaced with separation hf as indicated in Figure 17-18. Compare this with the uneven spacing of the energy levels for the particle in a box, shown in Figure 17-12. When a harmonic oscillator makes a transition from energy level n to the next lowest energy level $n - 1$, the energy of the photon emitted is

$$E_n - E_{n-1} = (n + \tfrac{1}{2})hf_0 - (n - 1 + \tfrac{1}{2})hf_0 = hf_0$$

The frequency of the emitted photon is therefore equal to the classical frequency of the oscillator.

The Hydrogen Atom

In the hydrogen atom, an electron is bound to a proton by the electrostatic force of attraction, which we shall study in Chapter 22. This force varies inversely as the square of the separation distance (exactly like the gravitational attraction of the earth and sun). The potential energy of the electron–proton system therefore varies inversely with separation distance like the gravitational potential energy described by Equation 11-18. As in the case of gravitational potential energy, the potential energy of the electron–proton system is chosen to be zero when the electron is an infinite distance from the proton. Then for all finite distances the potential energy is negative. Like the case of an object orbiting the earth, the electron–proton system is a bound system when its total energy is negative.

The allowed energies obtained by solving the Schrödinger equation for the hydrogen atom are described by a quantum number n, like the energies of a particle in a box and of a harmonic oscillator. As we shall see in Chapter 37, the allowed energies of the hydrogen atom are given by

$$E_n = -\frac{13.6 \text{ eV}}{n^2}, \qquad n = 1, 2, 3, \ldots \tag{17-27}$$

The lowest energy corresponds to $n = 1$. The ground-state energy is thus -13.6 eV. The energy of the first excited state is $-(13.6 \text{ eV}/2^2) = -3.40$ eV. Figure 17-19 shows the energy-level diagram for the hydrogen atom. Transitions from a higher state to a lower state with the emission of electromagnetic radiation are indicated by the vertical arrows. Only those transitions ending at the first excited state ($n = 2$) involve energy differences in the range of visible light of 1.77 to 3.1 eV, as calculated in Example 17-1.

Other atoms are more complicated than the hydrogen atom, but their energy levels are similar to those of hydrogen. The ground-state energies are of the order of -1 to -10 eV, and many transitions involve energies corresponding to photons in the visible range.

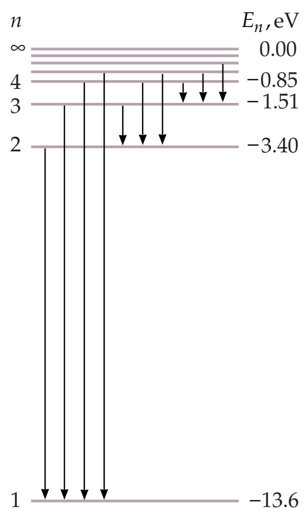


Figure 17-19 Energy-level diagram for the hydrogen atom. The energy of the ground state is -13.6 eV. As n approaches ∞, the energy approaches 0, the highest energy state.

Summary

1. All carriers of energy and momentum propagate like waves and exchange energy like particles.
2. The quantum of light is called a photon. It has energy $E = hf$, where h is Planck's constant.
3. The wavelength of electrons and other "particles" is given by the de Broglie relation $\lambda = h/p$.
4. Energy quantization in bound systems arises from standing wave conditions, which are equivalent to boundary conditions on the wave function.
5. The uncertainty principle is a fundamental law of nature that places theoretical restrictions on the precision of a simultaneous measurement of the position and momentum of a particle. The uncertainty principle follows from the general properties of waves.

Topic	Remarks and Relevant Equations	
1. Wave Nature of Light	The wave nature of light can be demonstrated by the interference of light from two narrow slits illuminated by a single source.	
2. Quantization of Radiation	Light and other electromagnetic energy is not continuous, but instead is quantized. The quantum of light energy is called a photon.	
Einstein equation for photon energy	$E = hf = \frac{hc}{\lambda}$	17-1
Planck's constant	$h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s} = 4.136 \times 10^{-15}\,\mathrm{eV \cdot s}$	17-2
Einstein's photoelectric equation	$K_{max} = (\frac{1}{2}mv^2)_{max} = hf - \phi$ where ϕ is the work function of the cathode	17-3
hc	$hc = 1240\,\mathrm{eV \cdot nm}$	17-5
Momentum of a photon	$p = \frac{h}{\lambda}$	17-7
Compton scattering equation	$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - \cos\theta)$	17-8
3. de Broglie Hypothesis	Electrons and other "particles" have wave properties.	
de Broglie wavelength	$\lambda = \frac{h}{p}$	17-10
de Broglie frequency	$f = \frac{E}{h}$	17-11
λ for nonrelativistic particles	$\lambda = \frac{hc}{\sqrt{2mc^2K}}$	17-12
λ for nonrelativistic electrons	$\lambda = \frac{1.23}{\sqrt{K}}\,\mathrm{nm}$ (K in electron volts)	17-13

optional

optional

4. Quantum Mechanics

The state of a particle such as an electron is described by its wave function ψ, which is the solution of the Schrödinger wave equation.

Probability density

The probability of finding the particle in some region of space dx is given by

$$P(x)\,dx = \psi^2(x)\,dx \qquad \textbf{17-14}$$

Normalization condition

$$\int_{-\infty}^{\infty} \psi^2\,dx = 1 \qquad \textbf{17-15}$$

Quantum number

The wave function for a particular energy state is characterized by a quantum number n. In three dimensions there are three quantum numbers, one associated with each dimension.

Bohr correspondence principle

In the limit of very large quantum numbers, the classical calculation and the quantum calculation must yield the same results.

Expectation value

The expectation value of x is the average value of x that we would expect to obtain from a measurement of the positions of a large number of particles with the same wave function $\psi(x)$.

$$\langle x \rangle = \int x\psi^2(x)\,dx \qquad \textbf{17-23}$$

$$\langle f(x) \rangle = \int f(x)\psi^2(x)\,dx \qquad \textbf{17-24}$$

5. Wave–Particle Duality

Light, electrons, neutrons, and all carriers of energy and momentum exhibit both wave and particle properties. Each propagates like a classical wave, exhibiting diffraction and interference, yet exchanges energy in discrete lumps like a classical particle. Because the wavelength of macroscopic objects is so small, diffraction and interference are not observed. Also, when a macroscopic amount of energy is exchanged, so many quanta are involved that the particle nature of the energy is not evident.

Uncertainty principle

The wave–particle duality of nature leads to the uncertainty principle, which states that the product of the uncertainty in a measurement of position and the uncertainty in a measurement of momentum must be greater than $\frac{1}{2}\hbar$.

$$\Delta x\,\Delta p \geq \tfrac{1}{2}\hbar \quad \text{where } \hbar = h/2\pi \qquad \textbf{17-16}$$

6. Particle in a Box

Allowed energy

$$E_n = \frac{p_n^2}{2m} = n^2\frac{h^2}{8mL^2} = n^2E_1 \qquad \textbf{17-18}$$

Ground-state energy

$$E_1 = \frac{h^2}{8mL^2} \qquad \textbf{17-19}$$

Wave function

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} \qquad \textbf{17-22}$$

Transitions between energy states

A system in state of energy E_i can make a transition to a state of energy E_f by emitting or absorbing a photon of frequency f given by

$$hf = |E_i - E_f| \qquad \textbf{17-20}$$

$$\lambda = \frac{c}{f} = \frac{hc}{|E_i - E_f|} \qquad \textbf{17-21}$$

7.	**Energy Quantization in Other Systems**		
	Harmonic oscillator	$E_n = (n + \frac{1}{2})hf_0, \quad n = 0, 1, 2, 3, \ldots$	**17-26**
	Hydrogen atom	$E_n = -\dfrac{13.6\text{eV}}{n^2}, \quad n = 1, 2, 3, \ldots$	**17-27**

Problem-Solving Guide

1. Begin by drawing a neat diagram that includes the important features of the problem.
2. Numerical calculations of the energies of photons or the wavelengths of electrons can often be simplified by using the combination $hc = 1240$ eV·nm.

Summary of Worked Examples

Type of Calculation		Procedure and Relevant Examples	
1.	**Photons**		
	Find the energy of a photon from its wavelength or the wavelength from its energy.	Use $E = hf = hc/\lambda$ with $hc = 1240$ eV·nm.	**Example 17-1**
	Find the number of photons in a light beam.	The intensity gives the energy per second per unit area. The energy is Nhf.	**Example 17-2**
2.	**Matter Waves**		
	Find the de Broglie wavelength of an electron.	Use $\lambda = h/p = h/mv$.	**Example 17-4**
3.	**Probability**		
	Calculate the classical probability density.	Classically, $P(x)\,dx$ is proportional to the time spent in dx which is dx/v. If the particle is confined, the total probability of finding it in the confined region must be 1.	**Example 17-5**
	Calculate the probability of finding a particle in some region of space Δx.	The probability of a particle being in dx is $\psi^2\,dx$. If Δx is very small, just replace dx with Δx. Otherwise integrate $\psi^2\,dx$ over the region of interest.	**Example 17-7**
	Calculate the expectation value of x or $f(x)$ for a particular state.	Use $\langle f(x) \rangle = \int f(x)\psi^2(x)\,dx$, where $\psi(x)$ is the wave function for that state.	**Example 17-8**
4.	**Energy Quantization**		
	Find the energy levels for a particle in a box.	Use $E_n = n^2h^2/8mL^2 = n^2(hc)^2/8mc^2L^2$.	**Example 17-6**
	Find the energy of a photon emitted by a system making a transition between two energy levels.	Use $\lambda = \dfrac{hc}{E_\text{i} - E_\text{f}}$	**Example 17-6**

optional

Problems

Conceptual Problems

Problems from Optional and Exploring sections

In a few problems, you are given more data than you actually need; in a few other problems, you are required to supply data from your general knowledge, outside sources, or informed estimates.

• Single-concept, single-step, relatively easy
•• Intermediate-level, may require synthesis of concepts
••• Challenging, for advanced students

Photons

1 • The quantized character of electromagnetic radiation is revealed by

(*a*) the Young double-slit experiment.
(*b*) diffraction of light by a small aperture.
(*c*) the photoelectric effect.
(*d*) the J. J. Thomson cathode-ray experiment.

2 •• Two monochromatic light sources, A and B, emit the same number of photons per second. The wavelength of A is $\lambda_A = 400$ nm, and that of B is $\lambda_B = 600$ nm. The power radiated by source B is

(*a*) equal to that of source A.
(*b*) less than that of source A.
(*c*) greater than that of source A.
(*d*) cannot be compared to that from source A using the available data.

3 • Find the photon energy in joules and in electron volts for an electromagnetic wave of frequency (*a*) 100 MHz in the FM radio band, and (*b*) 900 kHz in the AM radio band.

4 • An 80-kW FM transmitter operates at a frequency of 101.1 MHz. How many photons per second are emitted by the transmitter?

5 • What are the frequencies of photons having the following energies? (*a*) 1 eV, (*b*) 1 keV, and (*c*) 1 MeV.

6 • Find the photon energy for light of wavelength (*a*) 450 nm, (*b*) 550 nm, and (*c*) 650 nm.

7 • Find the photon energy if the wavelength is (*a*) 0.1 nm (about 1 atomic diameter), and (*b*) 1 fm (1 fm = 10^{-15} m, about 1 nuclear diameter).

8 •• The wavelength of light emitted by a 3-mW He-Ne laser is 632 nm. If the diameter of the laser beam is 1.0 mm, what is the density of photons in the beam?

The Photoelectric Effect

9 • True or false: In the photoelectric effect,

(*a*) the current is proportional to the intensity of the incident light.
(*b*) the work function of a metal depends on the frequency of the incident light.
(*c*) the maximum kinetic energy of electrons emitted varies linearly with the frequency of the incident light.
(*d*) the energy of a photon is proportional to its frequency.

10 • In the photoelectric effect, the number of electrons emitted per second is

(*a*) independent of the light intensity.
(*b*) proportional to the light intensity.
(*c*) proportional to the work function of the emitting surface.
(*d*) proportional to the frequency of the light.

11 • The work function of a surface is ϕ. The threshold wavelength for emission of photoelectrons from the surface is

(*a*) hc/ϕ.
(*b*) ϕ/hf.
(*c*) hf/ϕ.
(*d*) none of the above.

12 •• When light of wavelength λ_1 is incident on a certain photoelectric cathode, no electrons are emitted no matter how intense the incident light is. Yet when light of wavelength $\lambda_2 < \lambda_1$ is incident, electrons are emitted even when the incident light has low intensity. Explain.

13 • The work function for tungsten is 4.58 eV. (*a*) Find the threshold frequency and wavelength for the photoelectric effect. (*b*) Find the maximum kinetic energy of the electrons if the wavelength of the incident light is 200 nm, and (*c*) 250 nm.

14 • When light of wavelength 300 nm is incident on potassium, the emitted electrons have maximum kinetic energy of 2.03 eV. (*a*) What is the energy of an incident photon? (*b*) What is the work function for potassium? (*c*) What would be the maximum kinetic energy of the electrons if the incident light had a wavelength of 430 nm? (*d*) What is the threshold wavelength for the photoelectric effect with potassium?

15 • The threshold wavelength for the photoelectric effect for silver is 262 nm. (*a*) Find the work function for silver. (*b*) Find the maximum kinetic energy of the electrons if the incident radiation has a wavelength of 175 nm.

16 • The work function for cesium is 1.9 eV. (*a*) Find the threshold frequency and wavelength for the photoelectric effect. Find the maximum kinetic energy of the electrons if the wavelength of the incident light is (*b*) 250 nm, and (*c*) 350 nm.

17 •• When a surface is illuminated with light of wavelength 512 nm, the maximum kinetic energy of the emitted electrons is 0.54 eV. What is the maximum kinetic energy if the surface is illuminated with light of wavelength 365 nm?

Compton Scattering

18 • Find the shift in wavelength of photons scattered at $\theta = 60°$.

19 • When photons are scattered by electrons in carbon, the shift in wavelength is 0.33 pm. Find the scattering angle.

20 • The wavelength of Compton-scattered photons is measured at $\theta = 90°$. If $\Delta\lambda/\lambda$ is to be 1.5%, what should the wavelength of the incident photons be?

21 • Compton used photons of wavelength 0.0711 nm. (*a*) What is the energy of these photons? (*b*) What is the wavelength of the photon scattered at $\theta = 180°$? (*c*) What is the energy of the photon scattered at this angle?

22 • For the photons used by Compton, find the momentum of the incident photon and that of the photon scattered at 180°, and use the conservation of momentum to find the momentum of the recoil electron for this case (see Problem 21).

23 •• An X-ray photon of wavelength 6 pm that collides with an electron is scattered by an angle of 90°. (*a*) What is the change in wavelength of the photon? (*b*) What is the kinetic energy of the scattered electron?

24 •• How many head-on Compton scattering events are necessary to double the wavelength of a photon having initial wavelength 200 pm?

Matter Waves

25 • True or false:

(*a*) The de Broglie wavelength of an electron varies inversely with its momentum.
(*b*) Electrons can be diffracted.
(*c*) Neutrons can be diffracted.
(*d*) An electron microscope is used to look at electrons.

26 • If the de Broglie wavelength of an electron and a proton are equal, then

(*a*) the velocity of the proton is greater than that of the electron.
(*b*) the velocity of the proton and electron are equal.
(*c*) the velocity of the proton is less than that of the electron.
(*d*) the energy of the proton is greater than that of the electron.
(*e*) both (*a*) and (*d*) are correct.

27 • A proton and an electron have equal kinetic energies. It follows that the de Broglie wavelength of the proton is

(*a*) greater than that of the electron.
(*b*) equal to that of the electron.
(*c*) less than that of the electron.

28 • Use Equation 17-13 to calculate the de Broglie wavelength for an electron of kinetic energy (*a*) 2.5 eV, (*b*) 250 eV, (*c*) 2.5 keV, and (*d*) 25 keV.

29 • An electron is moving at $v = 2.5 \times 10^5$ m/s. Find its de Broglie wavelength.

30 • An electron has a wavelength of 200 nm. Find (*a*) its momentum, and (*b*) its kinetic energy.

31 • Find the energy of an electron in electron volts if its de Broglie wavelength is (*a*) 5 nm, and (*b*) 0.01 nm.

32 • A neutron in a reactor has kinetic energy of about 0.02 eV. Calculate the de Broglie wavelength of this neutron from Equation 17-12, where $mc^2 = 940$ MeV is the rest energy of the neutron.

33 • Use Equation 17-12 to find the de Broglie wavelength of a proton (rest energy $mc^2 = 938$ MeV) that has a kinetic energy of 2 MeV.

34 • A proton is moving at $v = 0.003c$, where c is the speed of light. Find its de Broglie wavelength.

35 • What is the kinetic energy of a proton whose de Broglie wavelength is (*a*) 1 nm, and (*b*) 1 fm?

36 • Find the de Broglie wavelength of a baseball of mass 0.145 kg moving at 30 m/s.

37 • The energy of the electron beam in Davisson and Germer's experiment was 54 eV. Calculate the wavelength for these electrons.

38 • The distance between Li^+ and Cl^- ions in a LiCl crystal is 0.257 nm. Find the energy of electrons that have a wavelength equal to this spacing.

39 • An electron microscope uses electrons of energy 70 keV. Find the wavelength of these electrons.

40 • What is the de Broglie wavelength of a neutron with speed 10^6 m/s?

Wave–Particle Duality

41 • Suppose you have a spherical object of mass 4 g moving at 100 m/s. What size aperture is necessary for the object to show diffraction? Show that no common objects would be small enough to squeeze through such an aperture.

42 • A neutron has a kinetic energy of 10 MeV. What size object is necessary to observe neutron diffraction effects? Is there anything in nature of this size that could serve as a target to demonstrate the wave nature of 10-MeV neutrons?

43 • What is the de Broglie wavelength of an electron of kinetic energy 200 eV? What are some common targets that could demonstrate the wave nature of such an electron?

Particle in a Box

44 •• Sketch the wave function $\psi(x)$ and the probability distribution $\psi^2(x)$ for the state $n = 4$ of a particle in a box.

45 •• (*a*) Find the energy of the ground state ($n = 1$) and the first two excited states of a proton in a one-dimensional box of length $L = 10^{-15}$ m $= 1$ fm. (These are of the order of magnitude of nuclear energies.) Make an energy-level diagram for this system and calculate the wavelength of electromagnetic radiation emitted when the proton makes a transition from (*b*) $n = 2$ to $n = 1$, (*c*) $n = 3$ to $n = 2$, and (*d*) $n = 3$ to $n = 1$.

46 •• (*a*) Find the energy of the ground state ($n = 1$) and the first two excited states of a proton in a one-dimensional box of length 0.2 nm (about the diameter of a H_2 molecule). Calculate the wavelength of electromagnetic radiation emitted when the proton makes a transition from (*b*) $n = 2$ to $n = 1$, (*c*) $n = 3$ to $n = 2$, and (*d*) $n = 3$ to $n = 1$.

47 •• (*a*) Find the energy of the ground state and the first two excited states of a small particle of mass 1 μg confined to a one-dimensional box of length 1 cm. (*b*) If the particle moves with a speed of 1 mm/s, calculate its kinetic energy and find the approximate value of the quantum number n.

Calculating Probabilities and Expectation Values

48 •• A particle is in the ground state of a box of length L. Find the probability of finding the particle in the interval $\Delta x = 0.002L$ at (*a*) $x = L/2$, (*b*) $x = 2L/3$, and (*c*) $x = L$. (Since Δx is very small, you need not do any integration because the wave function is slowly varying.)

49 •• Do Problem 48 for a particle in the first excited state ($n = 2$).

50 •• Do Problem 48 for a particle in the second excited state ($n = 3$).

51 •• The classical probability distribution function for a particle in a box of length L is given by $P(x) = 1/L$. Use this to find $\langle x \rangle$ and $\langle x^2 \rangle$ for a classical particle in such a box.

52 •• (*a*) Find $\langle x \rangle$ for the first excited state ($n = 2$) for a particle in a box of length L, and (*b*) find $\langle x^2 \rangle$.

53 •• (*a*) Find $\langle x \rangle$ for the second excited state ($n = 3$) for a particle in a box of length L, and (*b*) find $\langle x^2 \rangle$.

54 •• A particle in a one-dimensional box is in the first excited state ($n = 2$). (*a*) Sketch $\psi^2(x)$ versus x for this state. (*b*) What is the expectation value $\langle x \rangle$ for this state? (*c*) What is the probability of finding the particle in some small region dx centered at $x = \frac{1}{2}L$? (*d*) Are your answers for (*b*) and (*c*) contradictory? If not, explain.

55 •• A particle of mass m has a wave function given by $\psi(x) = Ae^{-|x|/a}$, where A and a are constants. (*a*) Find the normalization constant A. (*b*) Calculate the probability of finding the particle in the region $-a \le x \le a$.

56 •• A particle in a one-dimensional box of length L is in its ground state. Calculate the probability that the particle will be found in the region (*a*) $0 < x < \frac{1}{2}L$, (*b*) $0 < x < \frac{1}{3}L$, and (*c*) $0 < x < \frac{3}{4}L$.

57 •• Repeat Problem 56 for a particle in the first excited state of the box.

58 •• (*a*) For the wave functions

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}, \qquad n = 1, 2, 3, \ldots$$

corresponding to a particle in the nth state of a one-dimensional box of length L, show that

$$\langle x^2 \rangle = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}$$

(*b*) Compare this result for $n \gg 1$ with your answer for the classical distribution of Problem 51.

59 •• The wave functions for a particle of mass m in a one-dimensional box of length L *centered at the origin* (so that the ends are at $x = \pm L/2$) are given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \cos \frac{n\pi x}{L}, \qquad n = 1, 3, 5, 7, \ldots$$

and

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}, \qquad n = 2, 4, 6, 8, \ldots$$

Calculate $\langle x \rangle$ and $\langle x^2 \rangle$ for the ground state.

60 •• Calculate $\langle x \rangle$ and $\langle x^2 \rangle$ for the first excited state of the box described in Problem 59.

General Problems

61 • Can the expectation value of x ever equal a value that has zero probability of being measured?

62 • Explain why the maximum kinetic energy of electrons emitted in the photoelectric effect does not depend on the intensity of the incident light, but the total number of electrons emitted does.

63 •• A six-sided die has the number 1 painted on three sides and the number 2 painted on the other three sides. (*a*) What is the probability of a 1 coming up when the die is thrown? (*b*) What is the expectation value of the number that comes up when the die is thrown?

64 •• True or false:

(*a*) It is impossible in principle to know precisely the position of an electron.
(*b*) A particle that is confined to some region of space cannot have zero energy.
(*c*) All phenomena in nature are adequately described by classical wave theory.
(*d*) The expectation value of a quantity is the value that you expect to measure.

65 •• It was once believed that if two identical experiments are done on identical systems under the same conditions, the results must be identical. Explain why this is not true, and how it can be modified so that it is consistent with quantum physics.

66 • A light beam of wavelength 400 nm has an intensity of 100 W/m^2. (*a*) What is the energy of each photon in the beam? (*b*) How much energy strikes an area of 1 cm^2 perpendicular to the beam in 1 s? (*c*) How many photons strike this area in 1 s?

67 • A mass of 10^{-6} g is moving with a speed of about 10^{-1} cm/s in a box of length 1 cm. Treating this as a one-dimensional particle in a box, calculate the approximate value of the quantum number n.

68 • (*a*) For the classical particle of Problem 67, find Δx and Δp, assuming that these uncertainties are given by $\Delta x/L = 0.01\%$ and $\Delta p/p = 0.01\%$. (*b*) What is $(\Delta x\, \Delta p)/\hbar$?

69 • In 1987, a laser at Los Alamos National Laboratory produced a flash that lasted 1.0×10^{-12} s and had a power of 5.0×10^{15} W. Estimate the number of emitted photons if their wavelength was 400 nm.

70 • You can't see anything smaller than the wavelength λ used. What is the minimum energy of an electron needed in an electron microscope to see an atom, which has a diameter of about 0.1 nm?

71 • A common flea that has a mass of 0.008 g can jump vertically as high as 20 cm. Estimate the de Broglie wavelength for the flea immediately after takeoff.

72 • The work function for sodium is $\phi = 2.3$ eV. Find the minimum de Broglie wavelength for the electrons emitted by a sodium cathode illuminated by violet light with a wavelength of 420 nm.

73 •• Suppose that a 100-W source radiates light of wavelength 600 nm uniformly in all directions and that the eye can detect this light if only 20 photons per second enter a dark-adapted eye having a pupil 7 mm in diameter. How far from the source can the light be detected under these rather extreme conditions?

74 •• Data for maximum kinetic energy of the electrons versus wavelength for the photoelectric effect using sodium are

λ, nm	200	300	400	500	600
K_{max}, eV	4.20	2.06	1.05	0.41	0.03

Plot these data so as to obtain a straight line and from your plot find (*a*) the work function, (*b*) the threshold frequency, and (*c*) the ratio h/e.

75 •• The diameter of the pupil of the eye under room-light conditions is about 5 mm. (It can vary from about 1 to 8 mm.) Find the intensity of light of wavelength 600 nm such that 1 photon per second passes through the pupil.

76 •• A light bulb radiates 90 W uniformly in all directions. (*a*) Find the intensity at a distance of 1.5 m. (*b*) If the wavelength is 650 nm, find the number of photons per second that strike a surface of area 1 cm^2 oriented so that the line to the bulb is perpendicular to the surface.

77 •• When light of wavelength λ_1 is incident on the cathode of a photoelectric tube, the maximum kinetic energy of the emitted electrons is 1.8 eV. If the wavelength is reduced to $\lambda_1/2$, the maximum kinetic energy of the emitted electrons is 5.5 eV. Find the work function ϕ of the cathode material.

78 •• A photon of energy E is scattered at an angle of θ. Show that the energy E' of the scattered photon is given by

$$E' = \frac{E}{(E/m_ec^2)(1 - \cos\theta) + 1}$$

79 •• A particle is confined to a one-dimensional box. In making a transition from the state n to the state $n - 1$, radiation of 114.8 nm is emitted; in the transition from the state $n - 1$ to the state $n - 2$, radiation of wavelength 147.6 nm is emitted. The ground-state energy of the particle is 1.2 eV. Determine n.

80 •• A particle confined to a one-dimensional box has a ground-state energy of 0.4 eV. When irradiated with light of 206.7 nm it makes a transition to an excited state. When decaying from this excited state to the next lower state it emits radiation of 442.9 nm. What is the quantum number of the state to which the particle has decayed?

81 •• When a surface is illuminated with light of wavelength λ the maximum kinetic energy of the emitted electrons is 1.2 eV. If wavelength $\lambda' = 0.8\,\lambda$ is used the maximum kinetic energy increases to 1.76 eV, and for wavelength $\lambda'' = 0.6\lambda$ the maximum kinetic energy of the emitted electrons is 2.7 eV. Determine the work function of the surface and the wavelength λ.

82 •• A simple pendulum of length 1 m has a bob of mass 0.3 kg. The energy of this oscillator is quantized to the values $E_n = (n + \frac{1}{2})hf_0$, where n is an integer and f_0 is the frequency of the pendulum. (*a*) Find n if the angular amplitude is 10°. (*b*) Find Δn if the energy changes by 0.01%.

83 •• (*a*) Show that for large n, the fractional difference in energy between state n and state $n + 1$ for a particle in a box is given approximately by

$$\frac{E_{n+1} - E_n}{E_n} \approx \frac{2}{n}$$

(*b*) What is the approximate percentage energy difference between the states $n_1 = 1000$ and $n_2 = 1001$? (*c*) Comment on how this result is related to Bohr's correspondence principle.

84 •• In 1985, a light pulse of 1.8×10^{12} photons was produced in an AT&T laboratory during a time interval of 8×10^{-15} s. The wavelength of the produced light was $\lambda = 2400$ nm. Suppose all of the light was absorbed by the black surface of a screen. Estimate the force exerted by the photons on the screen.

85 •• This problem is one of estimating the time lag (expected classically but not observed) in the photoelectric effect. Let the intensity of the incident radiation be 0.01 W/m^2. (*a*) If the area of the atom is 0.01 nm^2, find the energy per second falling on an atom. (*b*) If the work function is 2 eV, how long would it take classically for this much energy to fall on one atom?