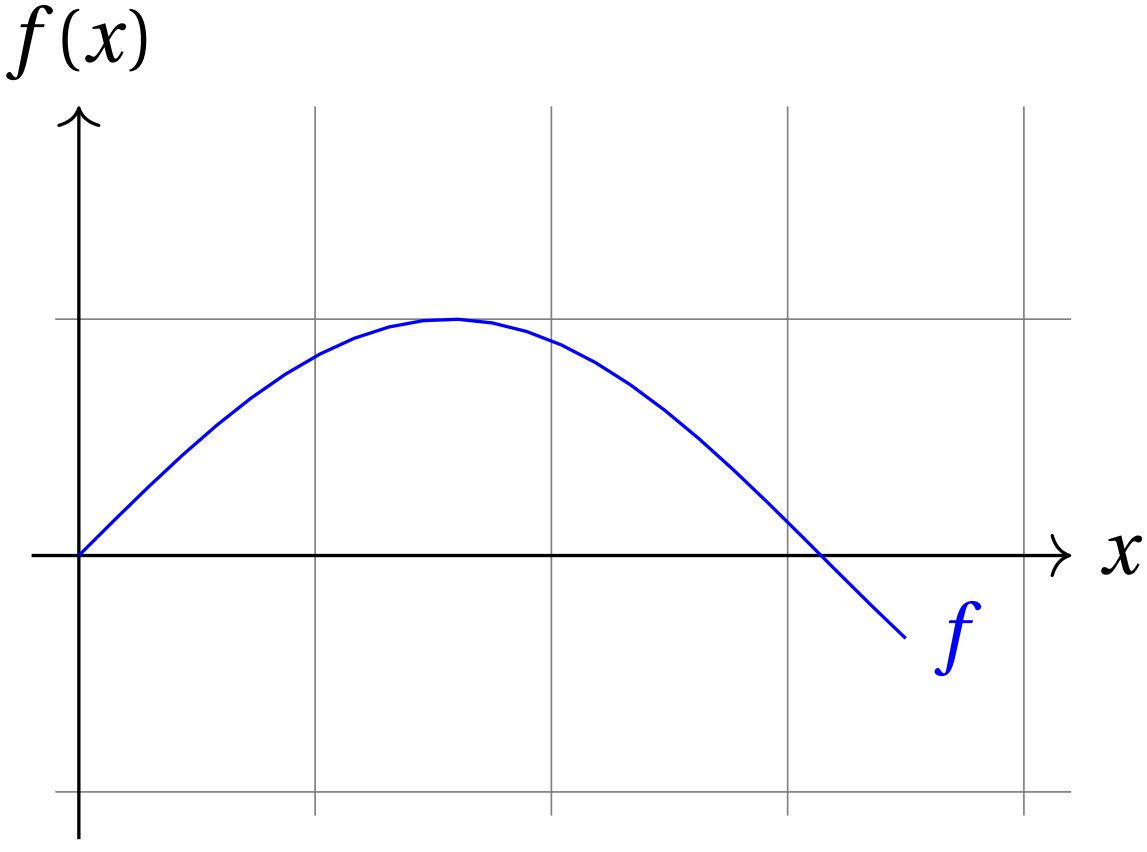
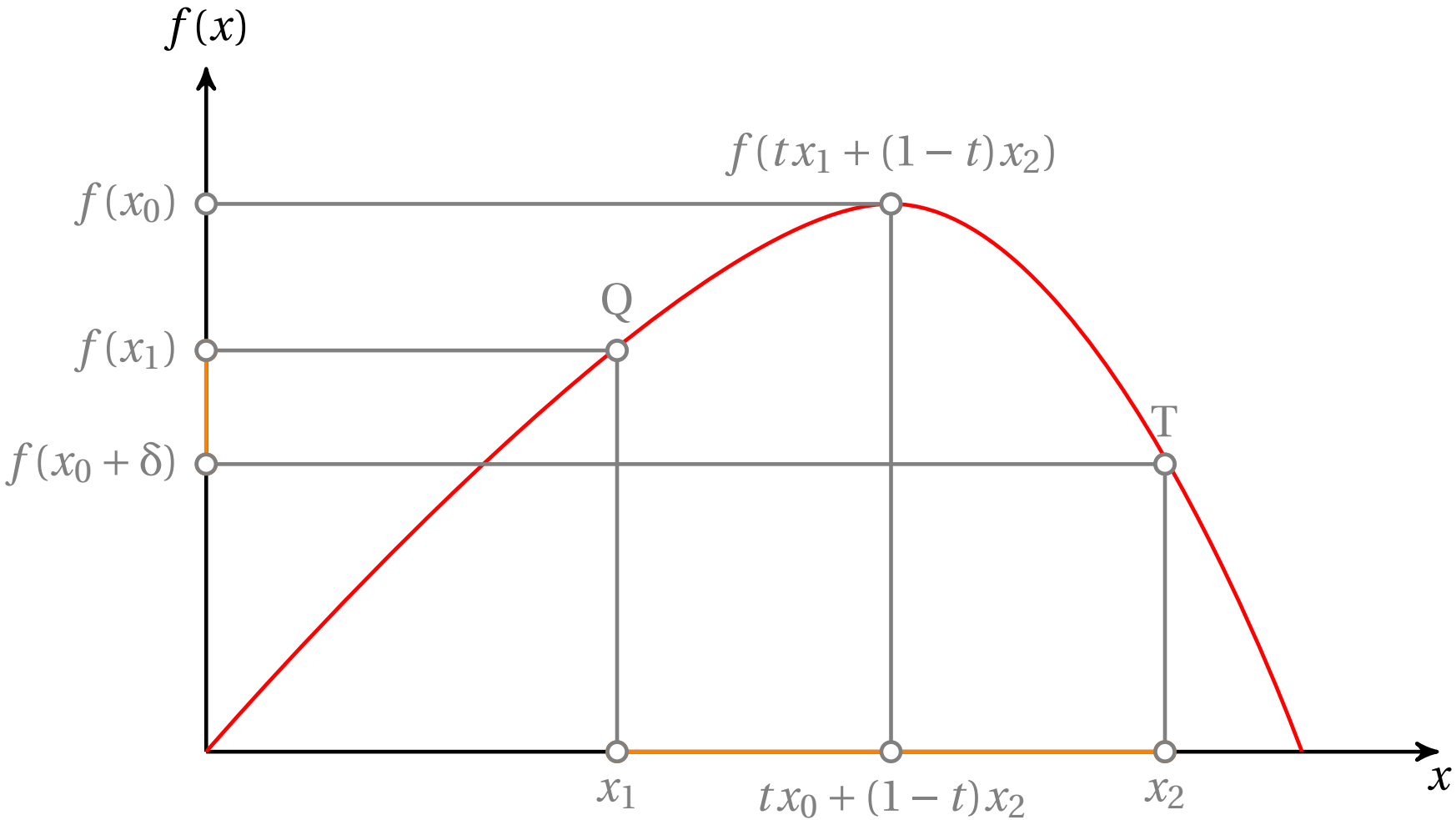
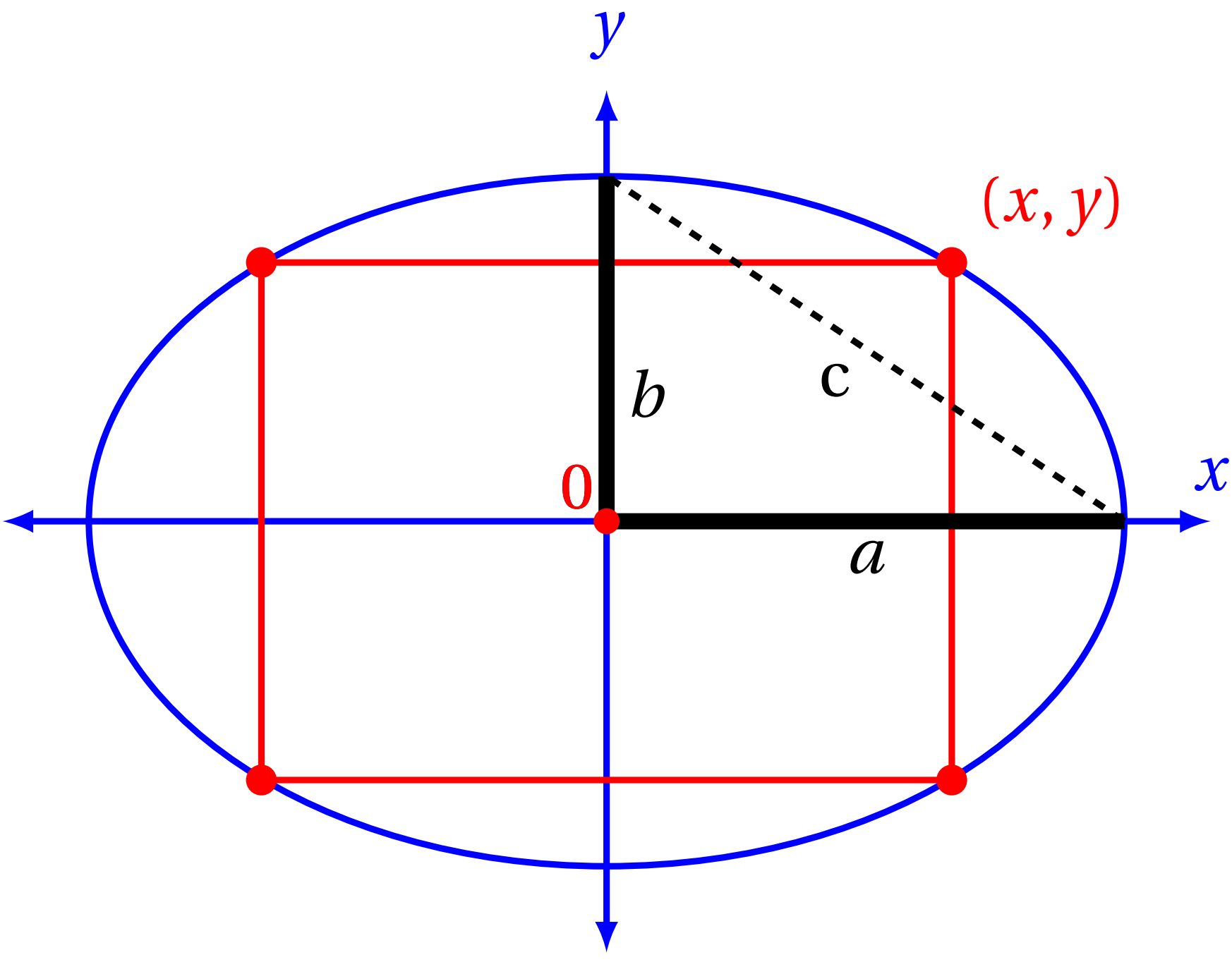
1 ./images-tikz/image-.tex


$f(x)$
x
f

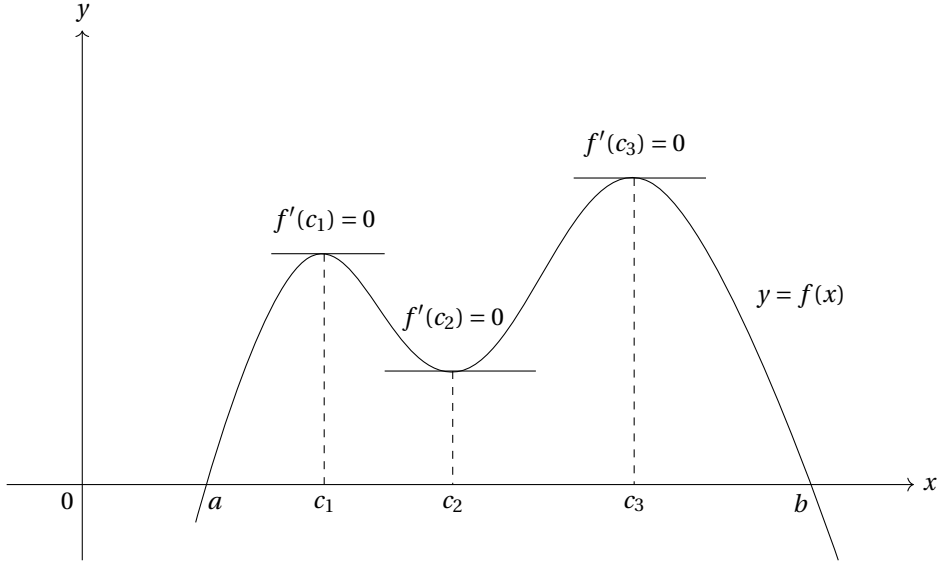
2 ./images-tikz/image-20.tex


$f(x)$
$f(tx_1+(1-t)x_2)$
$f(x_0)$
Q
$f(x_1)$
T
$f(x_0+\delta)$
x_1
$tx_0+(1-t)x_2$
x_2
x

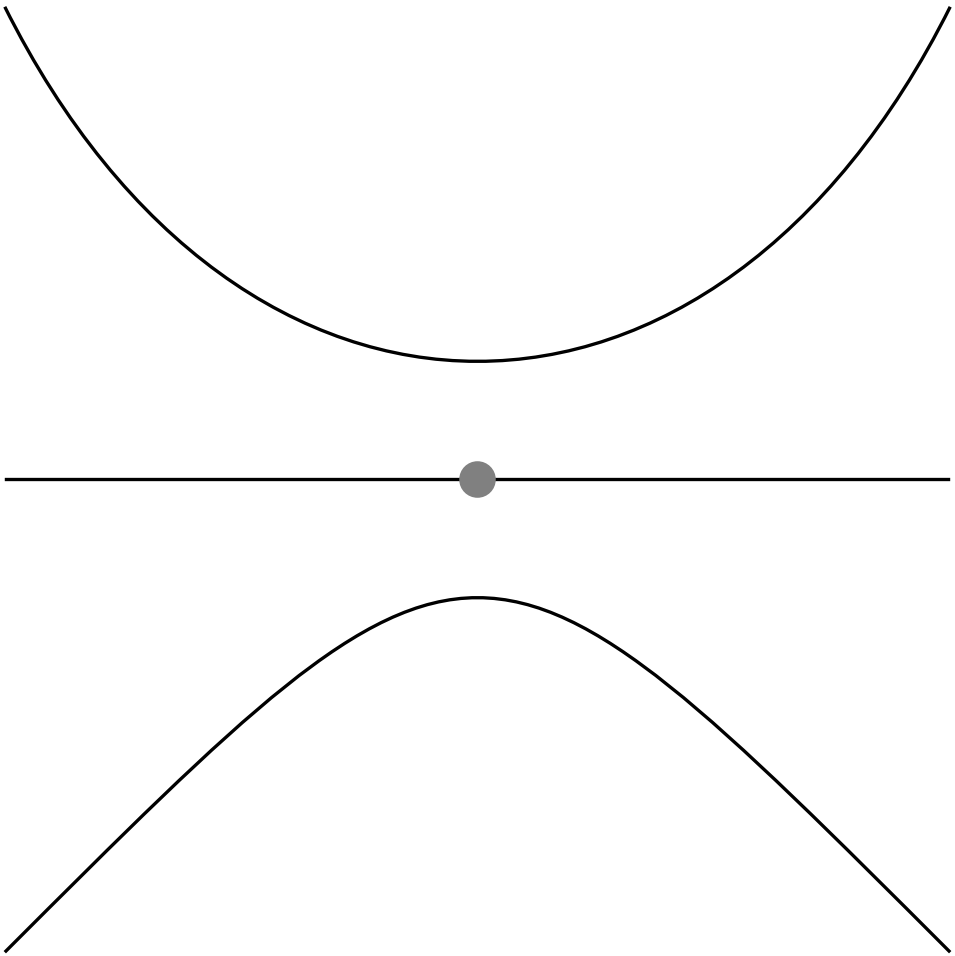
3 ./images-tikz/resuelto6-2.tex


y
(x, y)
b
c
0
x
a

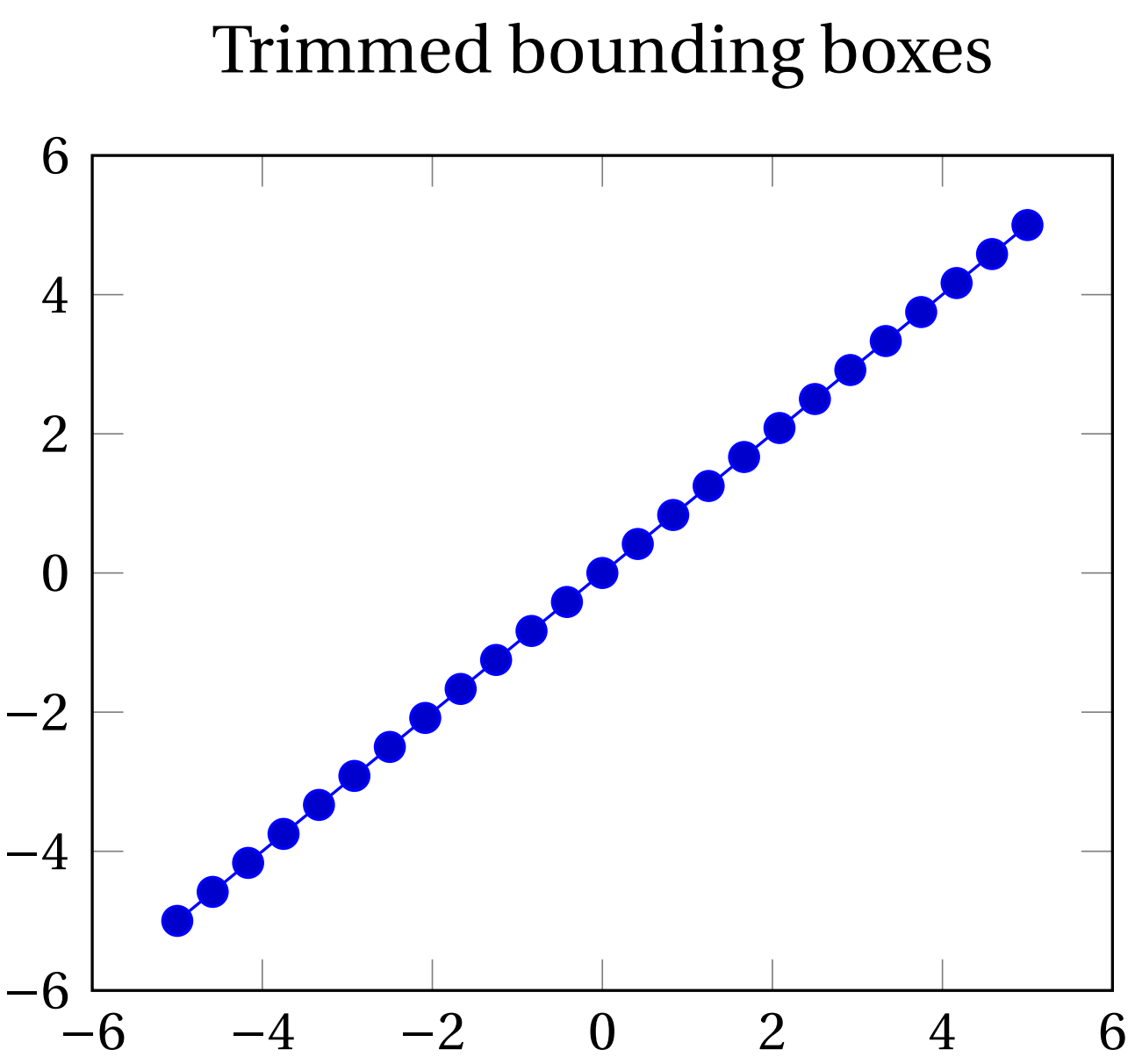
4 ./images-tikz/image-37.tex


y
x
0
a
c_1
c_2
c_3
b
$f'(c_1) = 0$
$f'(c_2) = 0$
$f'(c_3) = 0$
$y = f(x)$

5 ./images-tikz/image-2.tex

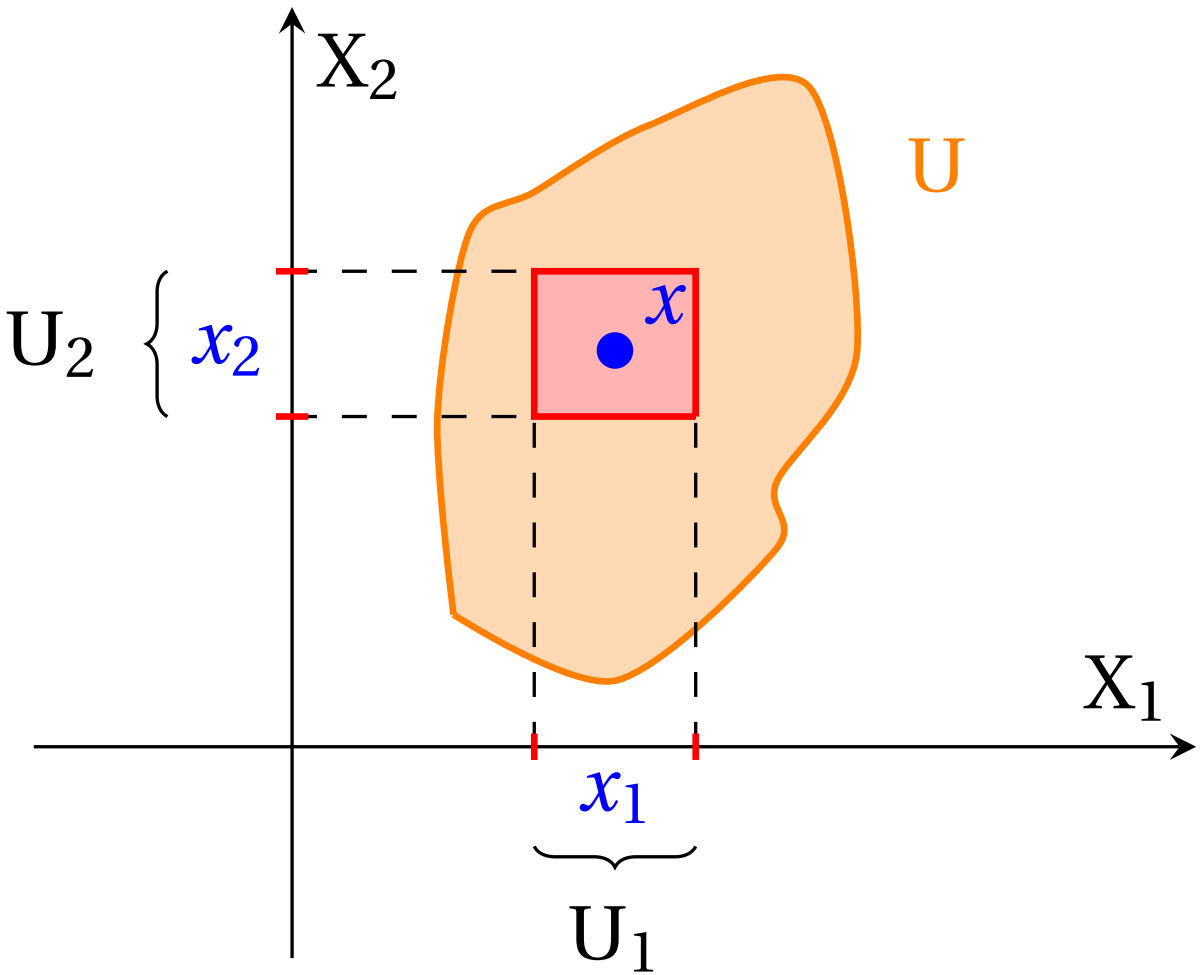


6 ./images-tikz/image-4.tex

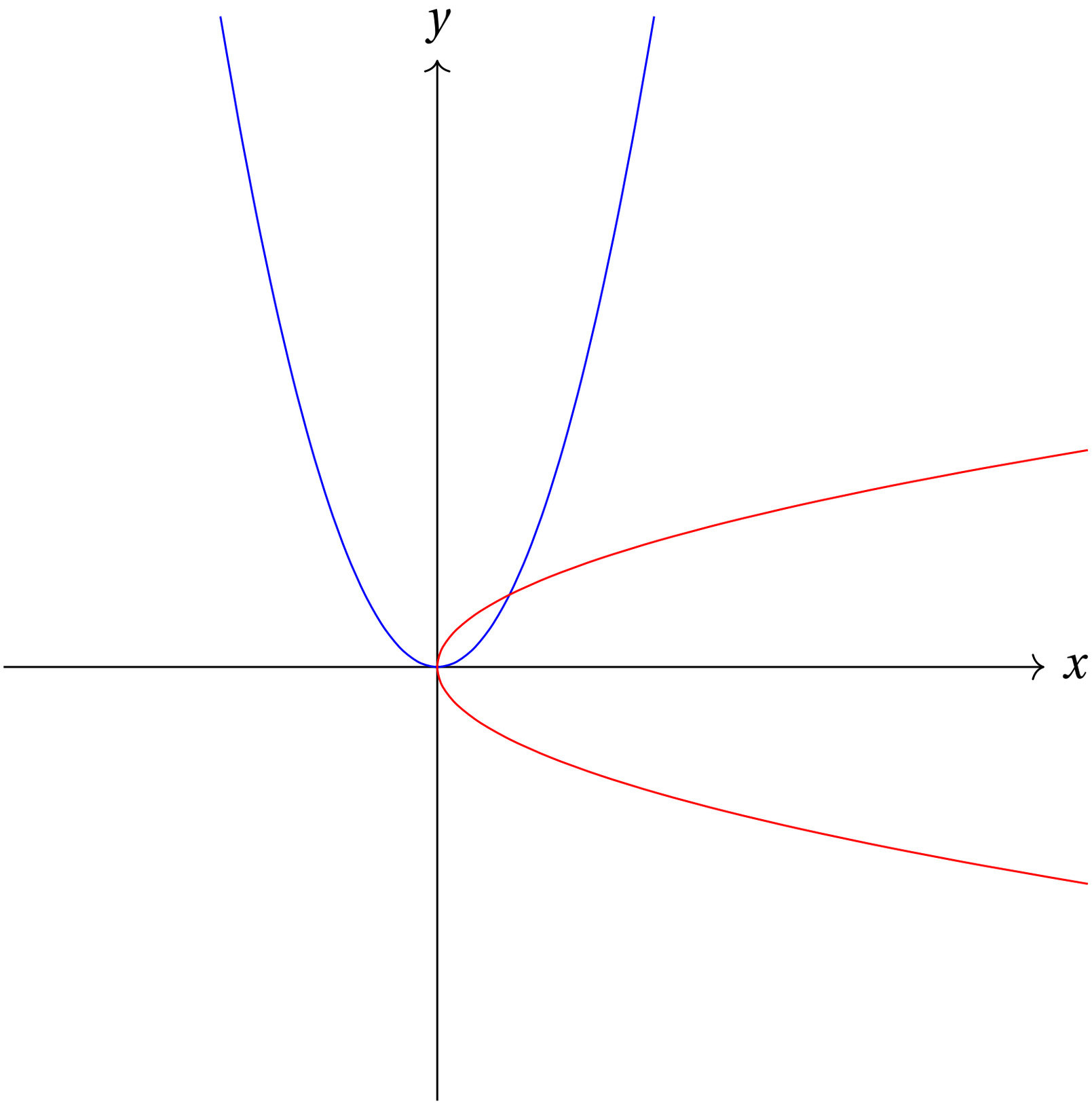

Trimmed bounding boxes
6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

7 ./images-tikz/image-35.tex

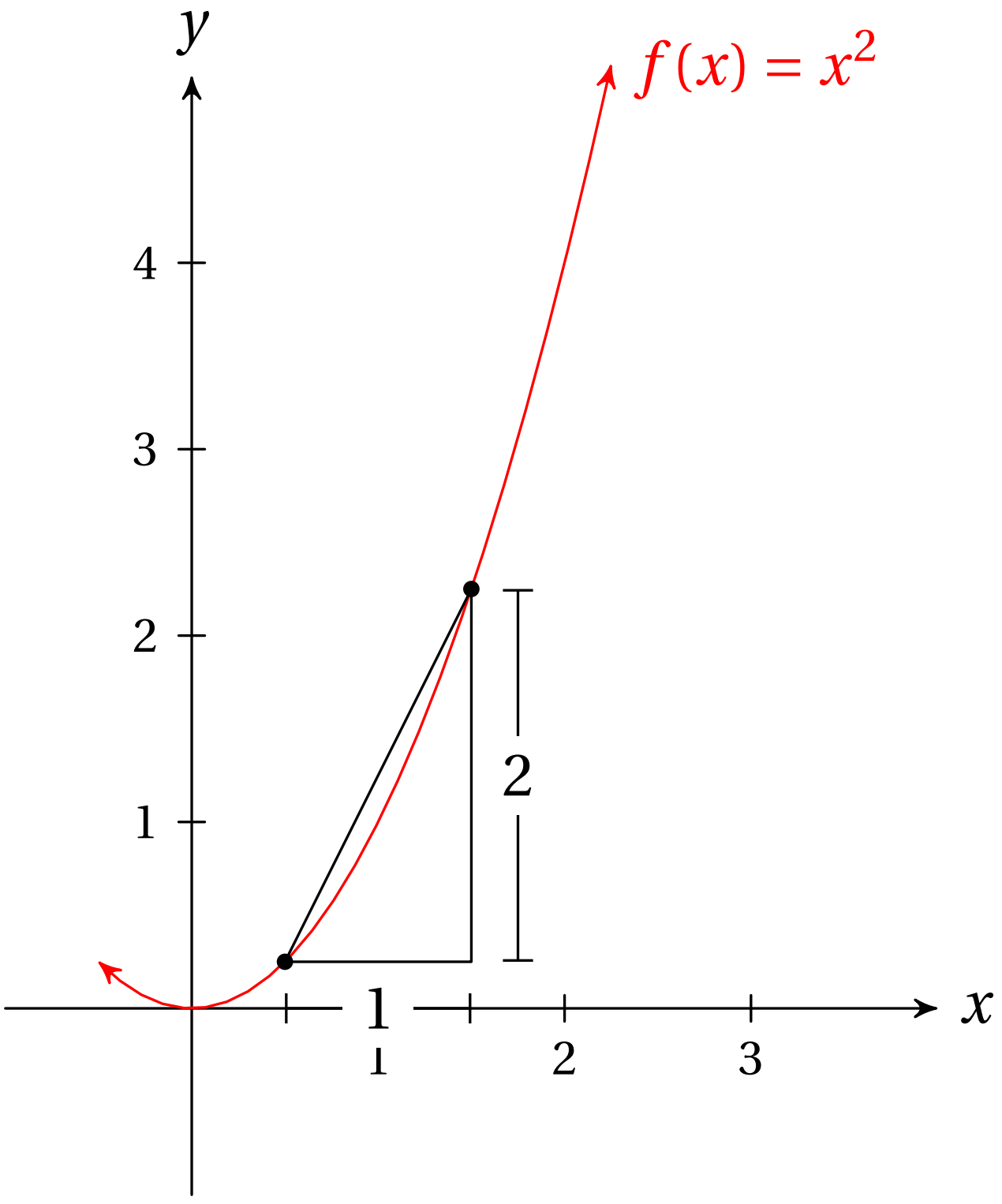
Trimmed bounding boxes



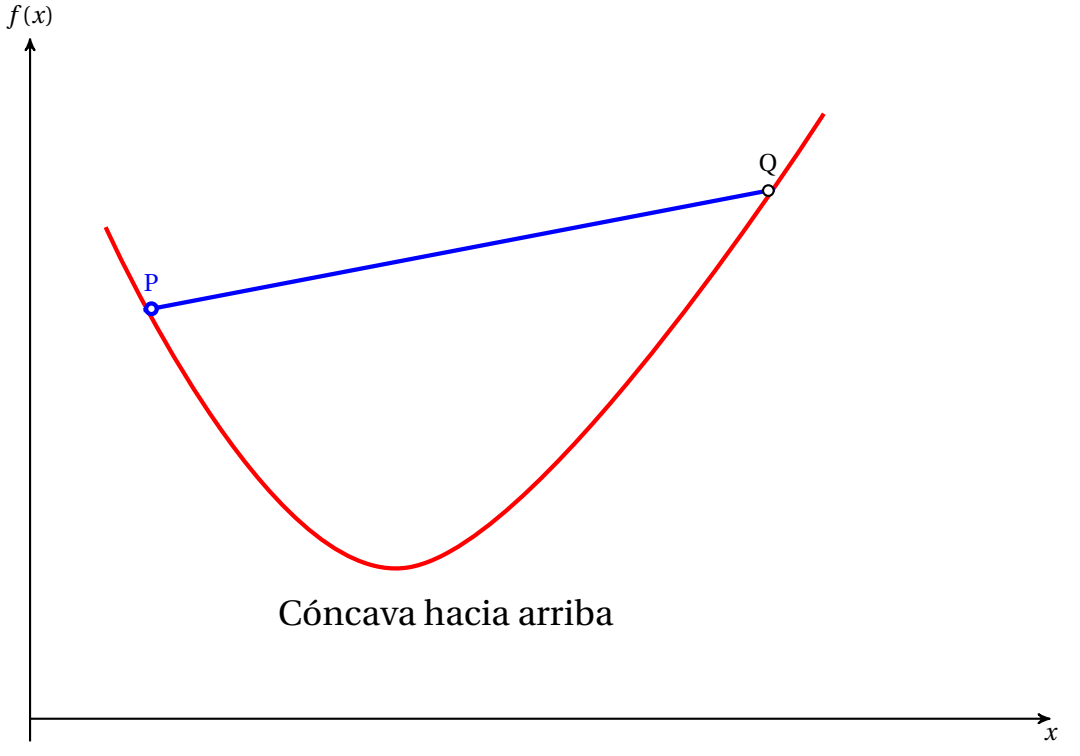
8 ./images-tikz/image-34.tex


y
x

9 ./images-tikz/image-36.tex

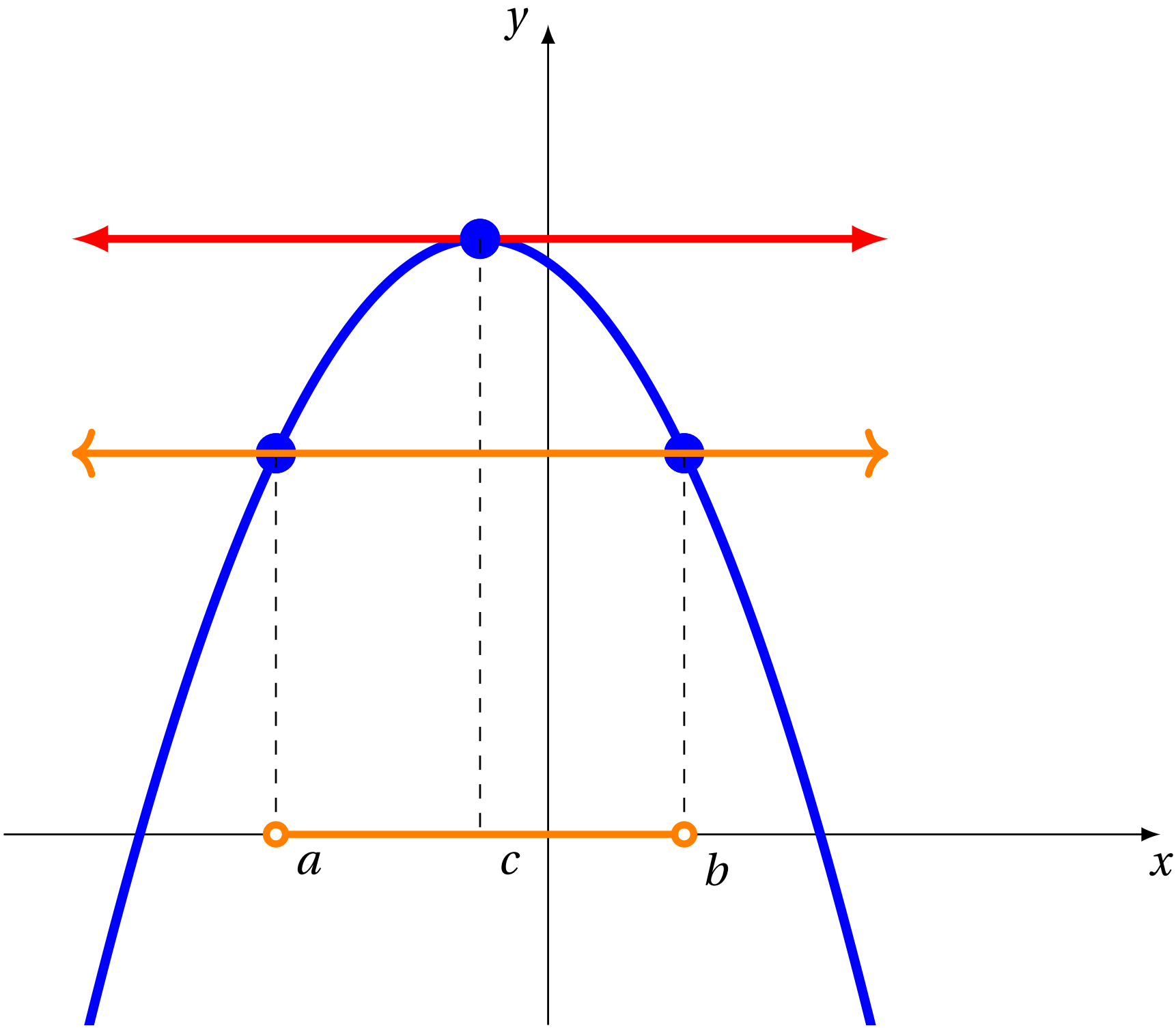

y
x
f(x) = x^2
4
3
2
1
2
1
1
2
3

10 ./images-tikz/concavidad-c.tex

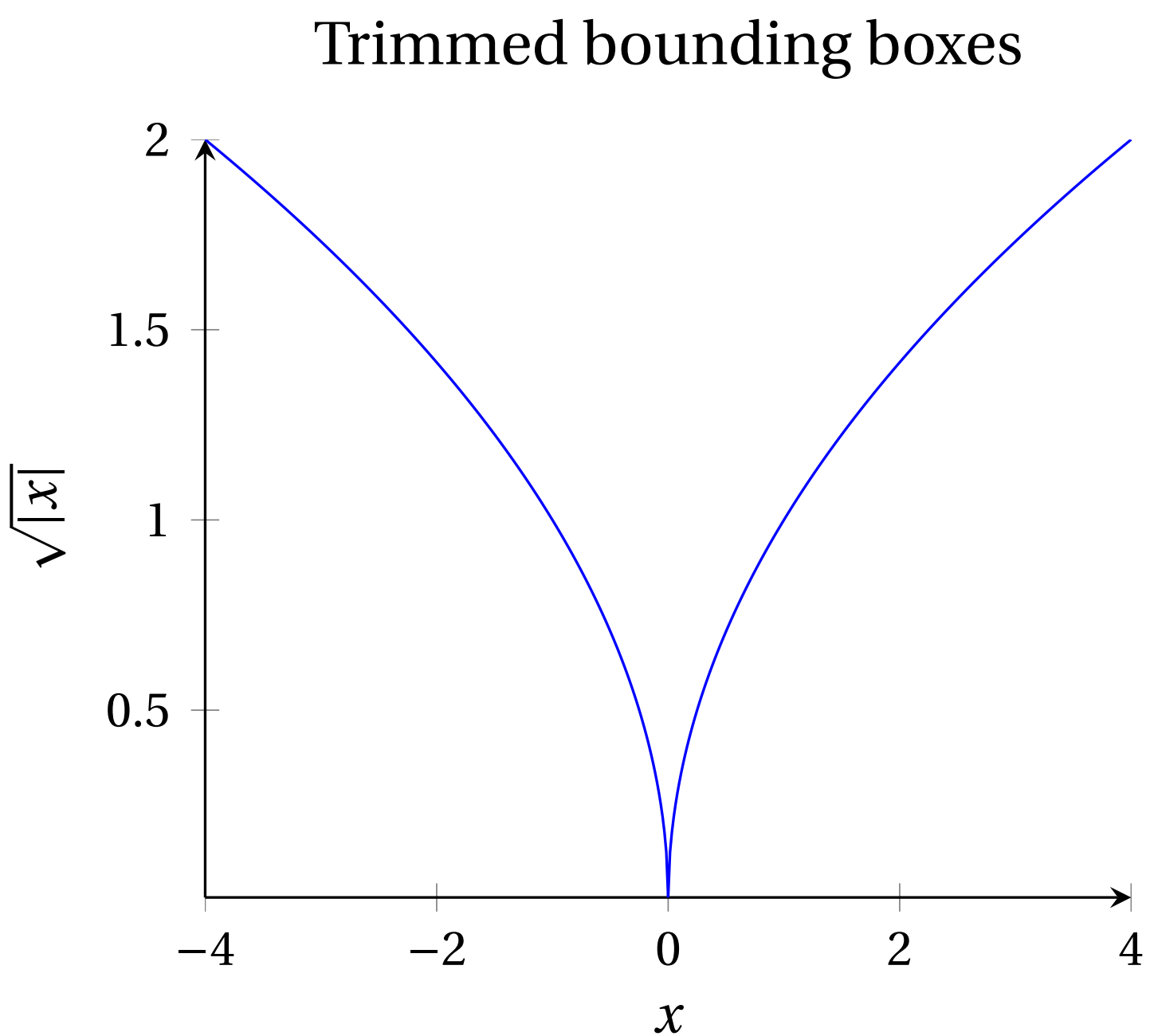


Cóncava hacia arriba

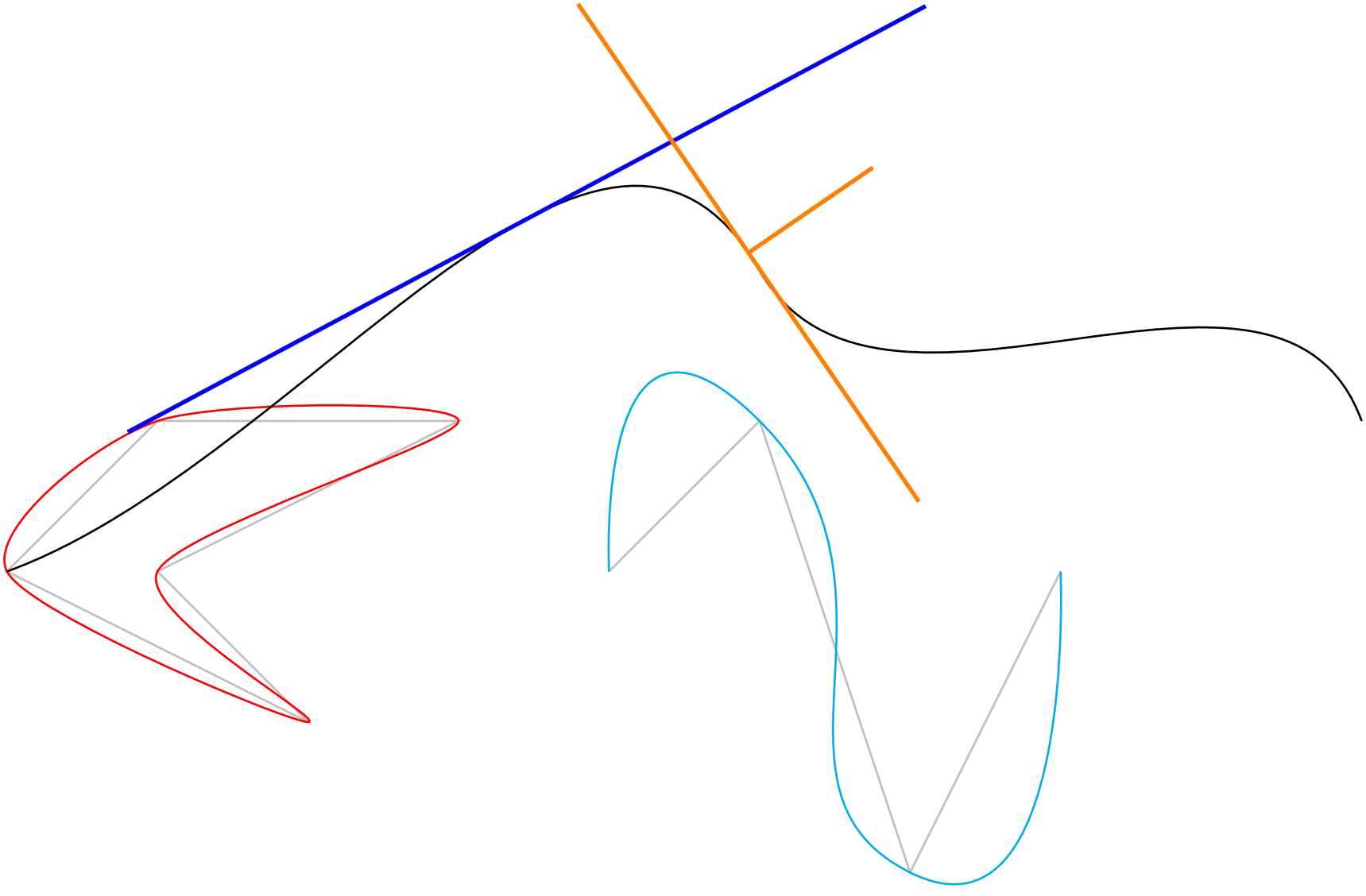
11 ./images-tikz/image-53.tex


y
x
a
c
b

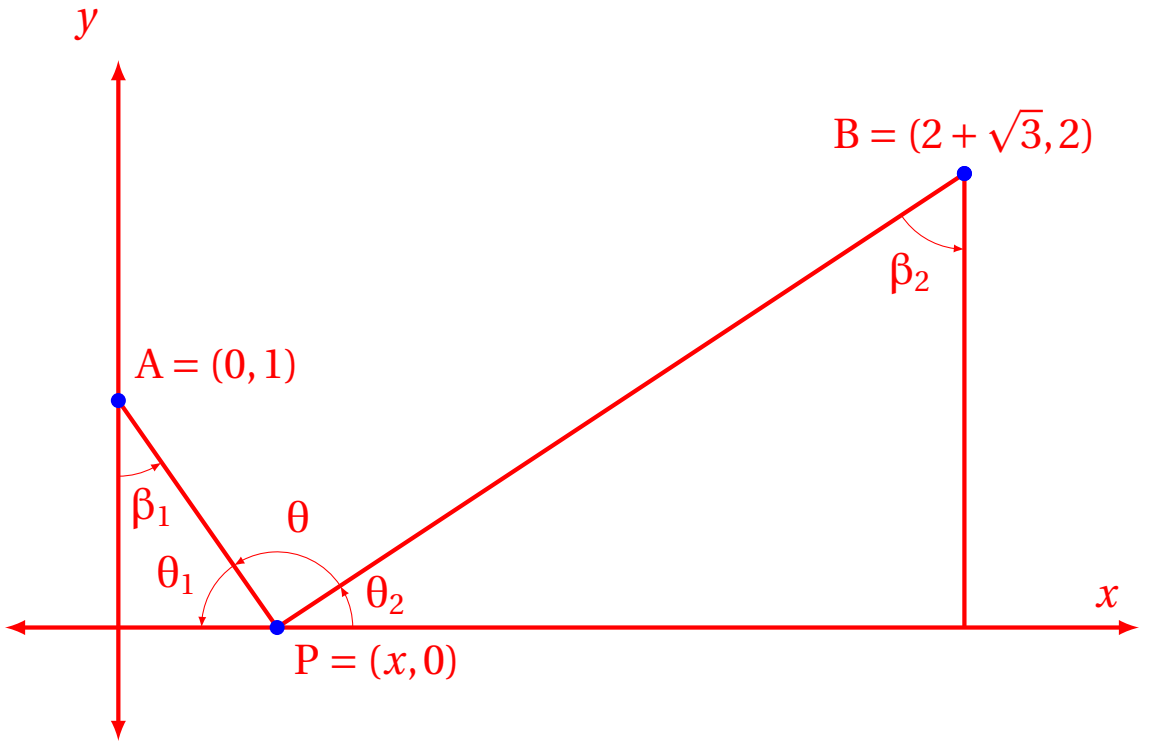
12 ./images-tikz/image-26.tex


Trimmed bounding boxes
2
1.5
1
0.5
$\sqrt{|x|}$
−4
−2
0
2
4
x

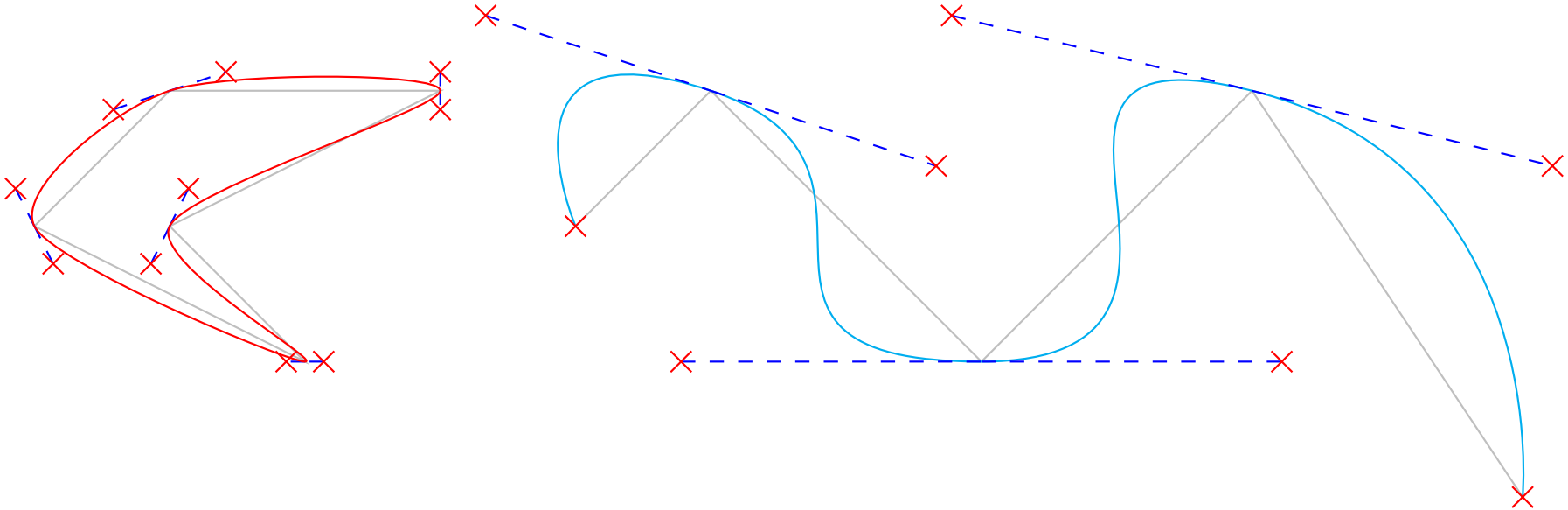
13 ./images-tikz/image-16.tex



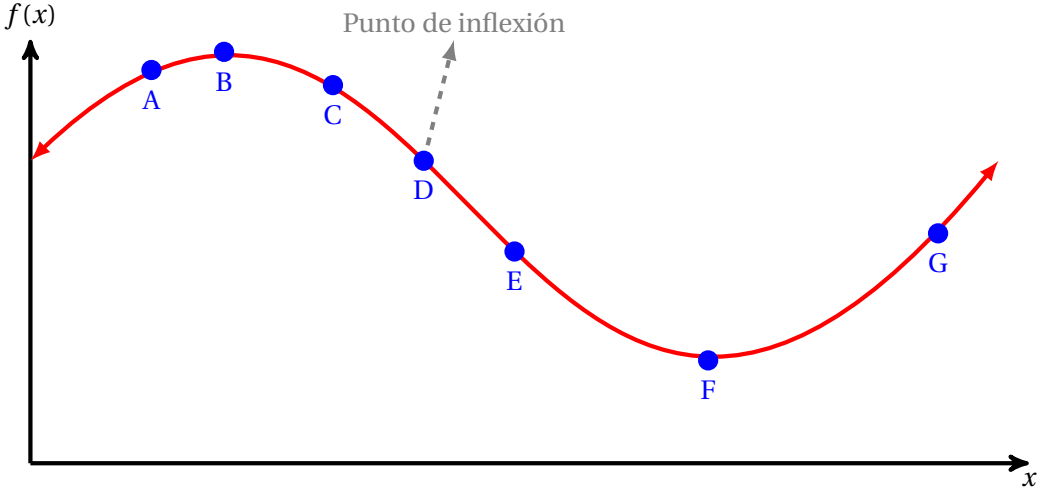
14 ./images-tikz/resuelto6-17.tex


y
B = (2 + √3, 2)
β2
A = (0, 1)
β1
θ
θ1
θ2
x
P = (x, 0)

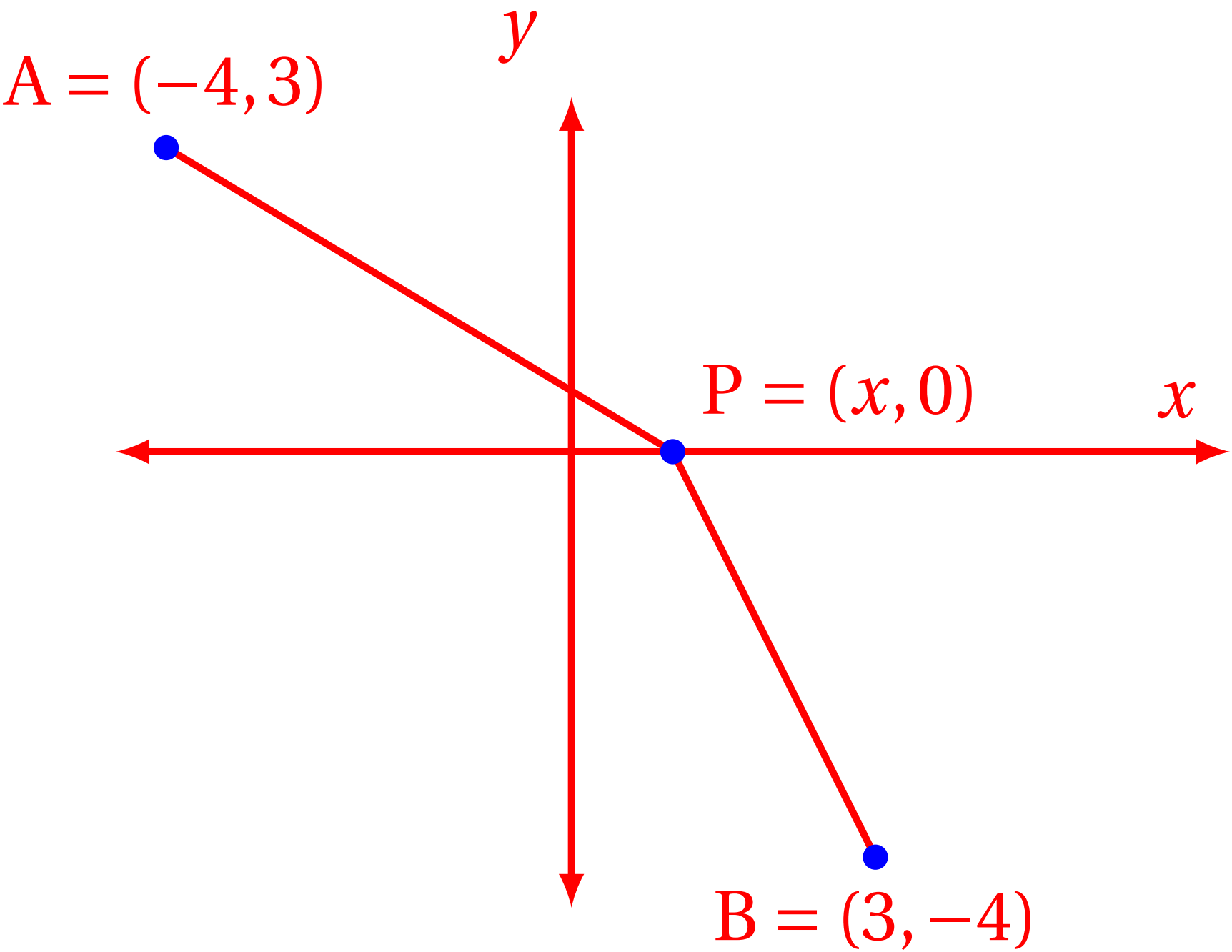
15 ./images-tikz/derivate7.tex



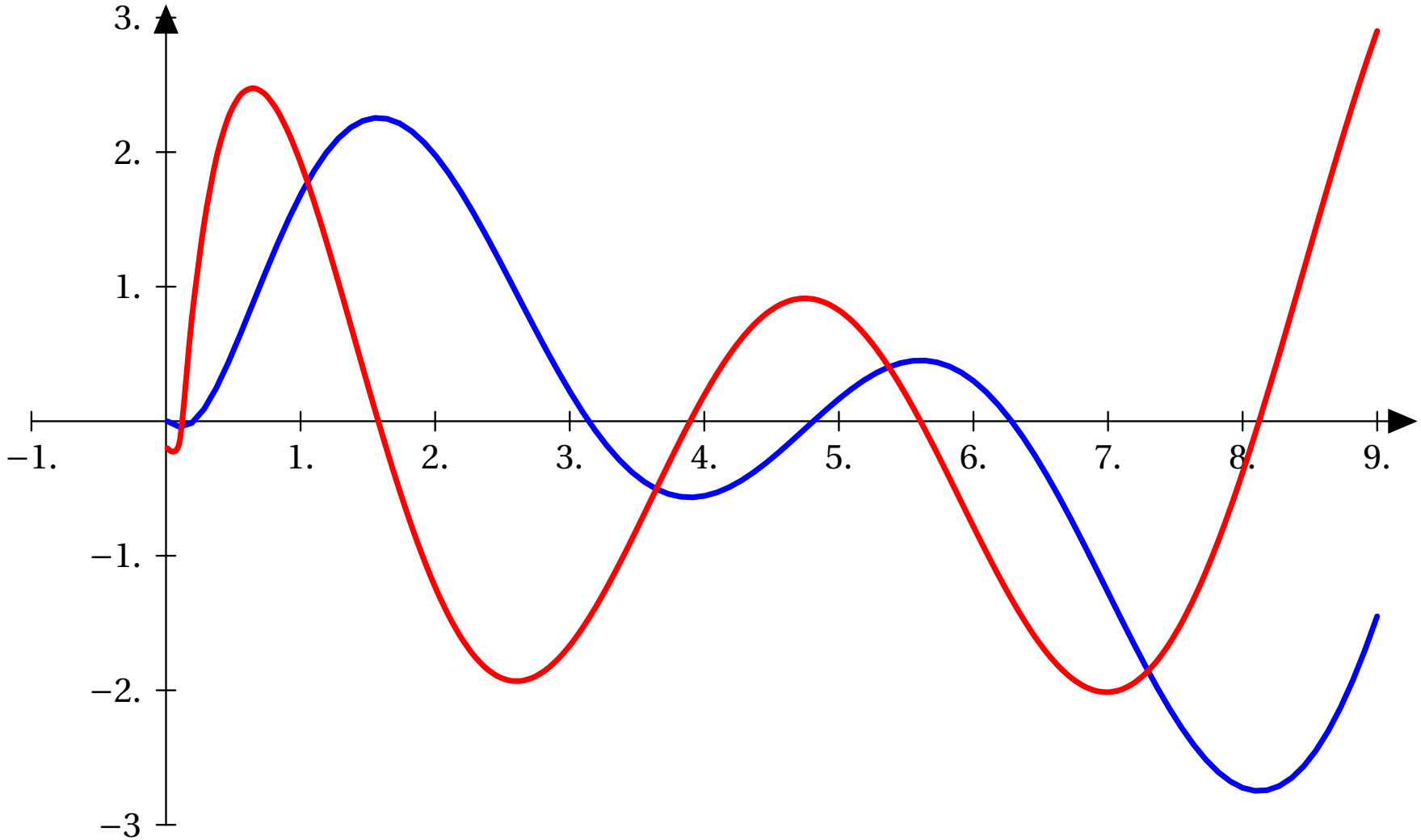
16 ./images-tikz/estudio.tex


$f(x)$
Punto de inflexión
A
B
C
D
E
F
G
x

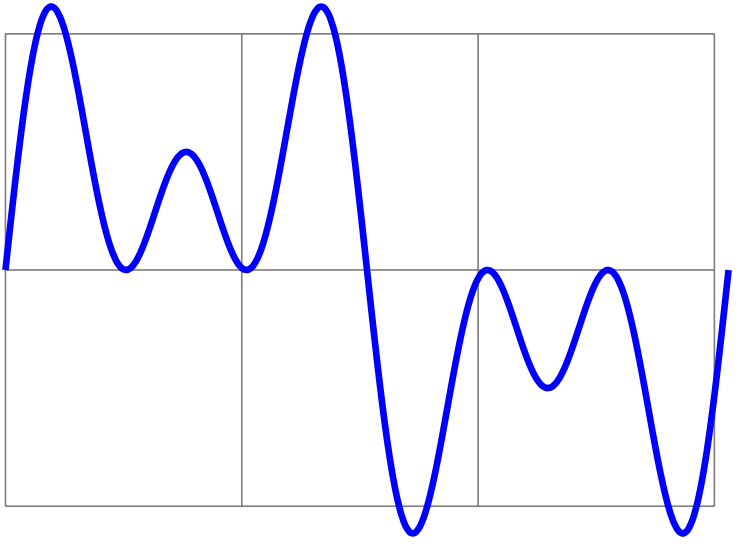
17 ./images-tikz/resuelto6-16.tex


A = (−4, 3)
y
P = (x, 0)
x
B = (3, −4)

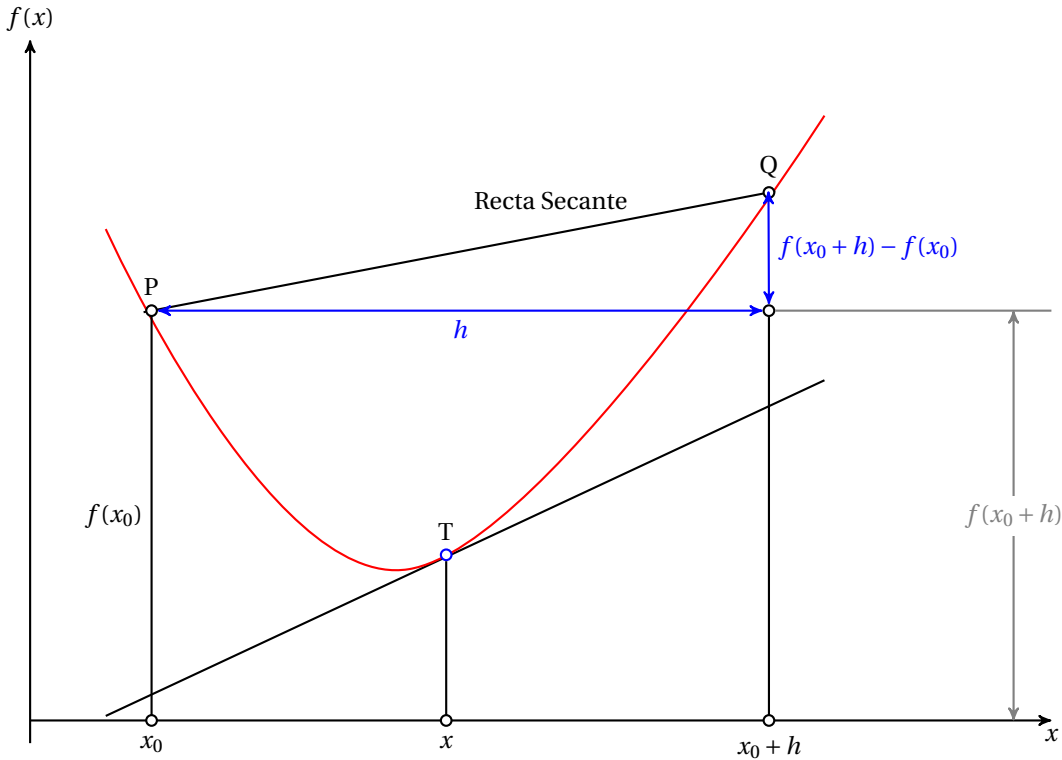
18 ./images-tikz/derivadas.tex


3.
2.
1.
−1.
−2.
−3
−1.
1.
2.
3.
4.
5.
6.
7.
8.
9.

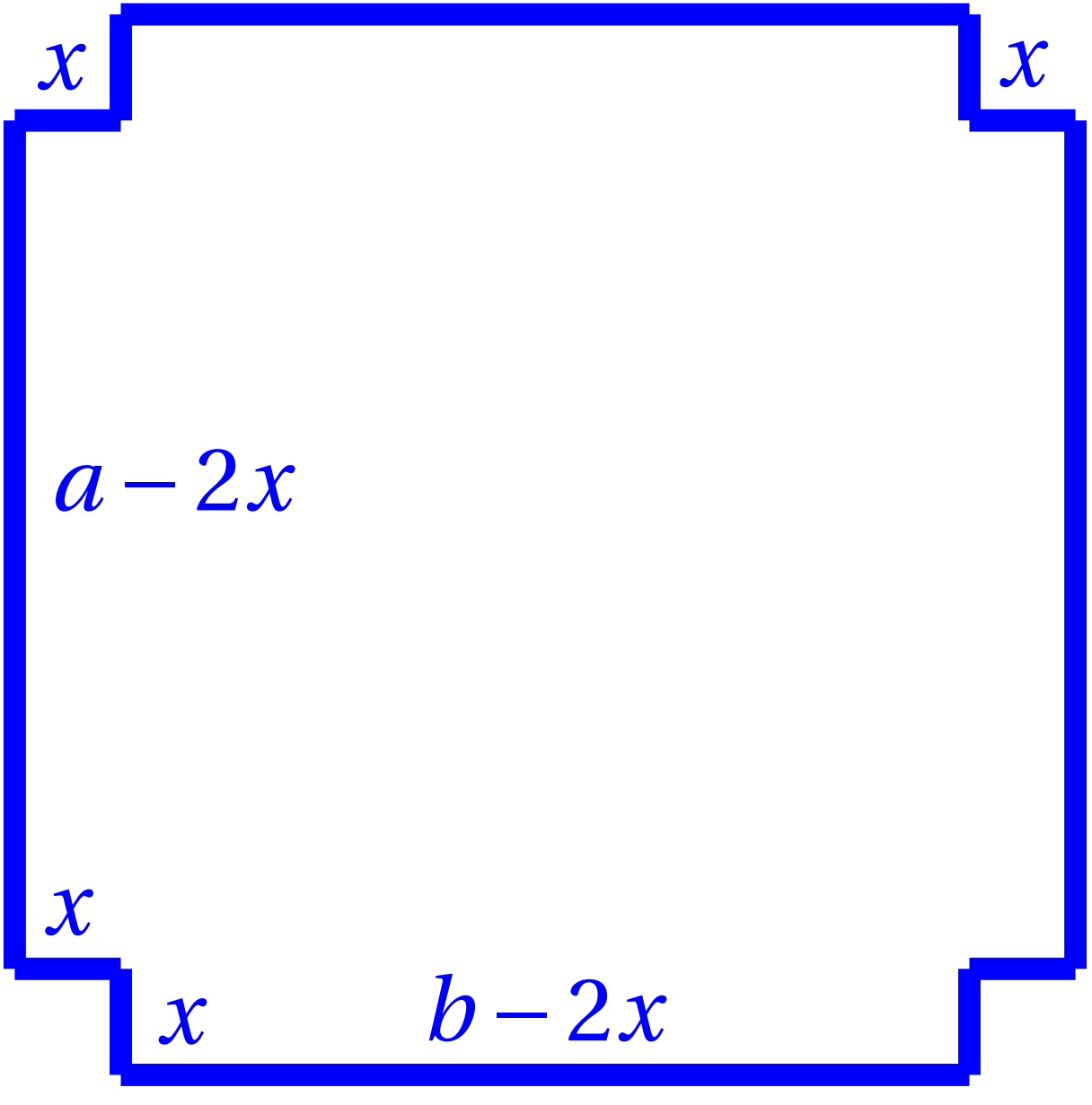
19 ./images-tikz/image-38.tex



20 ./images-tikz/image-19.tex

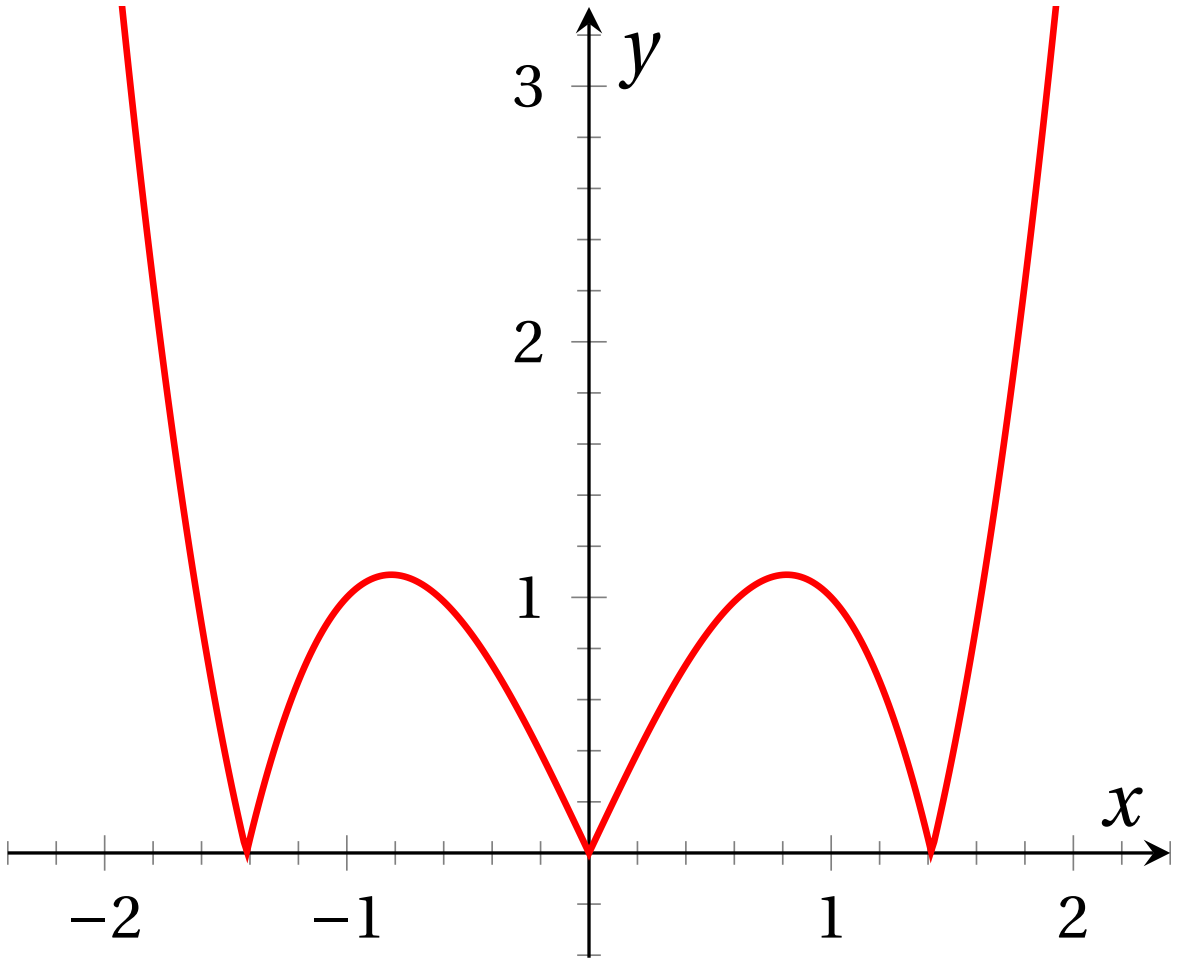

$f(x)$
Q
Recta Secante
$f(x_0+h)-f(x_0)$
P
h
$f(x_0)$
T
$f(x_0+h)$
x_0
x
x_0+h
x

21 ./images-tikz/resuelto6-5.tex


x
x
a − 2x
x
x
b − 2x

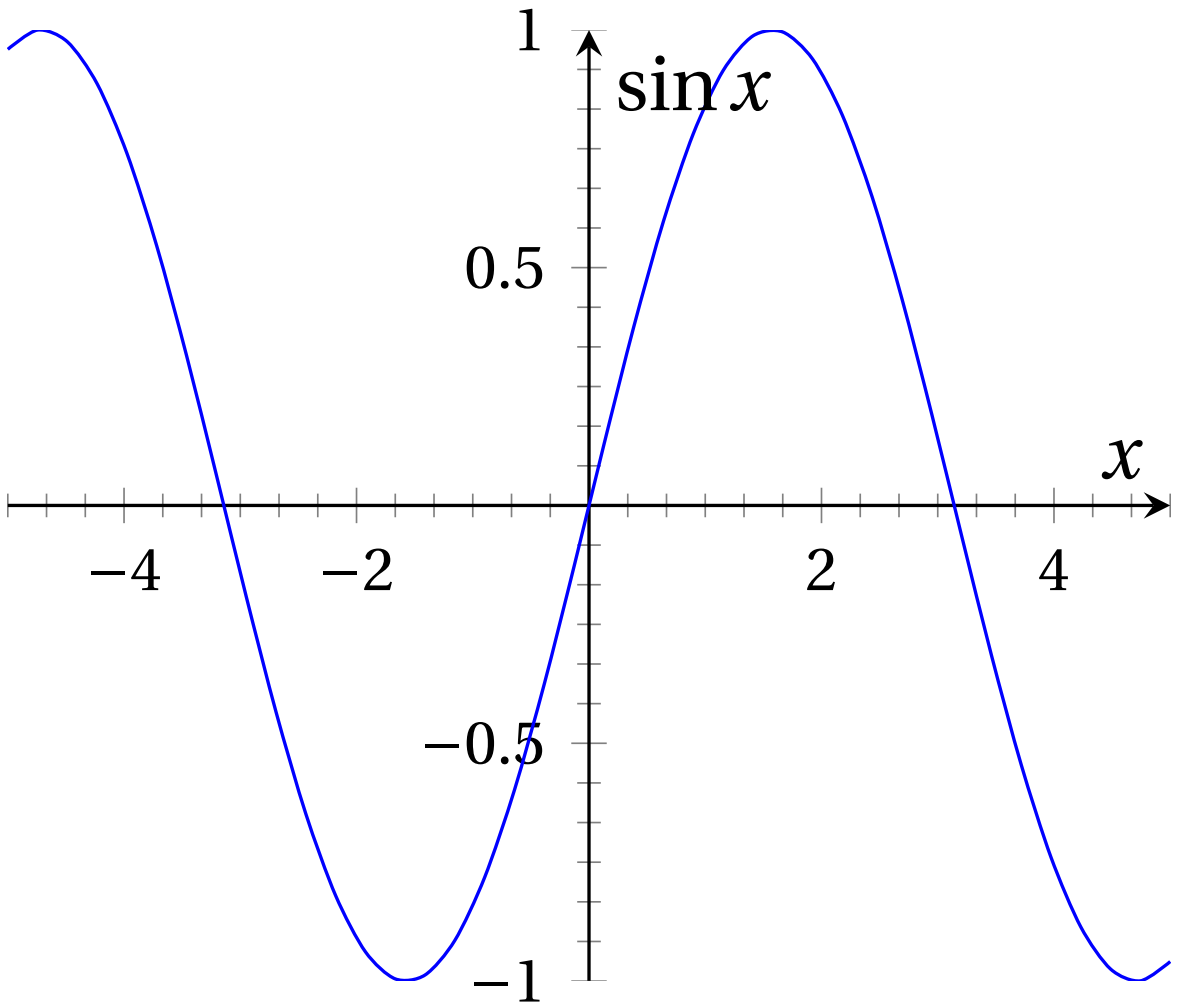
22 ./images-tikz/image-12.tex

Trimmed bounding boxes

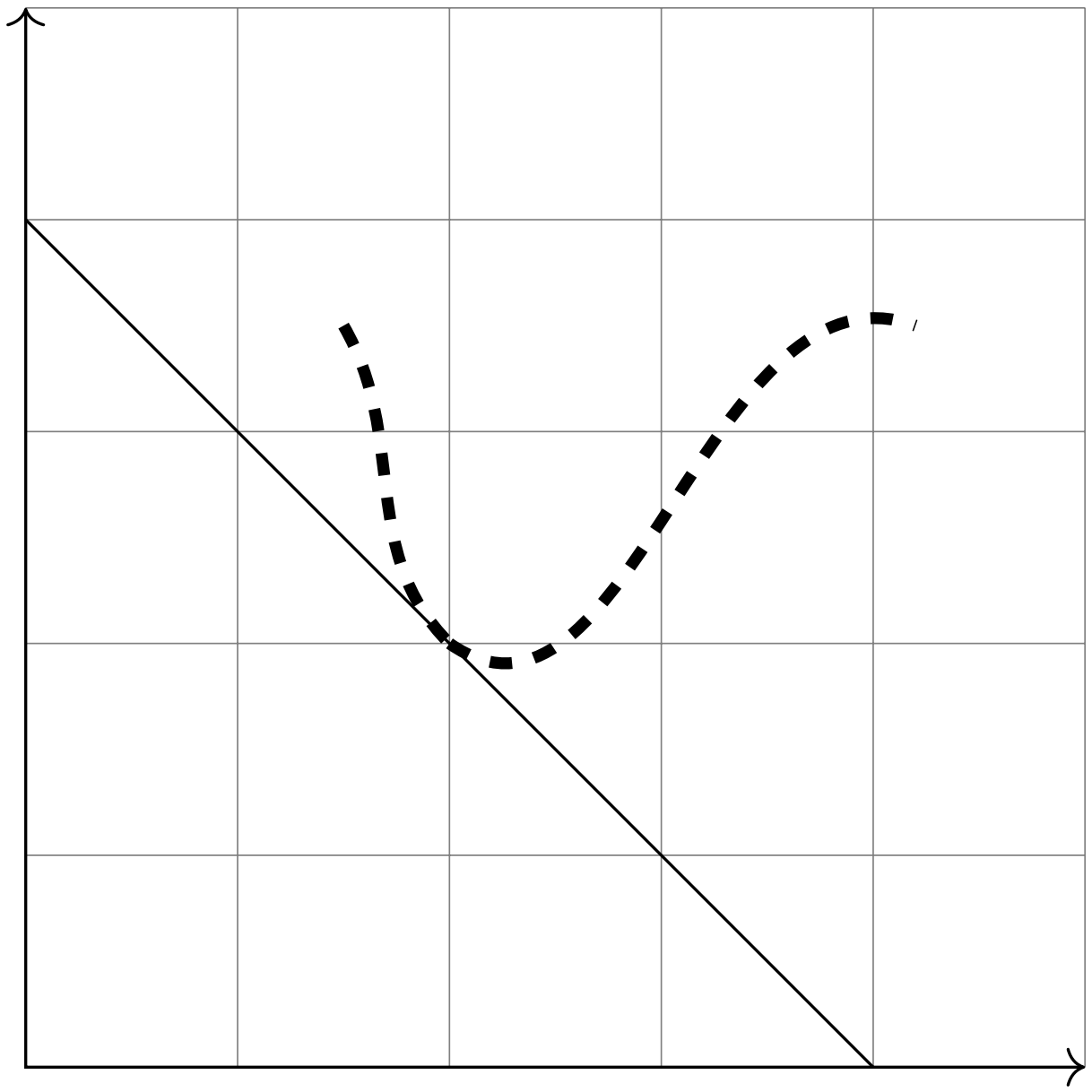


23 ./images-tikz/image-27.tex

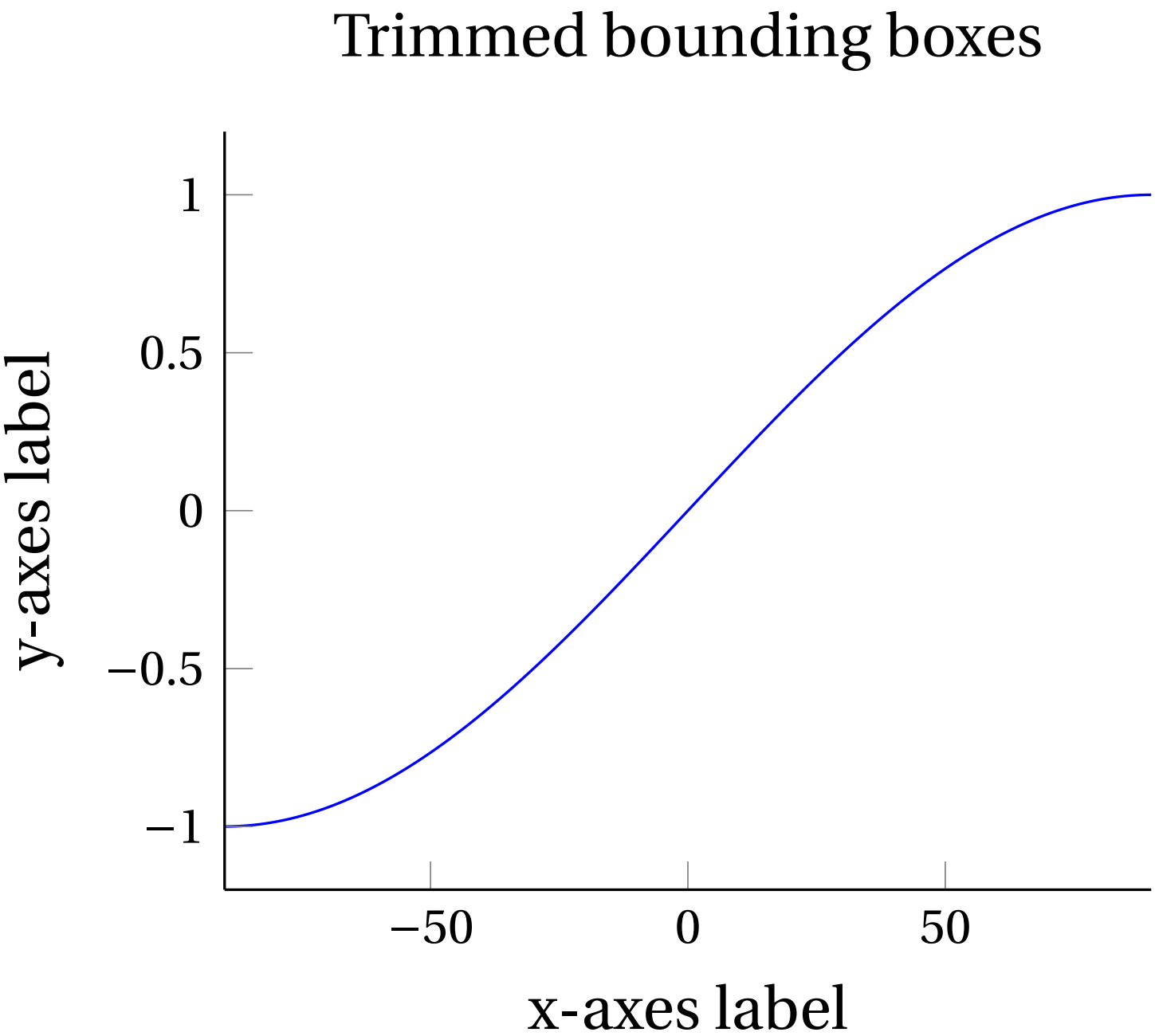
Trimmed bounding boxes



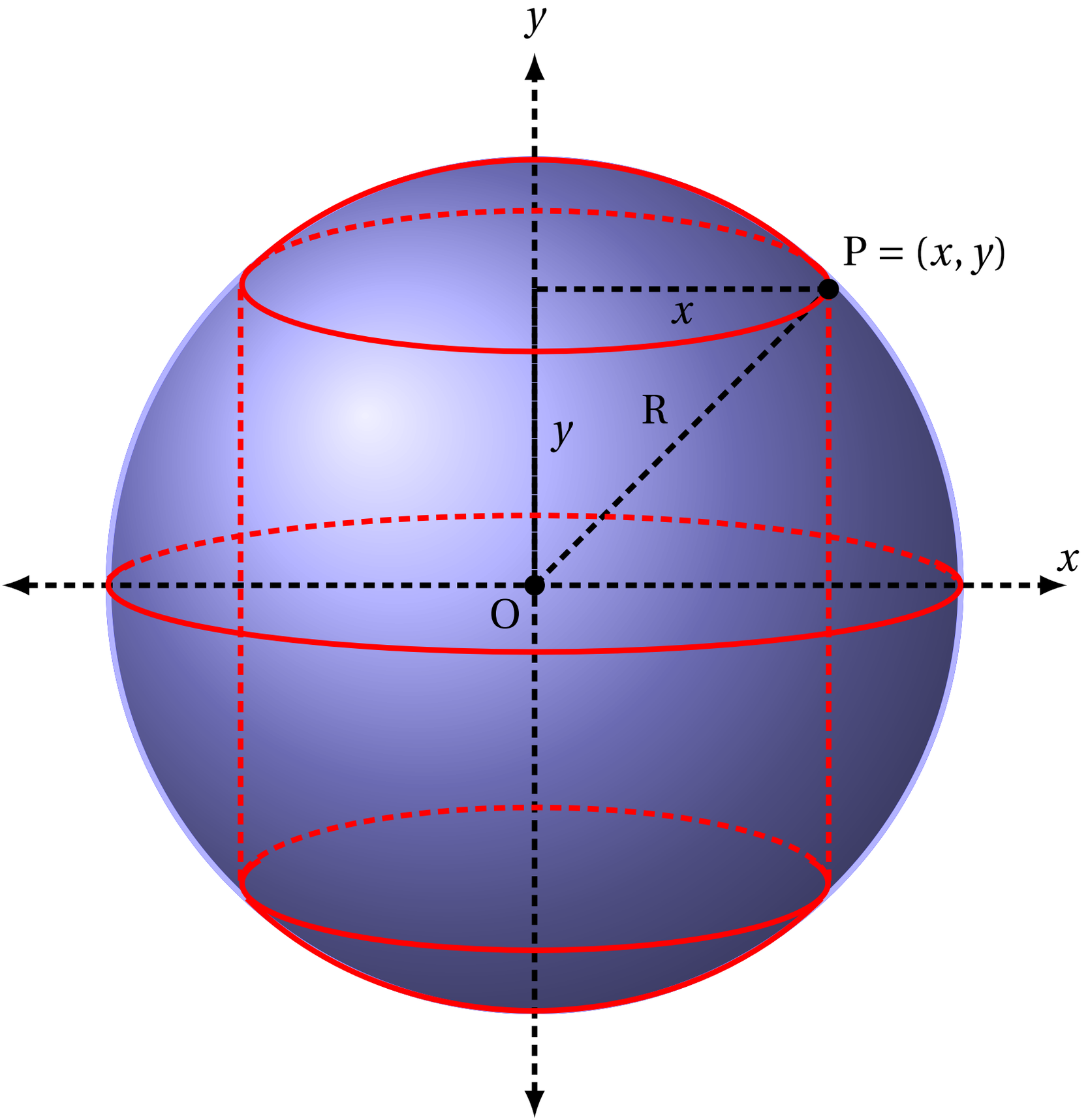
24 ./images-tikz/image-18.tex



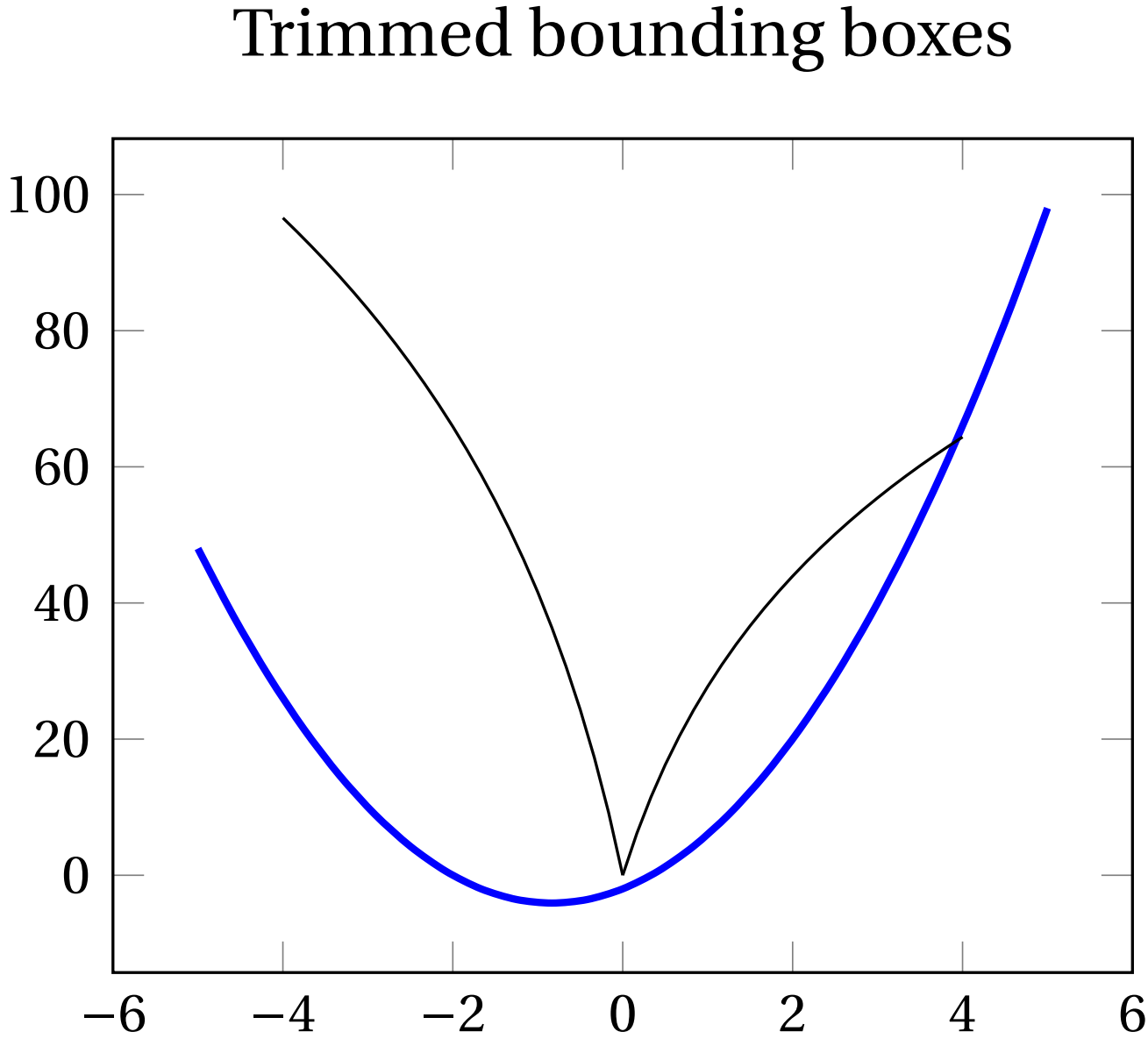
25 ./images-tikz/senx4.tex


Trimmed bounding boxes
y-axes label
1
0.5
0
−0.5
−1
−50
0
50
x-axes label

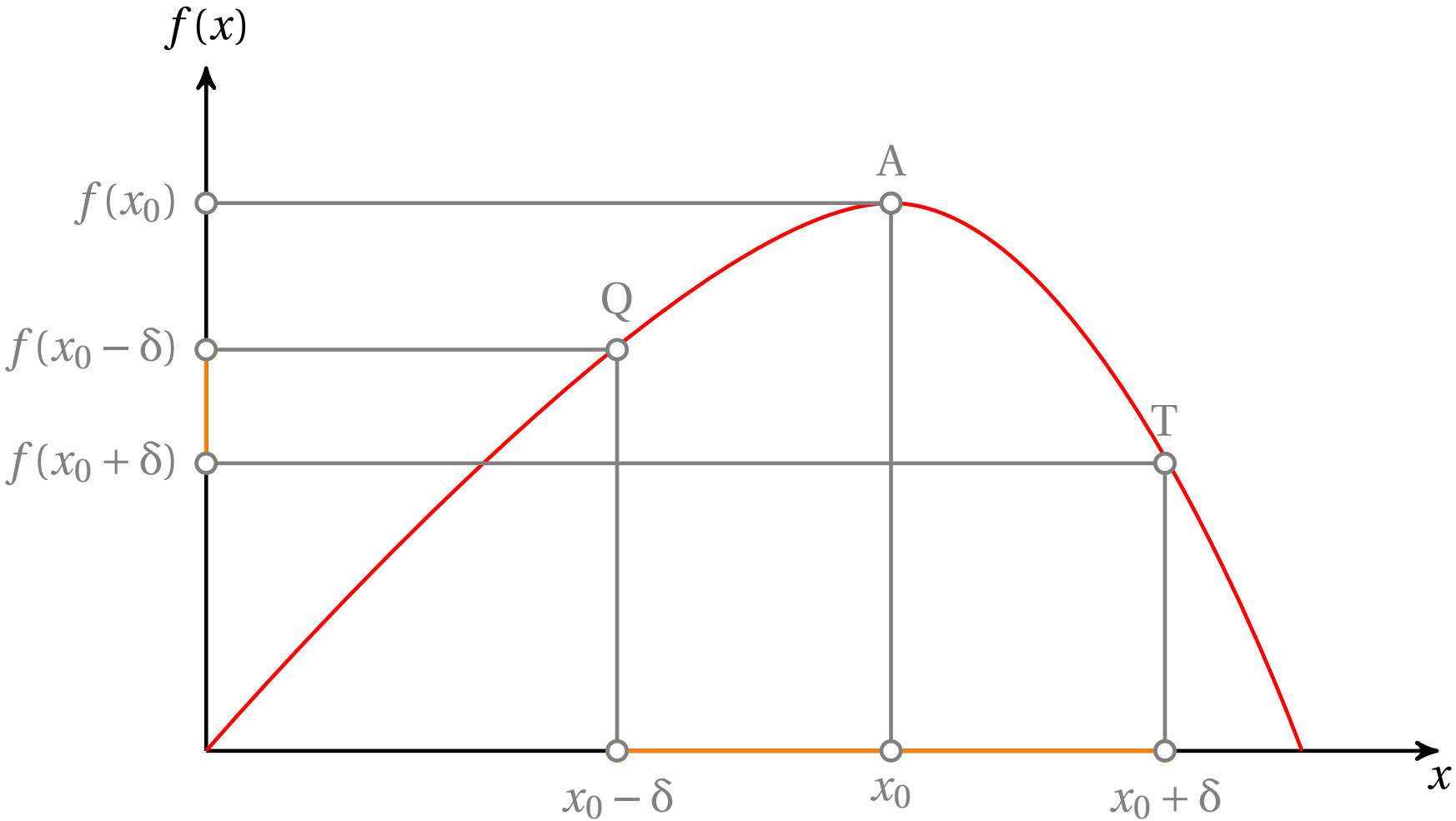
26 ./images-tikz/resuelto6-9.tex


y
P = (x, y)
x
R
y
x
O

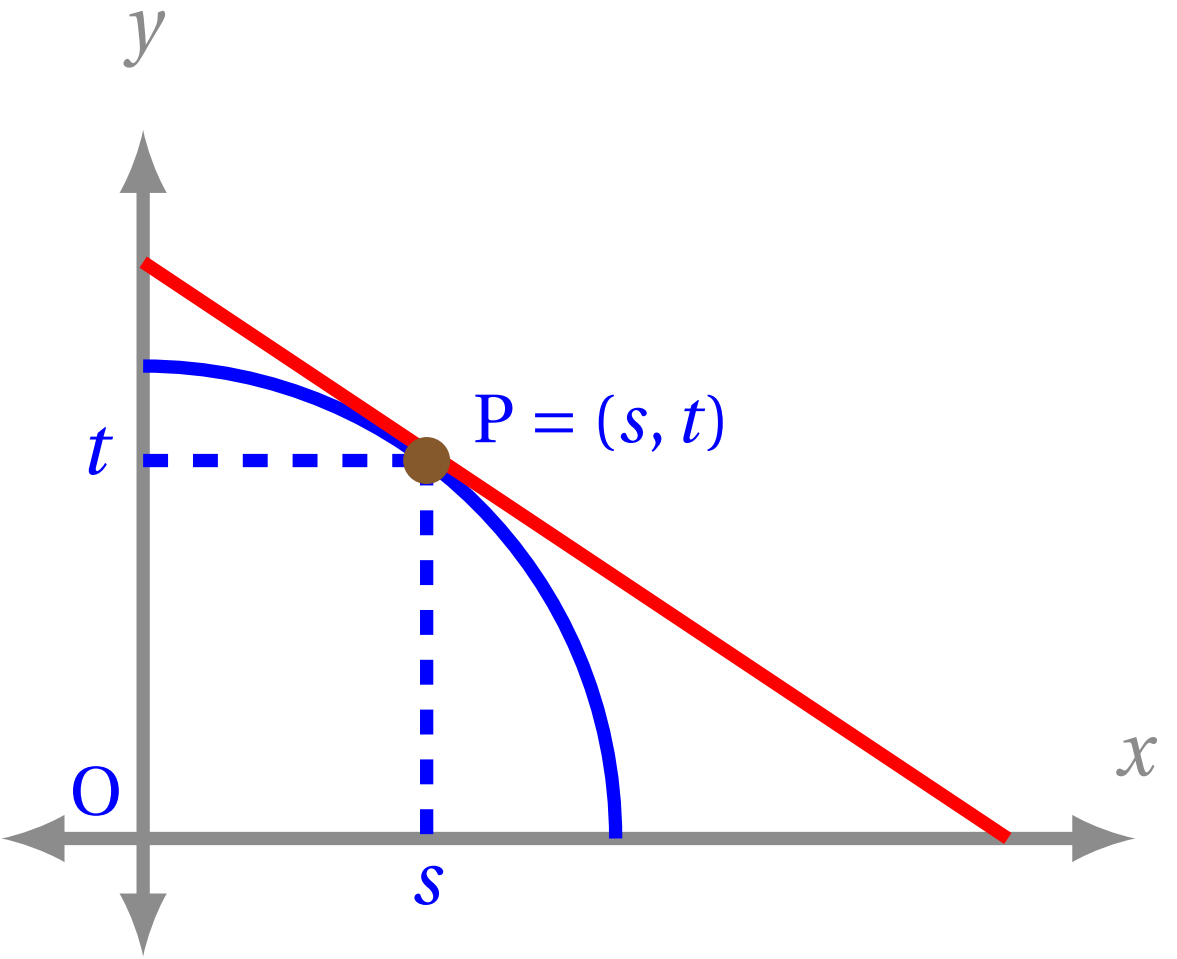
27 ./images-tikz/image-5.tex


Trimmed bounding boxes
100
80
60
40
20
0
−6
−4
−2
0
2
4
6

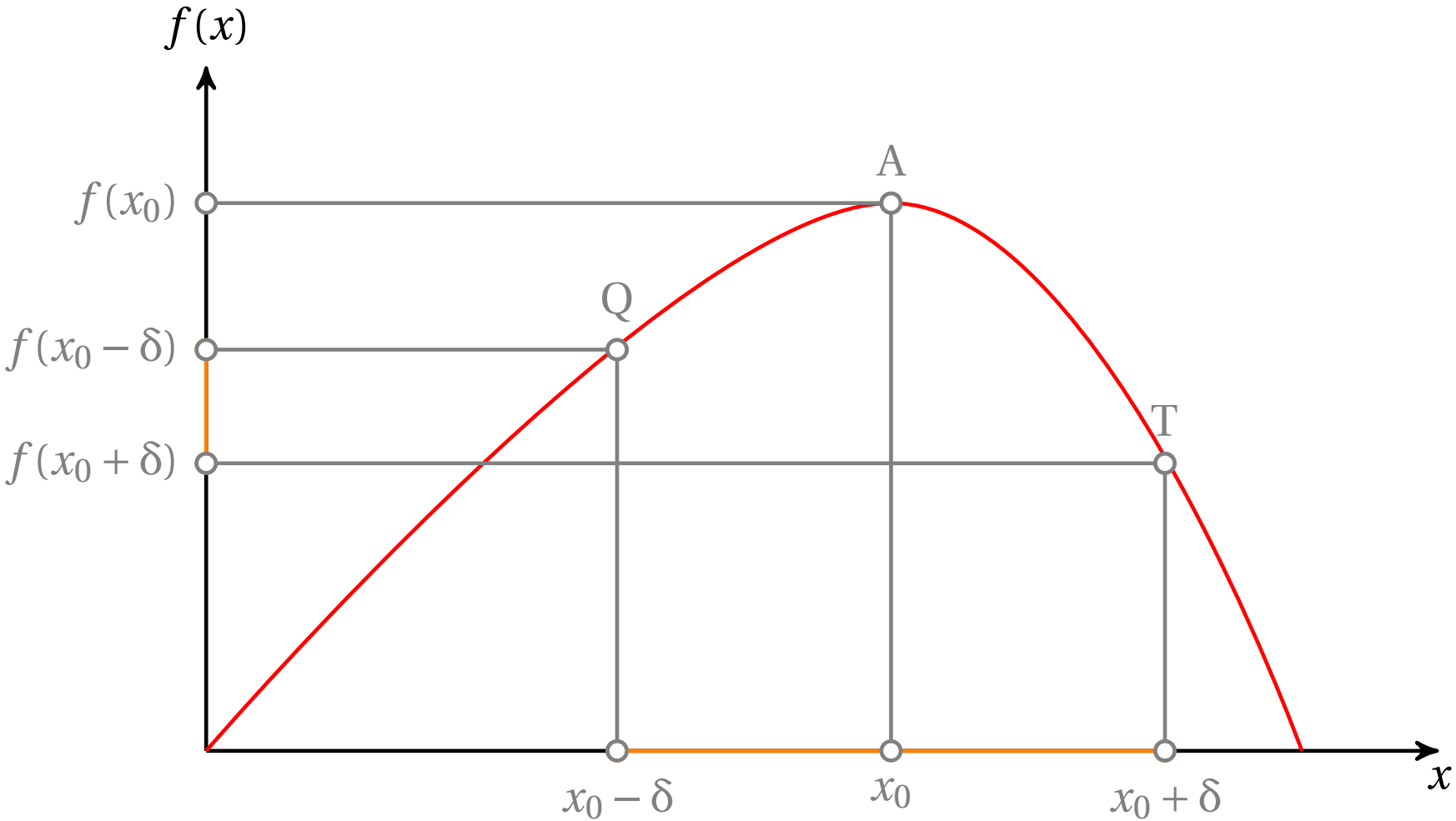
28 ./images-tikz/extremo-rel.tex


$f(x)$
$f(x_0)$
$f(x_0-\delta)$
$f(x_0+\delta)$
A
Q
T
$x_0-\delta$
x_0
$x_0+\delta$
x

29 ./images-tikz/resuelto6-4.tex

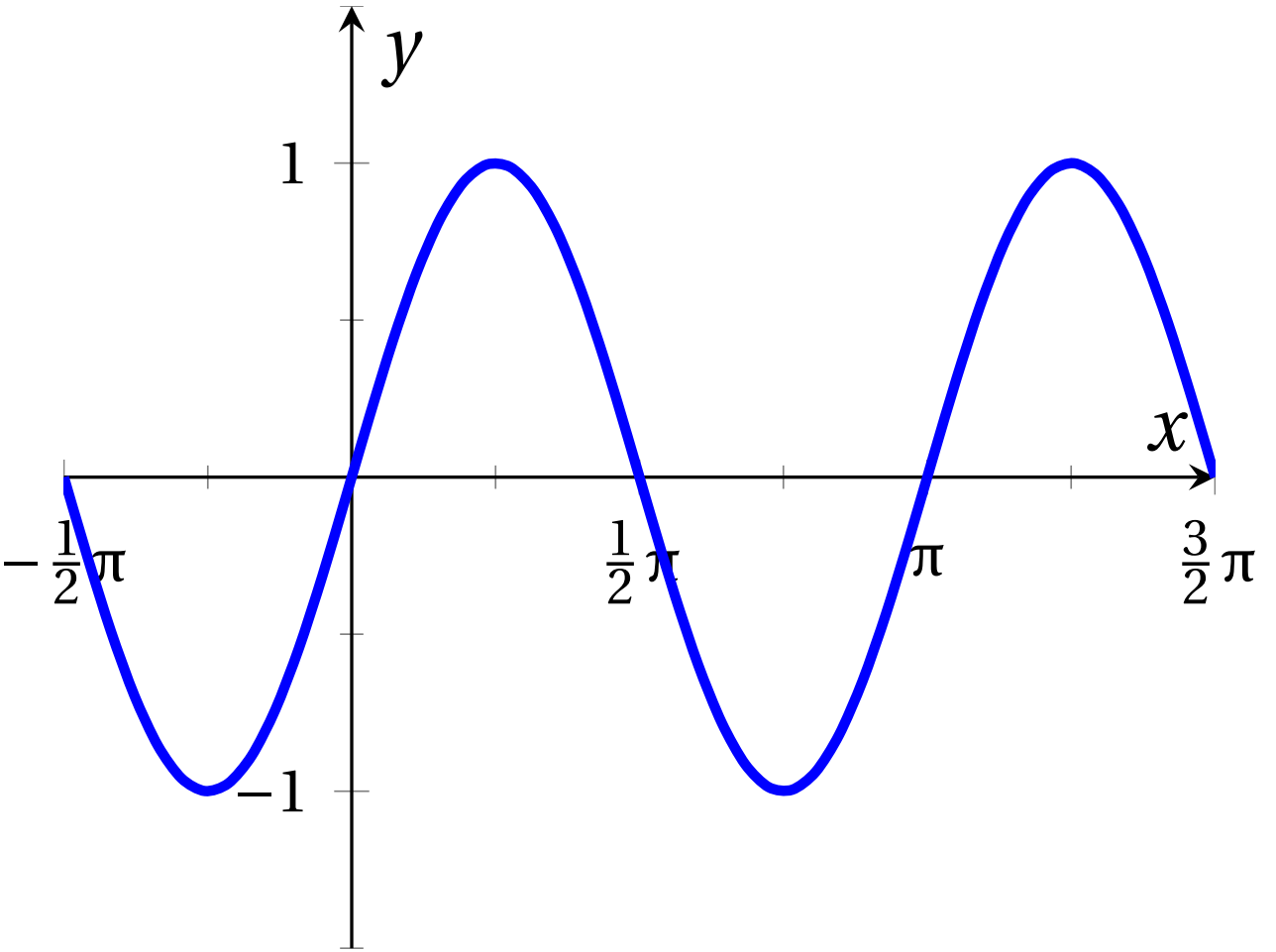

y
x
O
t
s
P = (s, t)

30 ./images-tikz/img-xx20.tex

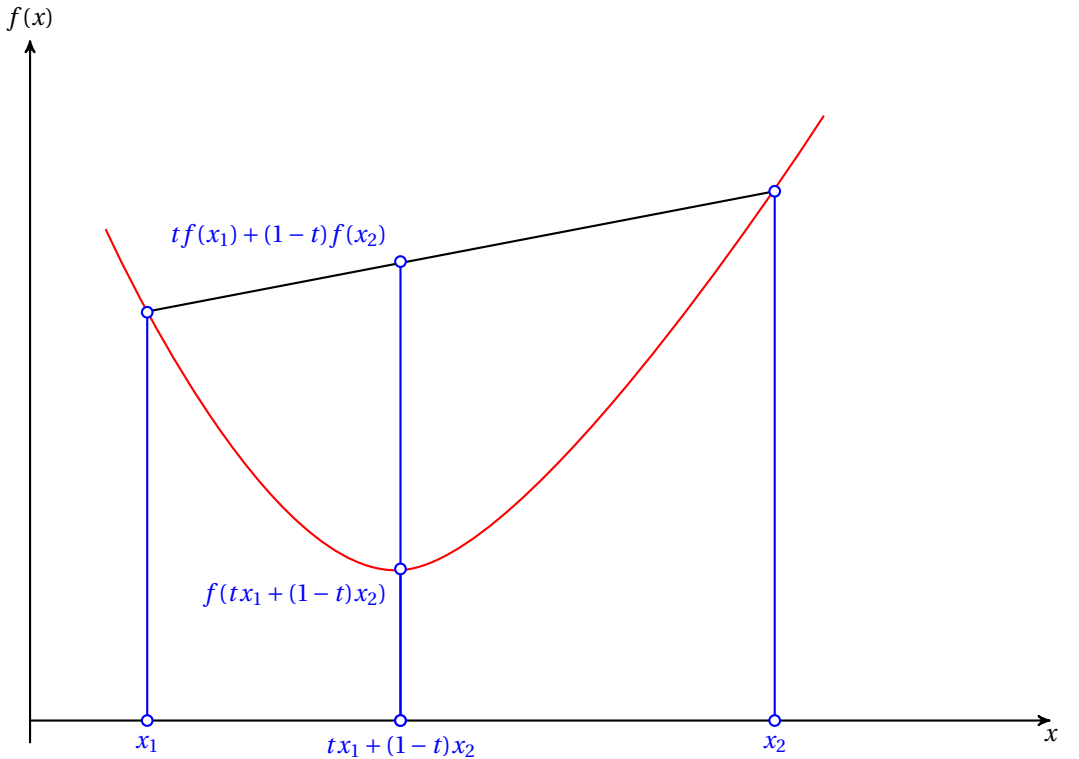

$f(x)$
$f(x_0)$
$f(x_0-\delta)$
$f(x_0+\delta)$
A
Q
T
$x_0-\delta$
x_0
$x_0+\delta$
x

31 ./images-tikz/senx.tex

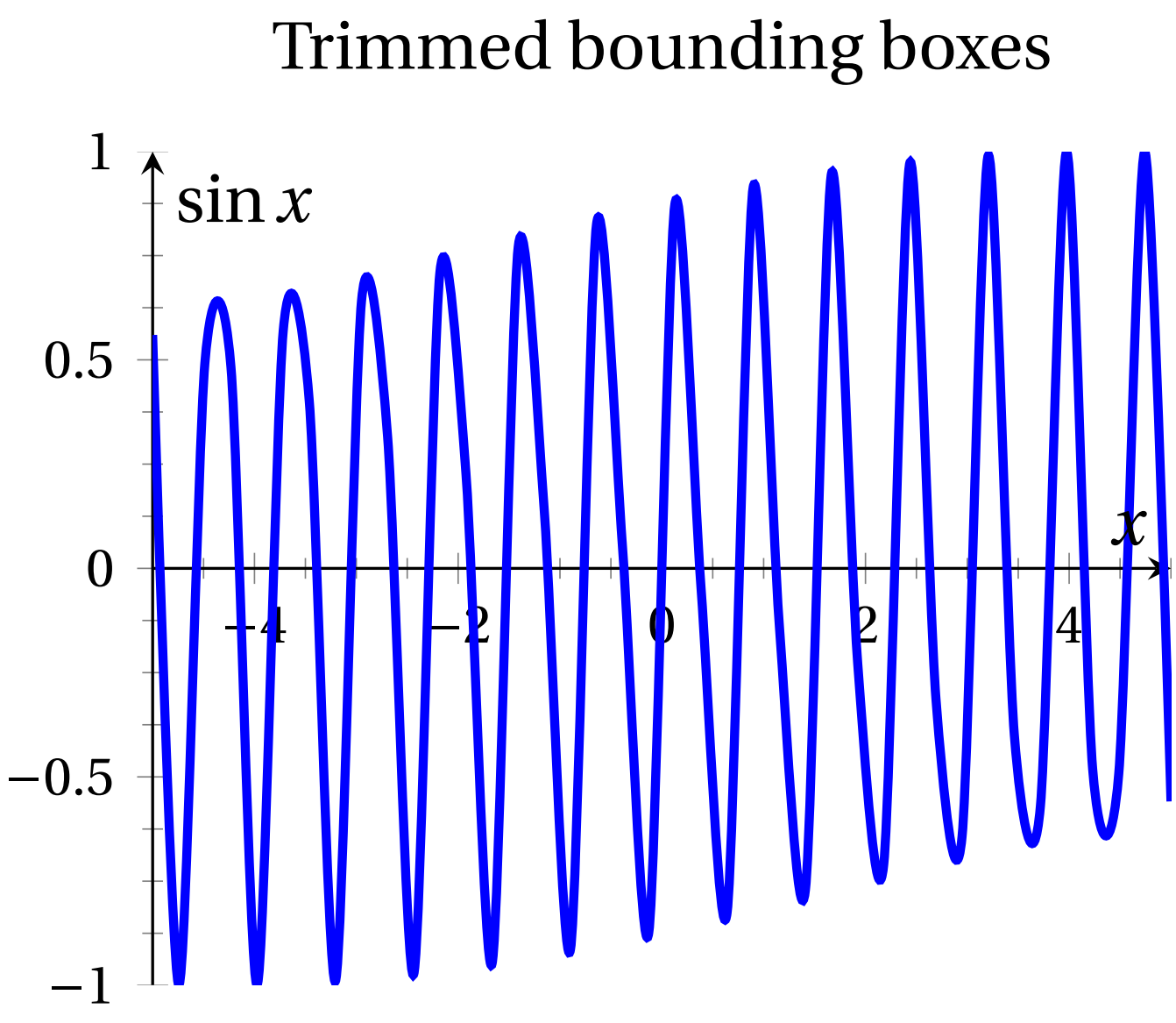
Trimmed bounding boxes



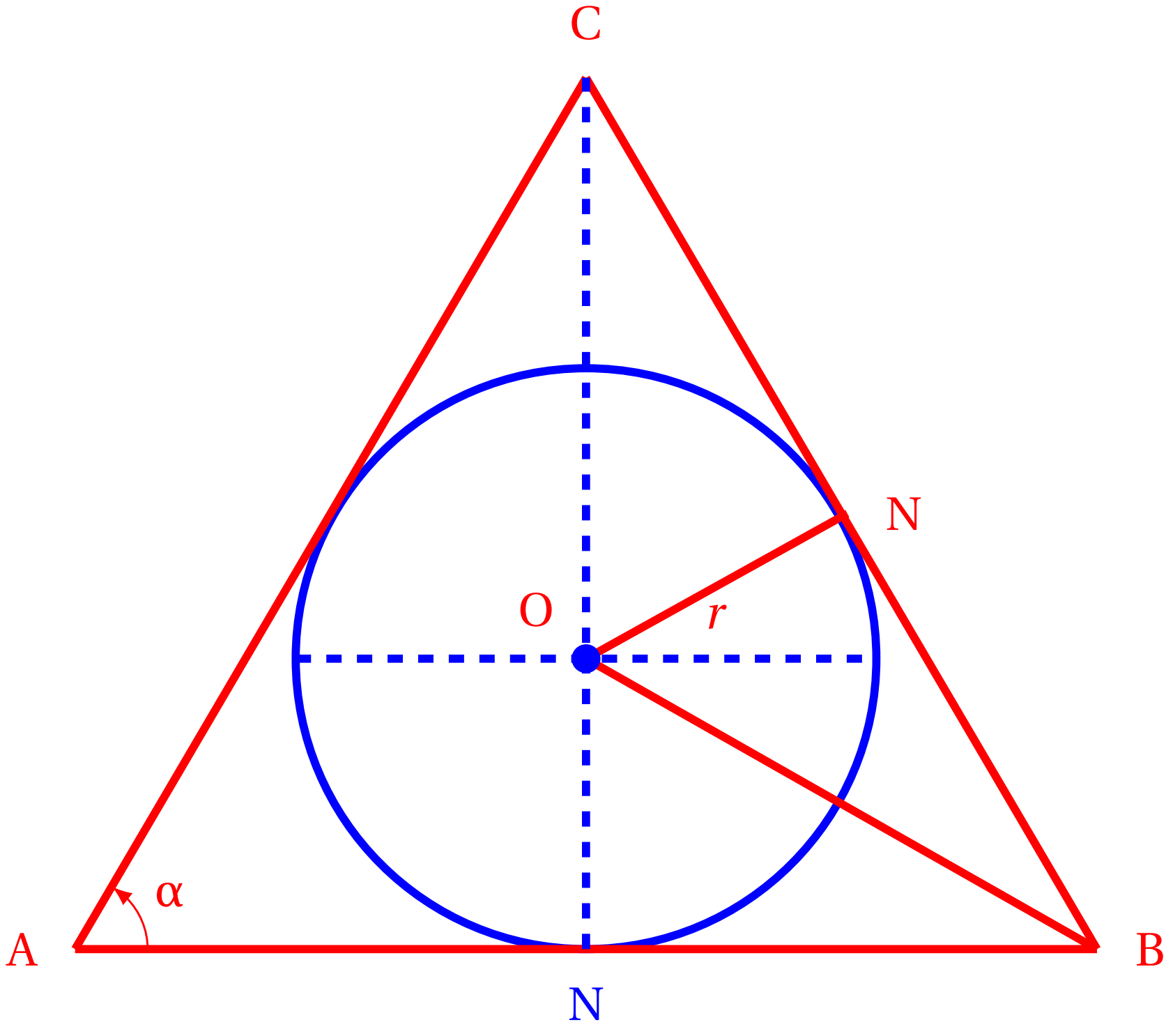
32 ./images-tikz/concavidad-arr.tex


$f(x)$
$tf(x_1) + (1-t)f(x_2)$
$f(tx_1 + (1-t)x_2)$
x_1
$tx_1 + (1-t)x_2$
x_2
x

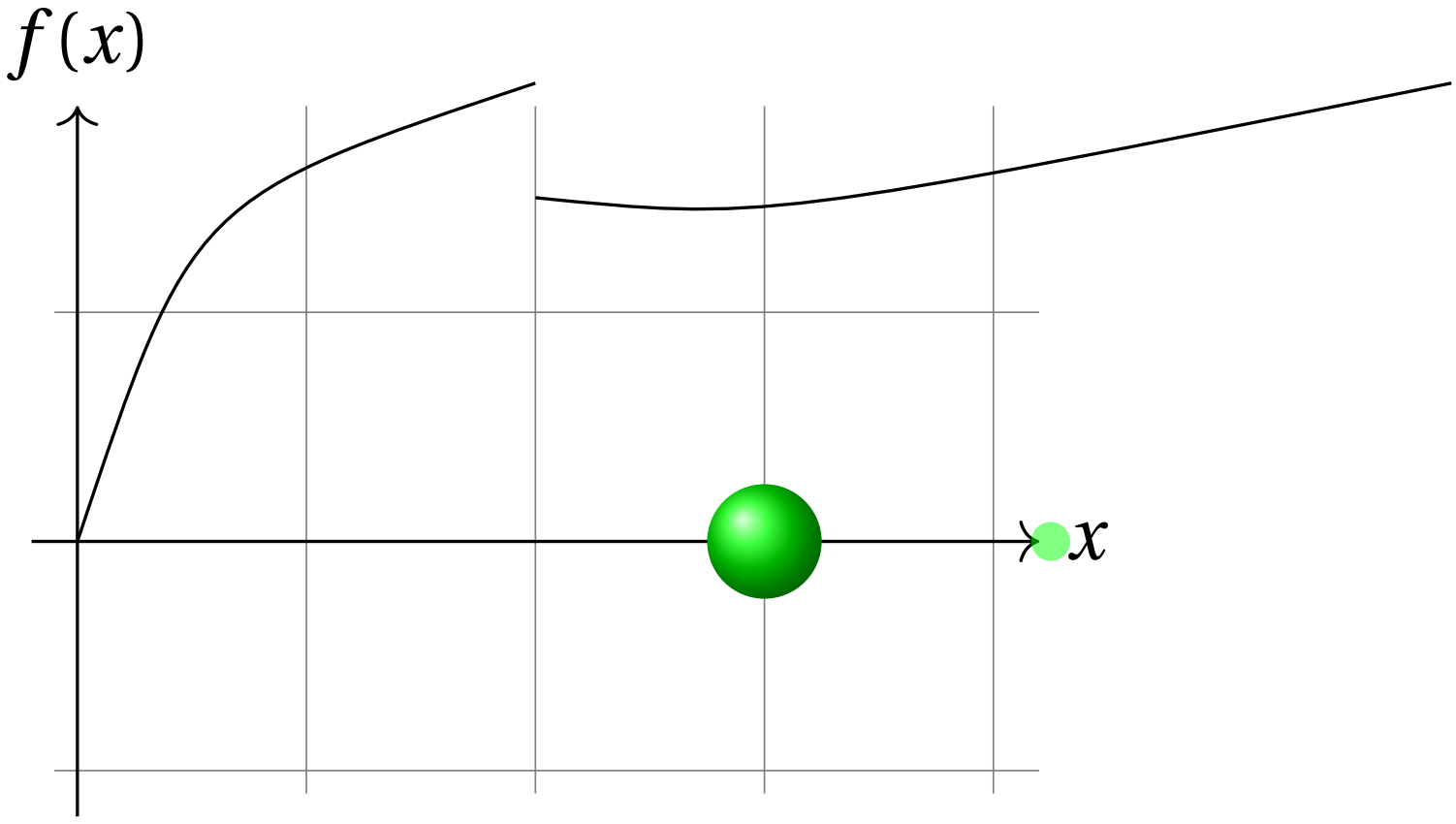
33 ./images-tikz/image-28.tex


Trimmed bounding boxes
1
sin x
0.5
0
−0.5
−1
x
−4
−2
0
2
4

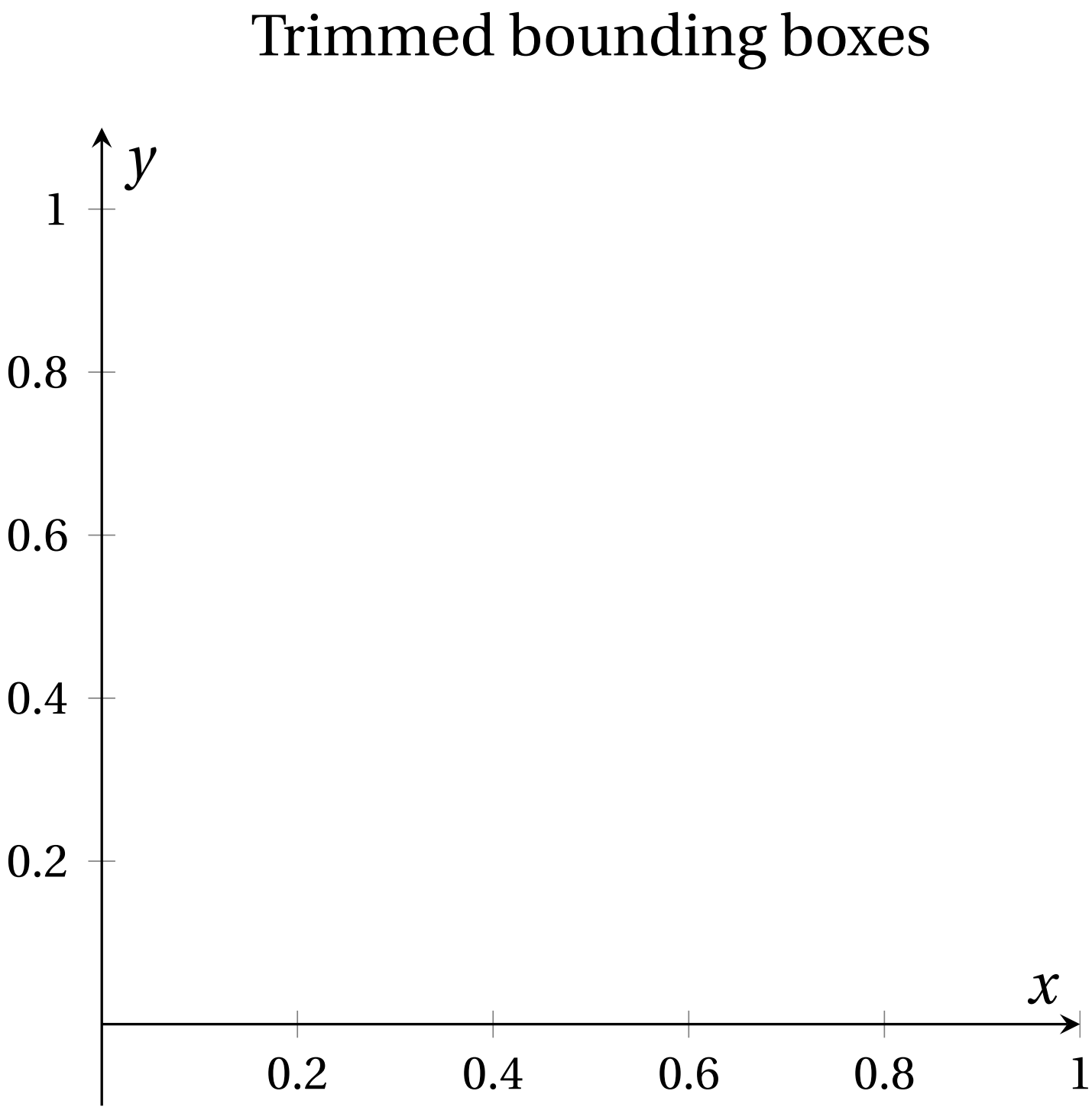
34 ./images-tikz/resuelto6-14.tex


C
N
O
r
α
A
B
N

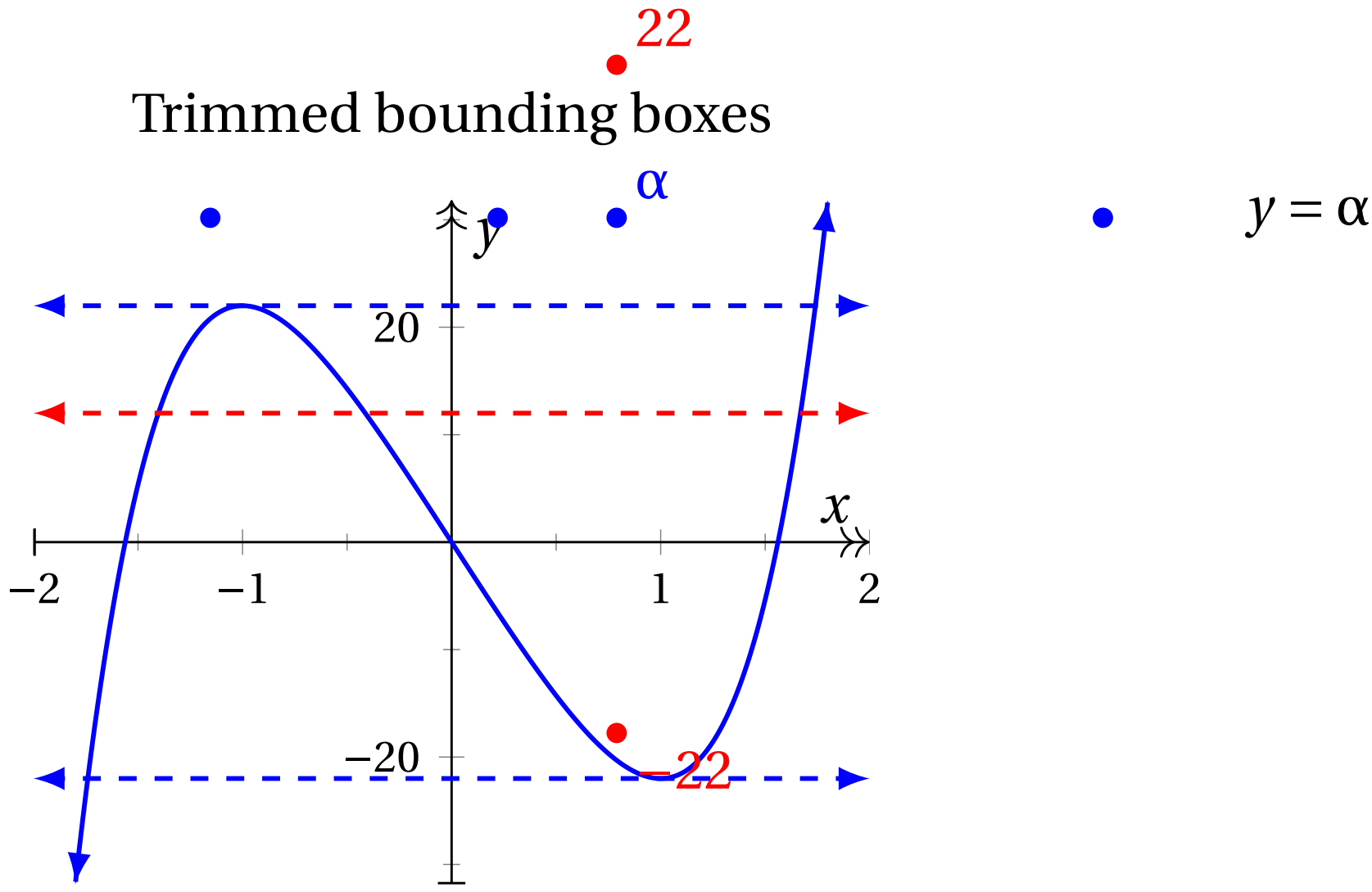
35 ./images-tikz/image-9.tex


f(x)
x

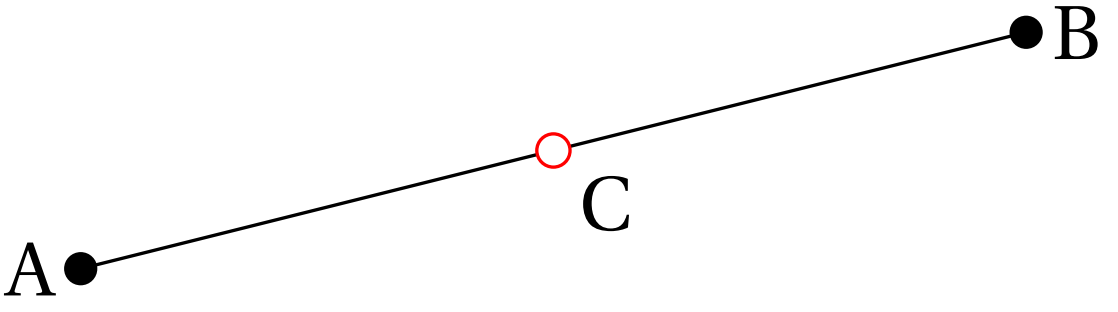
36 ./images-tikz/image-39.tex


Trimmed bounding boxes
y
1
0.8
0.6
0.4
0.2
x
0.2
0.4
0.6
0.8
1

37 ./images-tikz/resuelto6-18.tex

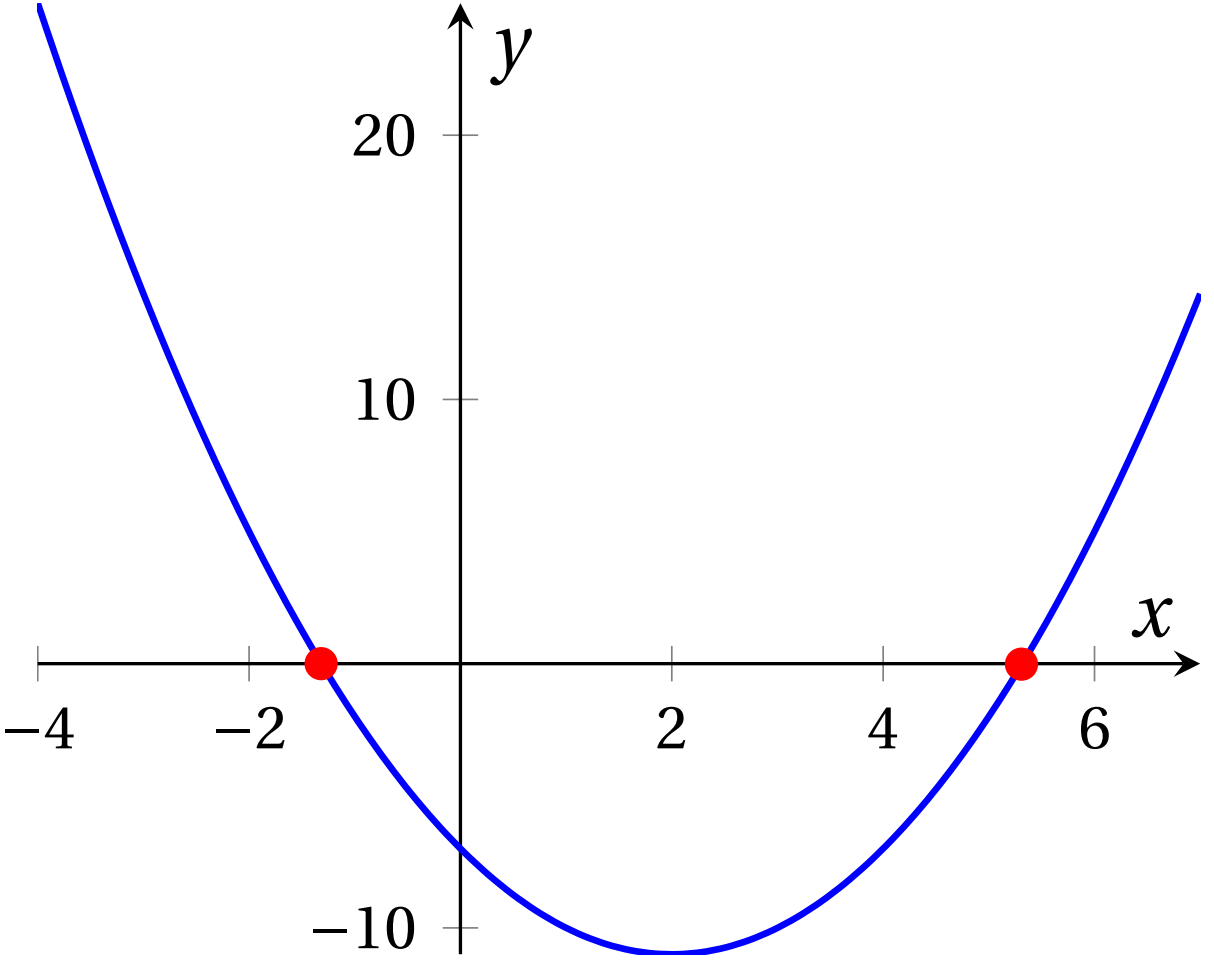

22
Trimmed bounding boxes
α
y = α
y
20
x
−2
−1
1
2
−20
−22

38 ./images-tikz/image-11.tex

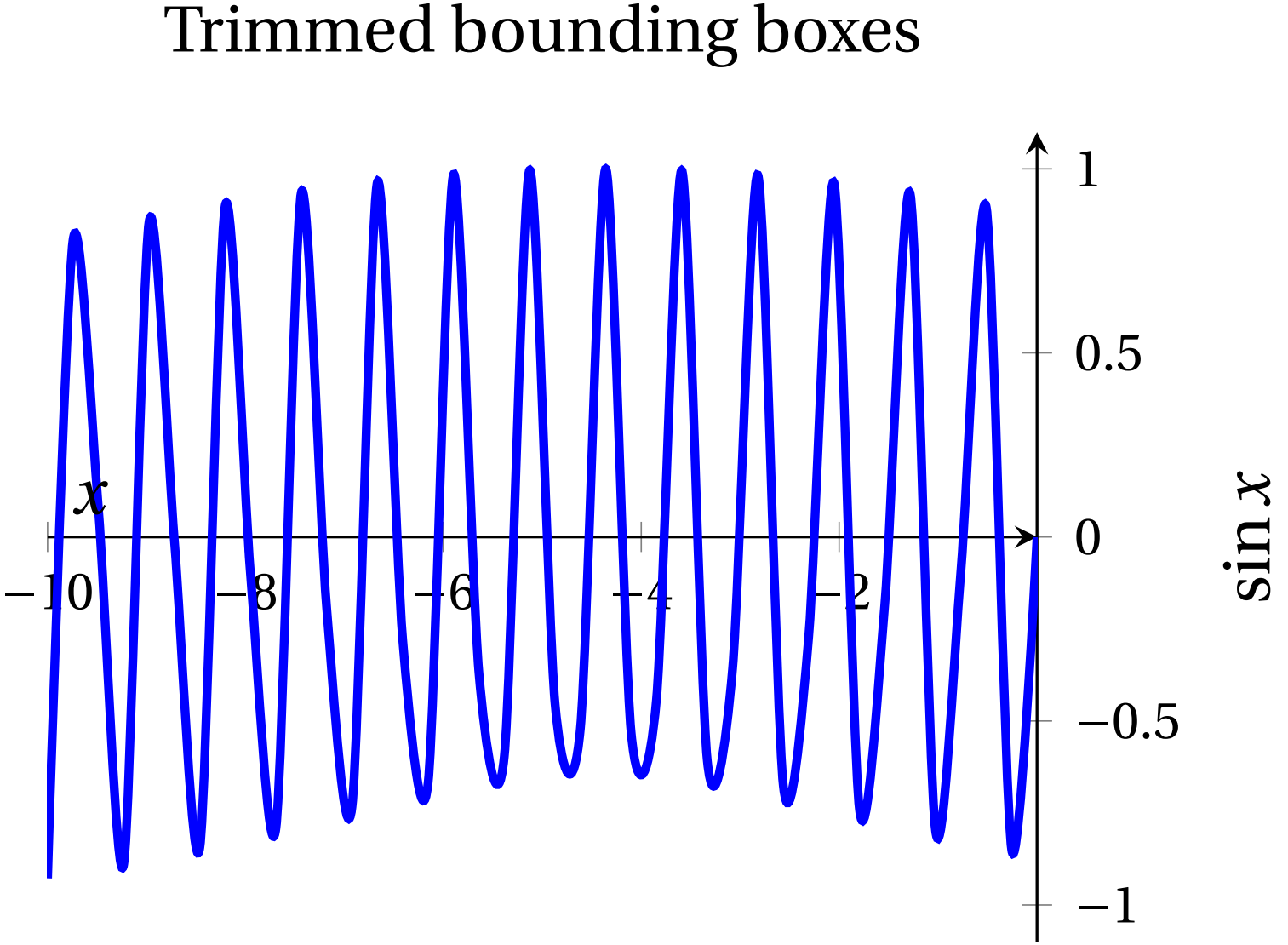

A
C
B

39 ./images-tikz/image-42.tex

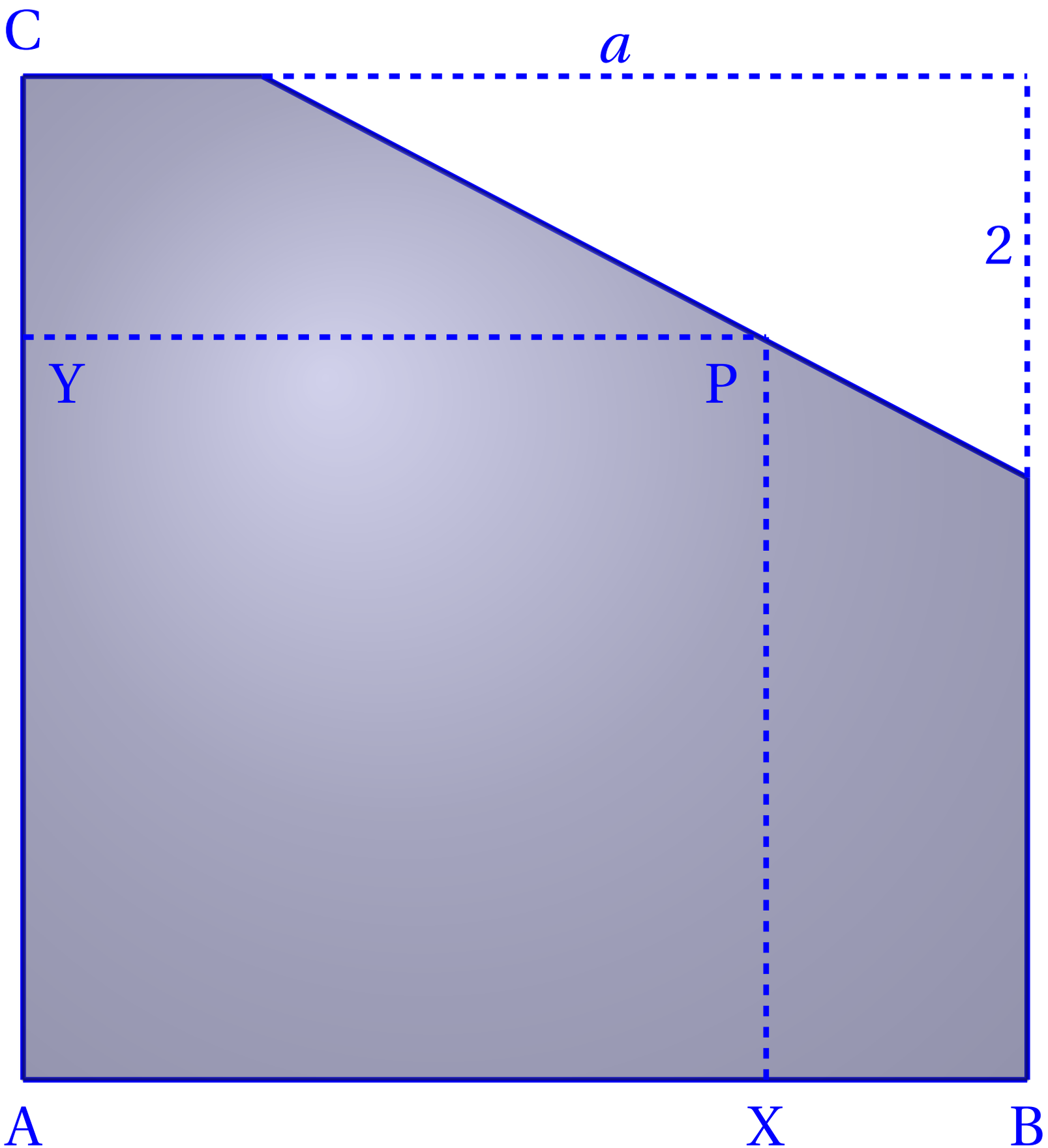
Trimmed bounding boxes



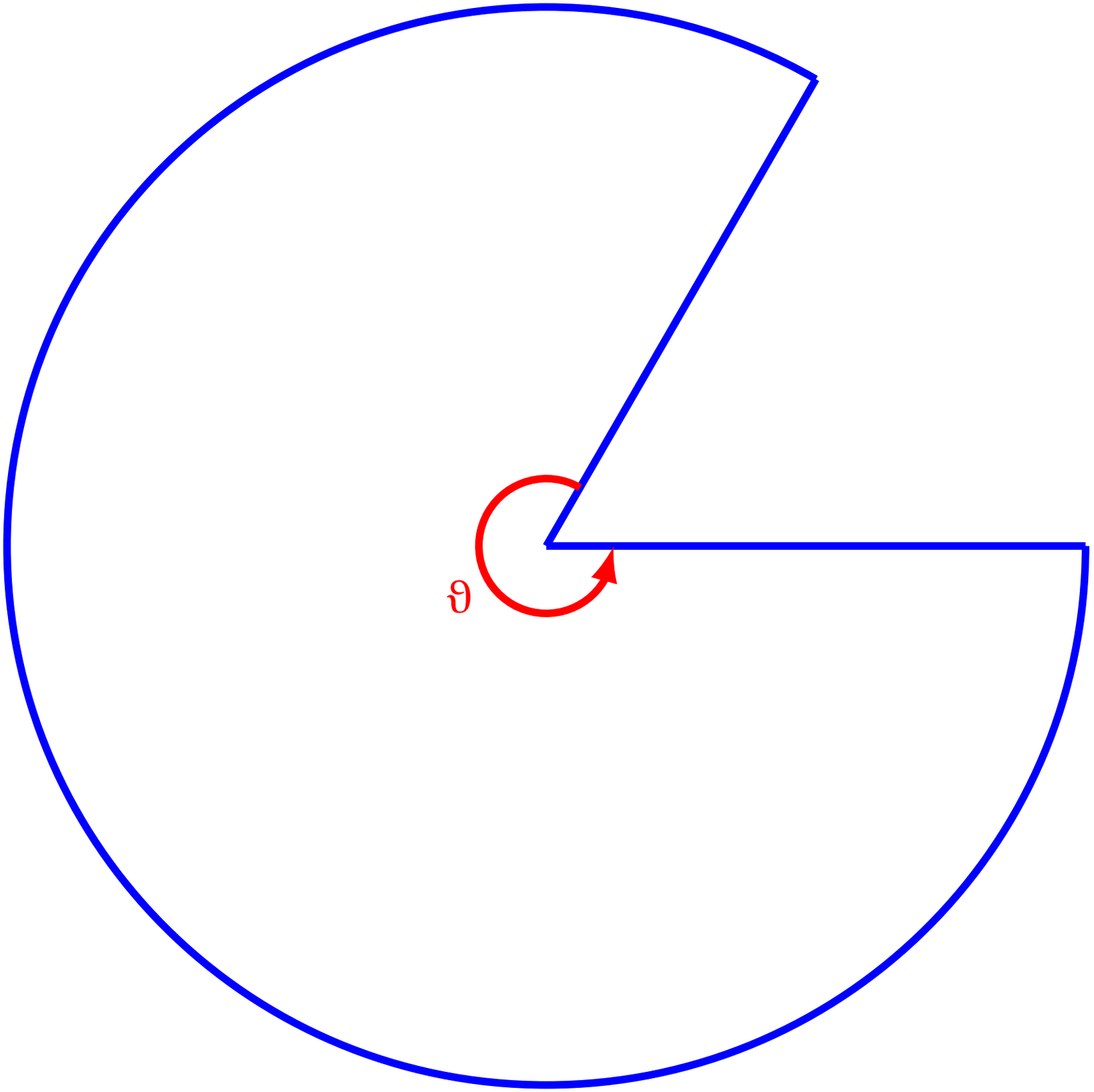
40 ./images-tikz/image-25.tex


Trimmed bounding boxes
x
−10
−8
−6
−4
−2
0
1
0.5
−0.5
−1
sin x

41 ./images-tikz/espejo.tex

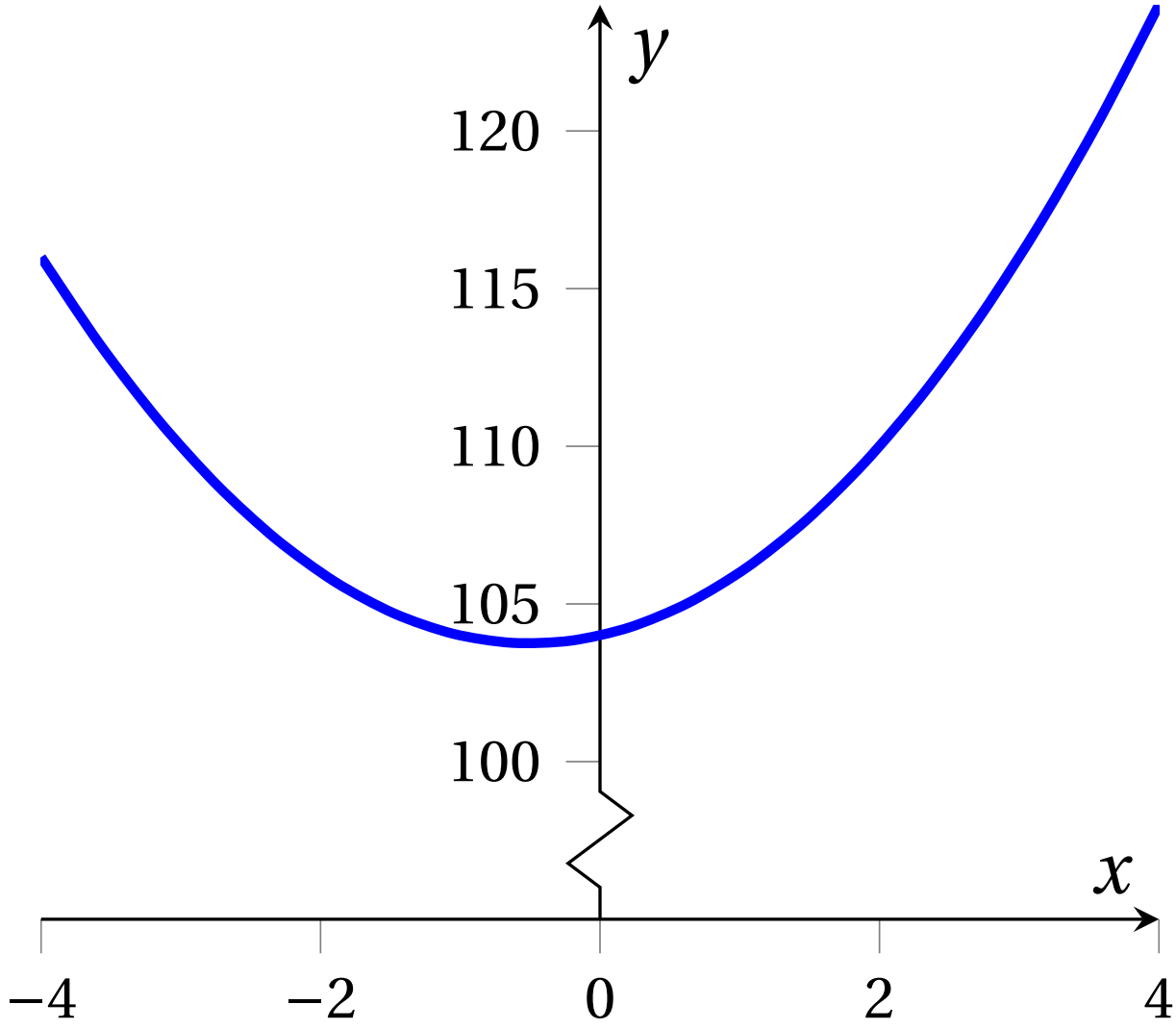

C
a
2
Y
P
A
X
B

42 ./images-tikz/resuelto6-11.tex

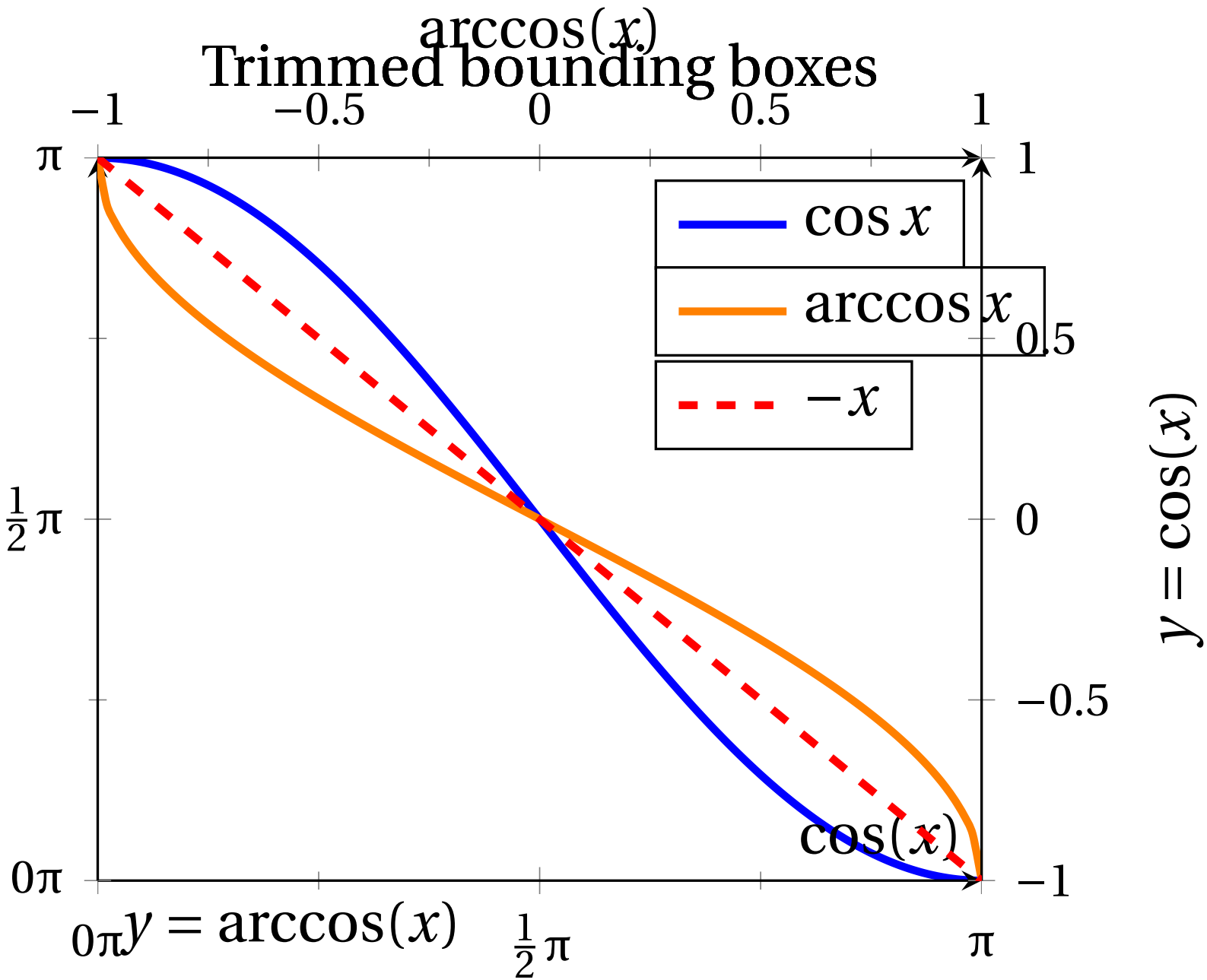

ϑ

43 ./images-tikz/image-30.tex

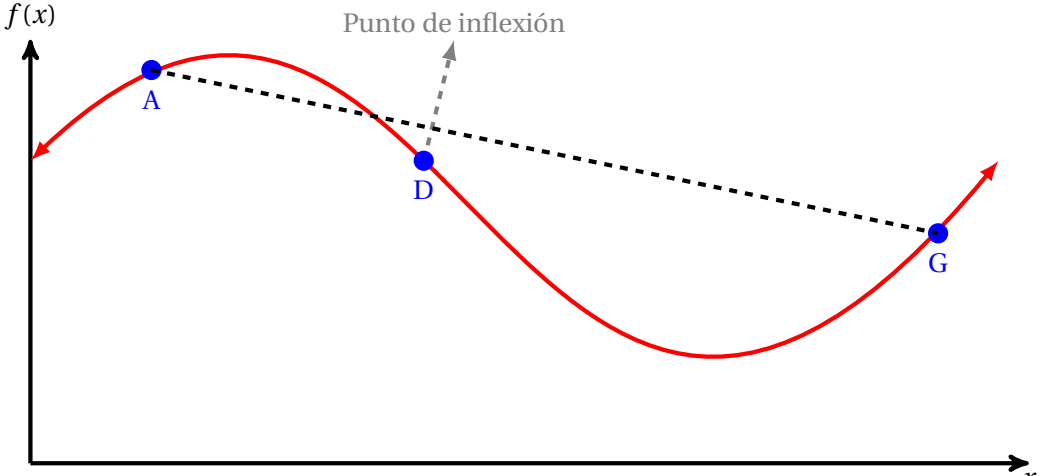
Trimmed bounding boxes



44 ./images-tikz/cos.tex

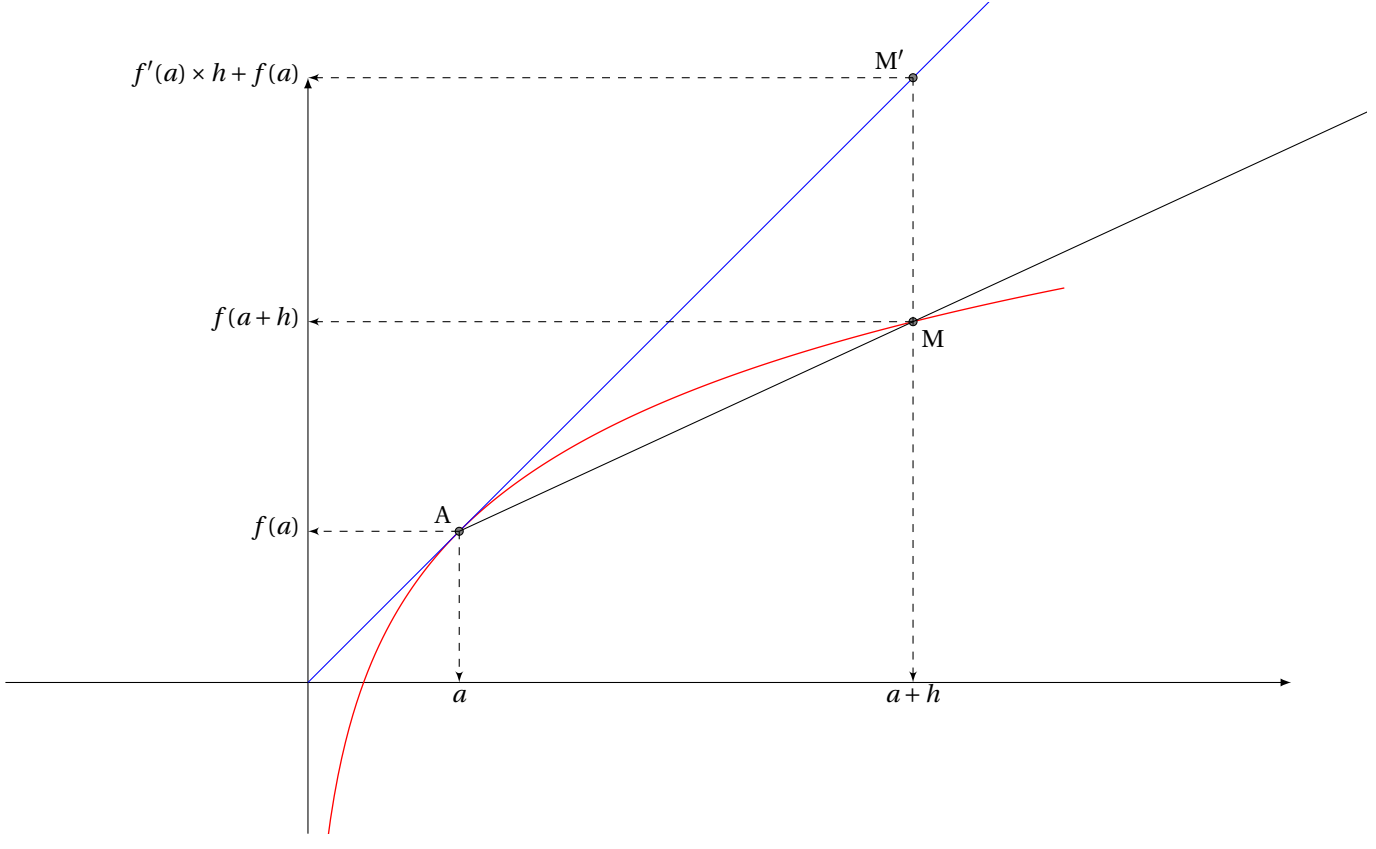

arccos(x)
Trimmed bounding boxes
−1
−0.5
0
0.5
1
π
1
cos x
arccos x
0.5
−x
½π
0
0
−0.5
cos(x)
−1
y = cos(x)
0π
0π
y = arccos(x)
½π
π

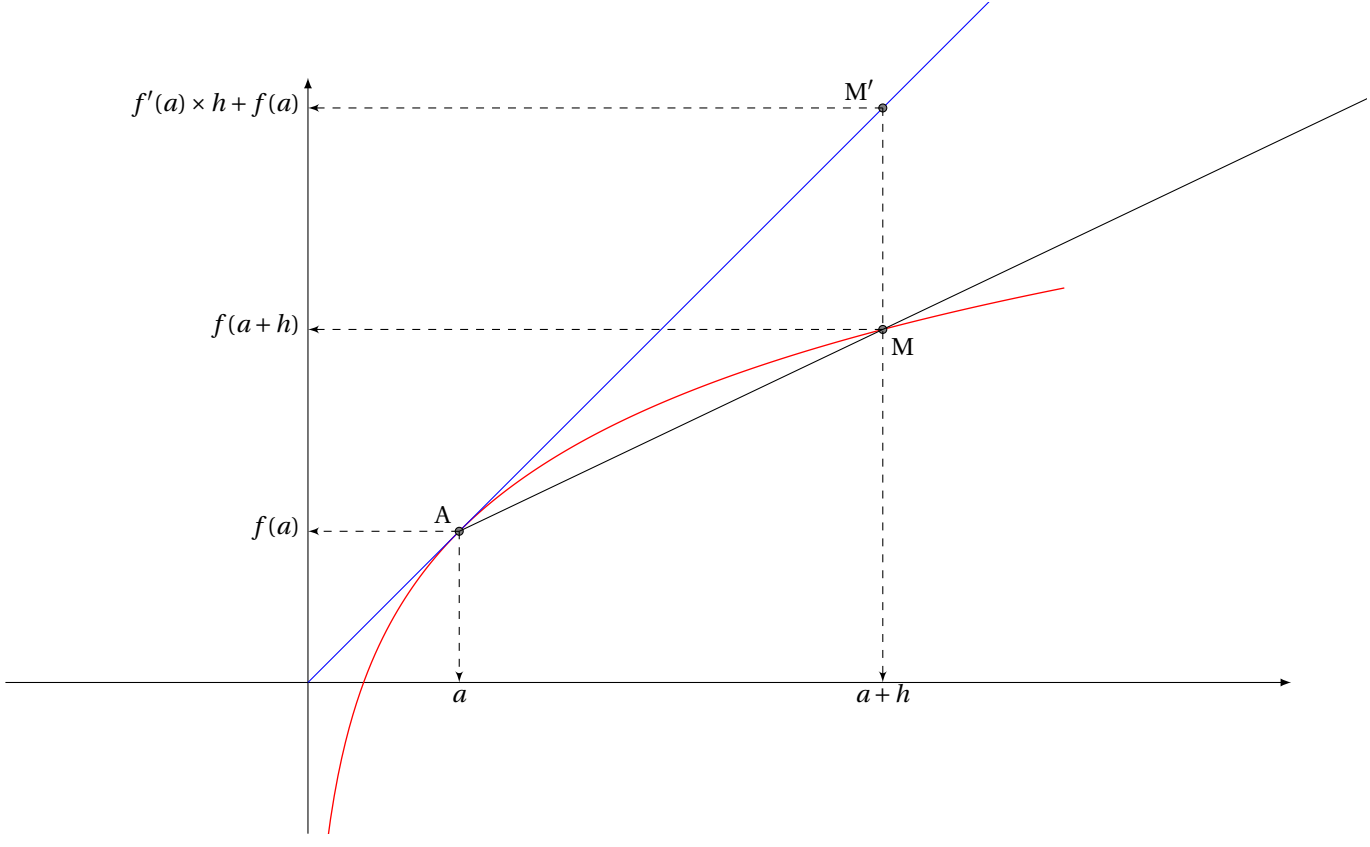
45 ./images-tikz/est-concavidad.tex

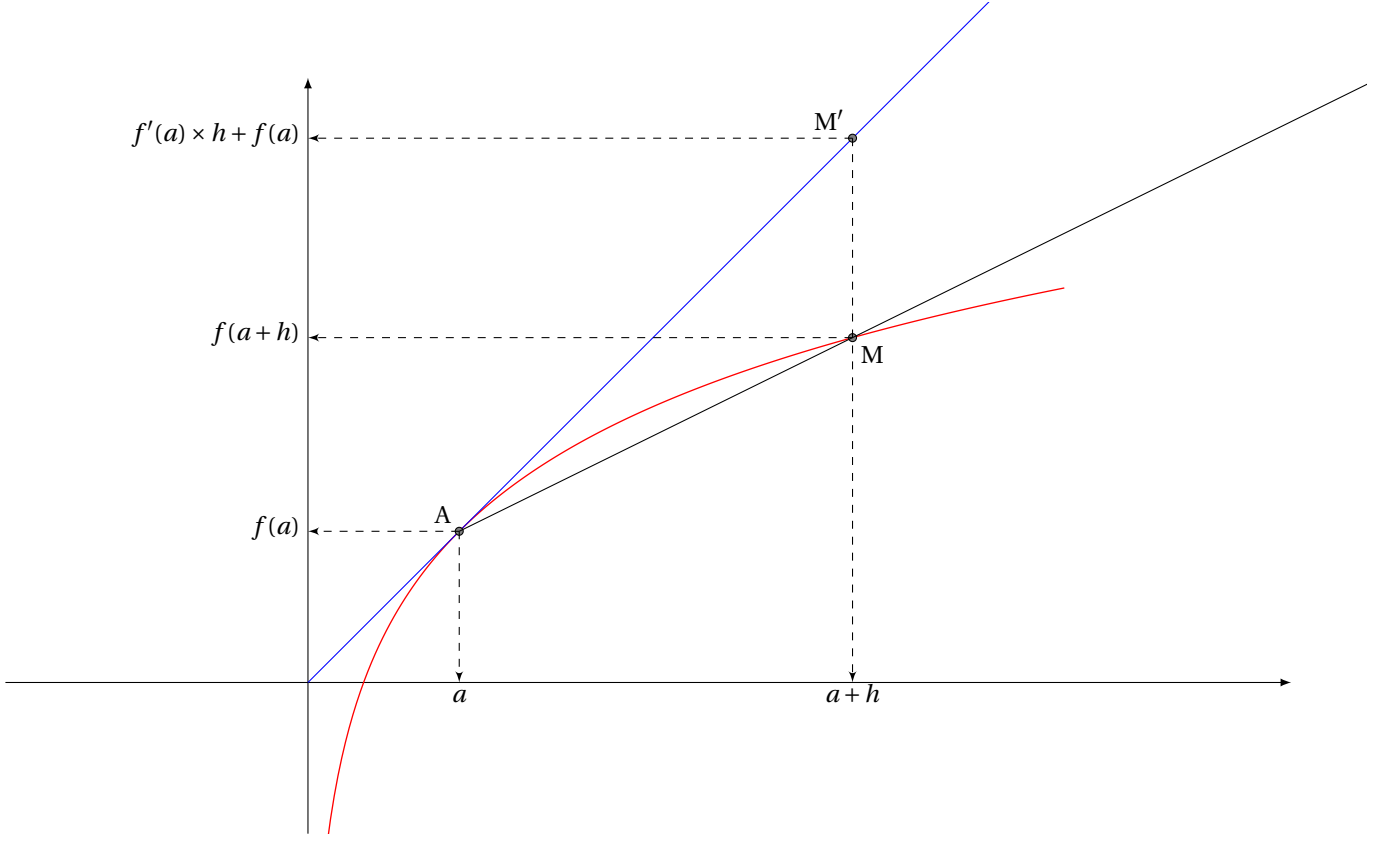


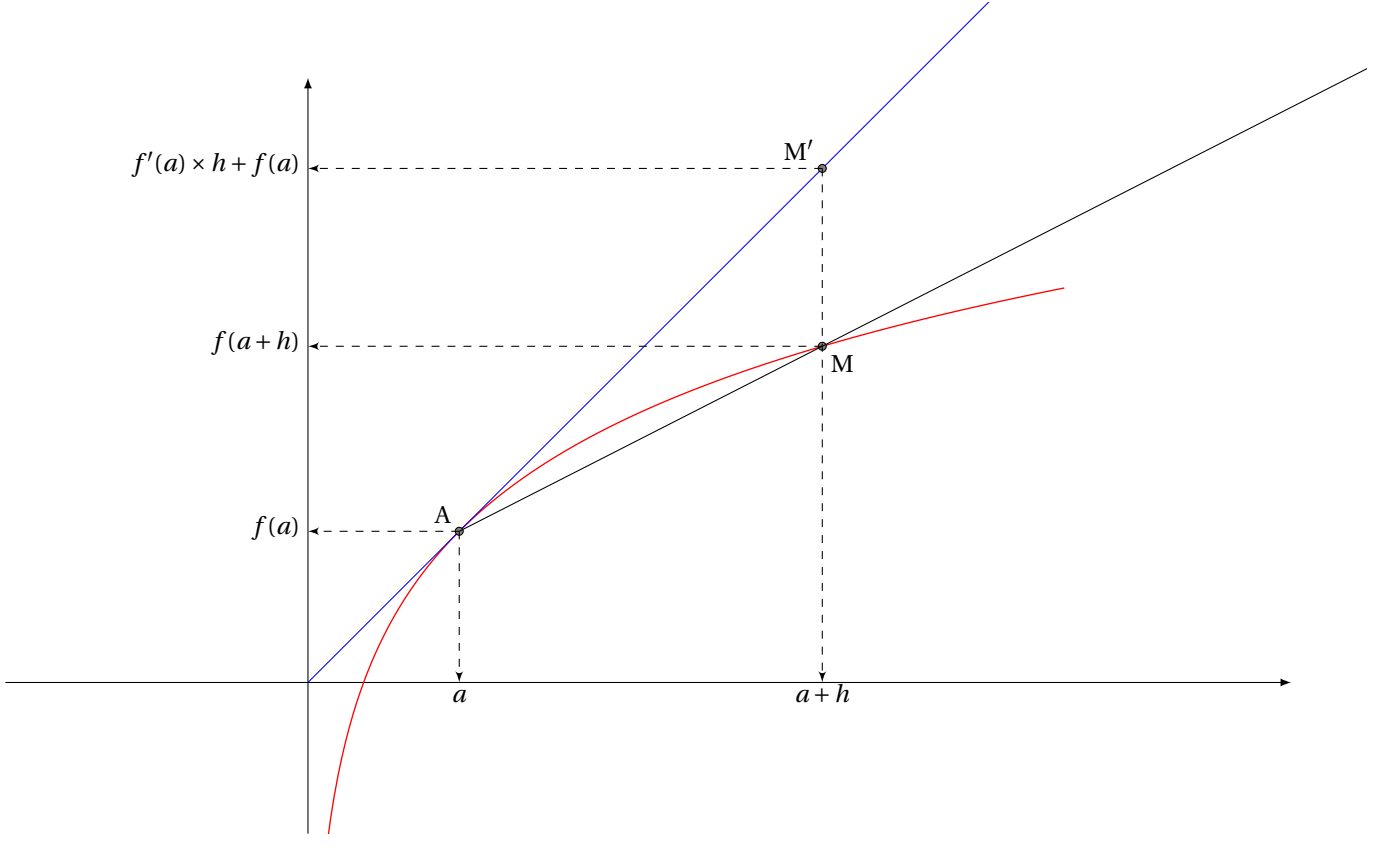
Estudio de la concavidad

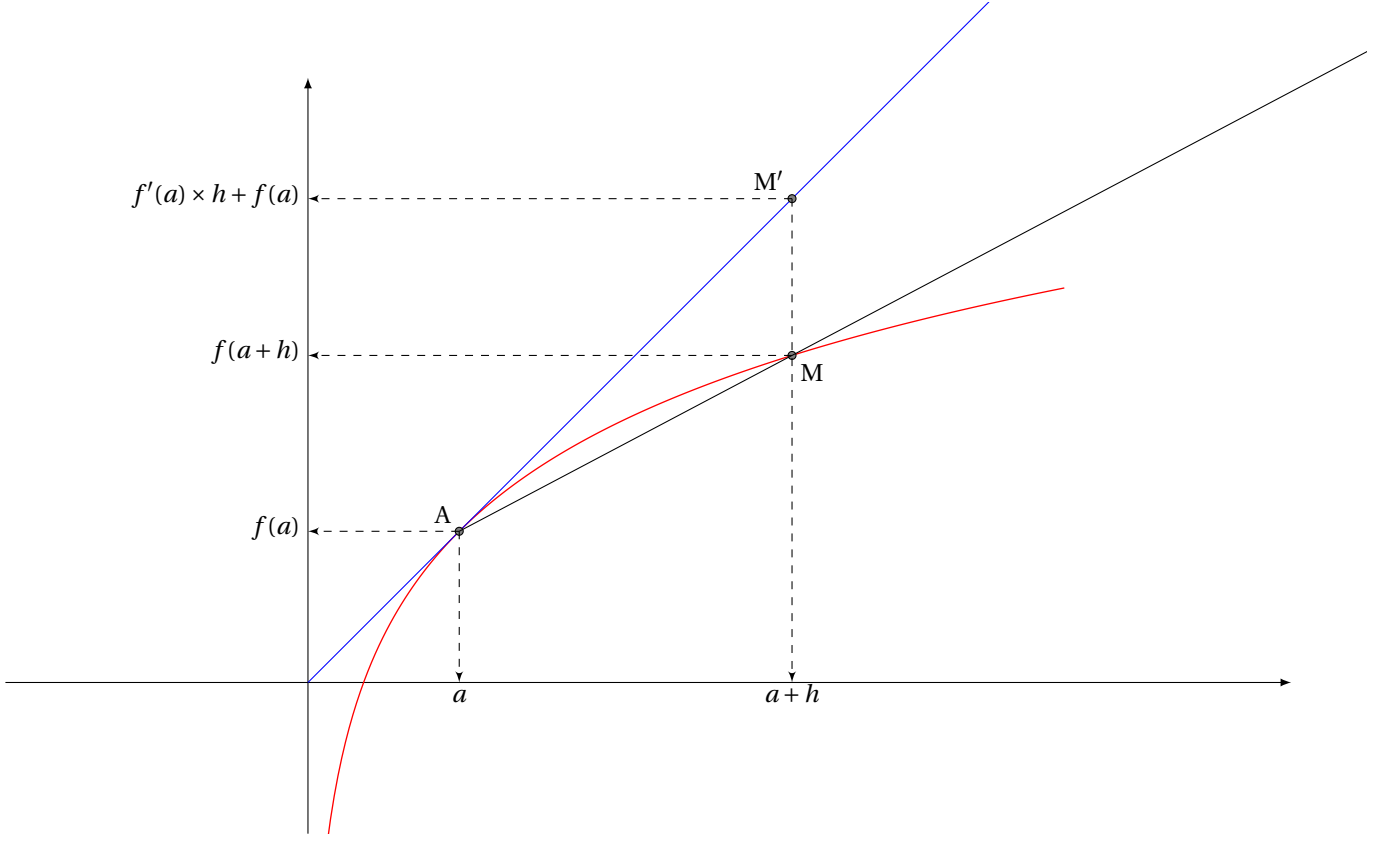
46 ./images-tikz/derivate2.tex

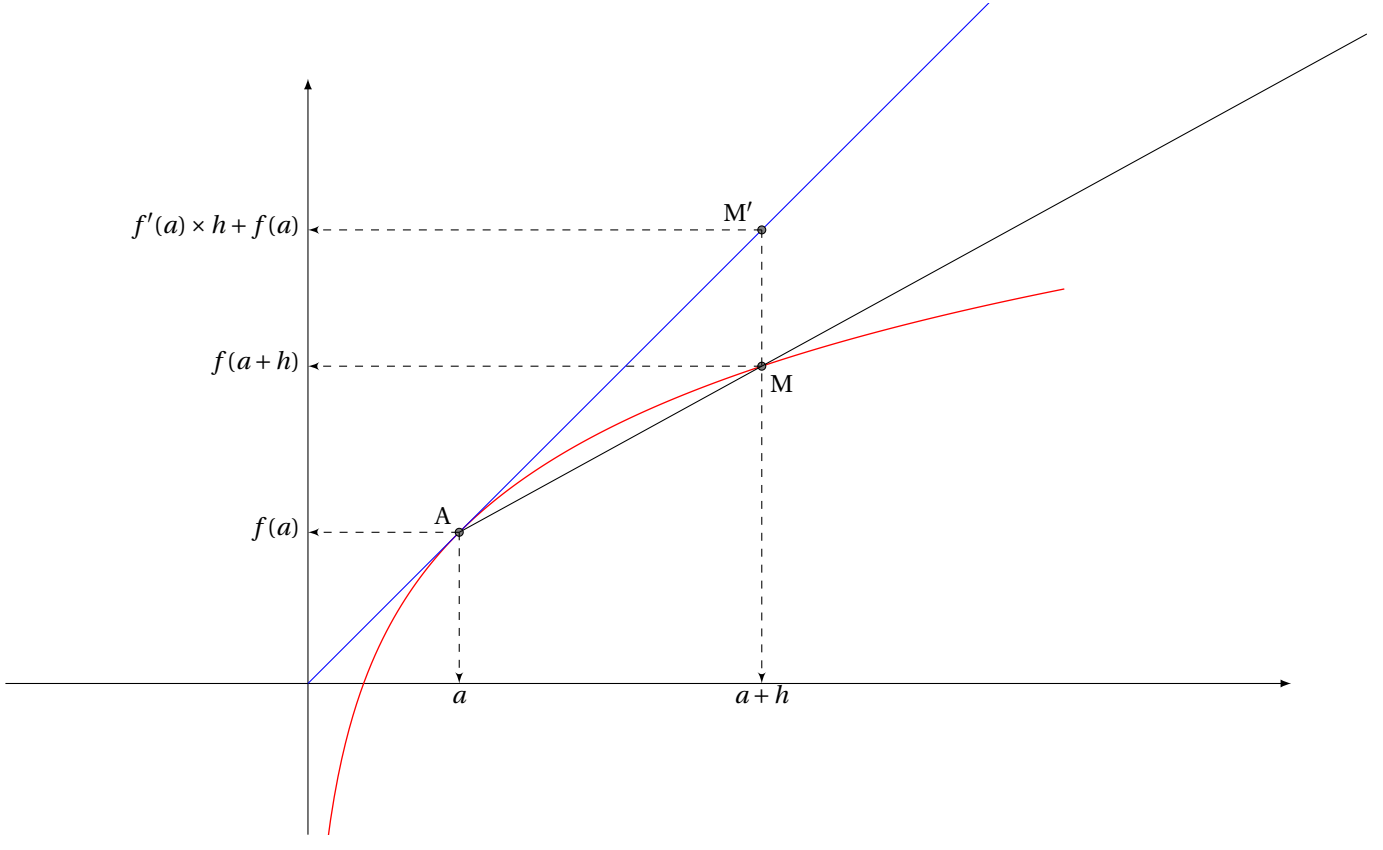

$f'(a) \times h + f(a)$
M′
$f(a+h)$
M
A
$f(a)$
a
$a+h$

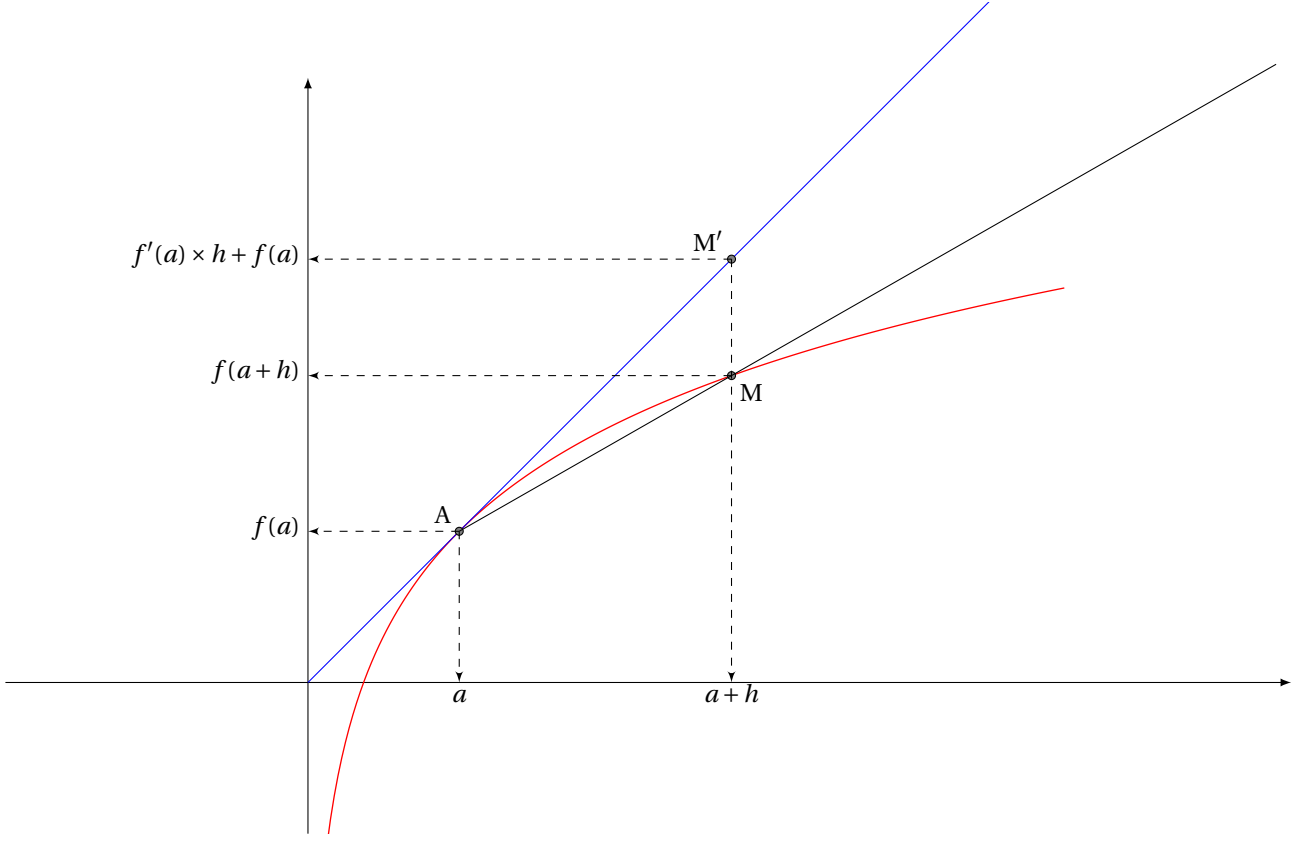

$f'(a) \times h + f(a)$
M′
$f(a+h)$
M
A
$f(a)$
a
$a+h$

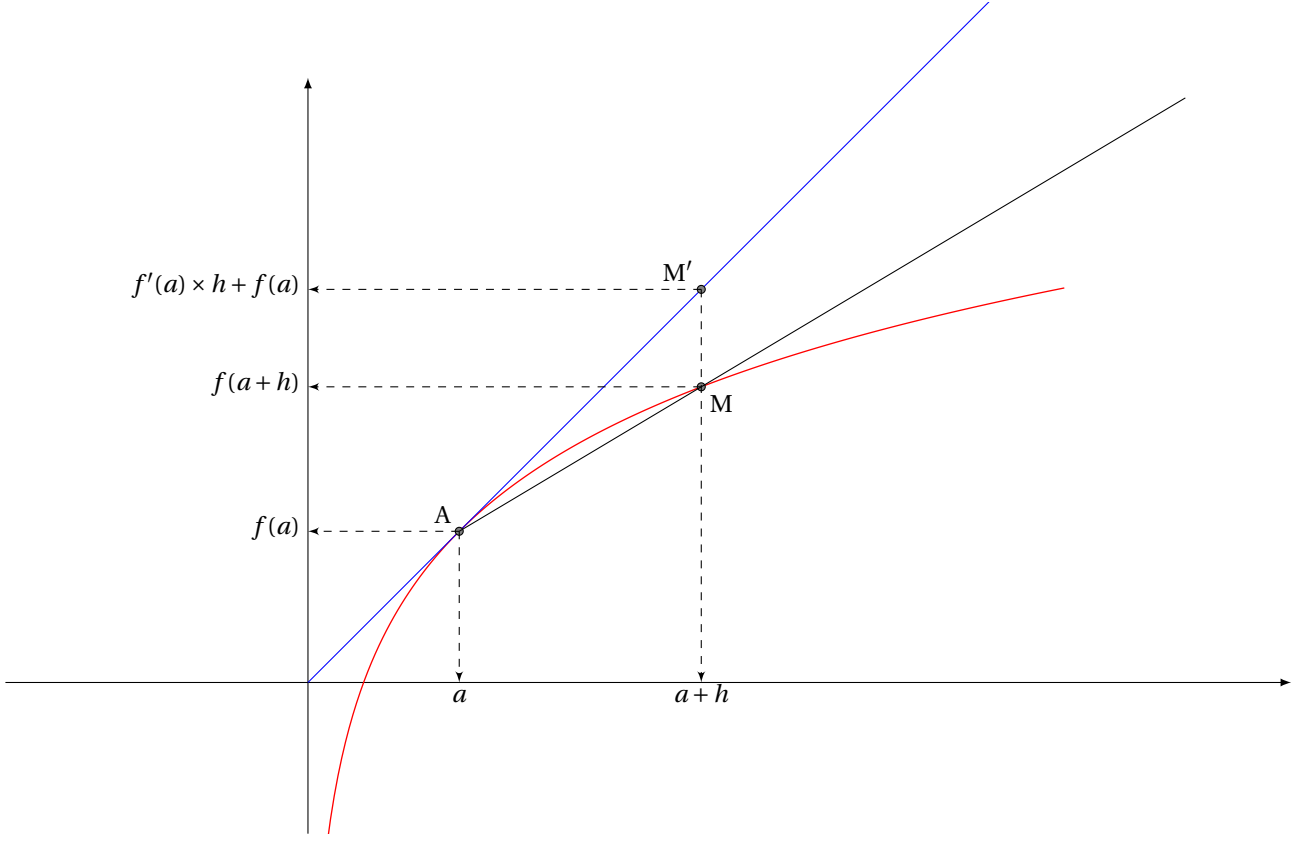

$f'(a) \times h + f(a)$
M′
$f(a+h)$
M
A
$f(a)$
a
$a+h$

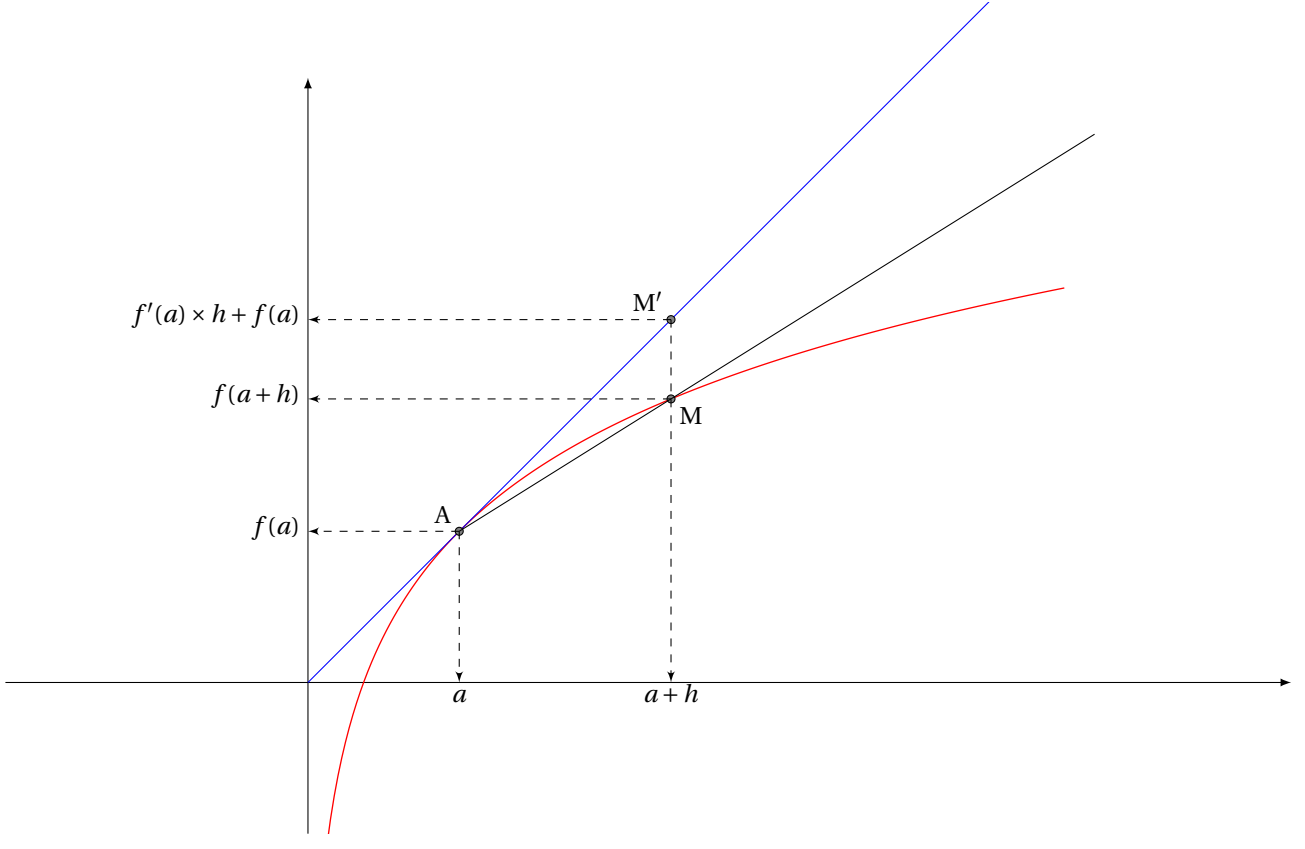

$f'(a) \times h + f(a)$
M′
$f(a+h)$
M
A
$f(a)$
a
$a+h$

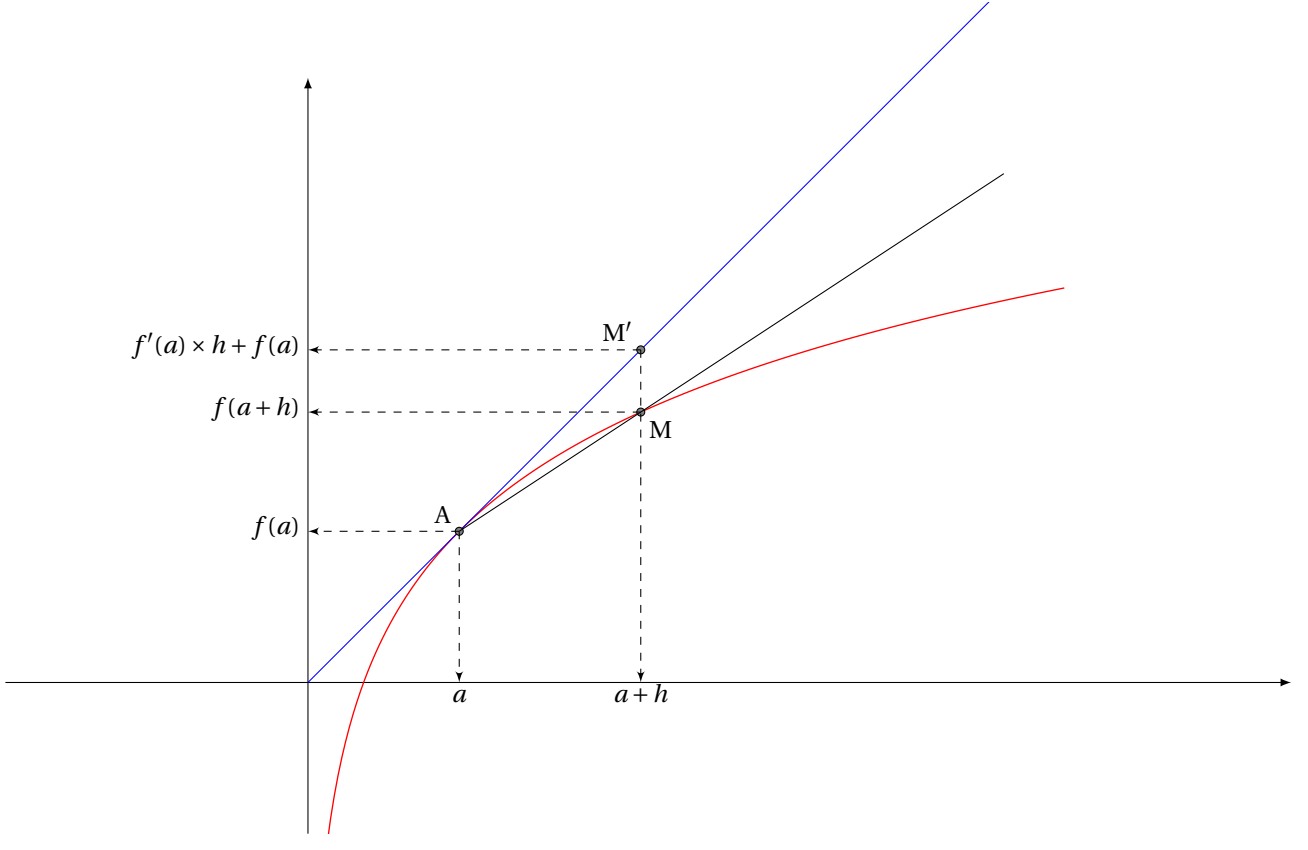

$f'(a) \times h + f(a)$
M′
$f(a+h)$
M
A
$f(a)$
a
$a+h$

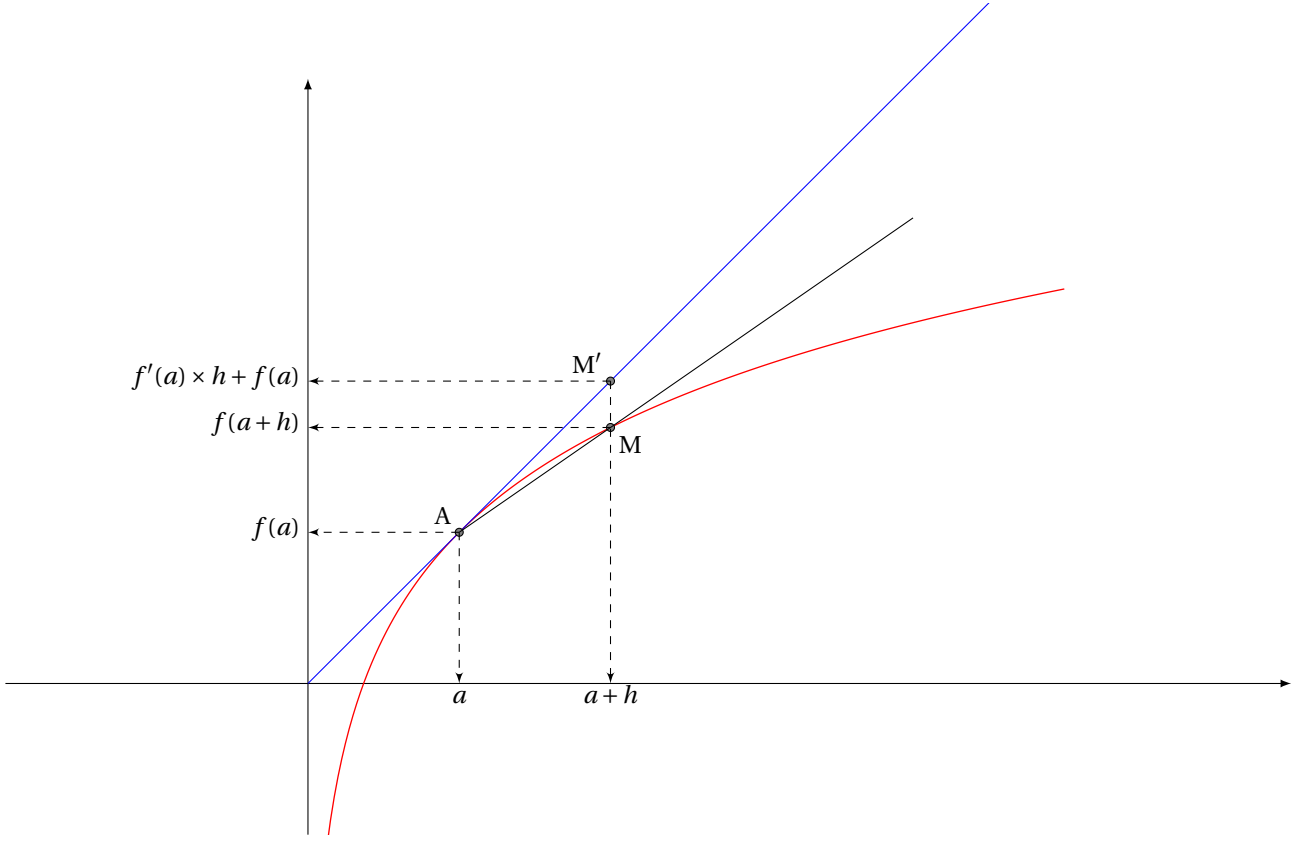

$f'(a) \times h + f(a)$
$f(a+h)$
$f(a)$
M′
M
A
a
$a+h$

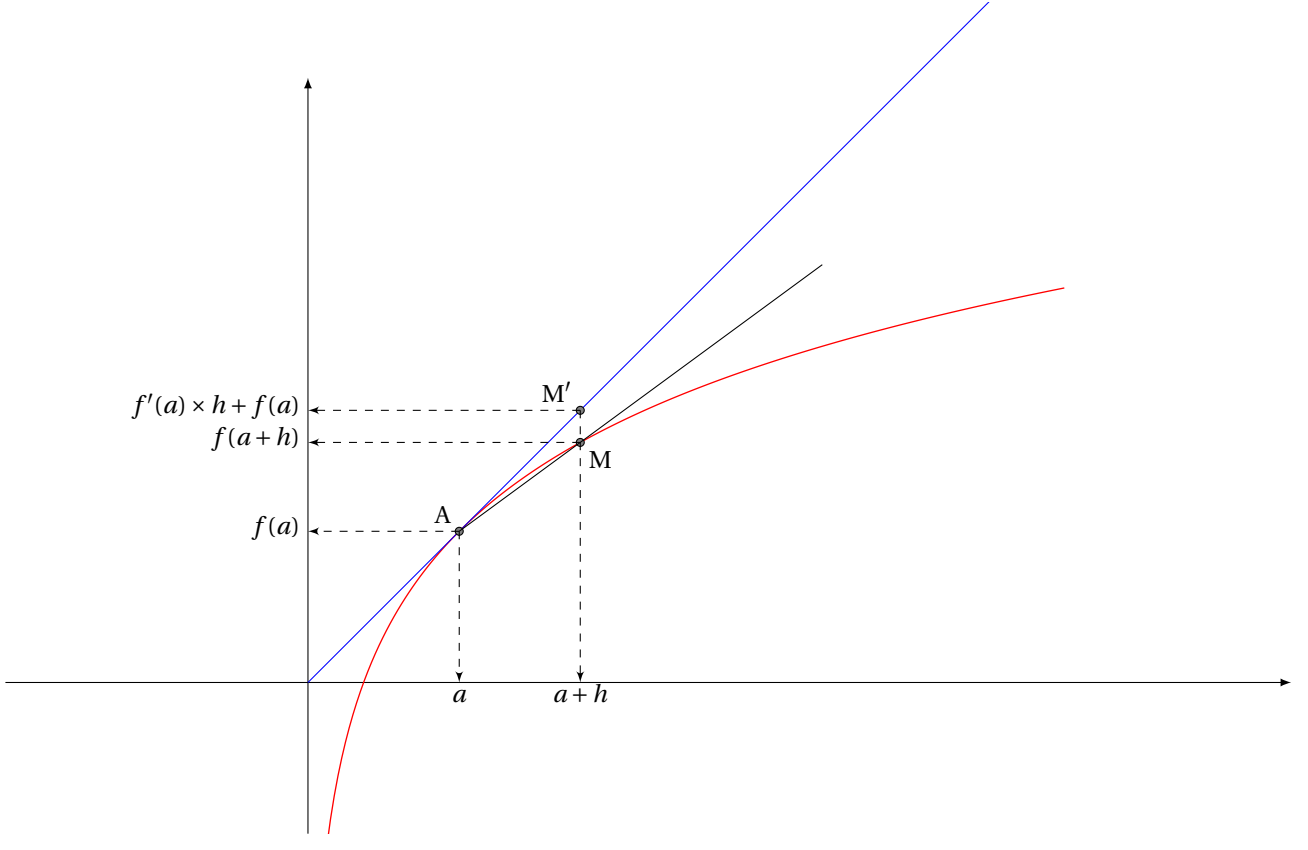

$f'(a) \times h + f(a)$
M′
$f(a+h)$
M
A
$f(a)$
a
$a+h$

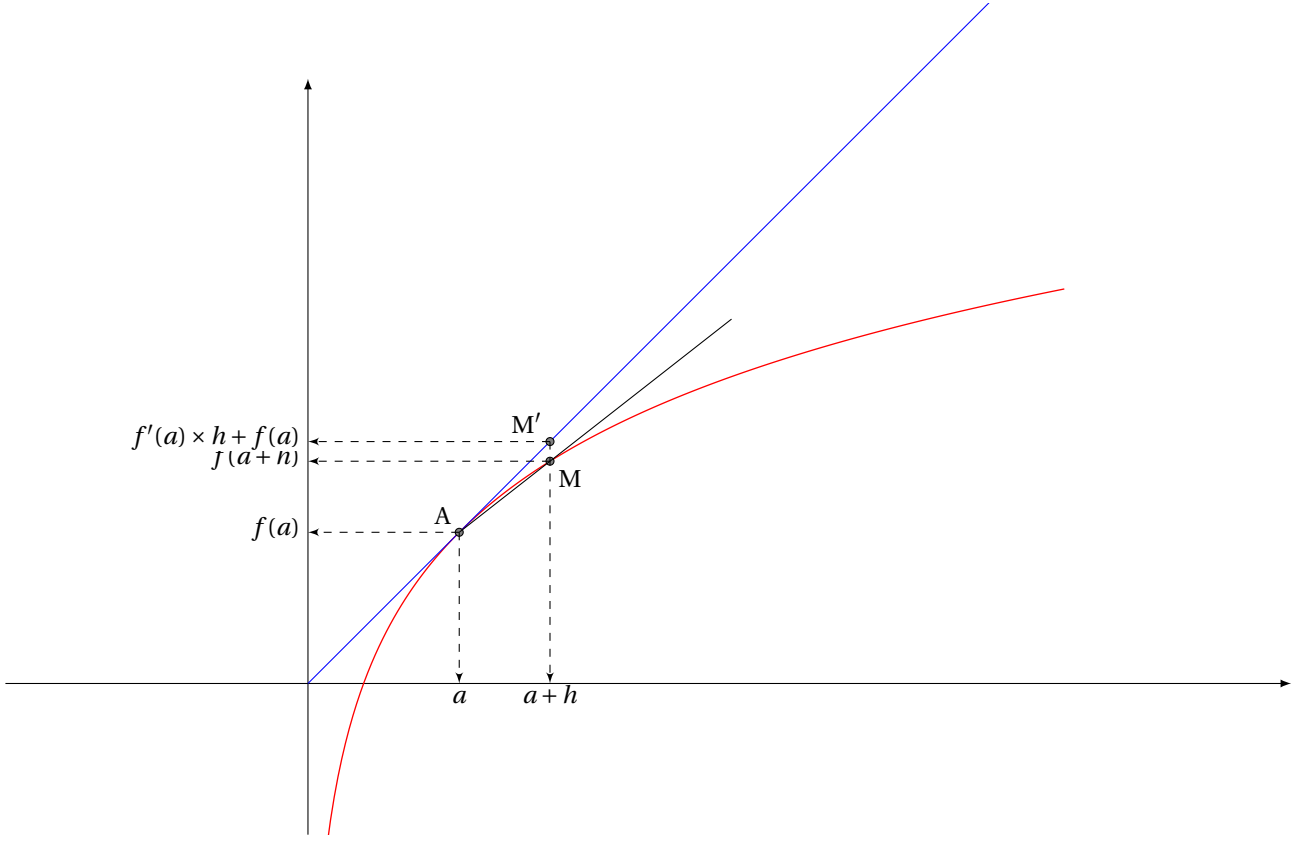

$f'(a) \times h + f(a)$
M′
$f(a+h)$
M
A
$f(a)$
a
$a+h$

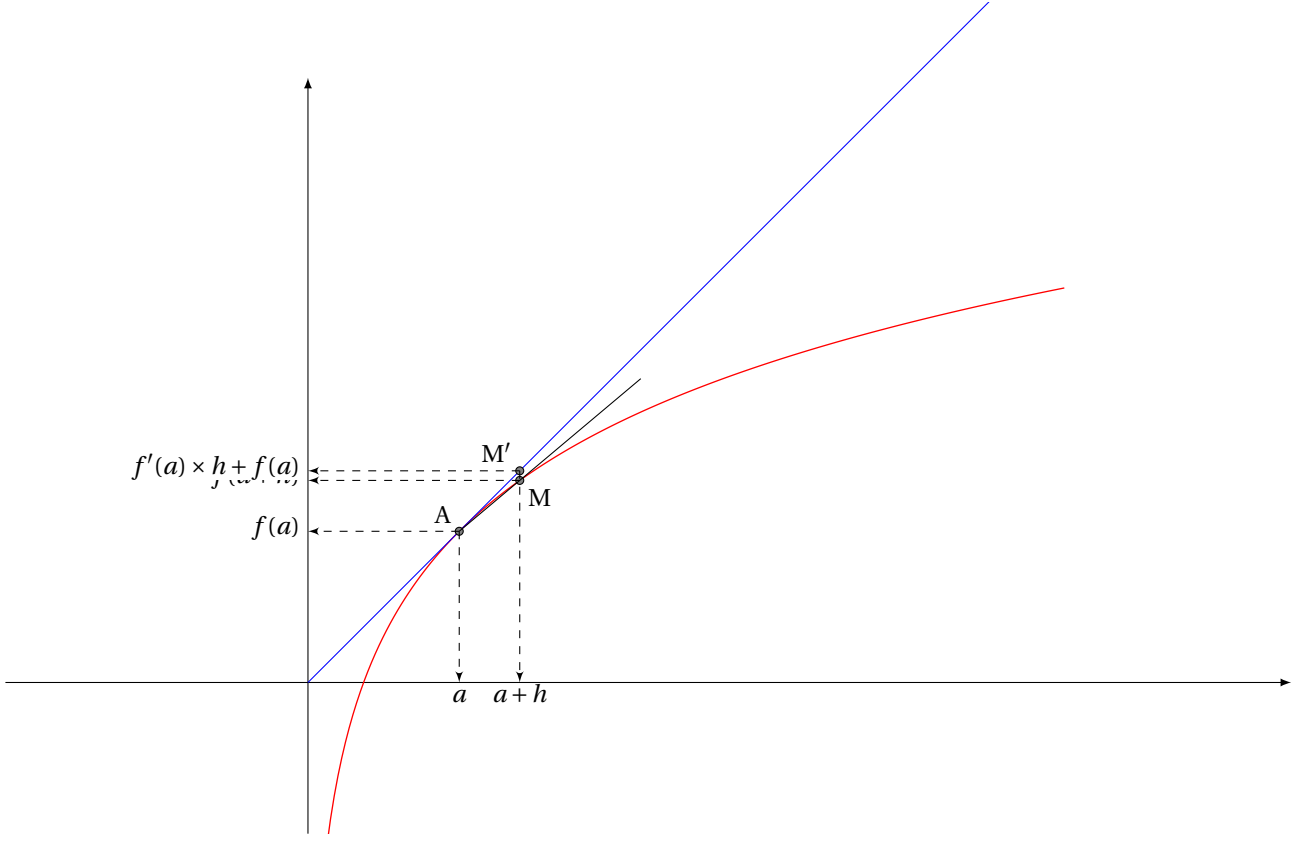

$f'(a) \times h + f(a)$
M′
$f(a+h)$
M
A
$f(a)$
a
$a+h$

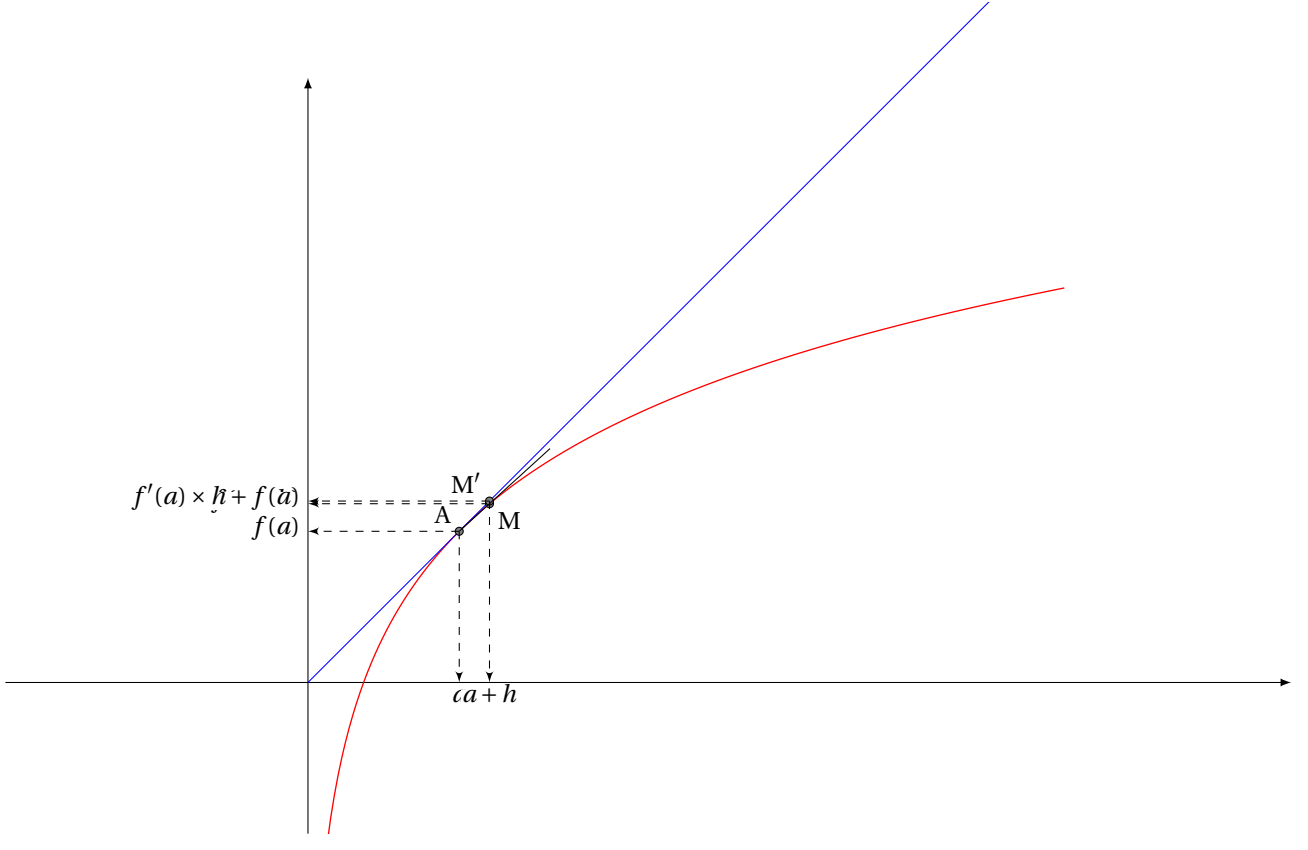

$f'(a) \times h + f(a)$
$f(a+h)$
$f(a)$
M′
M
A
a
$a+h$

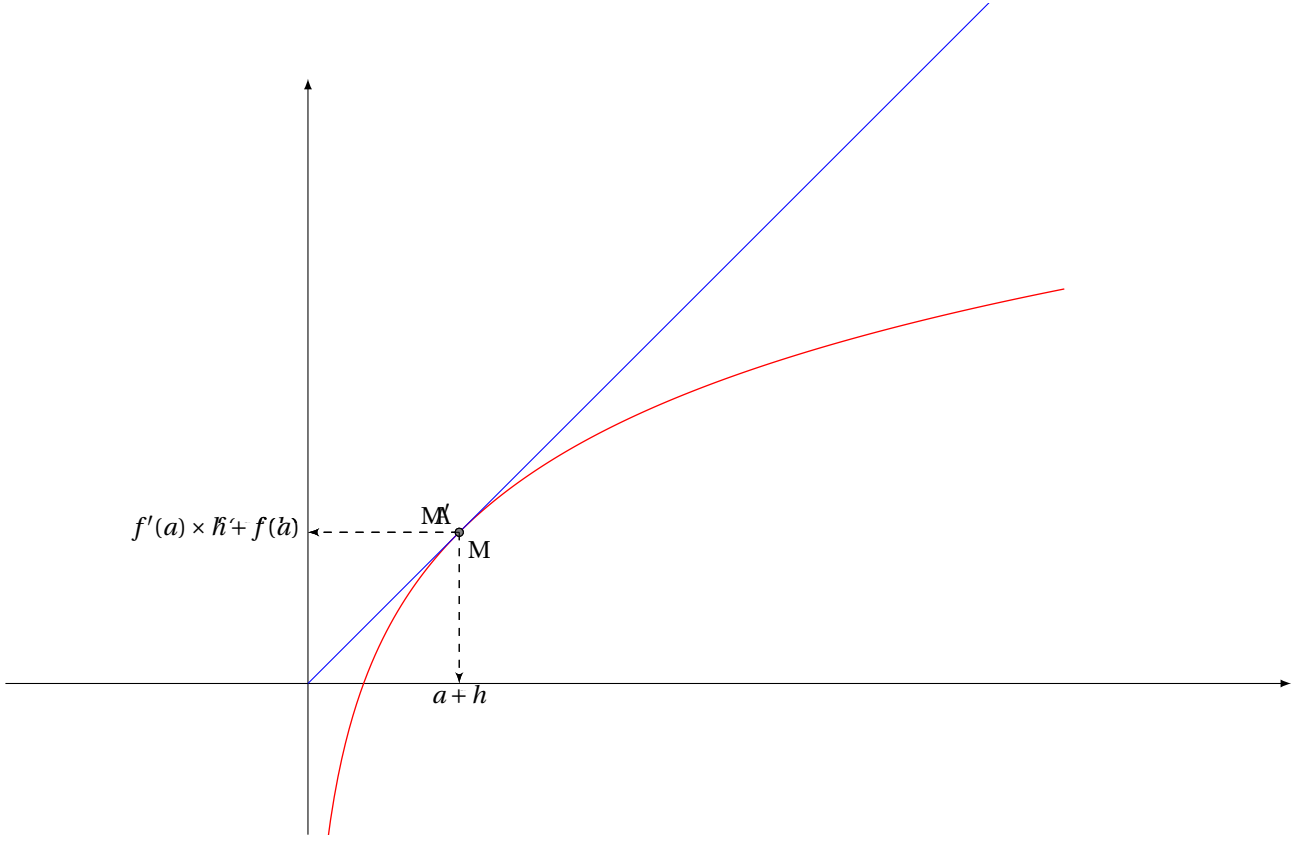

$f'(a) \times h + f(a)$
$f(a+h)$
$f(a)$
M′
M
A
a
$a+h$


$f'(a) \times h + f(a)$
$f(a+h)$
$f(a)$
M′
M
A
a
$a+h$

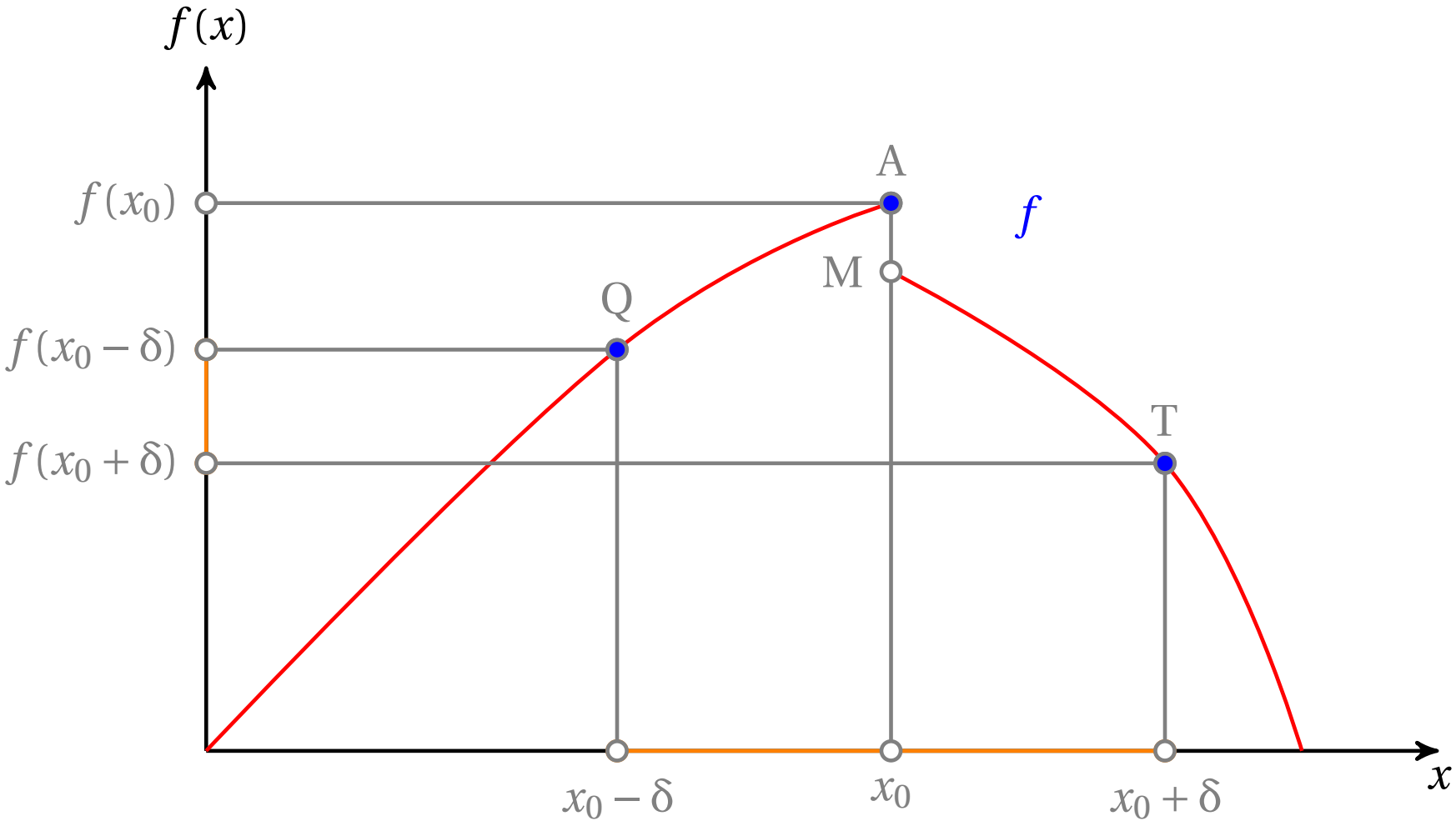

$f'(a) \times h + f(a)$
$f(a+h)$
$f(a)$
M′
M
A
a
$a+h$


$f'(a) \times h + f(a)$
M′
M
A
$f(a)$
a
$a+h$


$f'(a) \times h + f(a)$
$f(a)$
M′
A
M
a
$a + h$

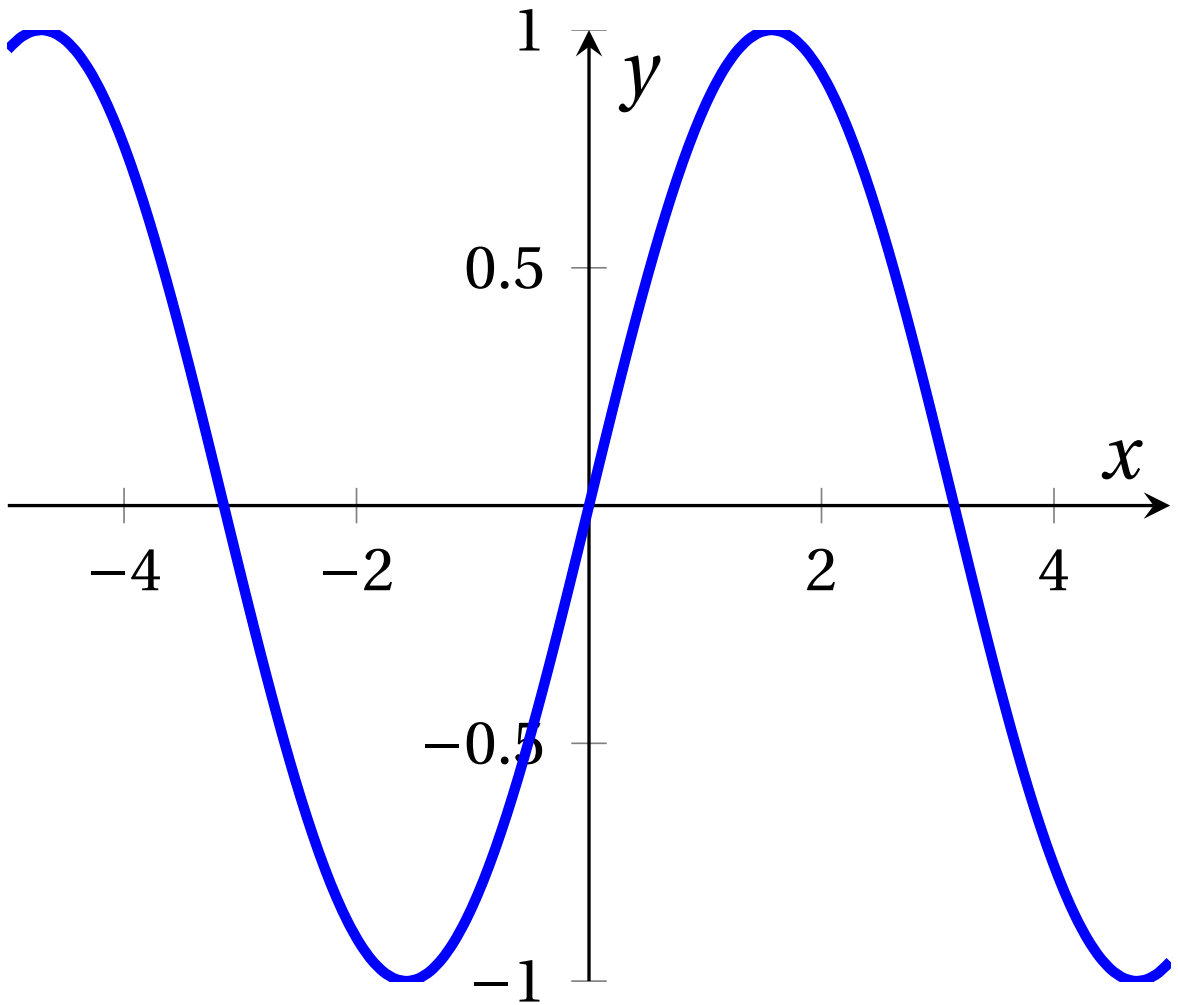

$f'(a) \times h + f(a)$
M'
M
$a + h$

47 ./images-tikz/image-21.tex

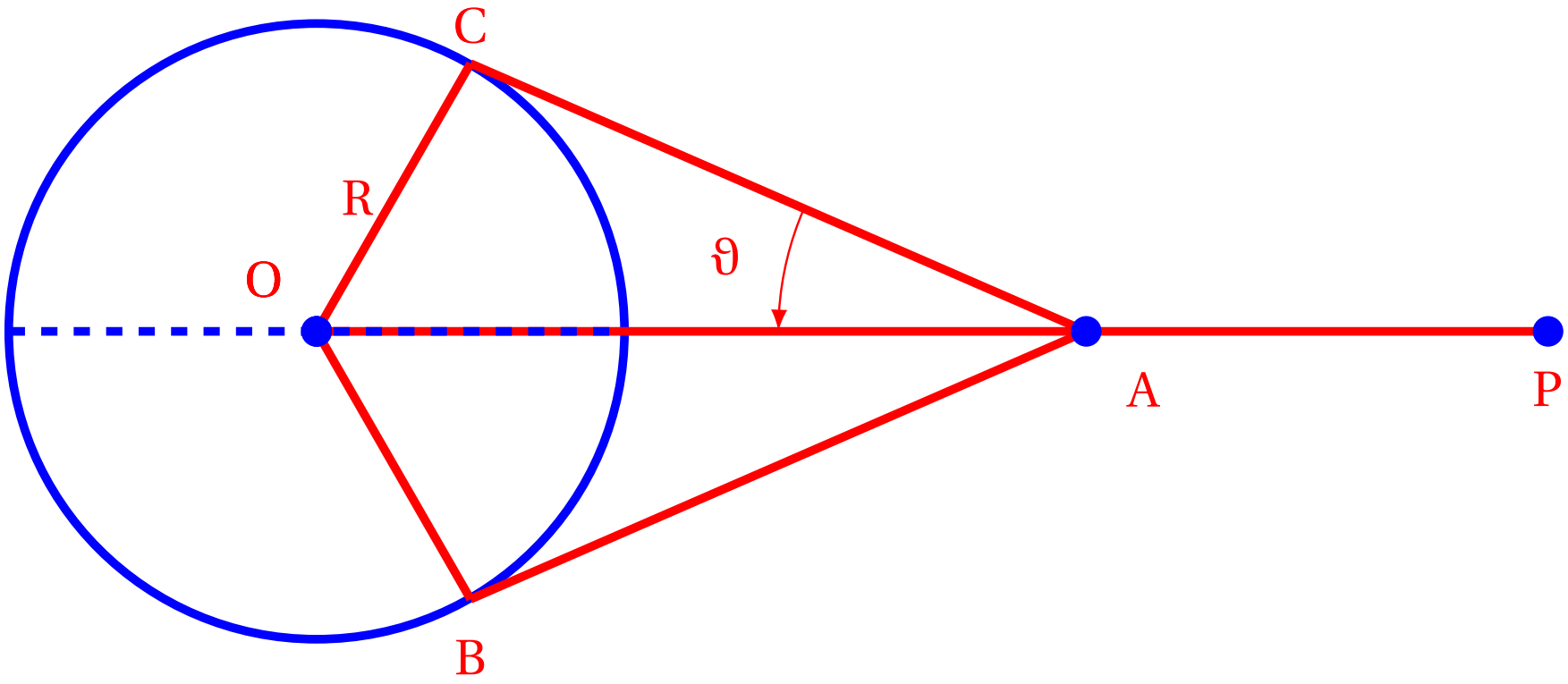

$f(x)$
$f(x_0)$
$f(x_0-\delta)$
$f(x_0+\delta)$
A
M
Q
T
f
$x_0-\delta$
x_0
$x_0+\delta$
x

48 ./images-tikz/image-13.tex

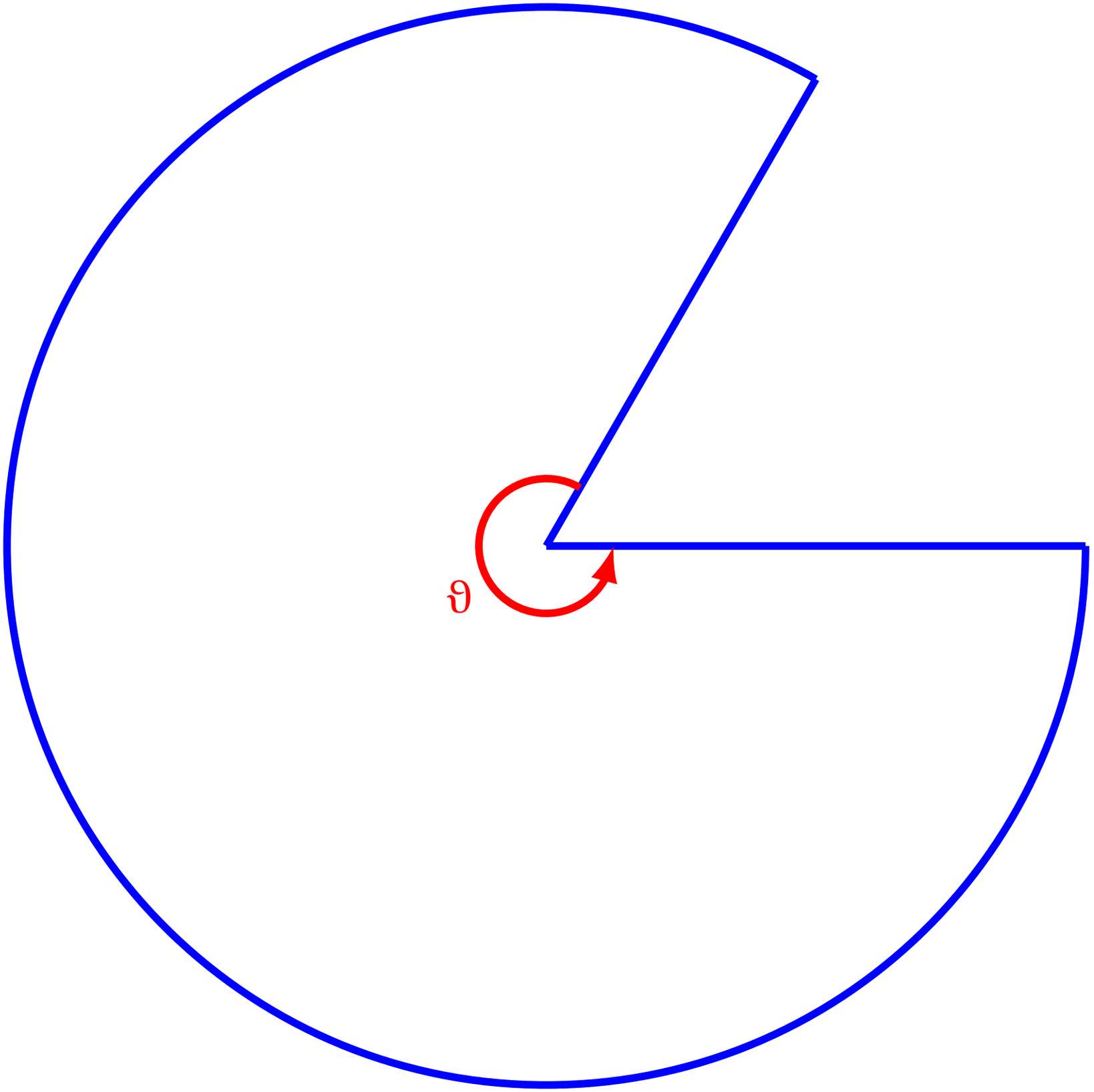
Trimmed bounding boxes



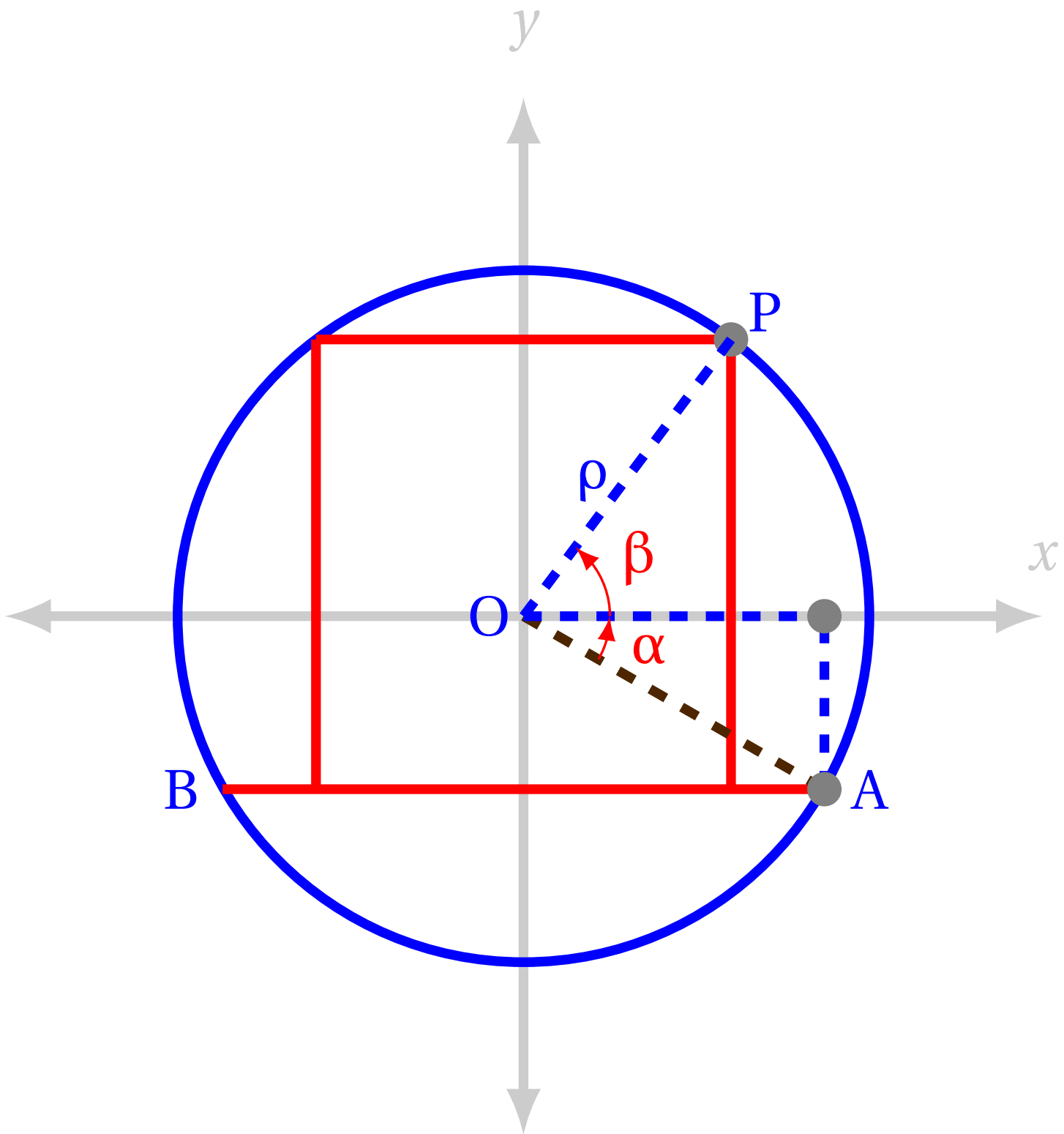
49 ./images-tikz/resuelto6-15.tex


C
R
O
ϑ
A
P
B

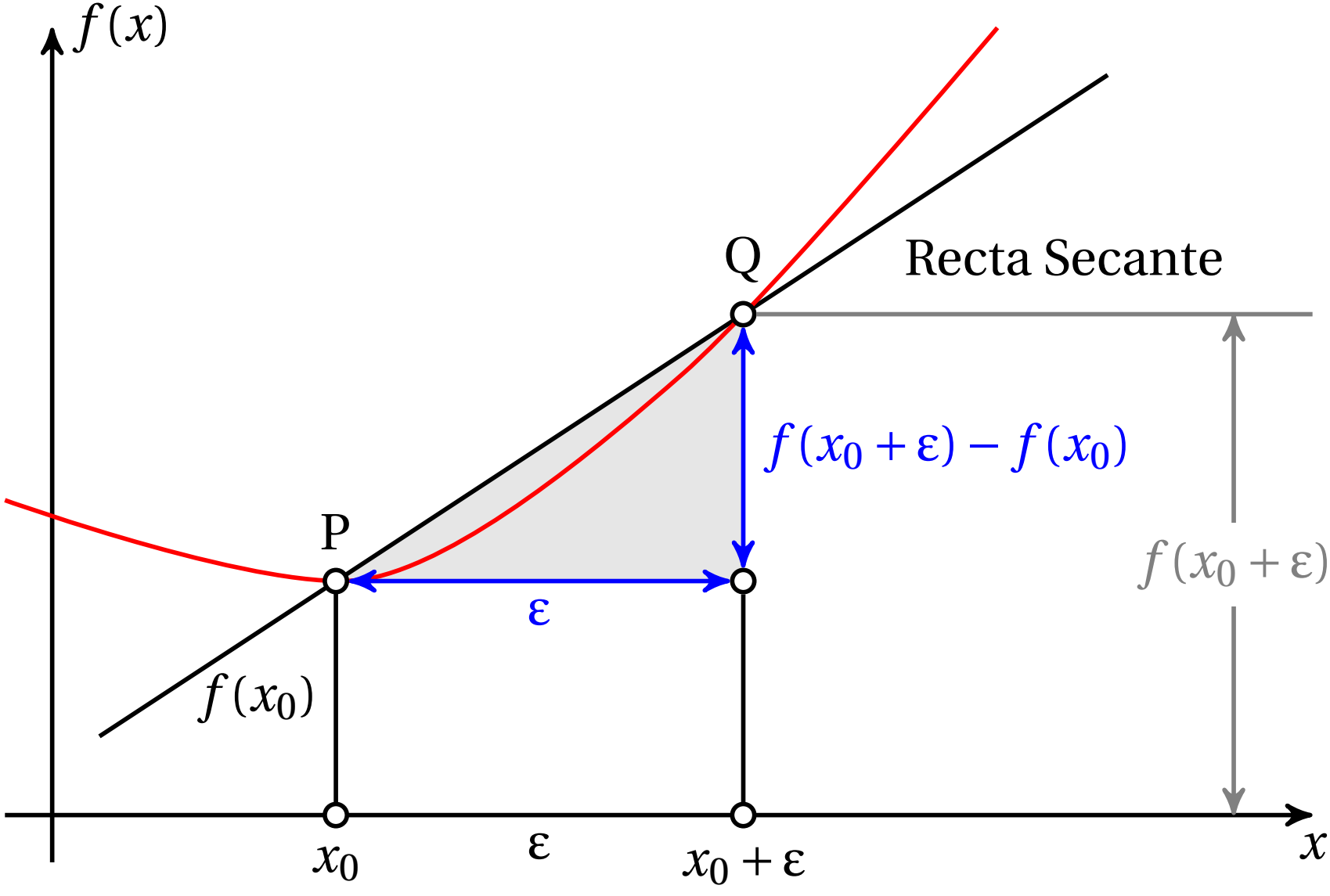
50 ./images-tikz/resuelto6-10.tex


ϑ

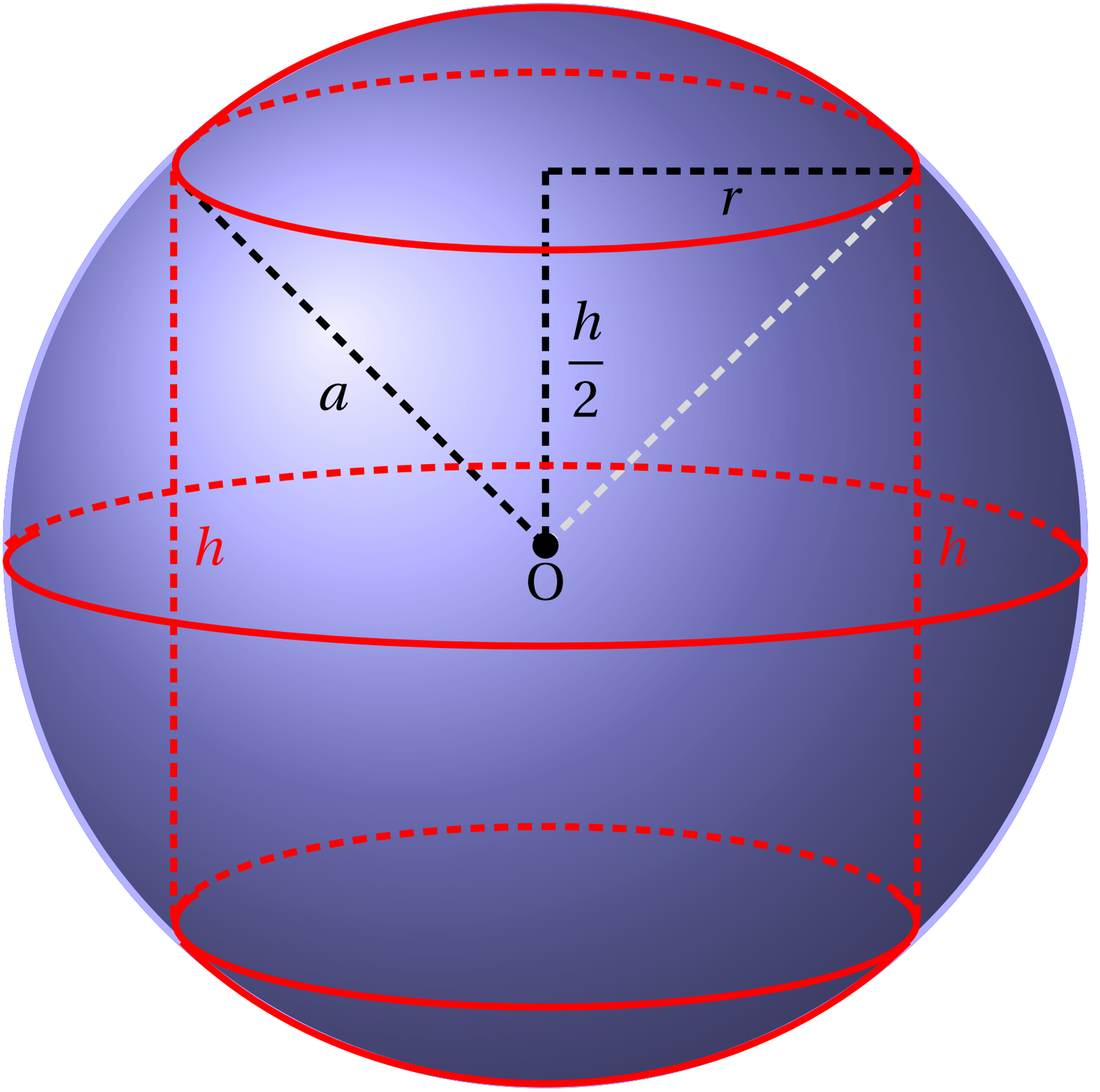
51 ./images-tikz/resuelto6-3.tex


y
x
P
ρ
β
O
α
B
A

52 ./images-tikz/image-15.tex

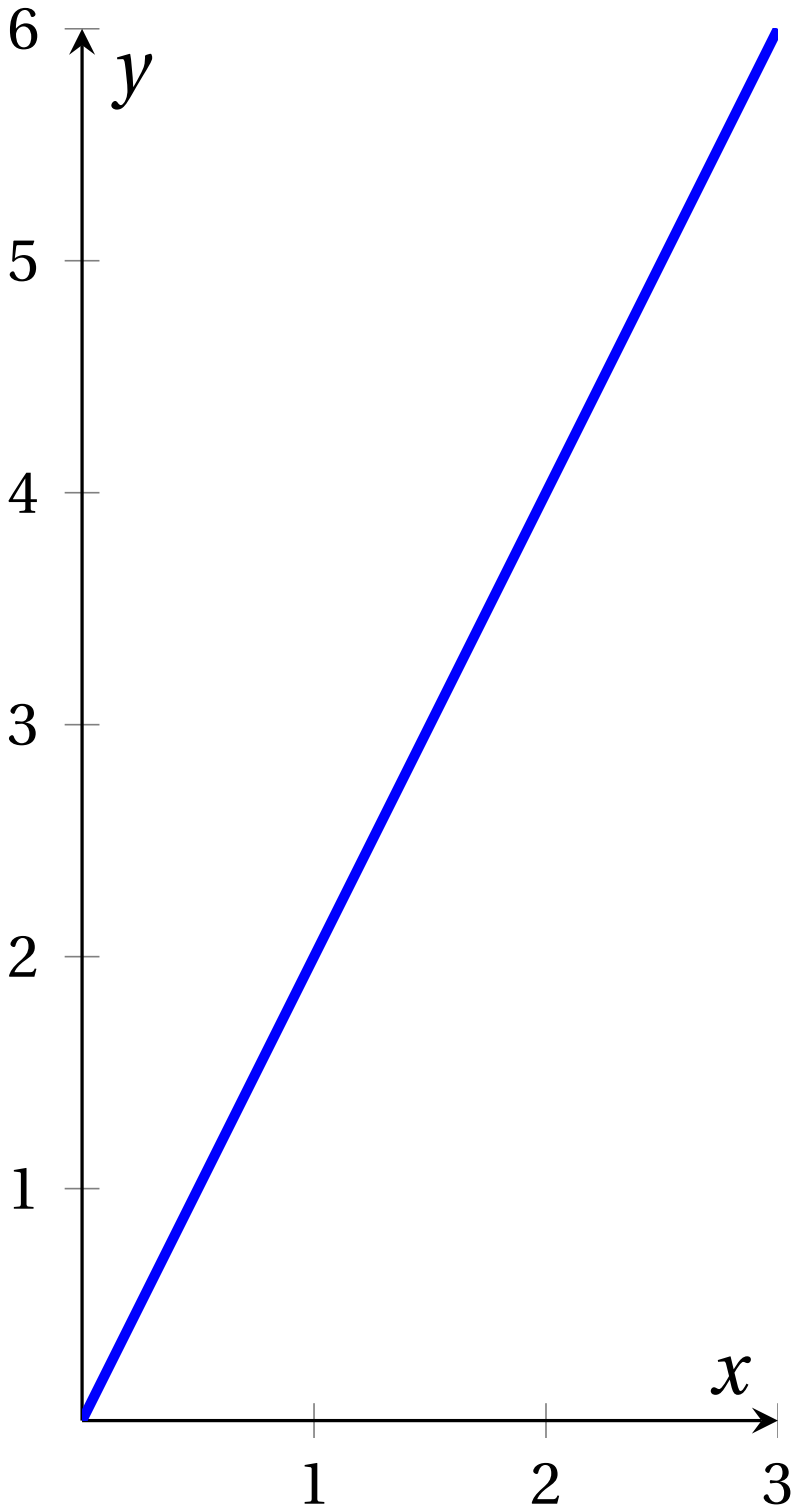

$f(x)$
Q
Recta Secante
$f(x_0+\varepsilon)-f(x_0)$
P
$f(x_0+\varepsilon)$
ε
$f(x_0)$
x_0
ε
$x_0+\varepsilon$
x

53 ./images-tikz/resuelto6-6.tex

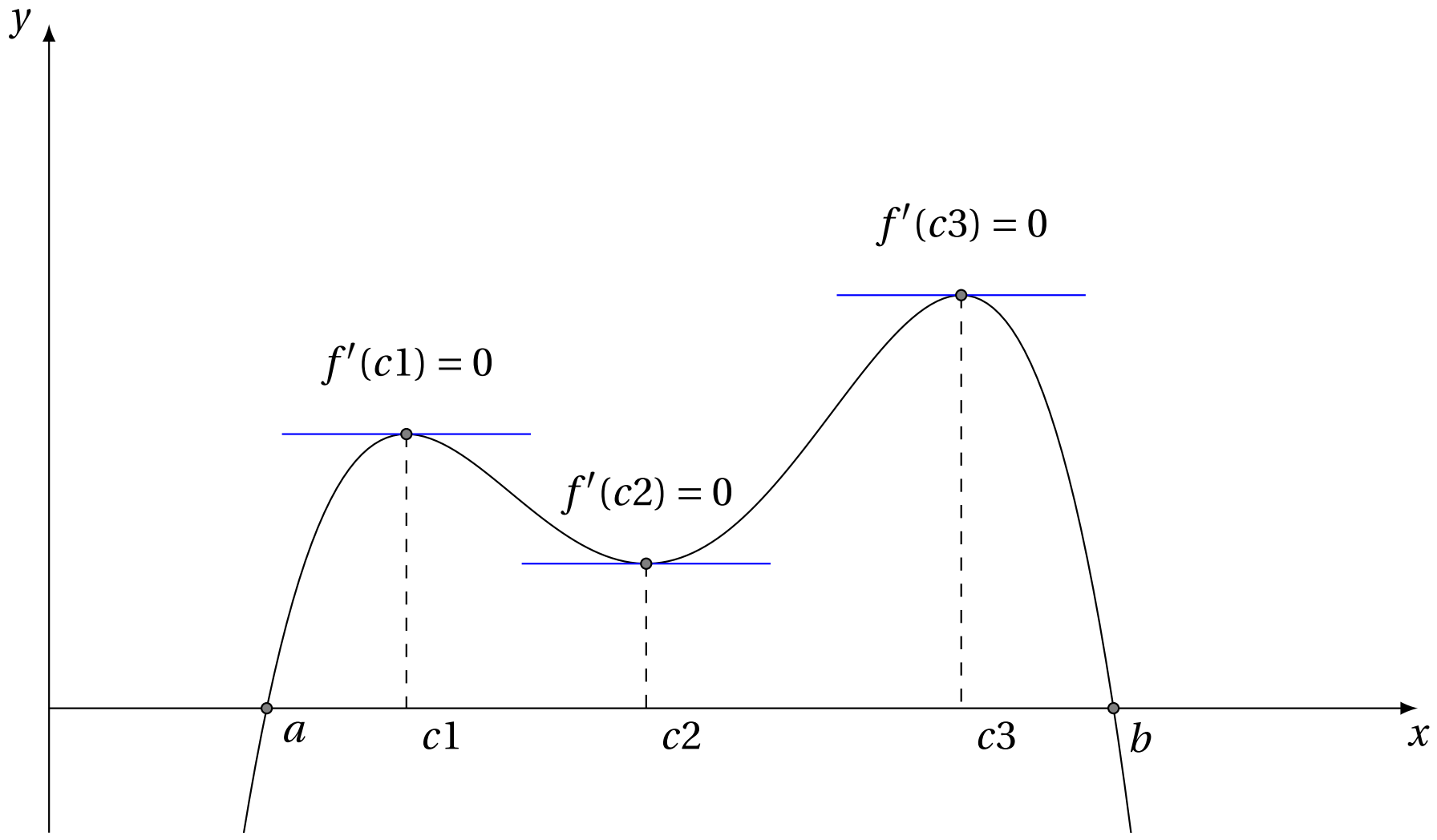

r
$\frac{h}{2}$
a
h
h
O

54 ./images-tikz/image-31.tex

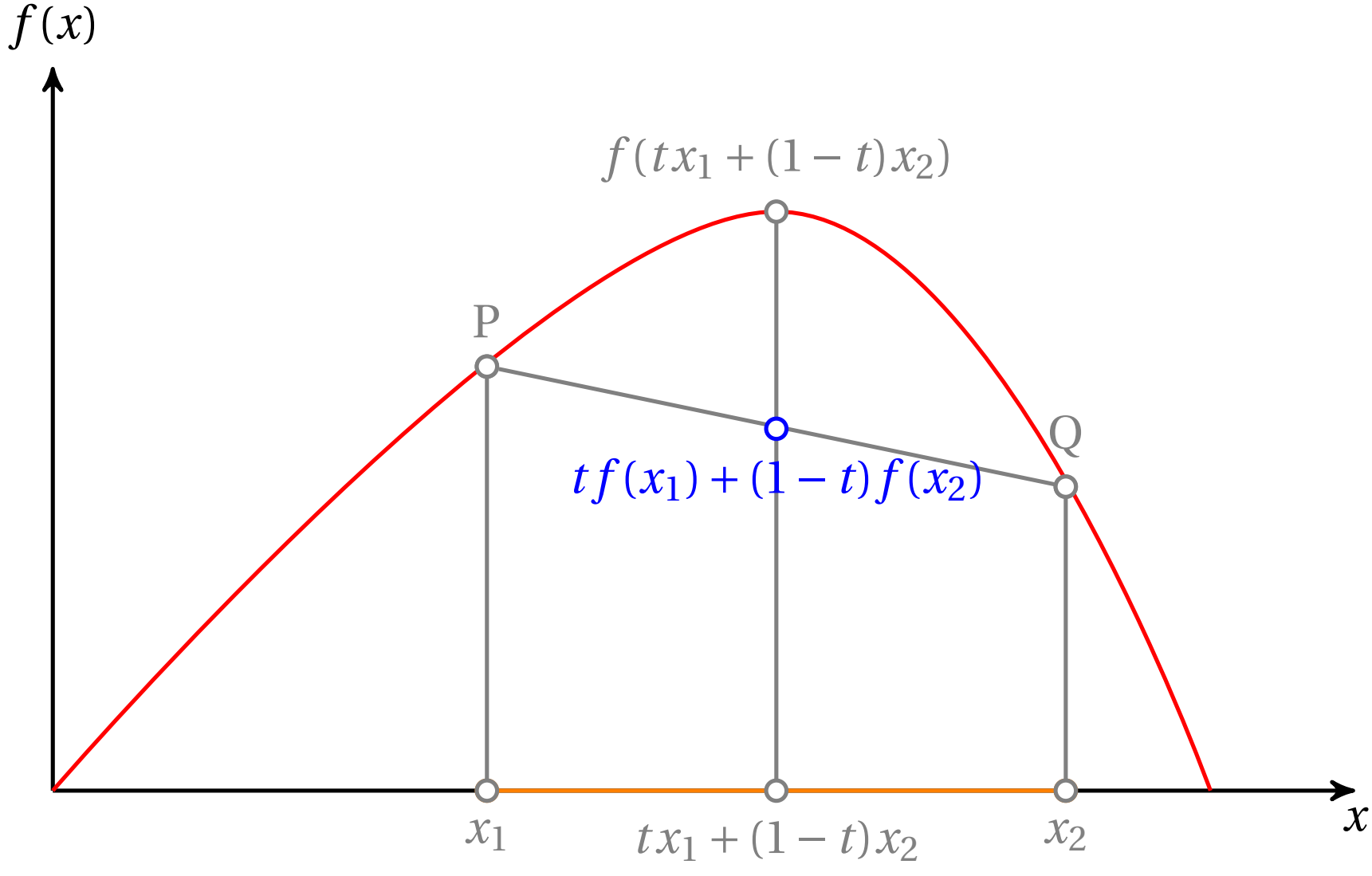
Trimmed bounding boxes



55 ./images-tikz/derivate5.tex

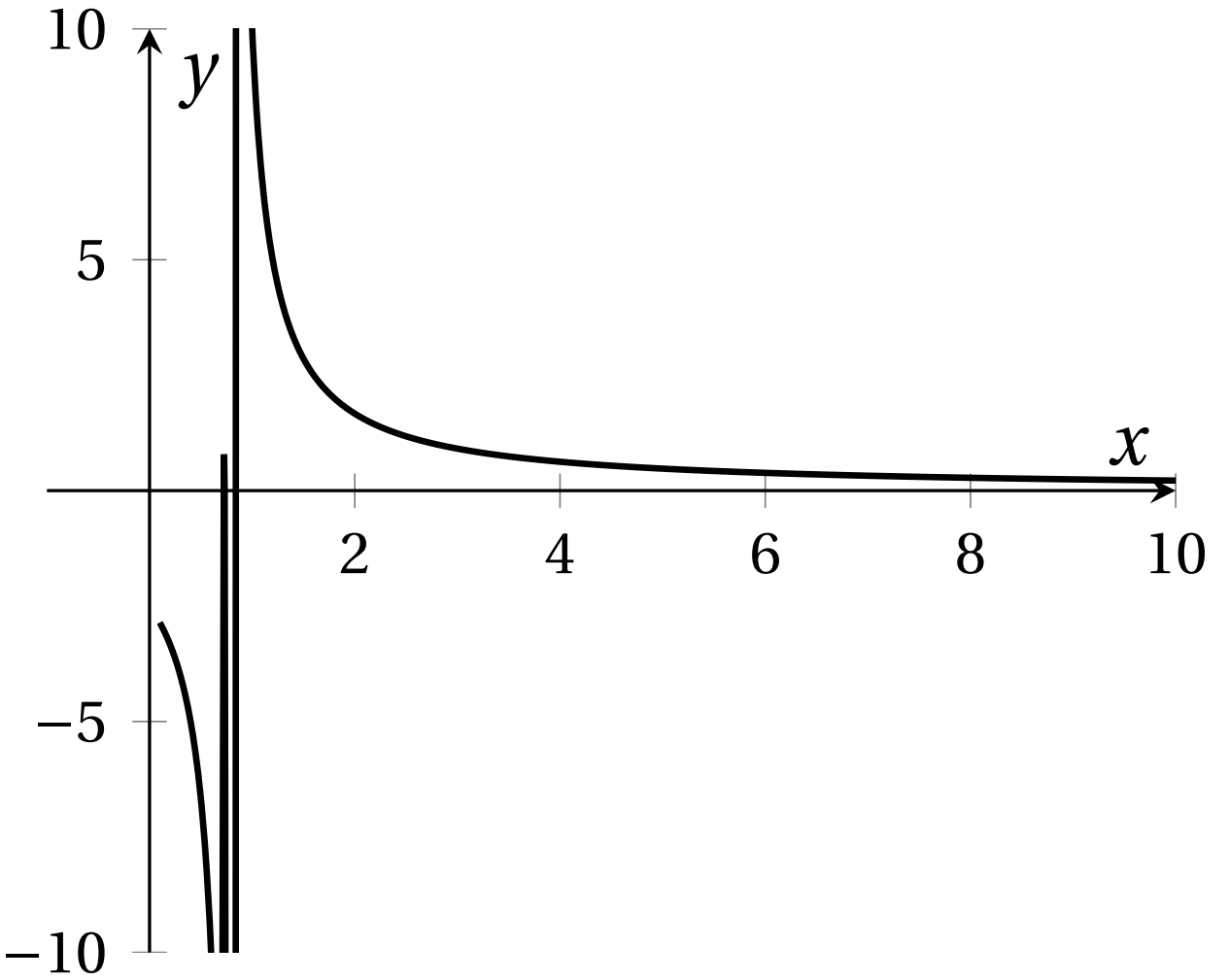

y
f′(c3) = 0
f′(c1) = 0
f′(c2) = 0
a
c1
c2
c3
b
x

56 ./images-tikz/concavidad-abj.tex


$f(x)$
$f(tx_1 + (1-t)x_2)$
P
Q
$tf(x_1) + (1-t)f(x_2)$
x_1
$tx_1 + (1-t)x_2$
x_2
x

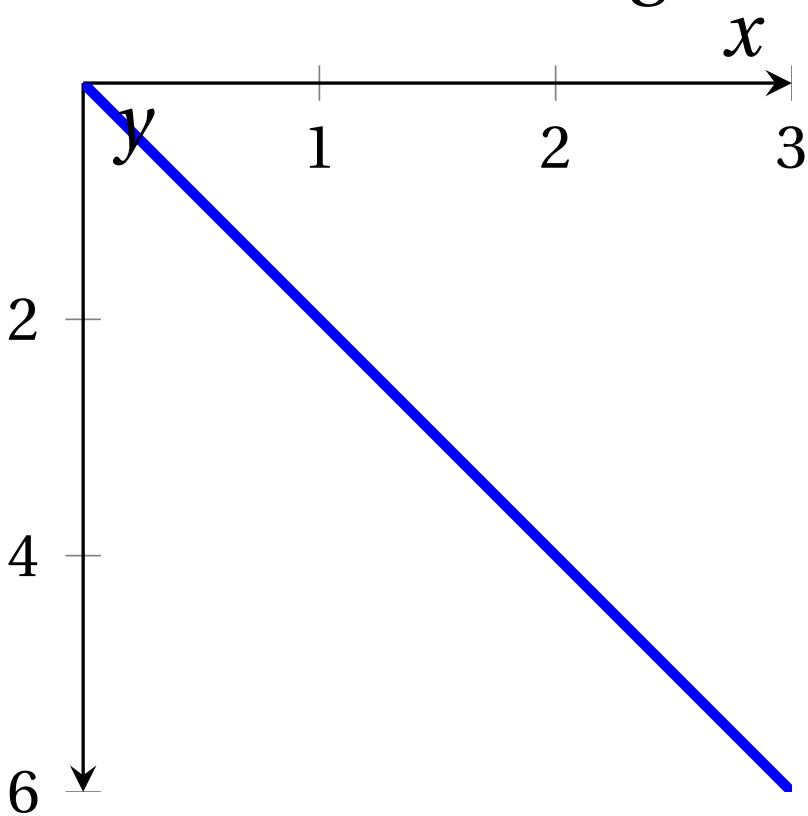
57 ./images-tikz/image-41.tex

Trimmed bounding boxes

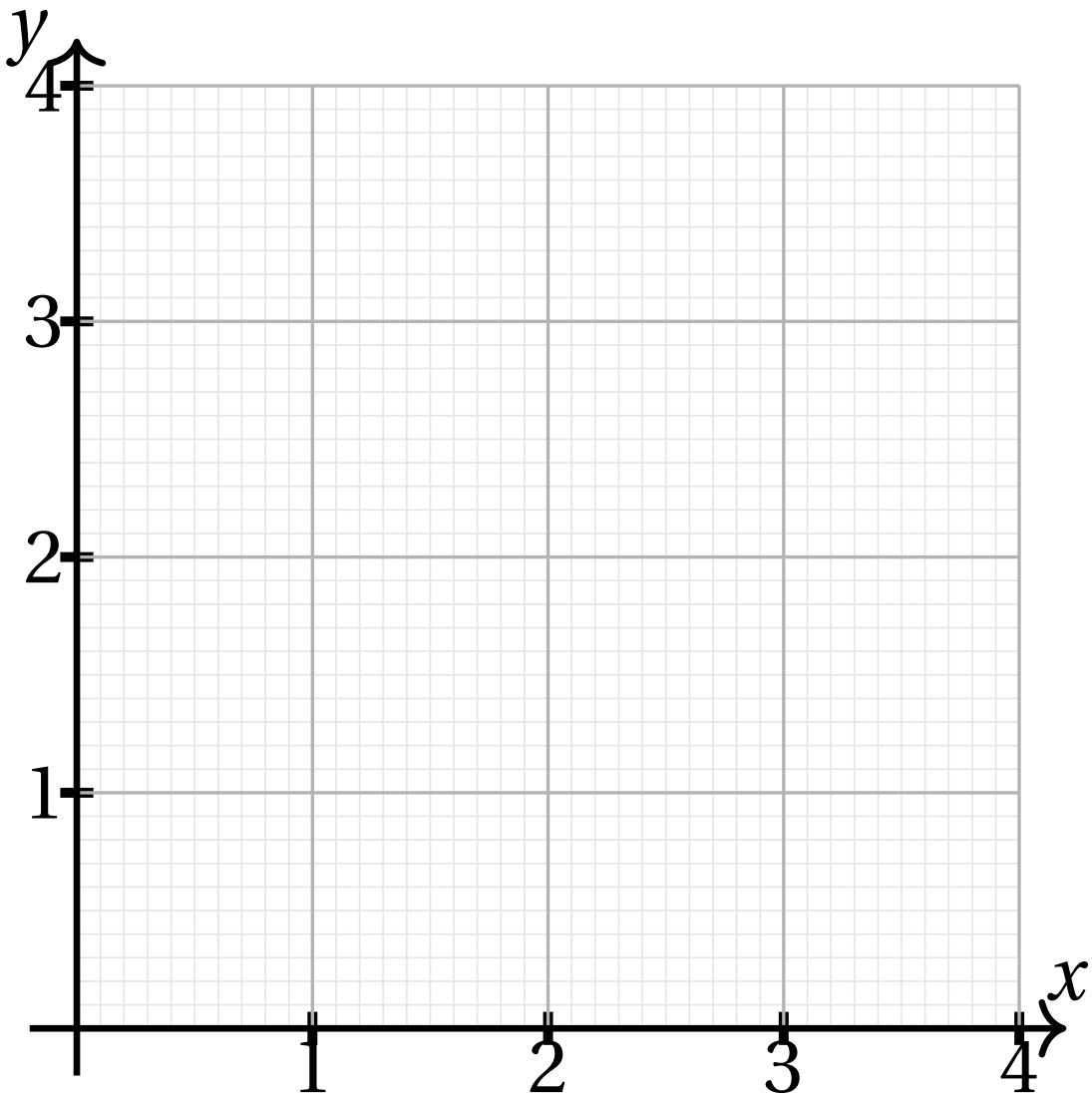


58 ./images-tikz/image-32.tex

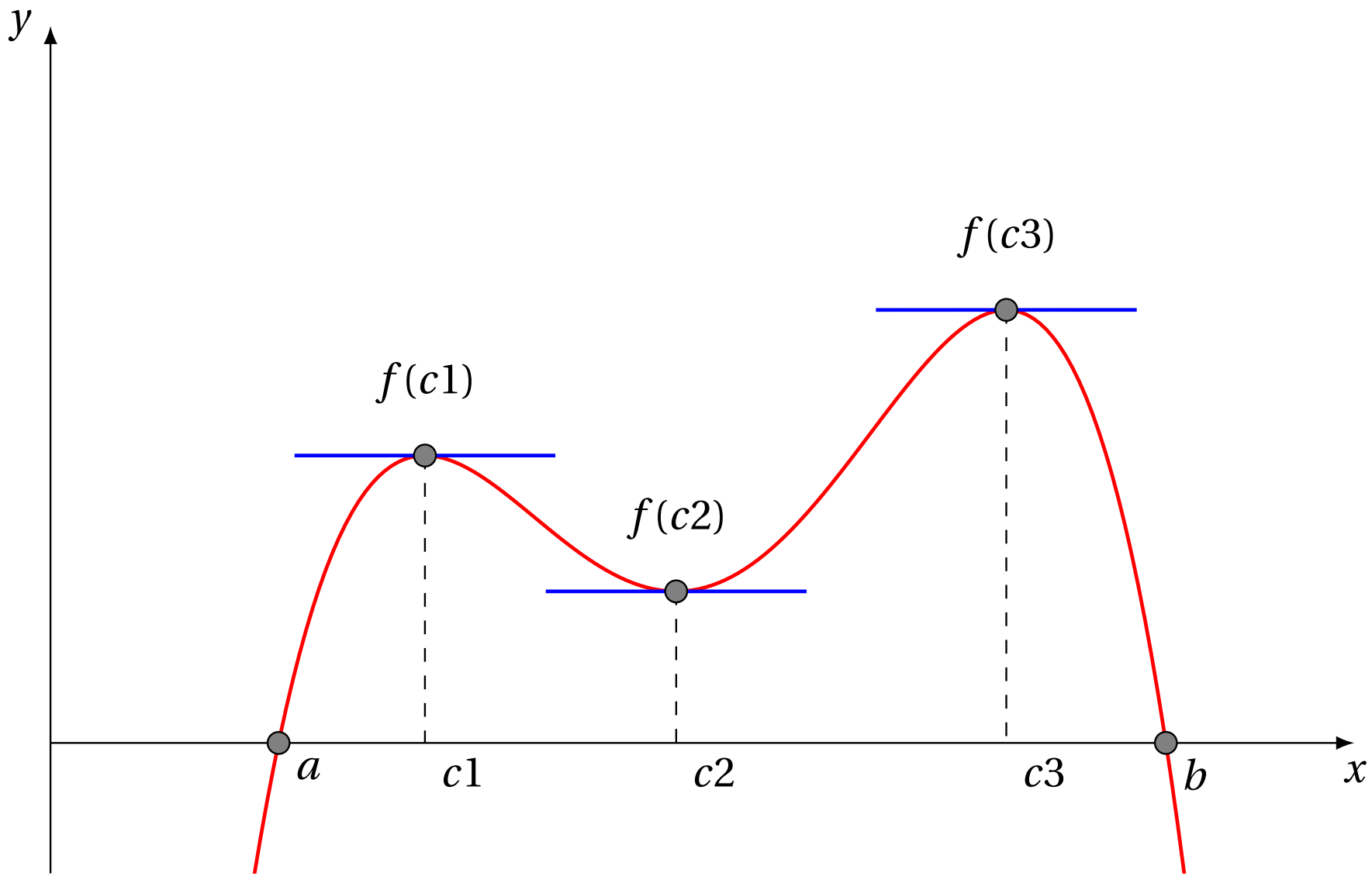
Trimmed bounding boxes



59 ./images-tikz/image-7.tex

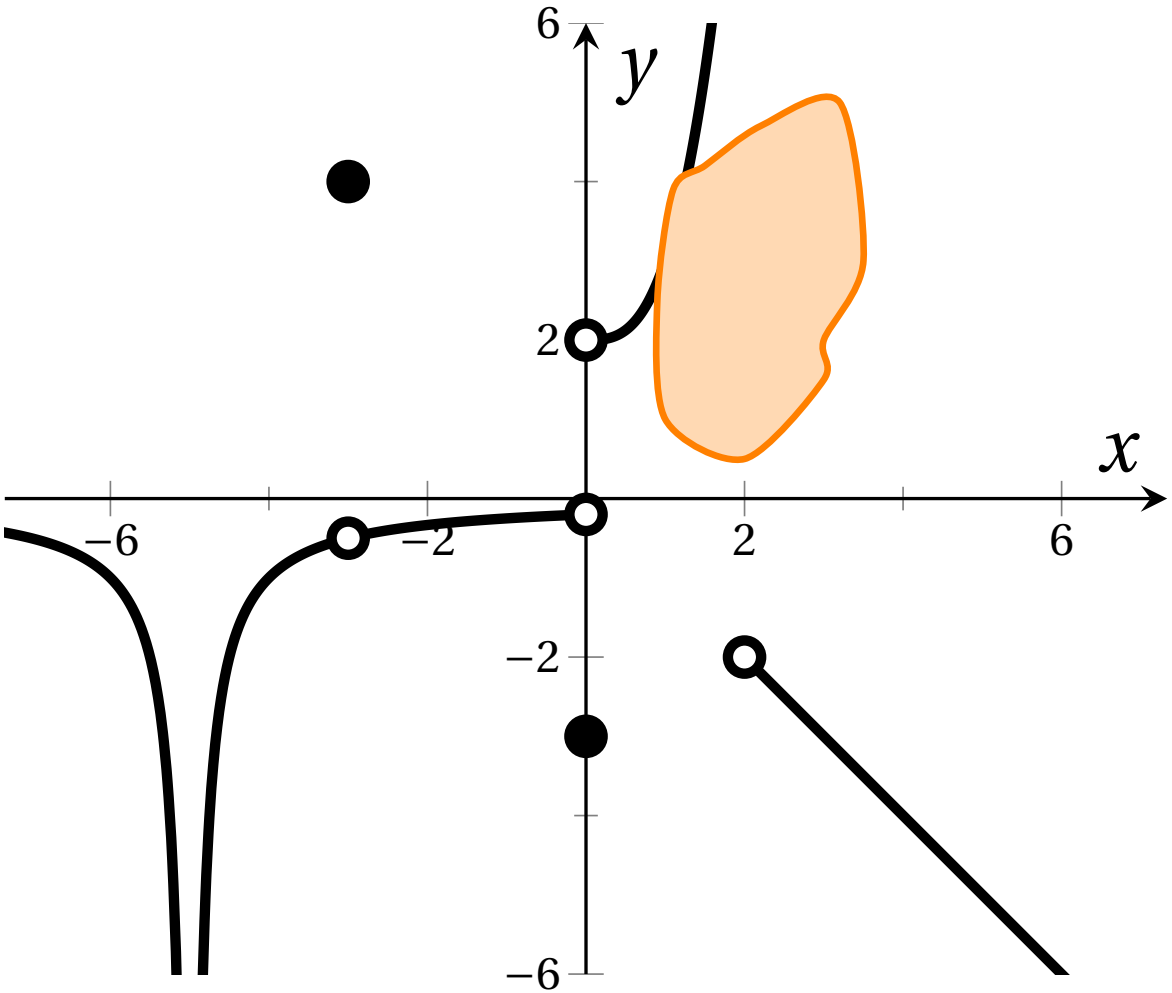

y
4
3
2
1
x
1
2
3
4

60 ./images-tikz/image-47.tex

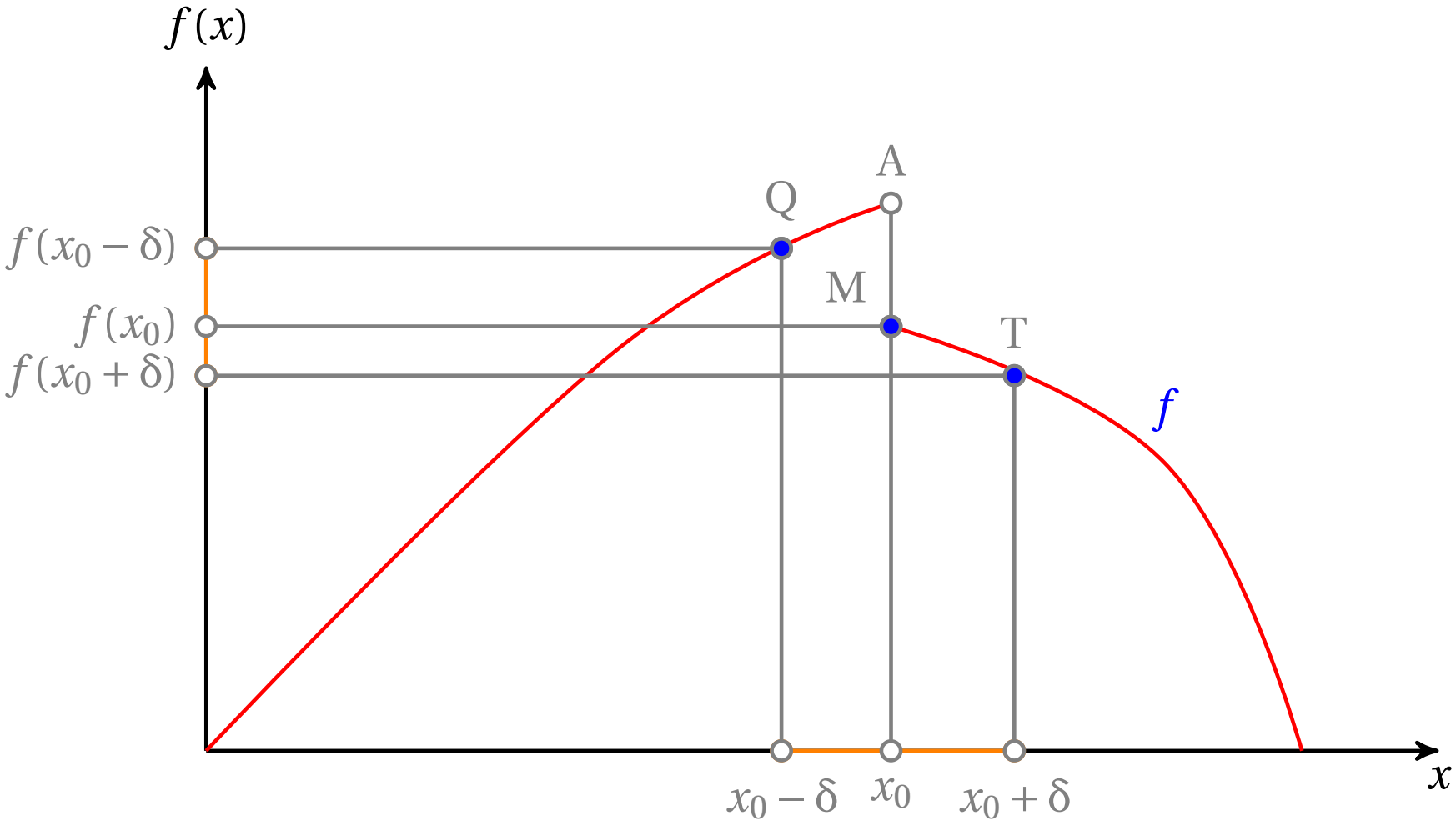

y
f(c3)
f(c1)
f(c2)
a
c1
c2
c3
b
x

61 ./images-tikz/image-14.tex

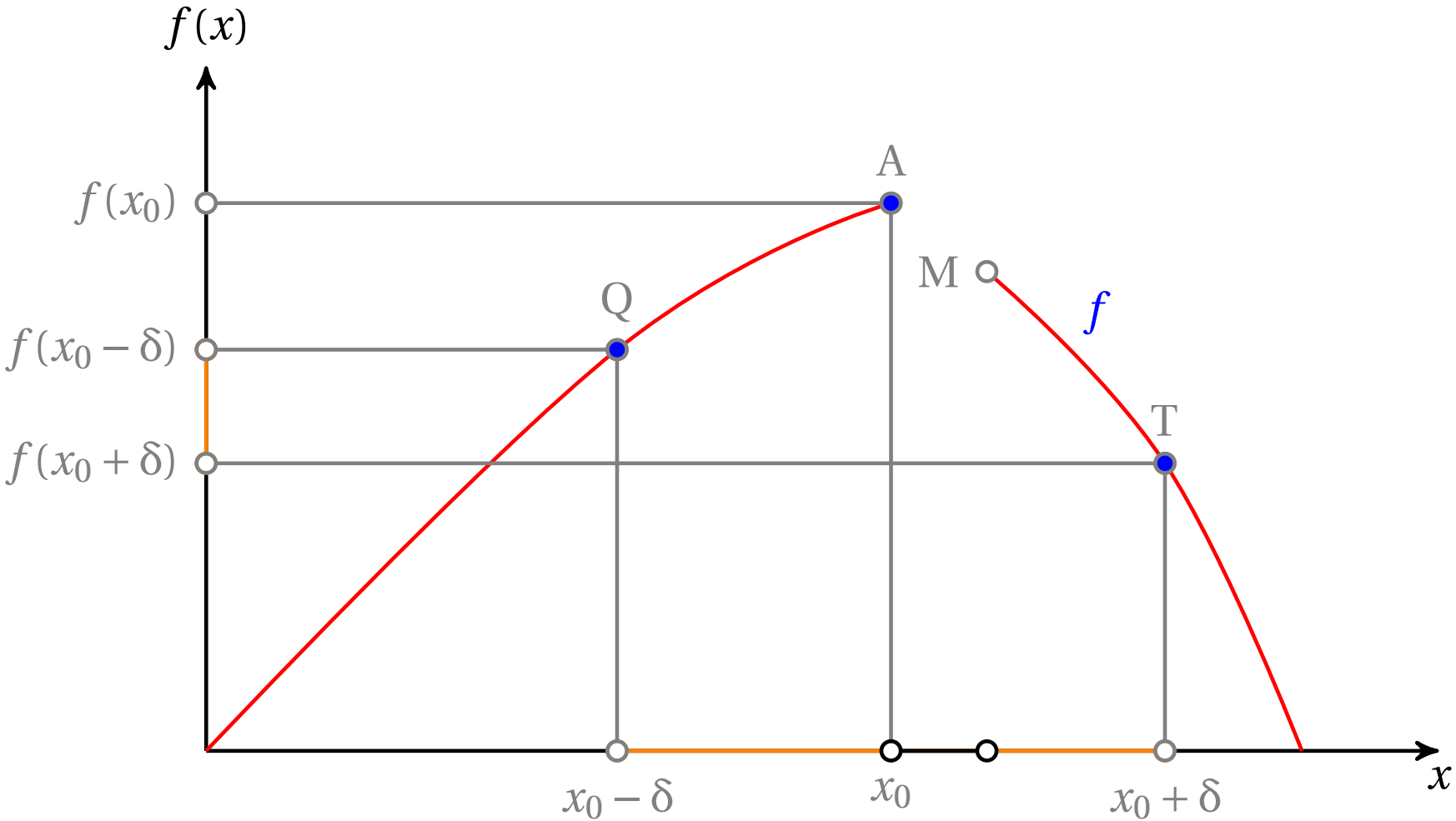
Trimmed bounding boxes



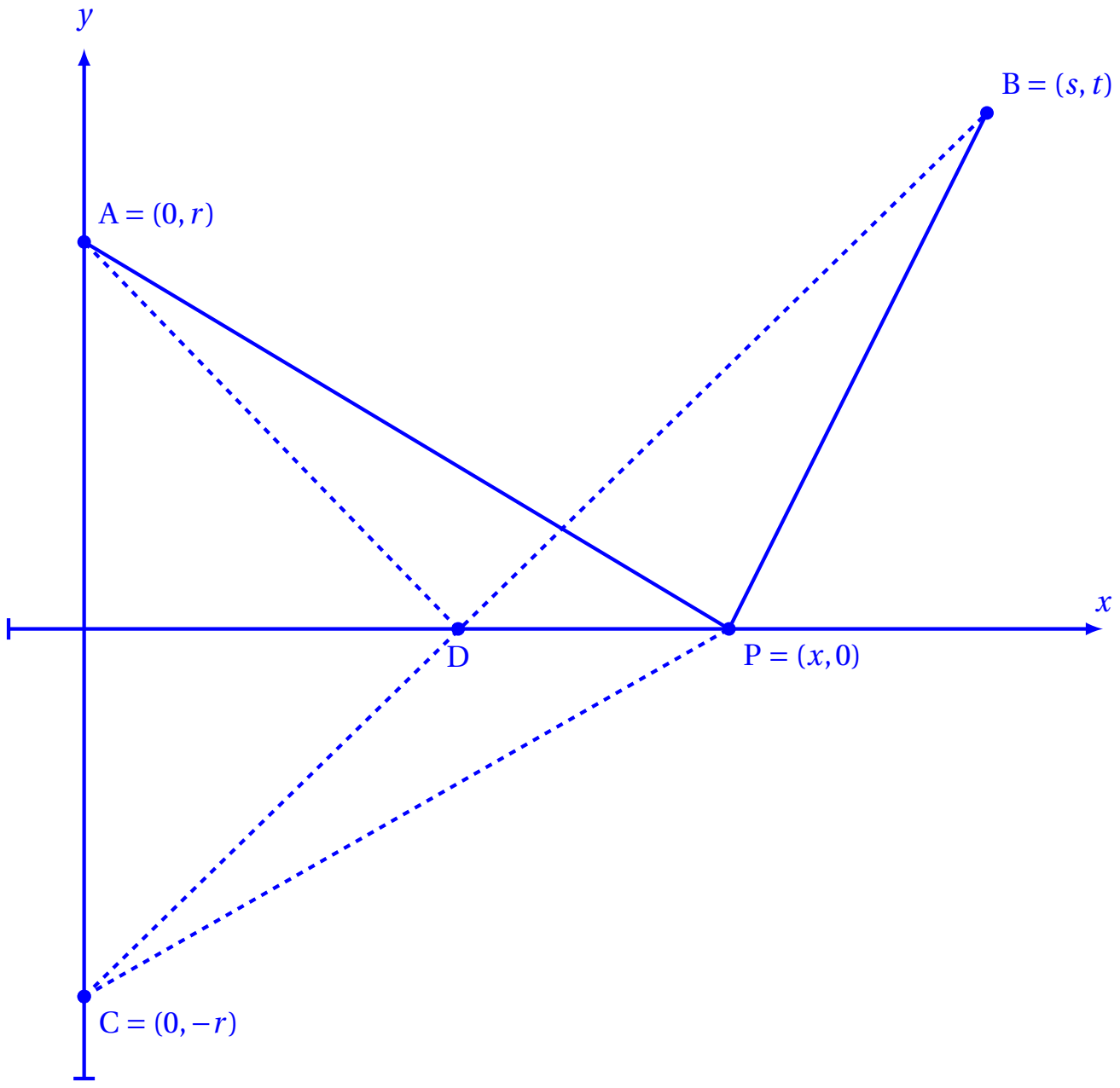
62 ./images-tikz/image-23.tex


$f(x)$
A
Q
M
T
f
$f(x_0-\delta)$
$f(x_0)$
$f(x_0+\delta)$
$x_0-\delta$
x_0
$x_0+\delta$
x

63 ./images-tikz/image-22.tex

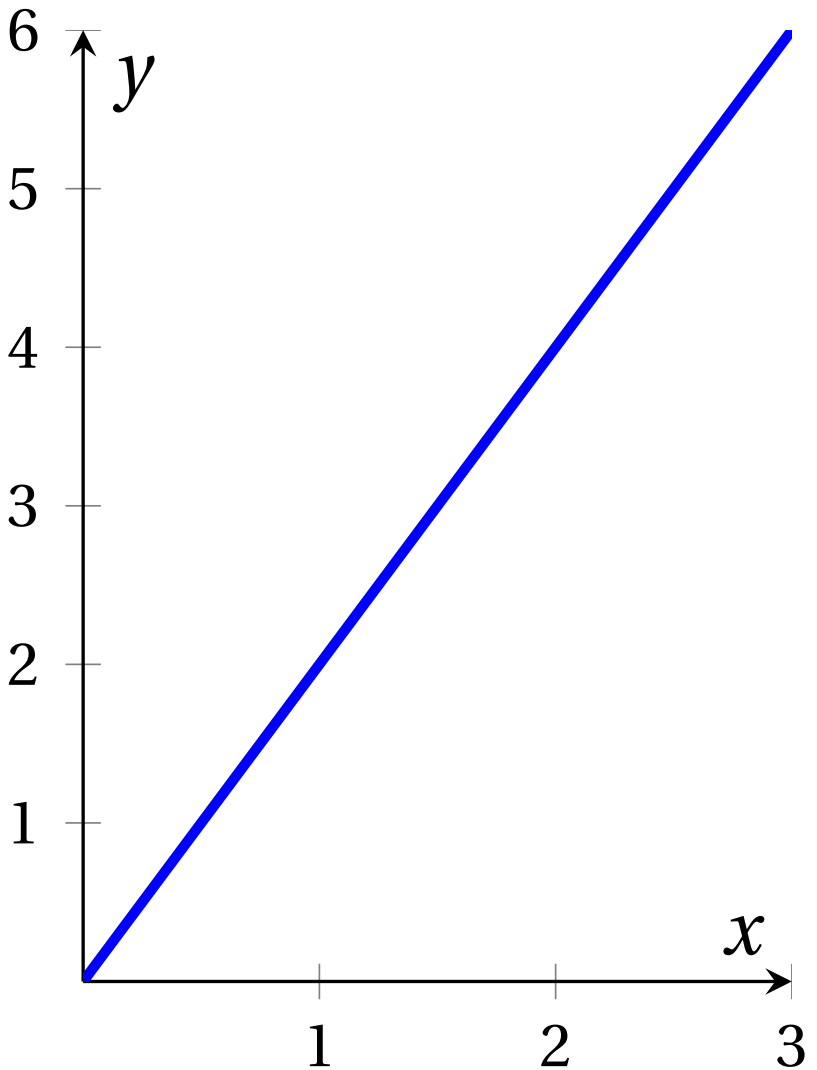

$f(x)$
$f(x_0)$
$f(x_0-\delta)$
$f(x_0+\delta)$
A
Q
M
f
T
x
$x_0-\delta$
x_0
$x_0+\delta$

64 ./images-tikz/resuelto6-12.tex

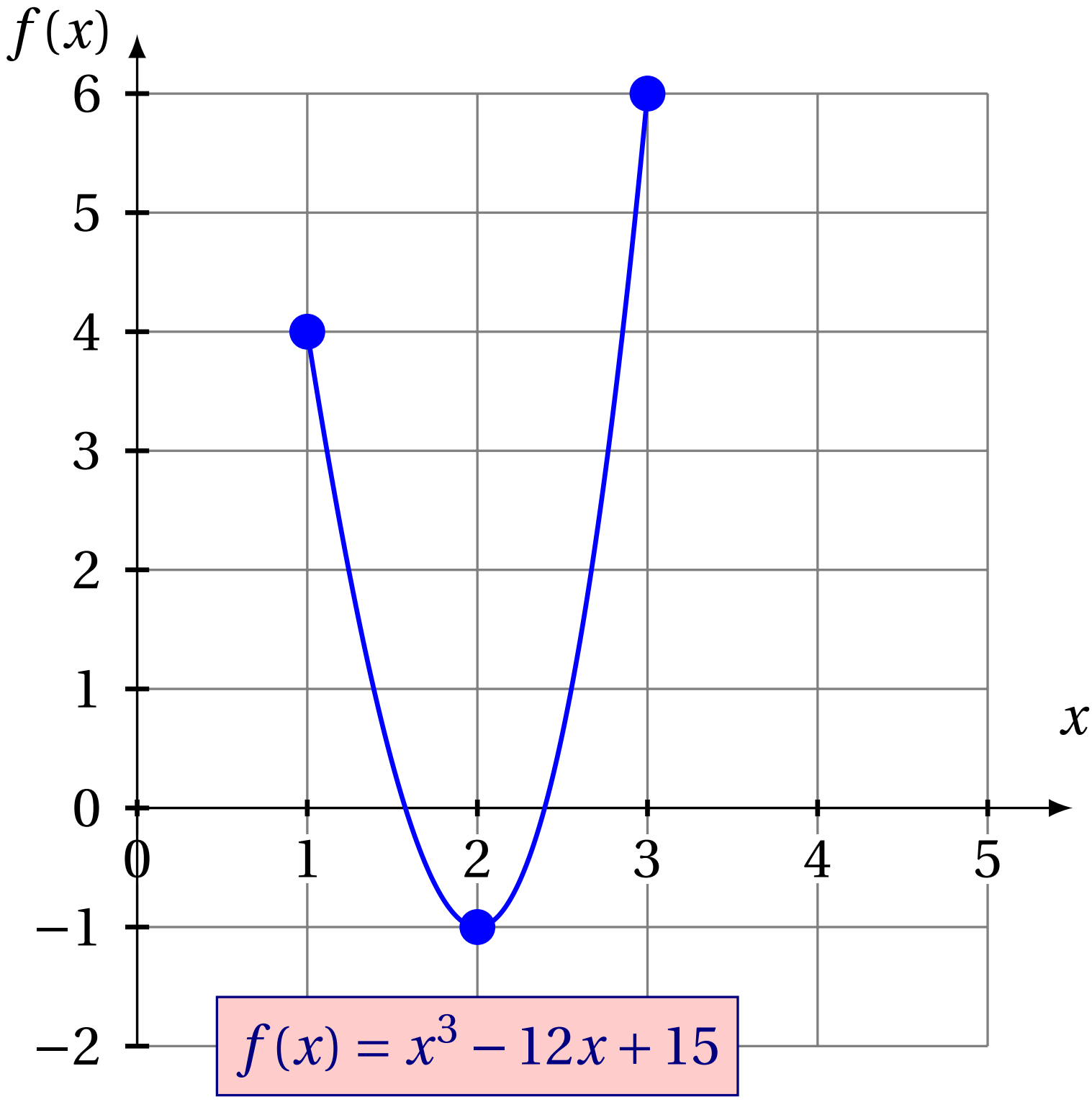

y
B = (s, t)
A = (0, r)
x
D
P = (x, 0)
C = (0, −r)

65 ./images-tikz/image-33.tex

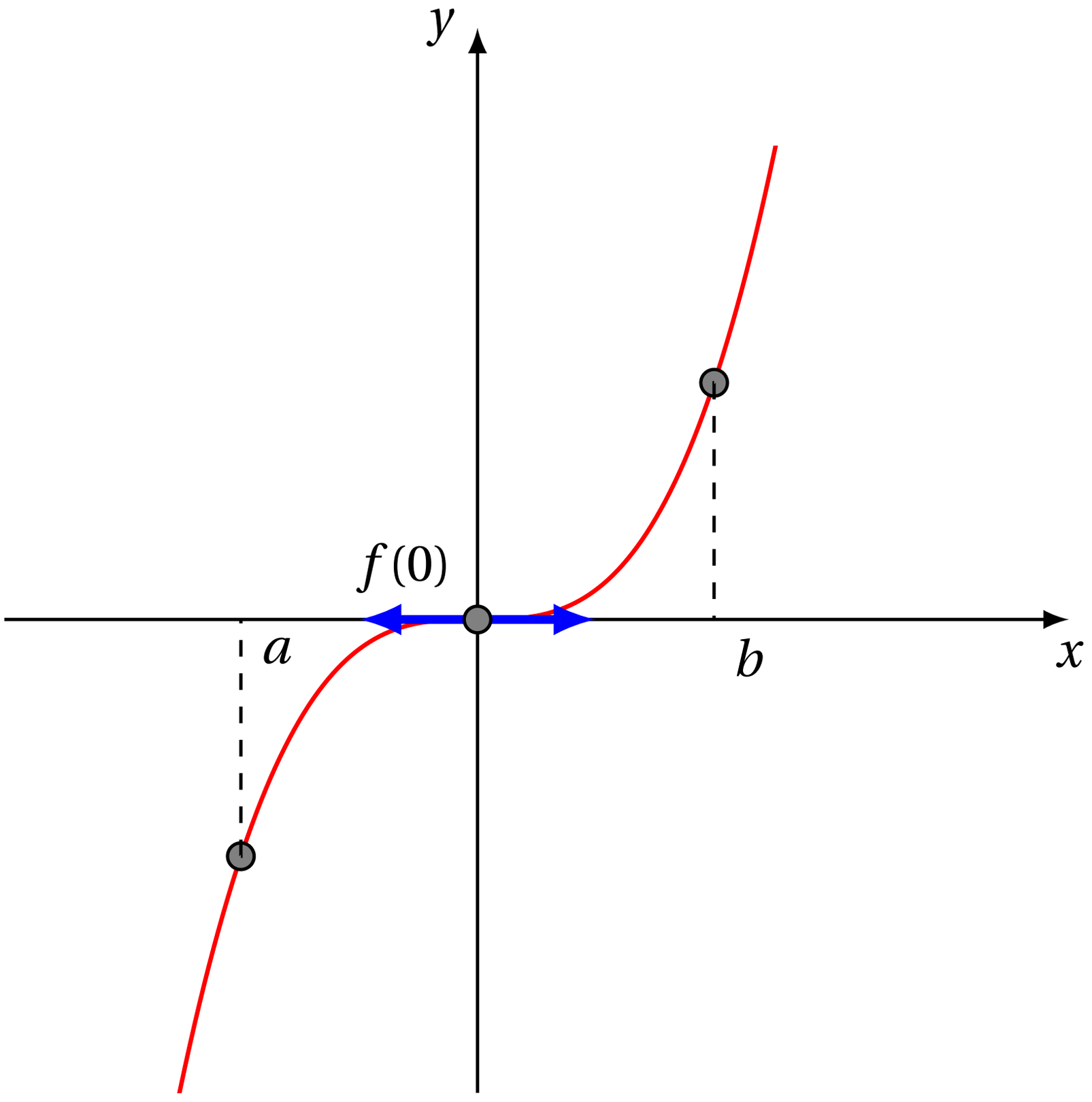
Height is deduced from height option



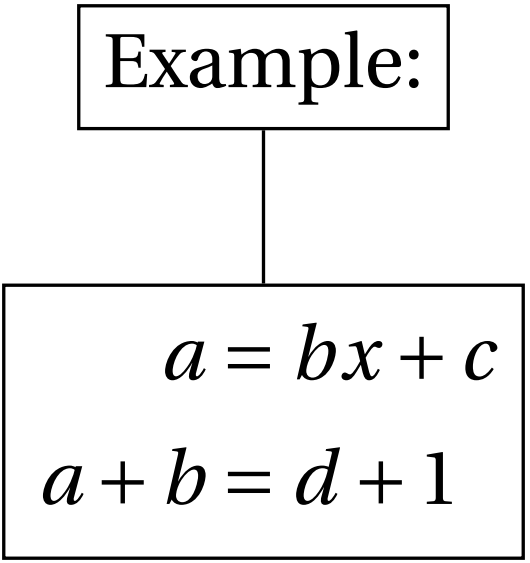
66 ./images-tikz/image-52.tex


f(x)
6
5
4
3
2
1
0
−1
−2
0
1
2
3
4
5
x
f(x) = x^3 − 12x + 15

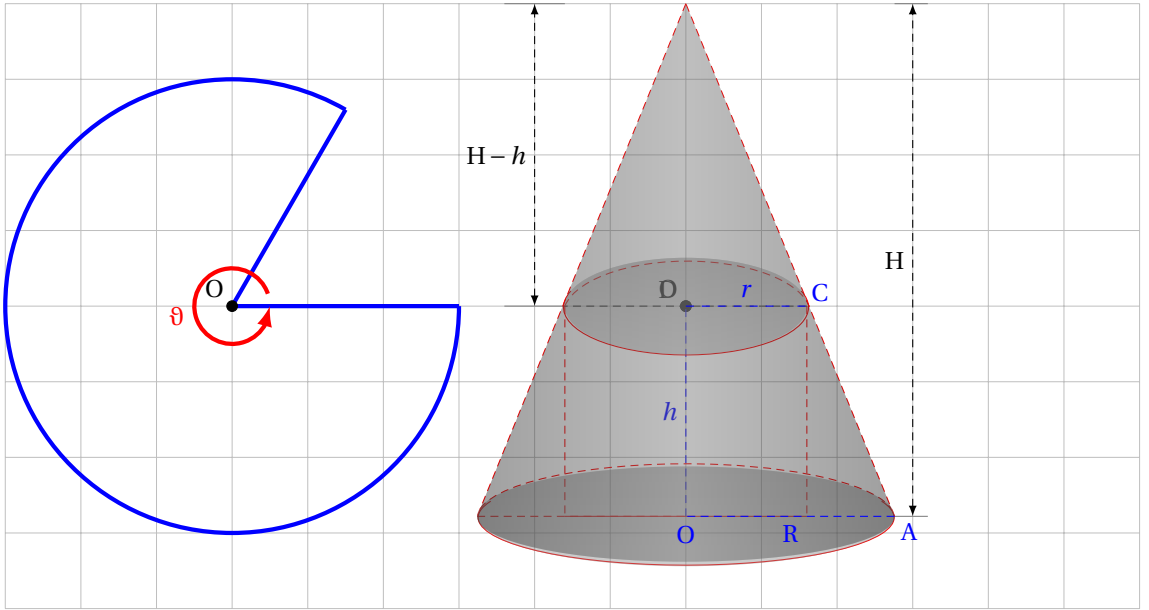
67 ./images-tikz/image-49.tex

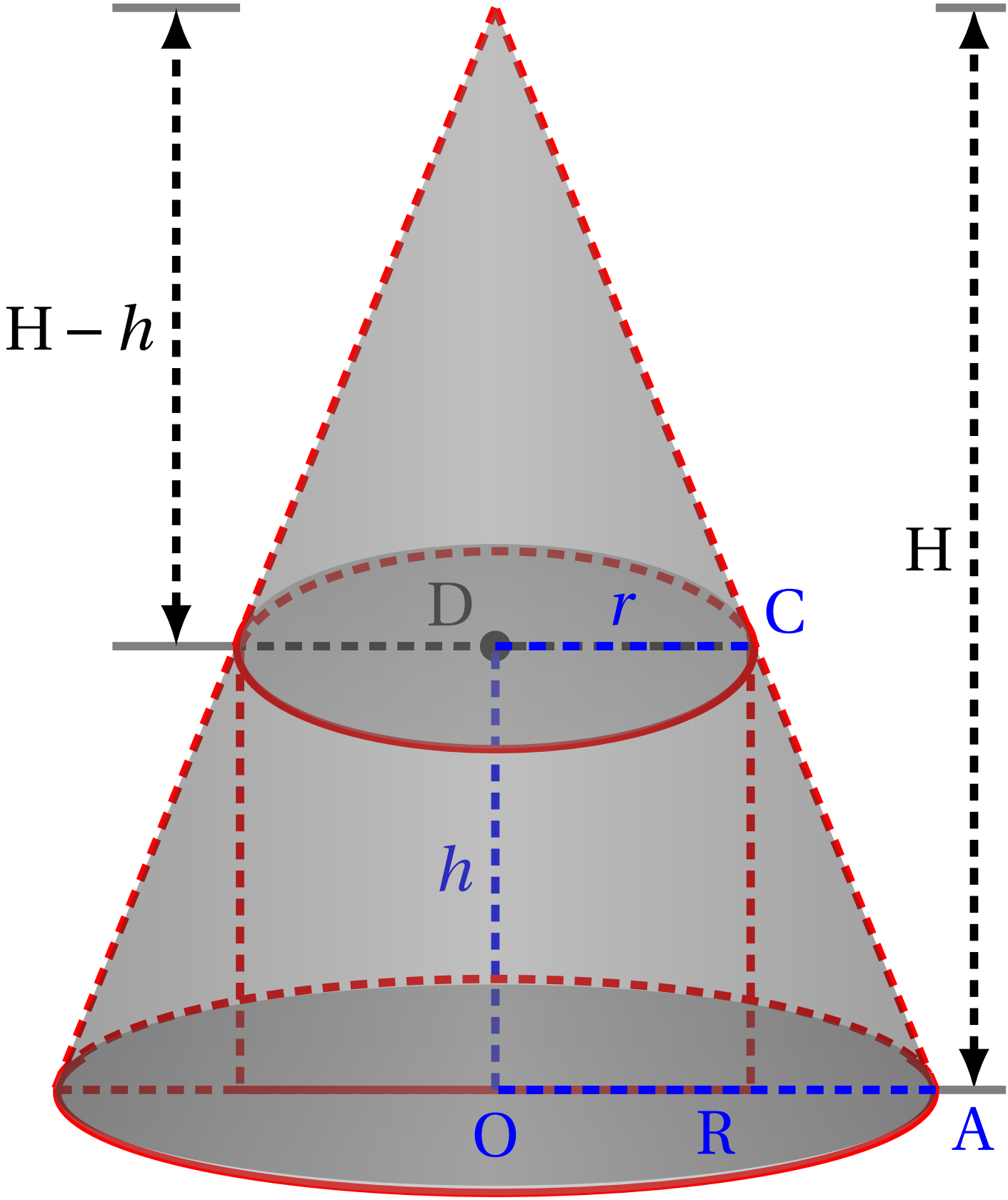

y
f(0)
a
b
x

68 ./images-tikz/image-6.tex

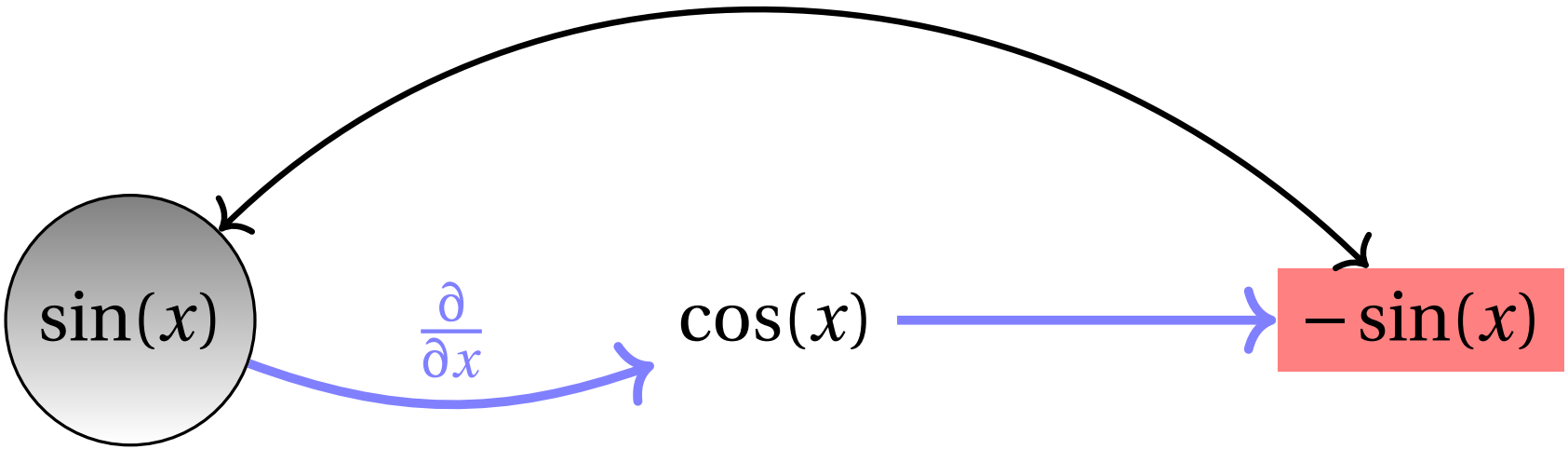

Example:
$a = bx + c$
$a + b = d + 1$

69 ./images-tikz/resuelto6-13.tex

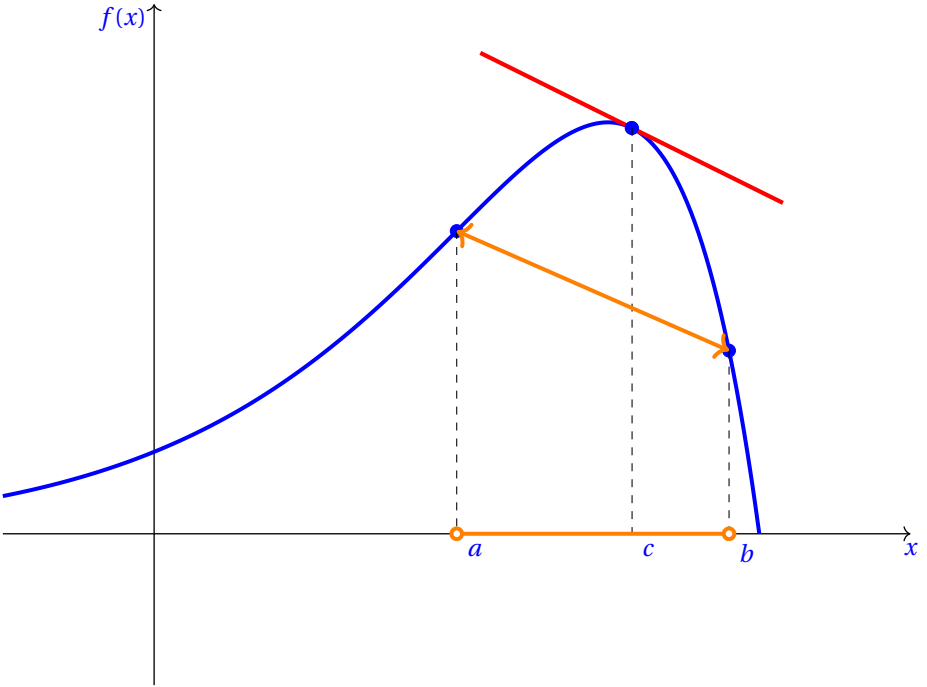

O
ϑ
H − h
H
D
r
C
h
O
R
A


H − h
H
D
r
C
h
O
R
A

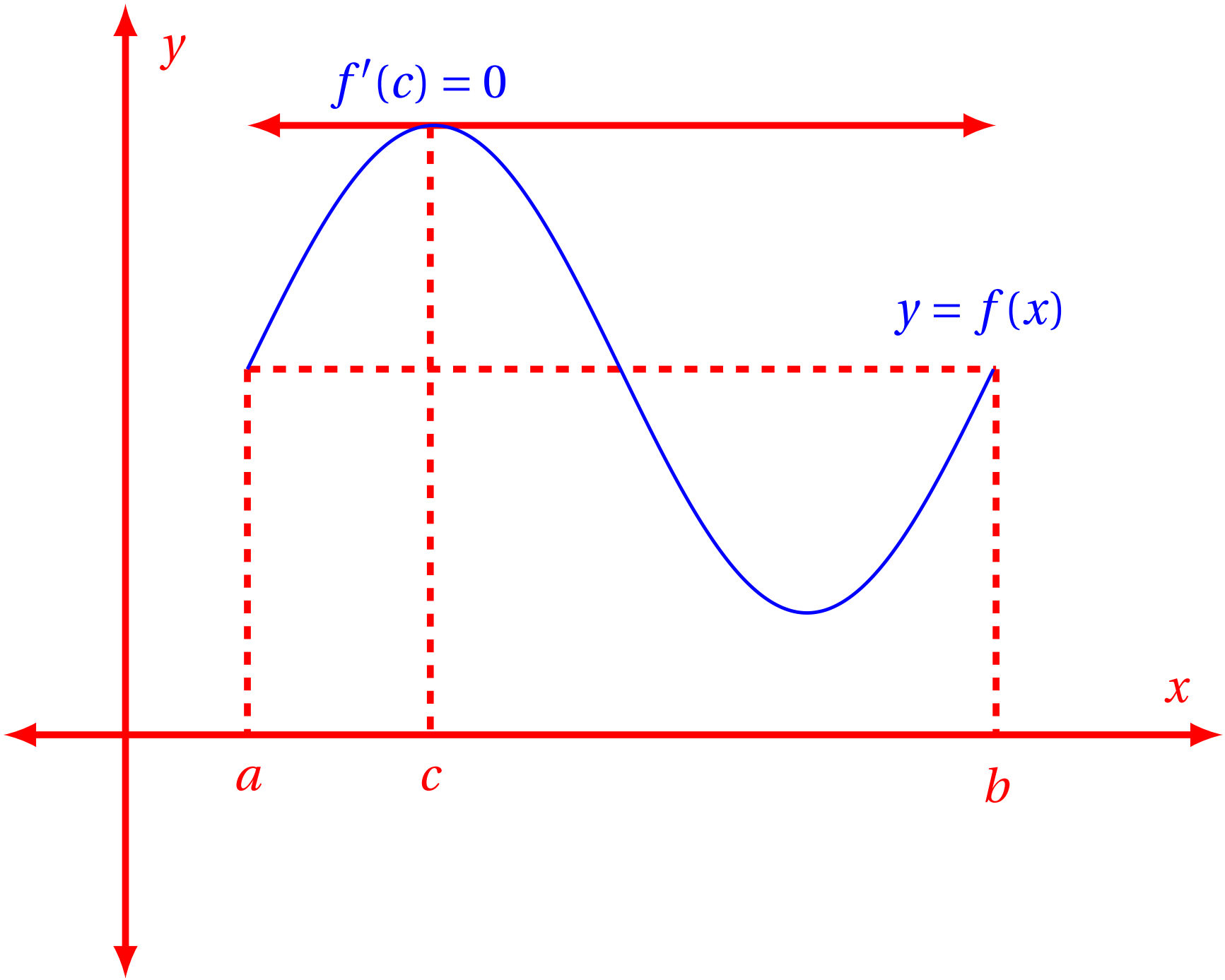
70 ./images-tikz/image-8.tex


$\sin(x)$
$\frac{\partial}{\partial x}$
$\cos(x)$
$-\sin(x)$

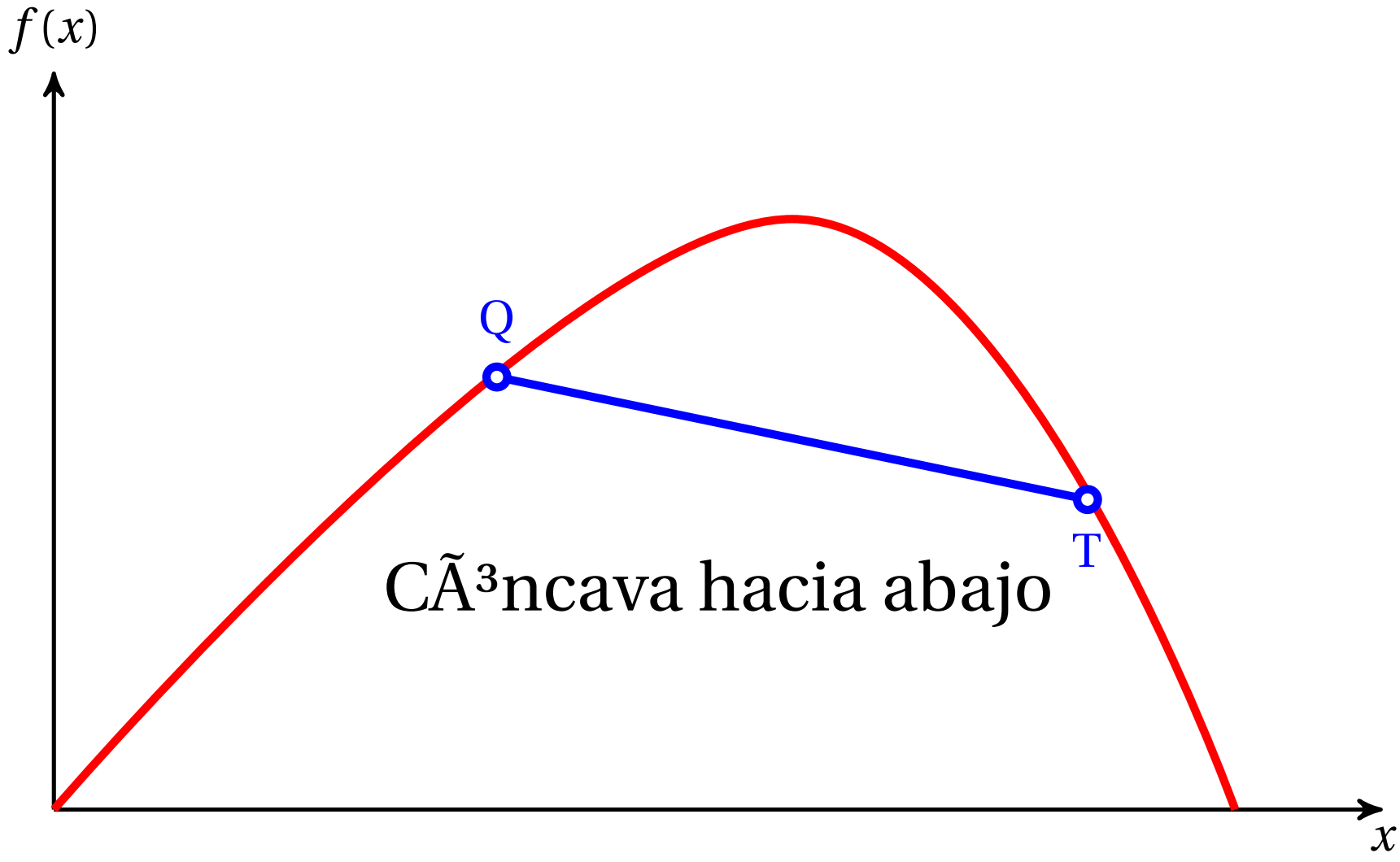
71 ./images-tikz/img-xx54.tex


f(x)
a
c
b
x

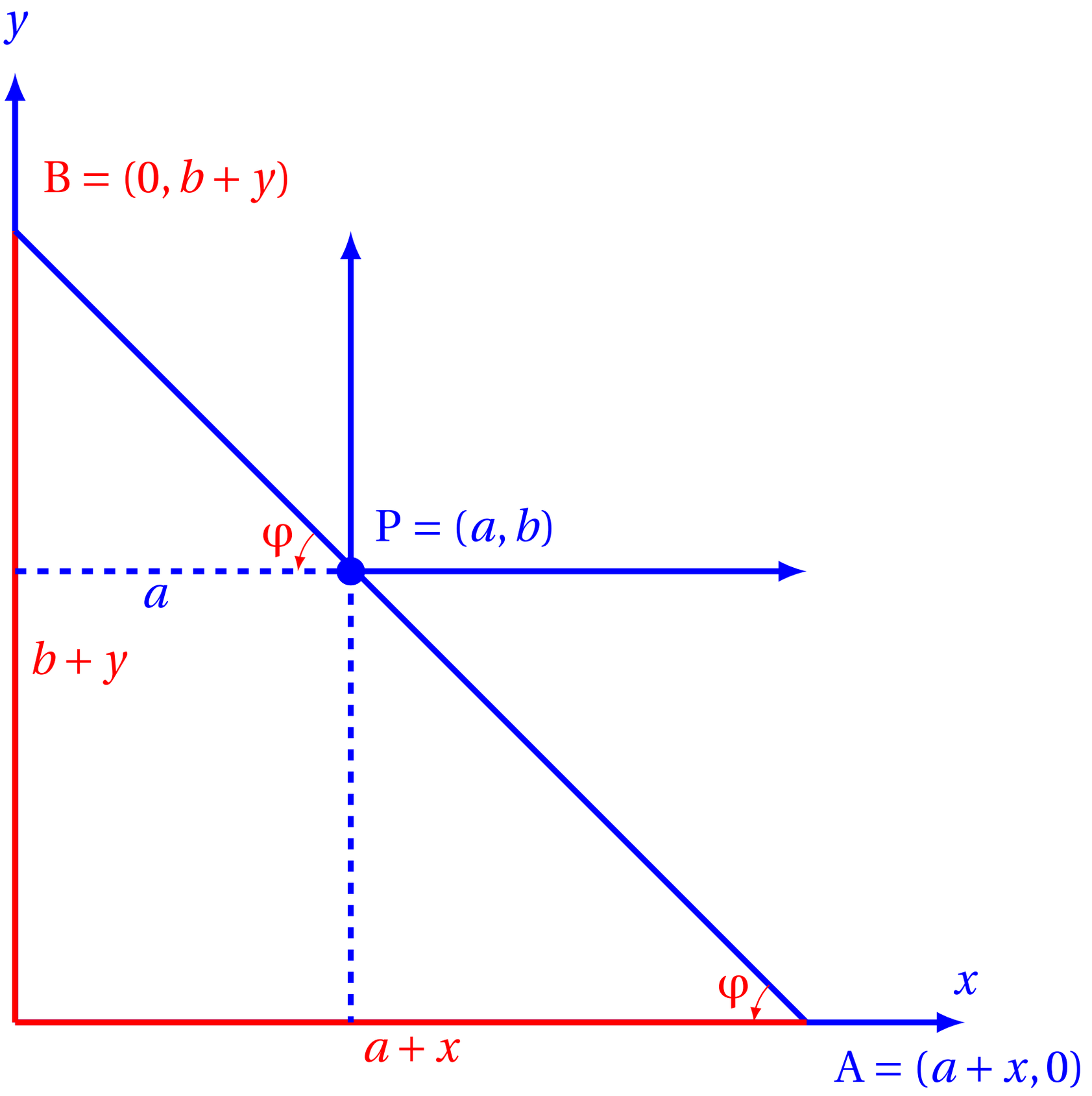
72 ./images-tikz/image-40.tex


y
f′(c) = 0
y = f(x)
x
a
c
b

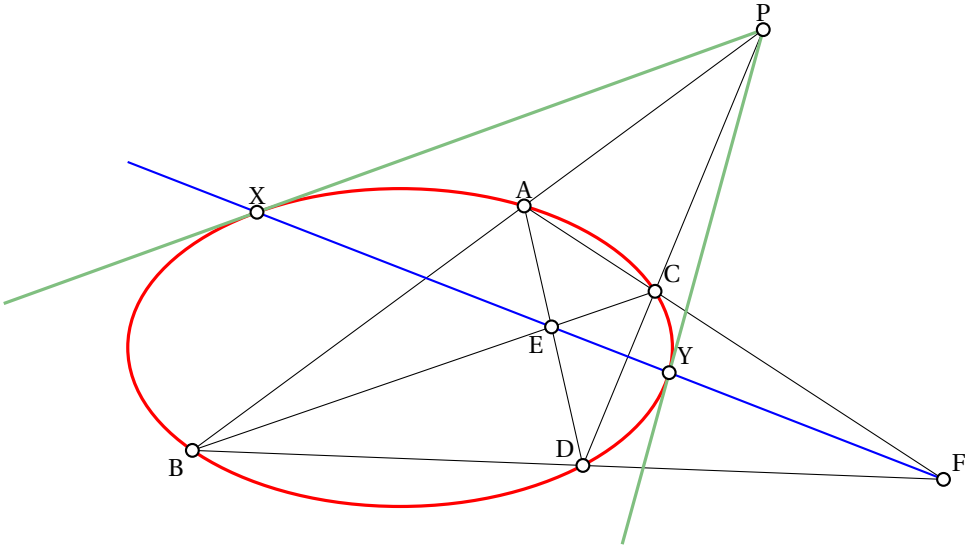
73 ./images-tikz/concavidad-d.tex


$f(x)$
Q
T
CÃ³ncava hacia abajo
x

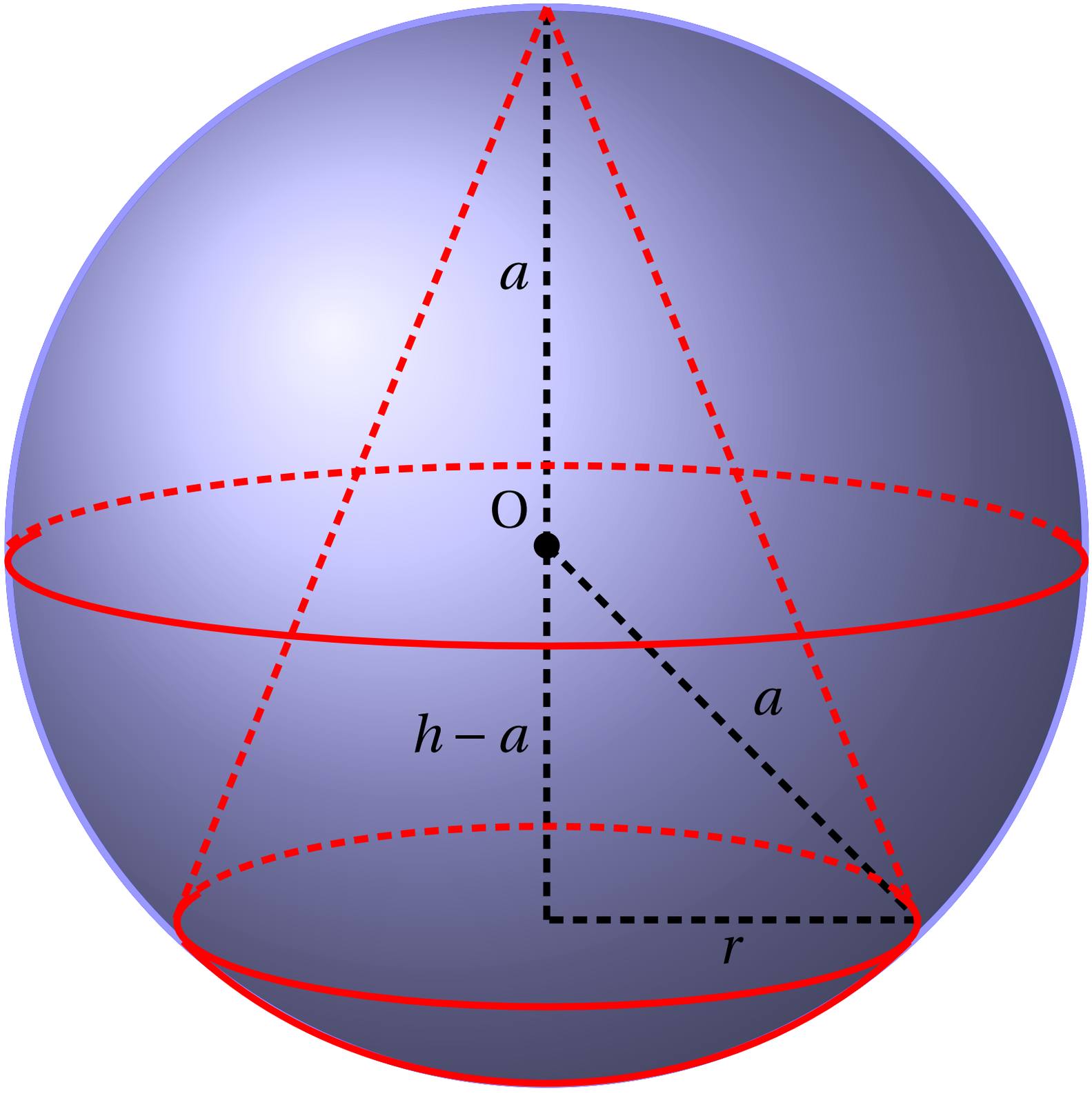
74 ./images-tikz/resuelto6-1.tex


y
B = (0, b + y)
P = (a, b)
φ
a
b + y
φ
x
a + x
A = (a + x, 0)

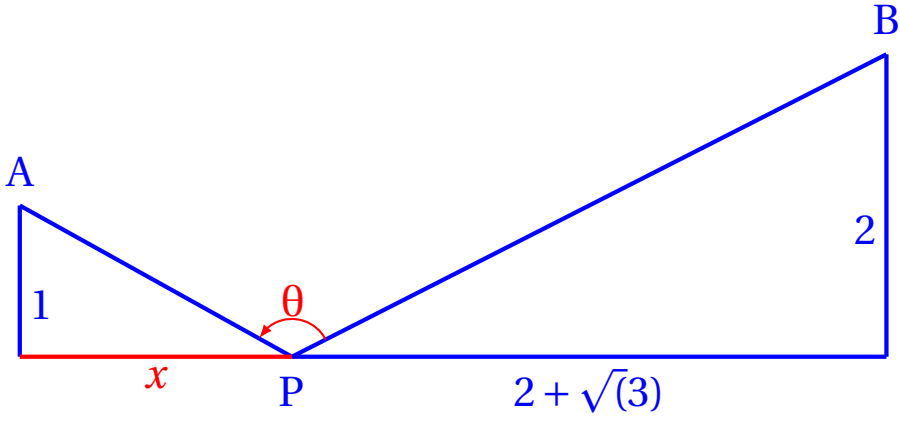
75 ./images-tikz/polars-tangents-conic.tex


P
X
A
C
E
Y
D
B
F

76 ./images-tikz/resuelto6-7.tex

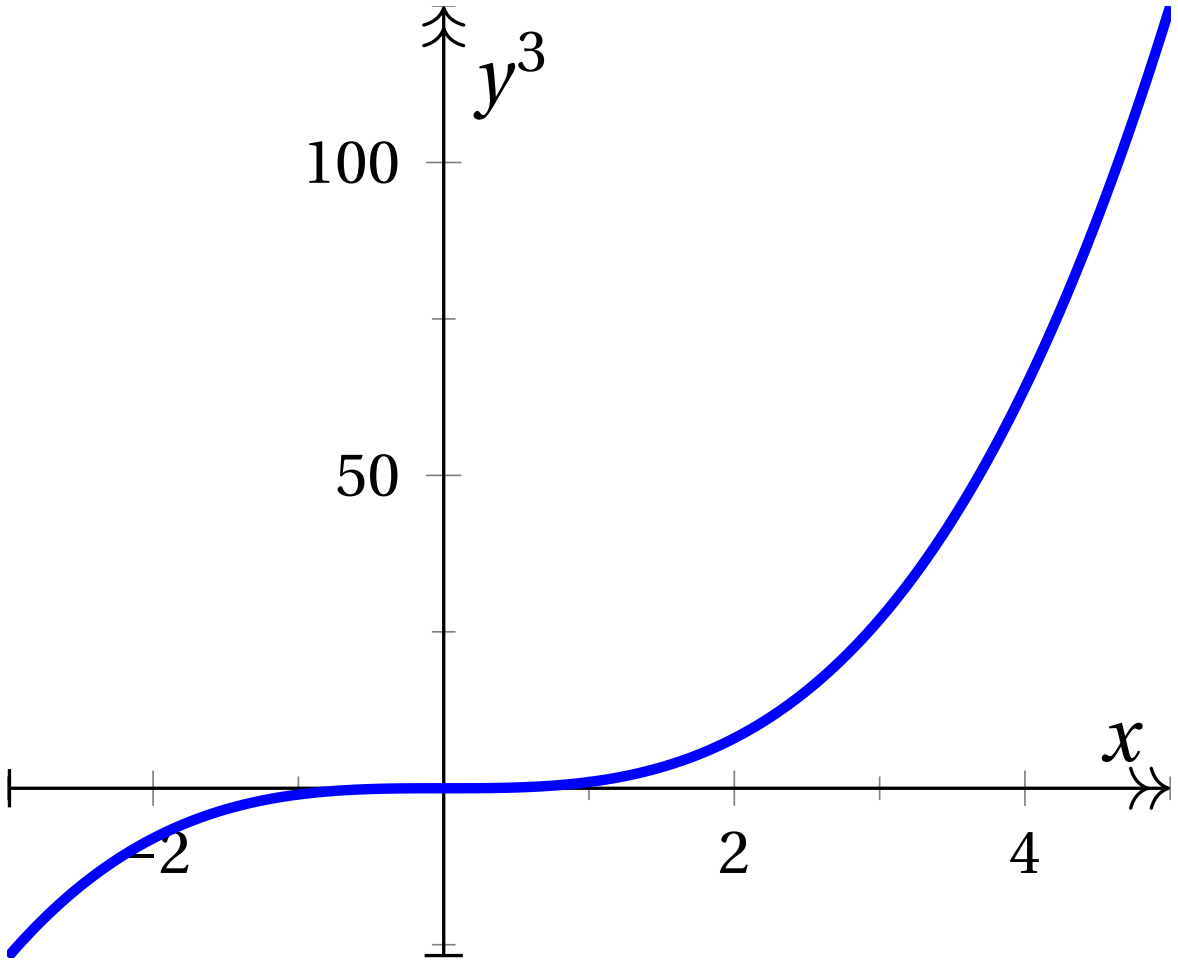

a
O
a
h – a
r

77 ./images-tikz/punto-p.tex

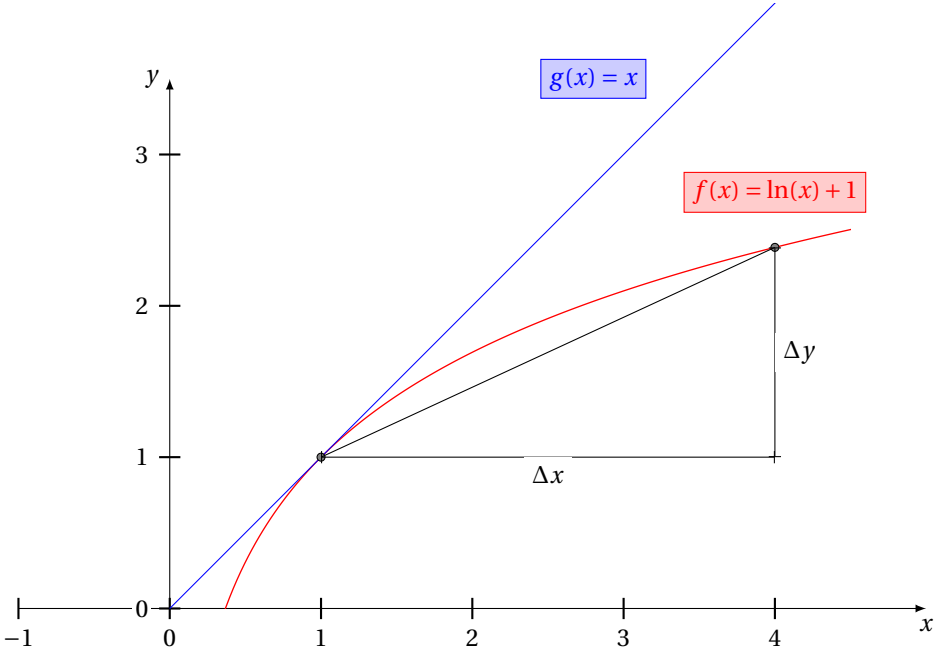

B
A
2
1
θ
x
P
2 + √(3)

78 ./images-tikz/image-29.tex

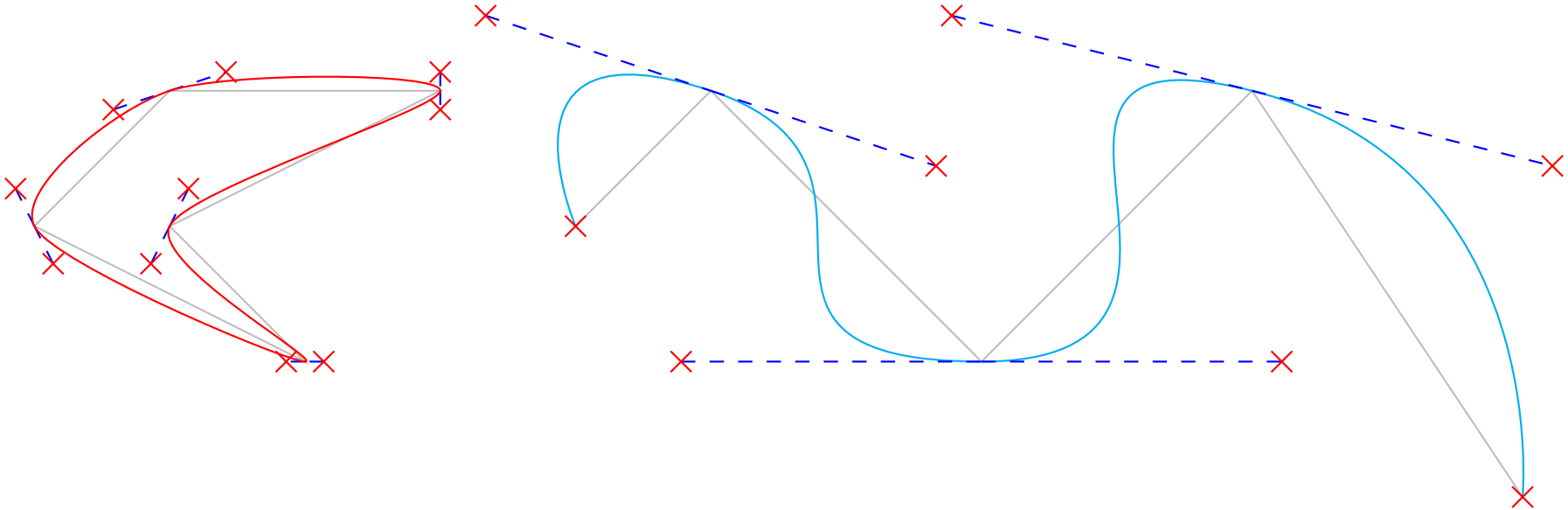
Trimmed bounding boxes



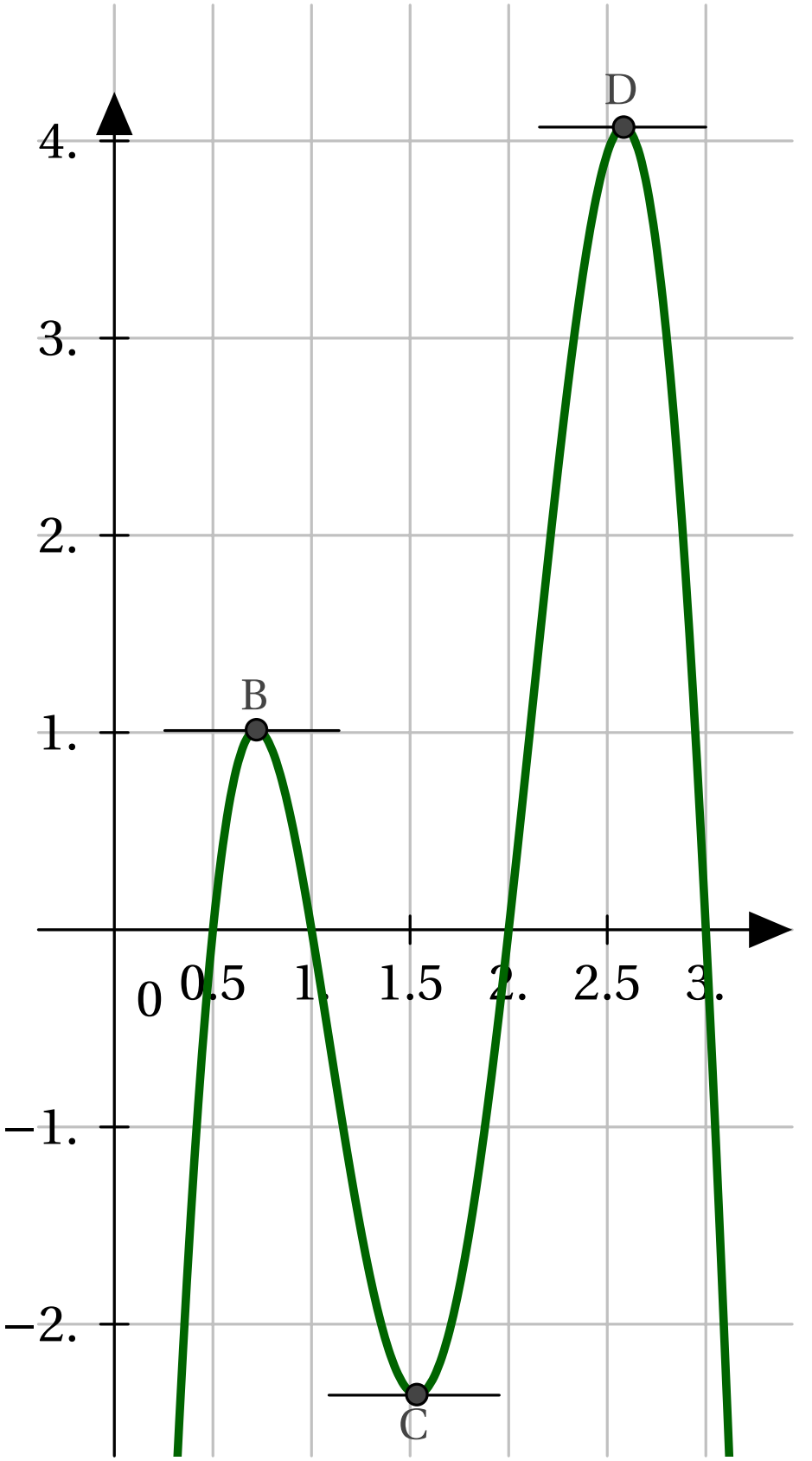
79 ./images-tikz/derivate1.tex


$g(x) = x$
$f(x) = \ln(x) + 1$
y
x
3
2
1
0
−1
0
1
2
3
4
Δy
Δx

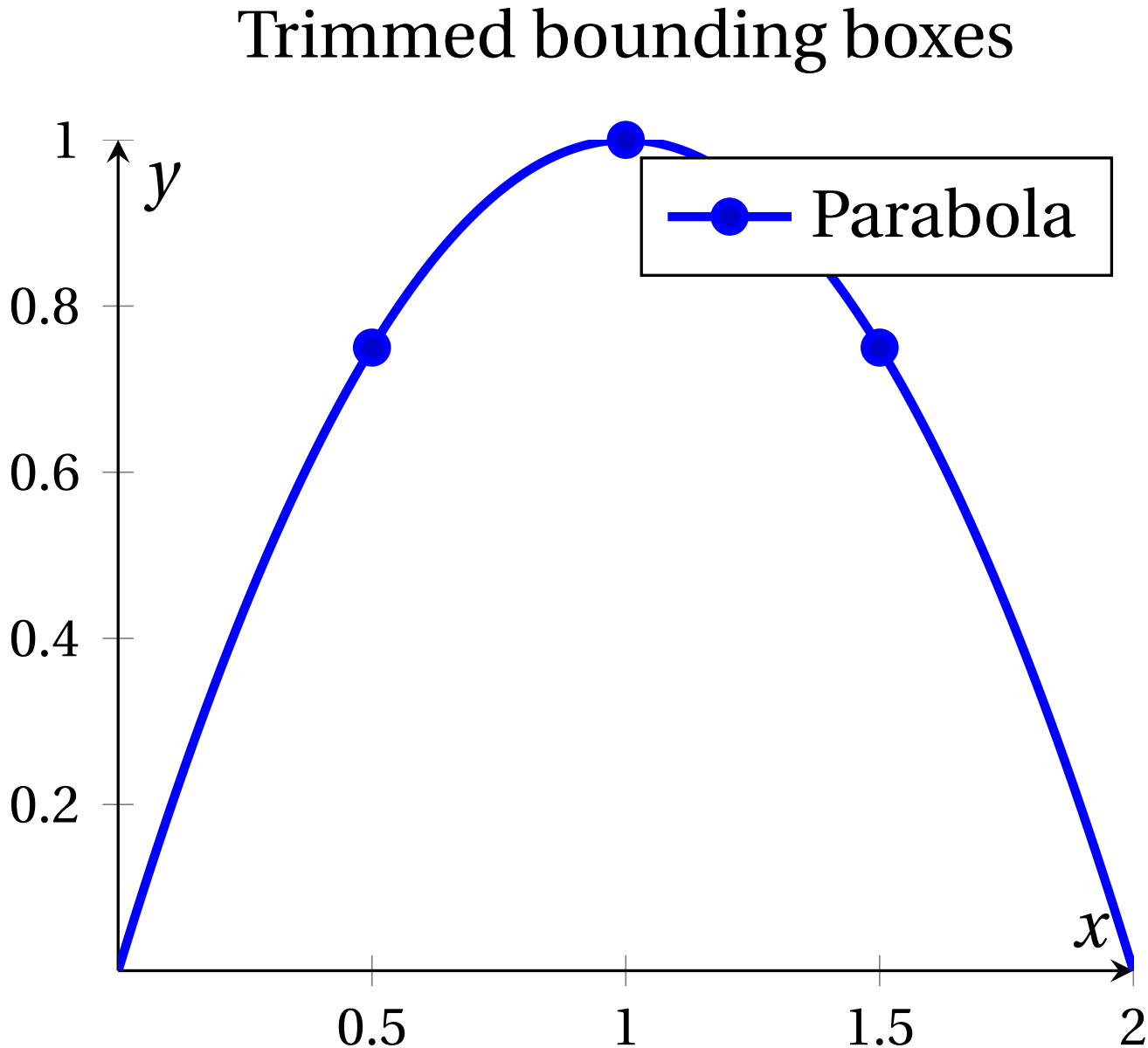
80 ./images-tikz/image-17.tex



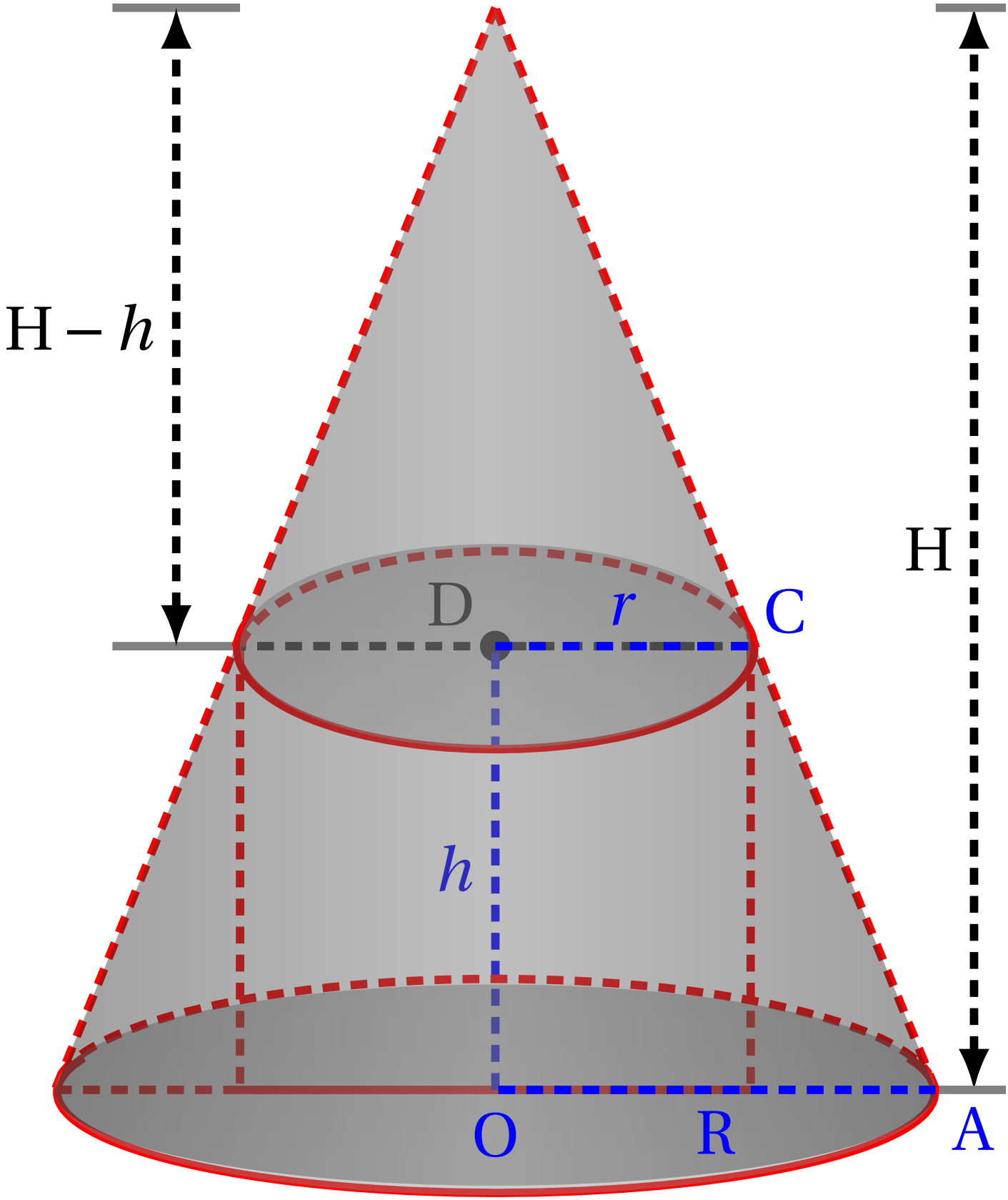
81 ./images-tikz/figad1.tex


D
4.
3.
2.
B
1.
0
0.5
1.
1.5
2.
2.5
3.
−1.
−2.
C

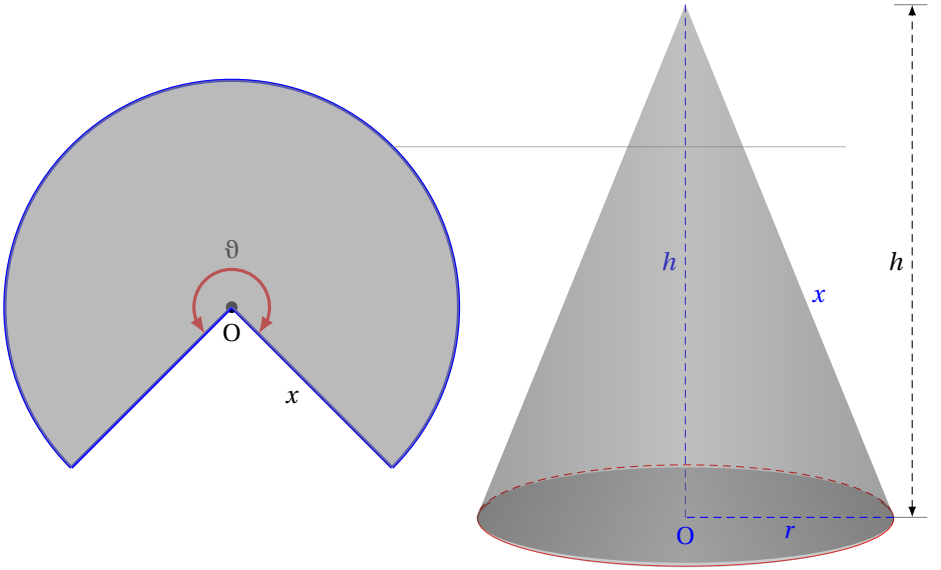
82 ./images-tikz/image-24.tex


Trimmed bounding boxes
1
0.8
0.6
0.4
0.2
y
x
0.5
1
1.5
2
Parabola

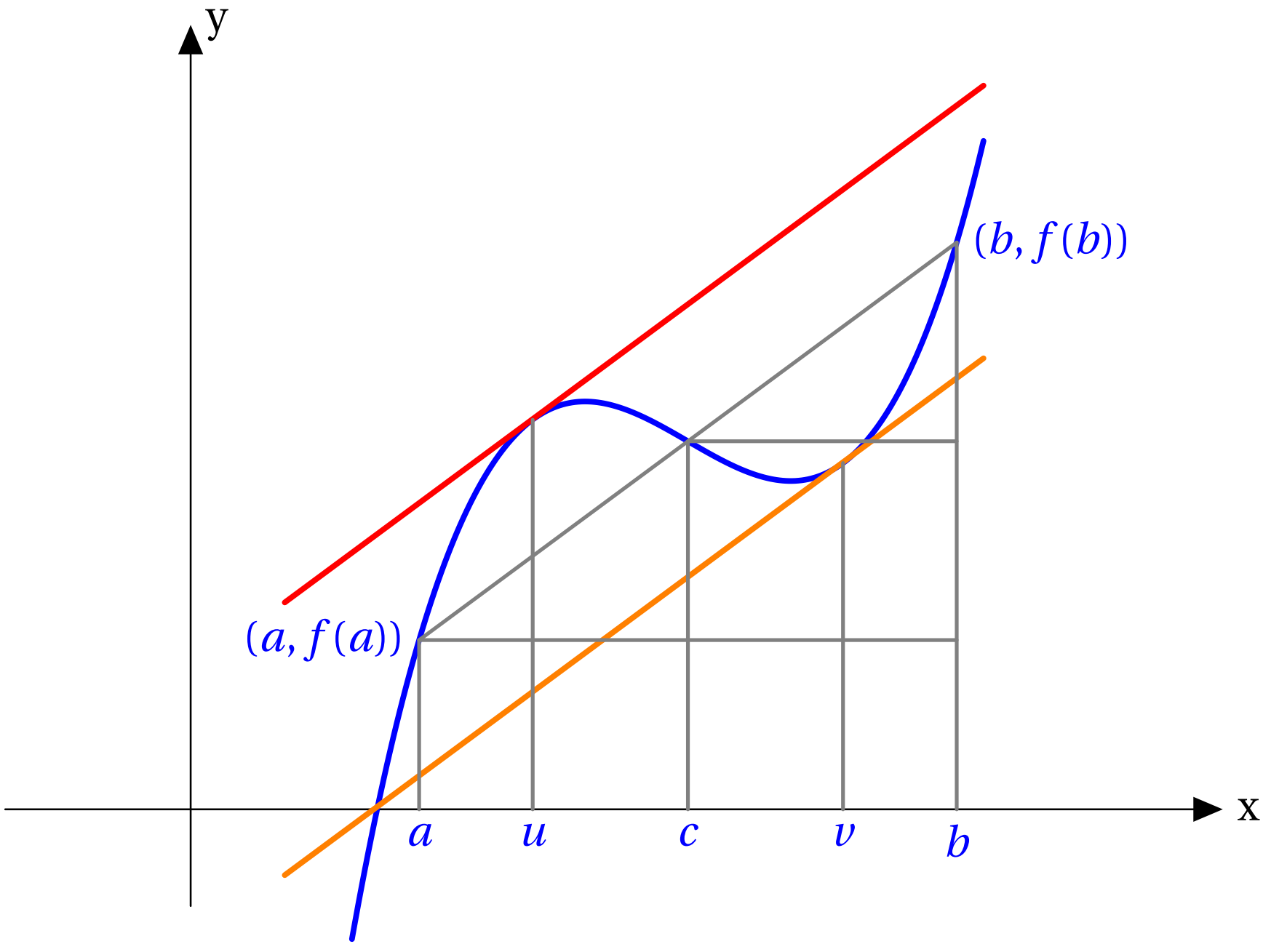
83 ./images-tikz/resuelto6-8.tex


H − h
H
D
r
C
h
O
R
A

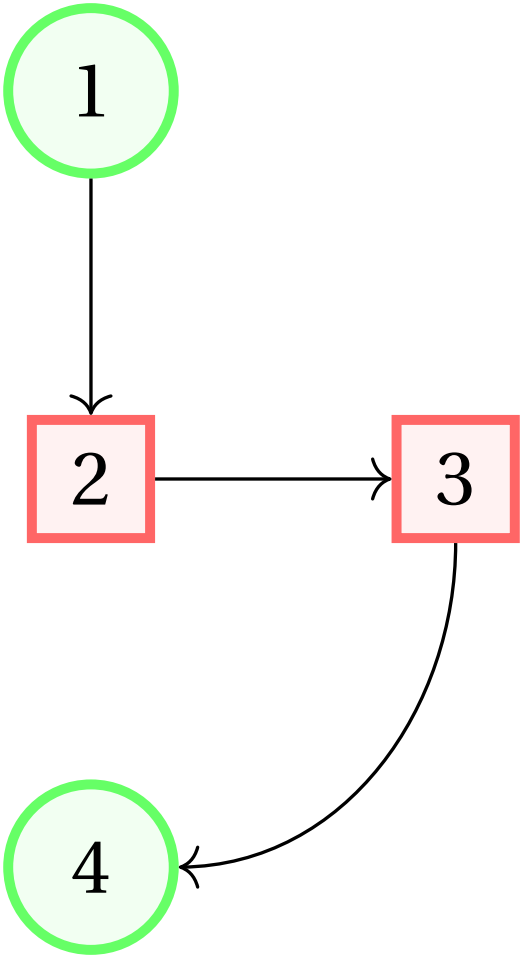
84 ./images-tikz/resuelto6-13a.tex


ϑ
O
x
h
x
h
O
r

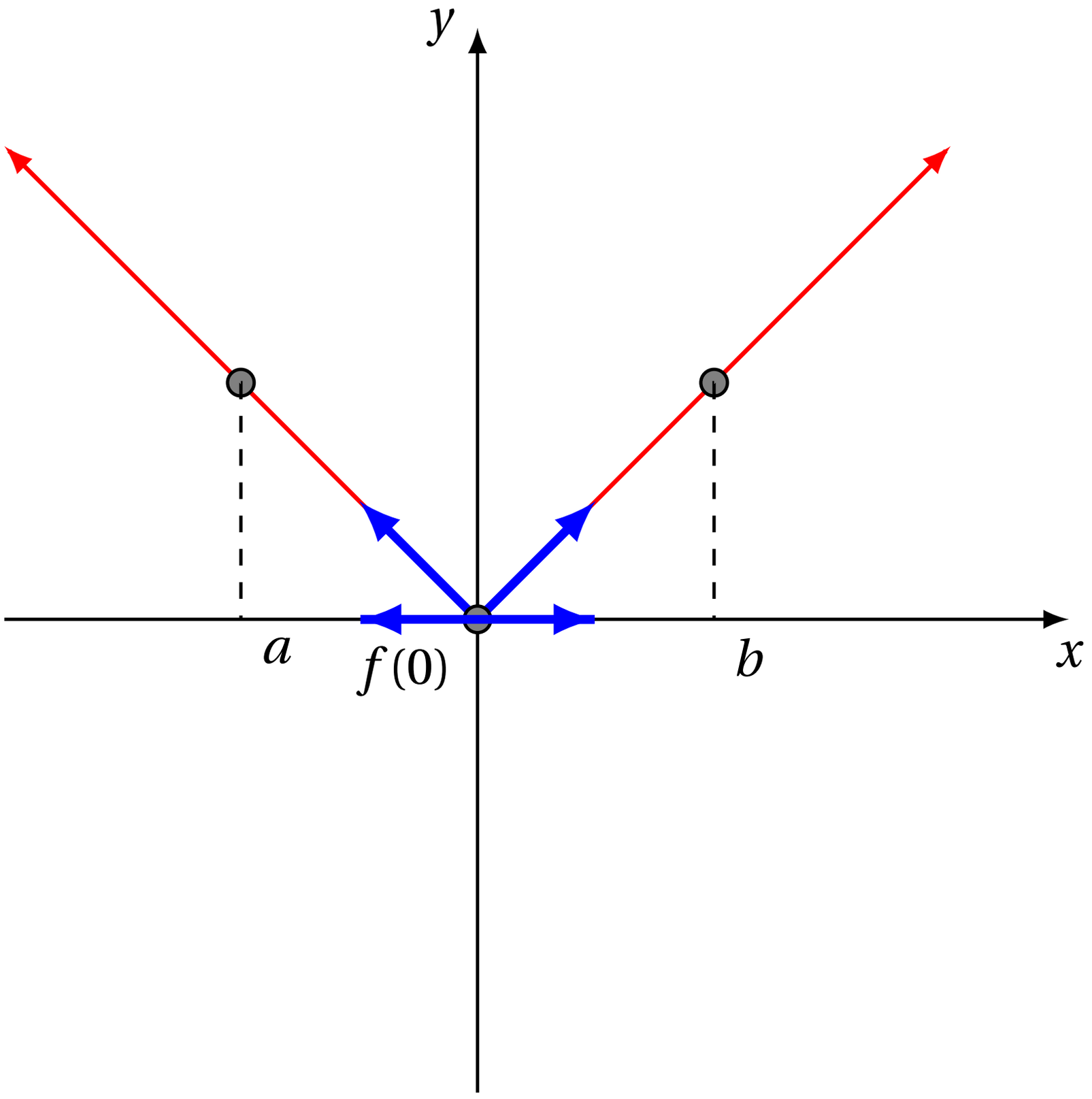
85 ./images-tikz/resuelto6-19.tex


y
x
$(b, f(b))$
$(a, f(a))$
a
u
c
v
b

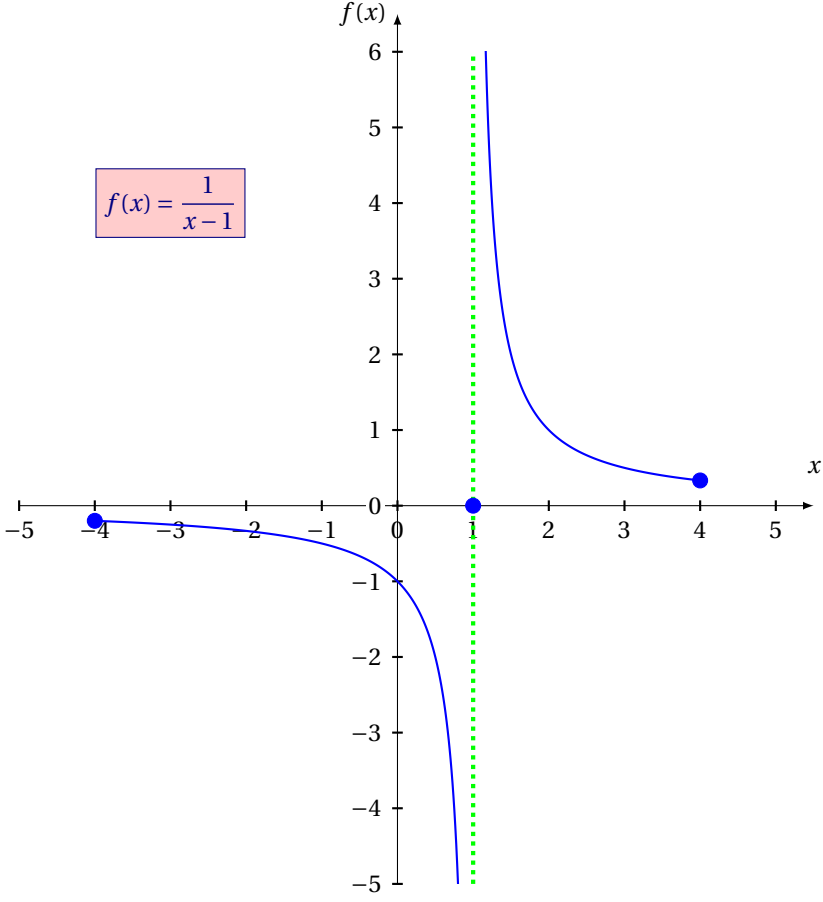
86 ./images-tikz/image-3.tex


1
2
3
4

87 ./images-tikz/image-48.tex

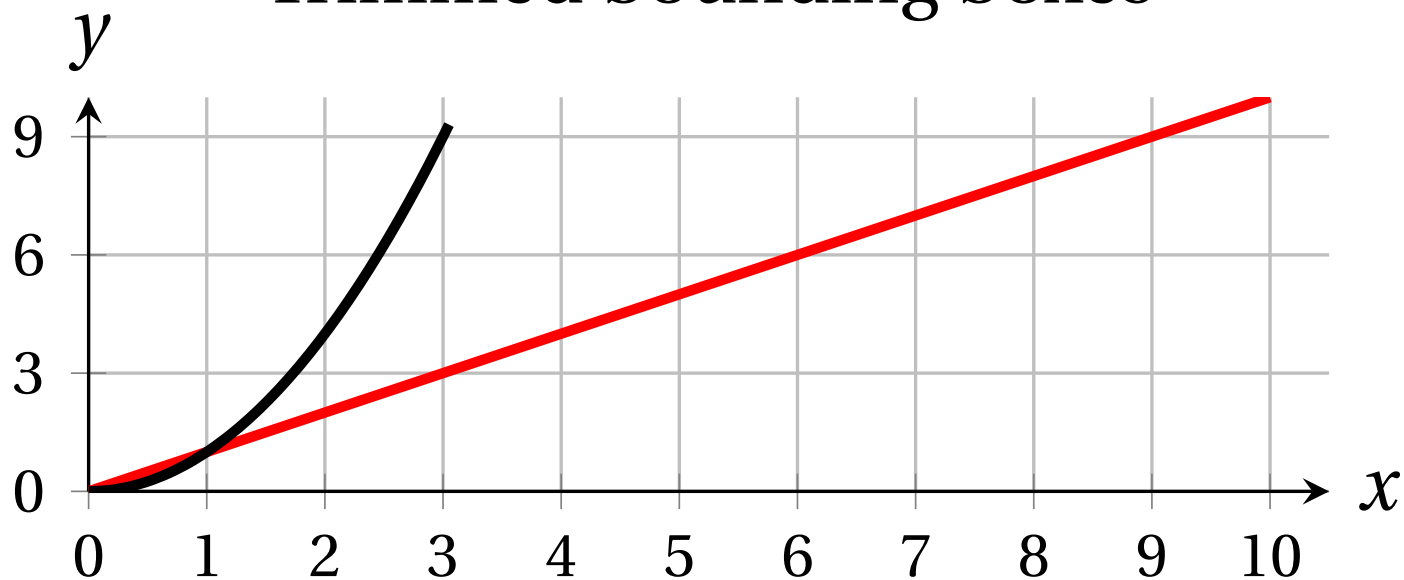

y
a
f(0)
b
x

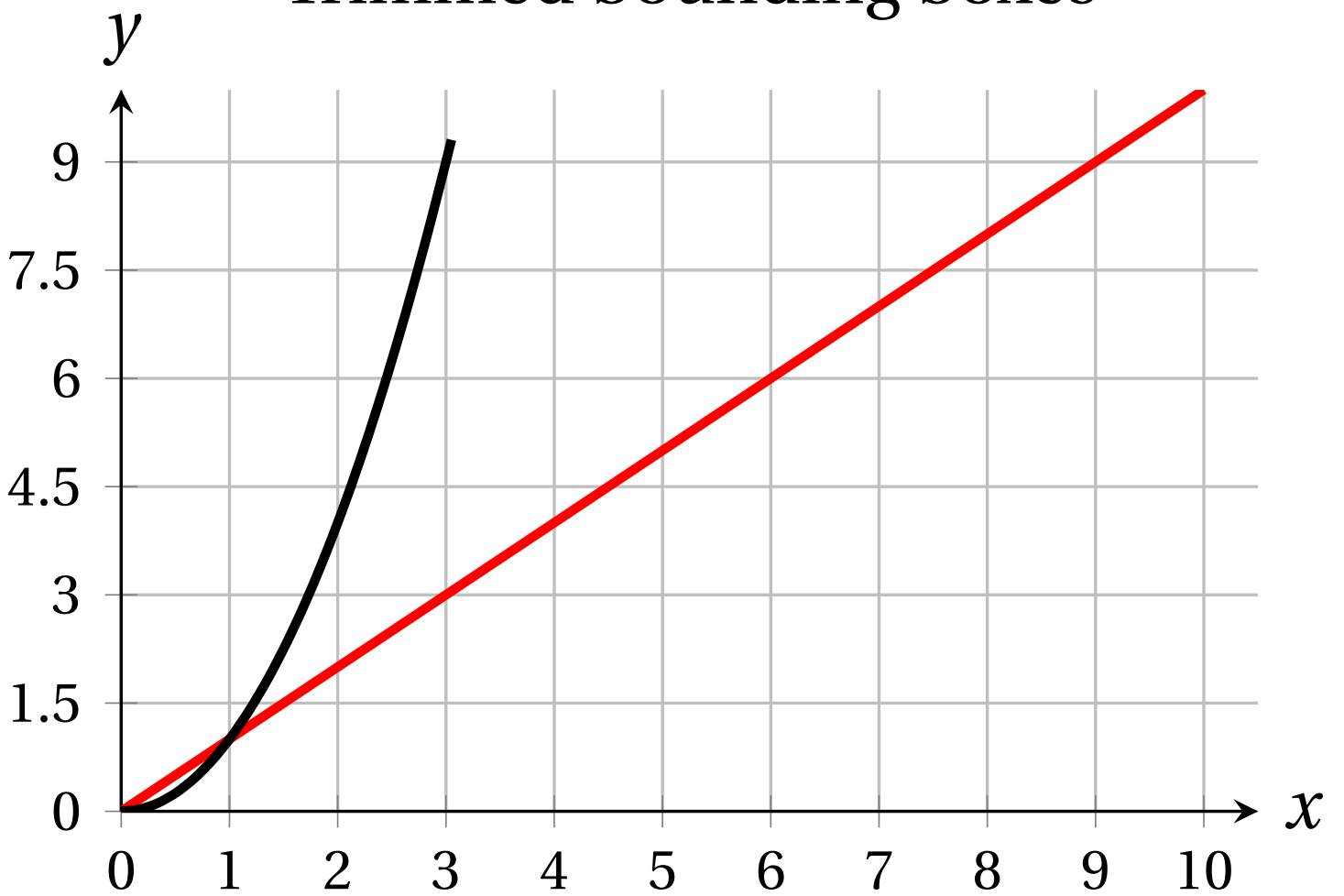
88 ./images-tikz/image-51.tex


$f(x) = \frac{1}{x-1}$
$f(x)$
x
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5
−4
−3
−2
−1
0
1
2
3
4
5

89 ./images-tikz/tikznew3.pgf

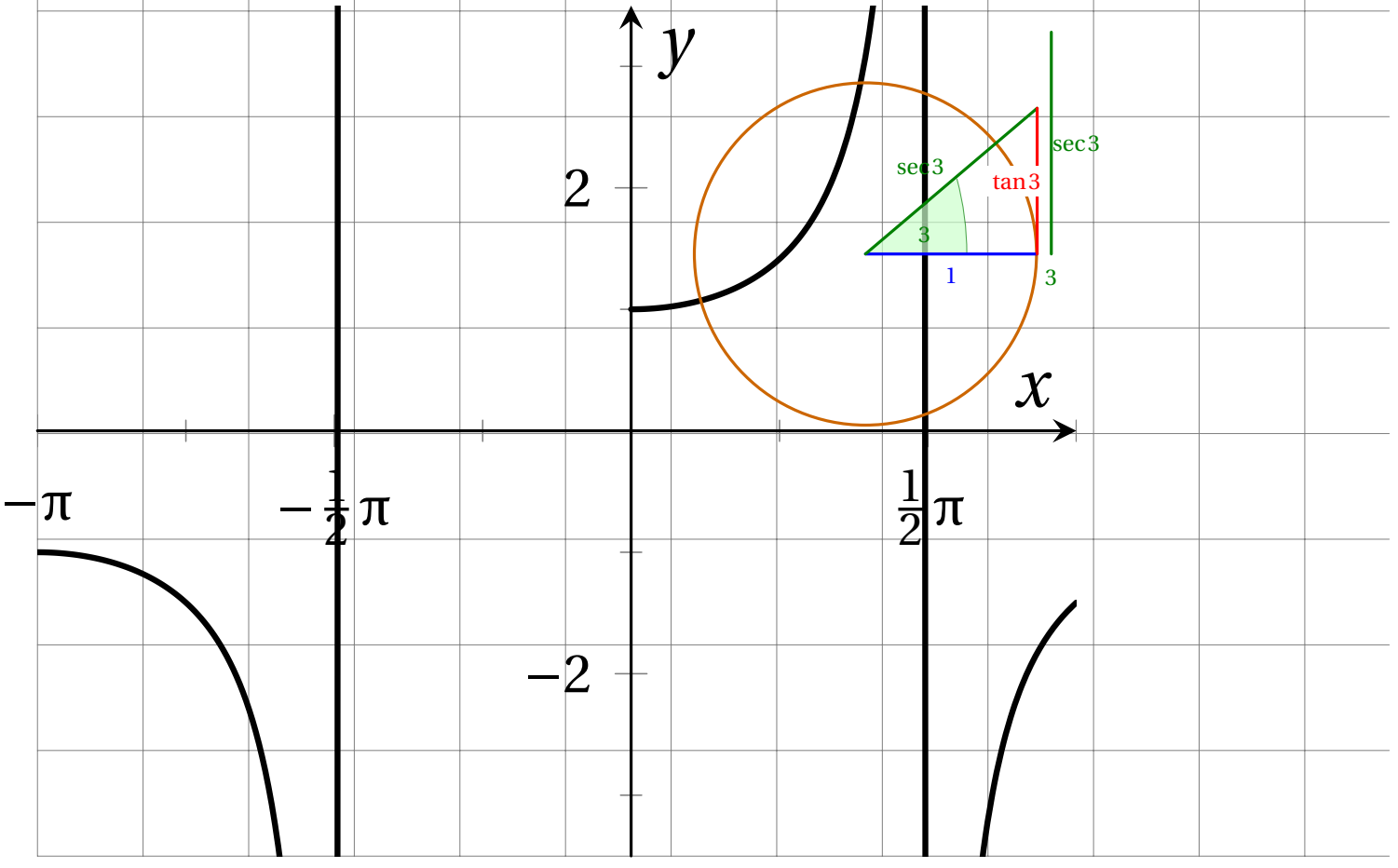
Trimmed bounding boxes



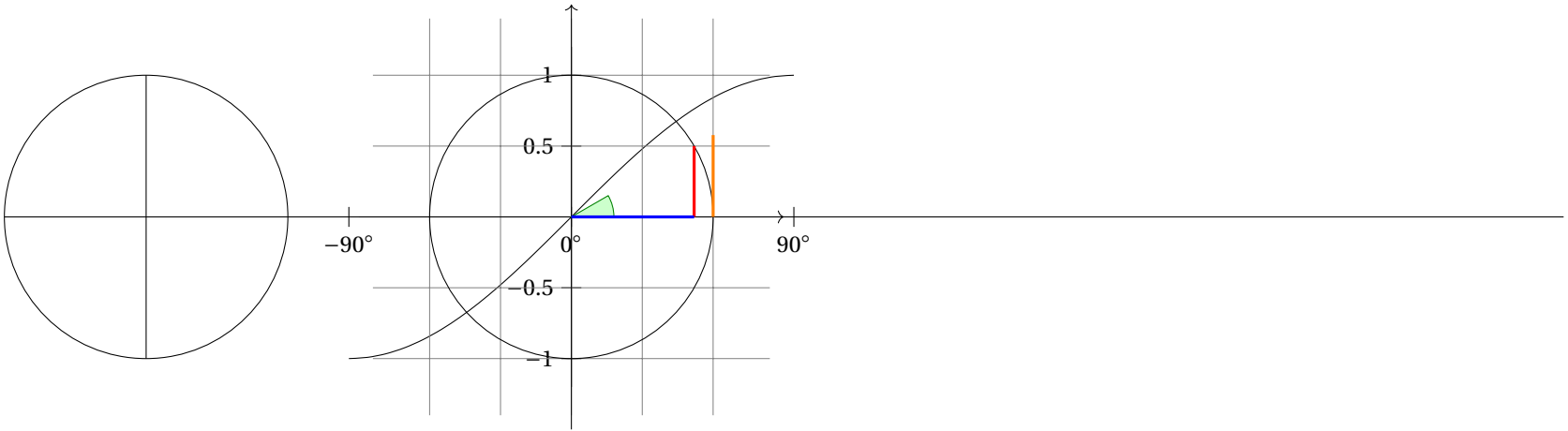

Trimmed bounding boxes
y
x
0
1.5
3
4.5
6
7.5
9
0
1
2
3
4
5
6
7
8
9
10

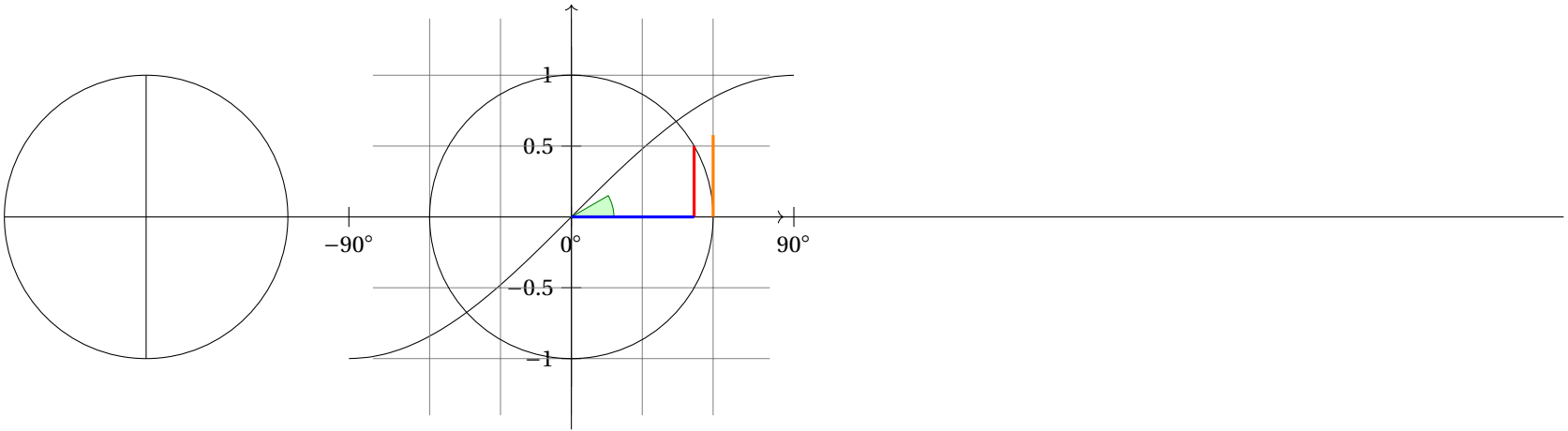
90 ./images-tikz/trig3b.pgf

Trimmed bounding boxes

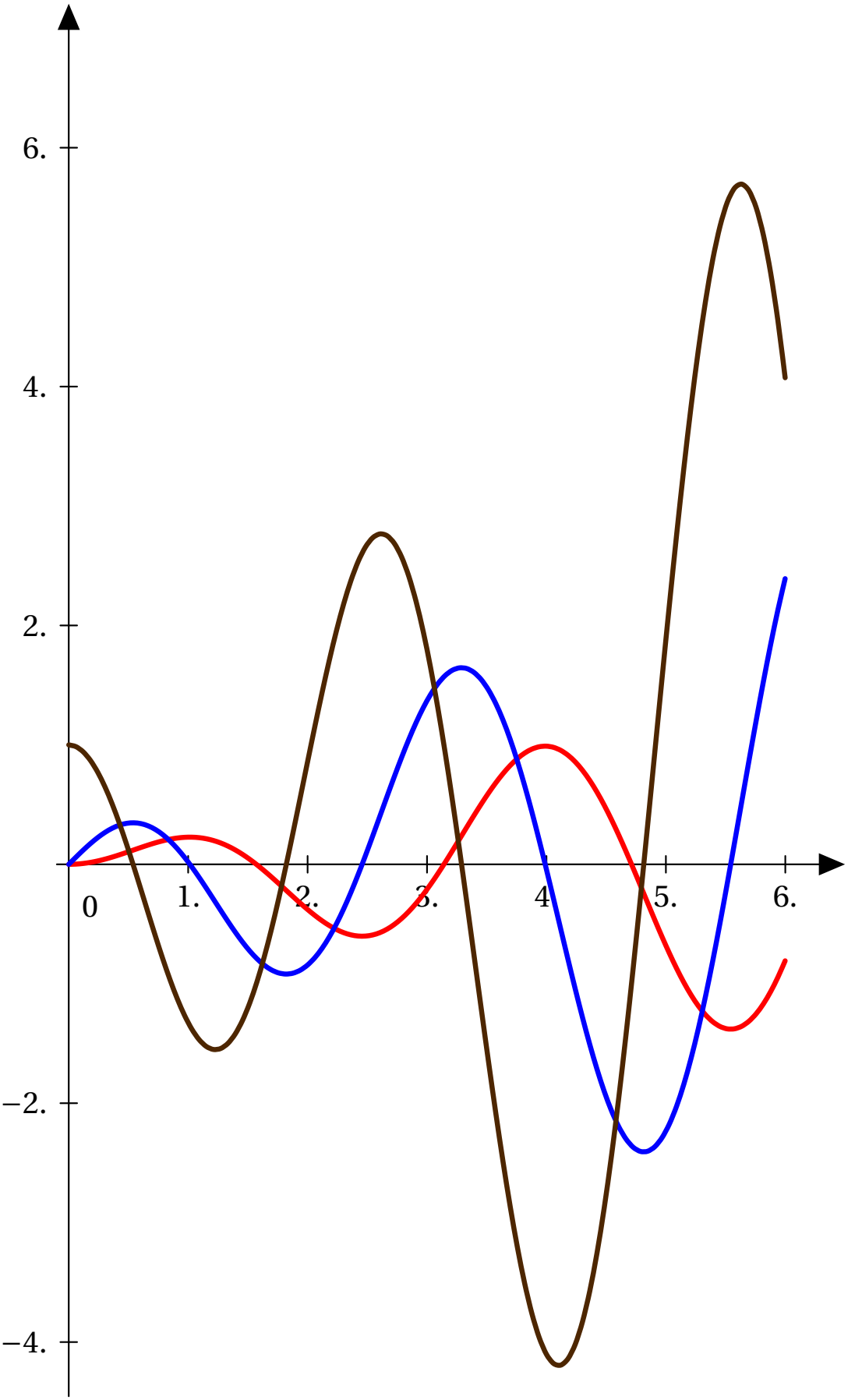


91 ./images-tikz/sin-arsin.pgf

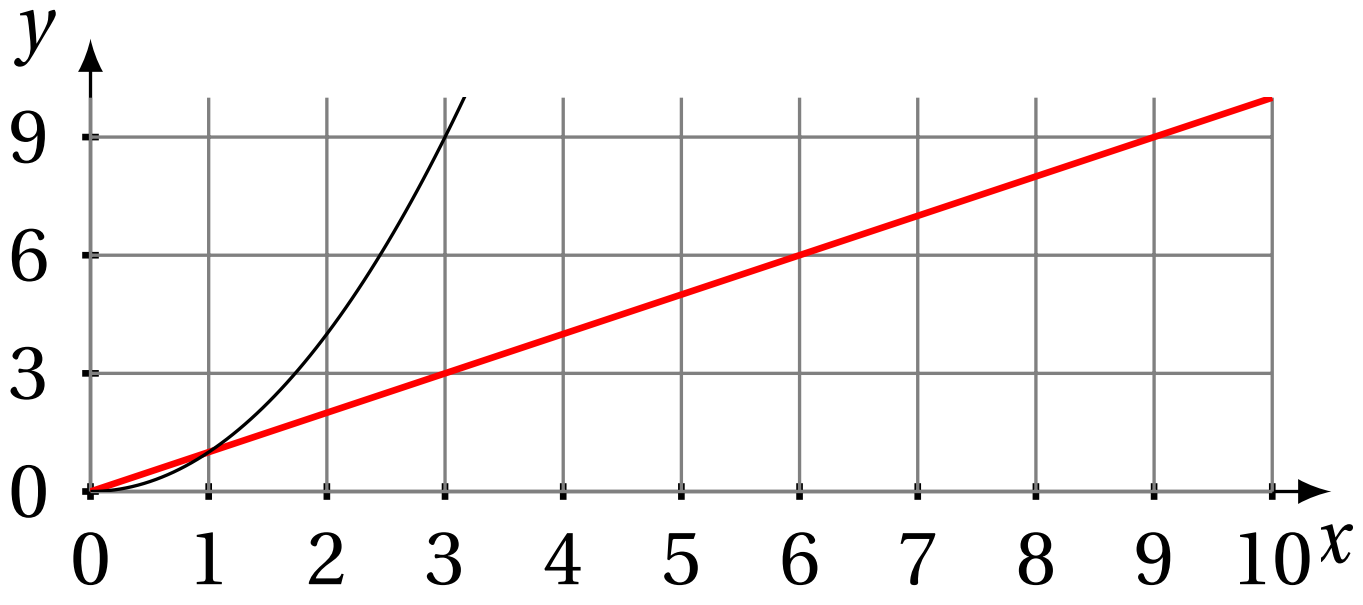

1
0.5
−0.5
−1
−90°
0°
90°

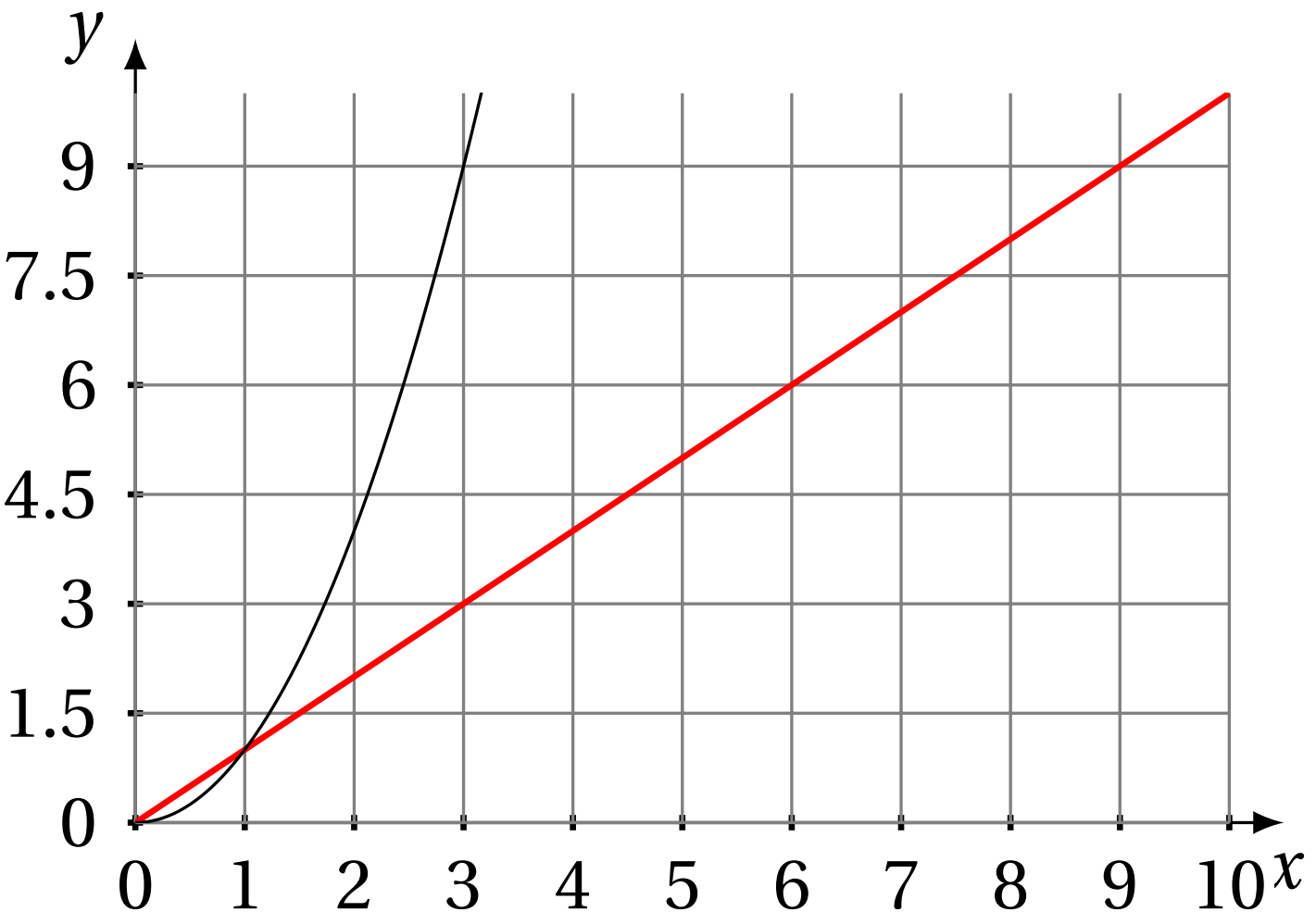

1
0.5
−0.5
−1
−90°
0°
90°

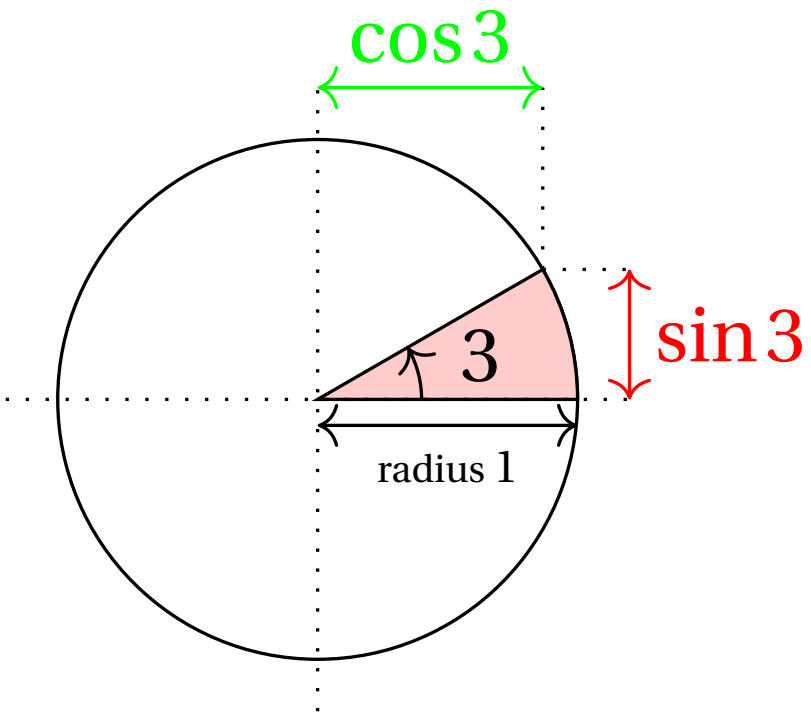
92 ./images-tikz/derivadas2.pgf


6.
4.
2.
−2.
−4.
0
1.
2.
3.
4
5.
6.

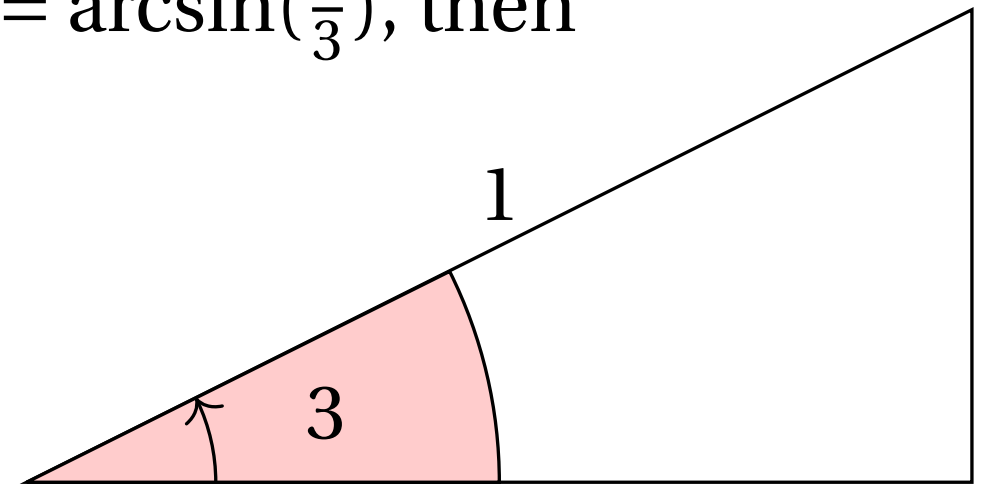
93 ./images-tikz/tikznew5.pgf


y
9
6
3
0
0
1
2
3
4
5
6
7
8
9
10
x


y
9
7.5
6
4.5
3
1.5
0
0
1
2
3
4
5
6
7
8
9
10
x


cos 3
sin 3
3
radius 1

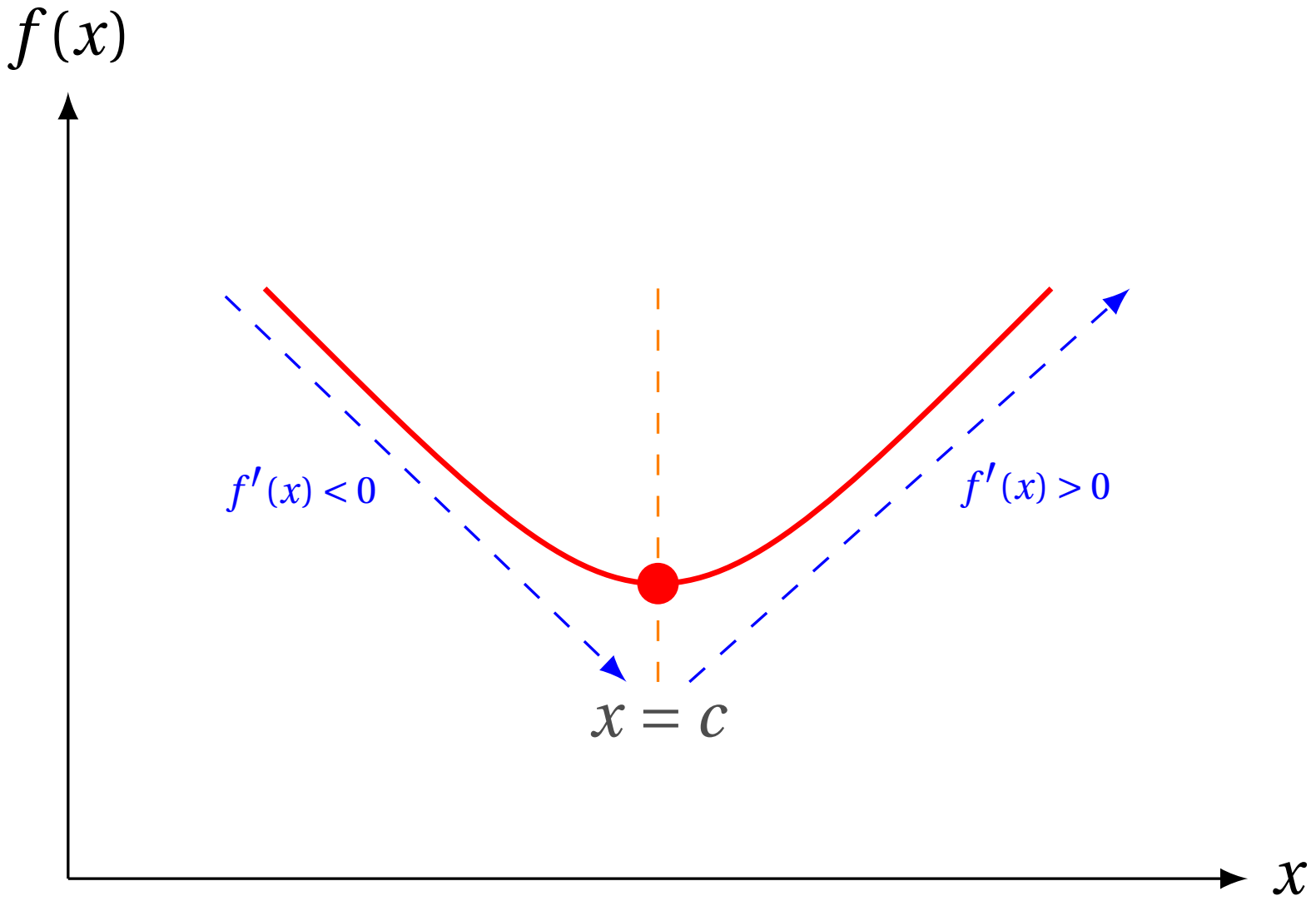
Let $3 = \arcsin(\frac{1}{3})$, then



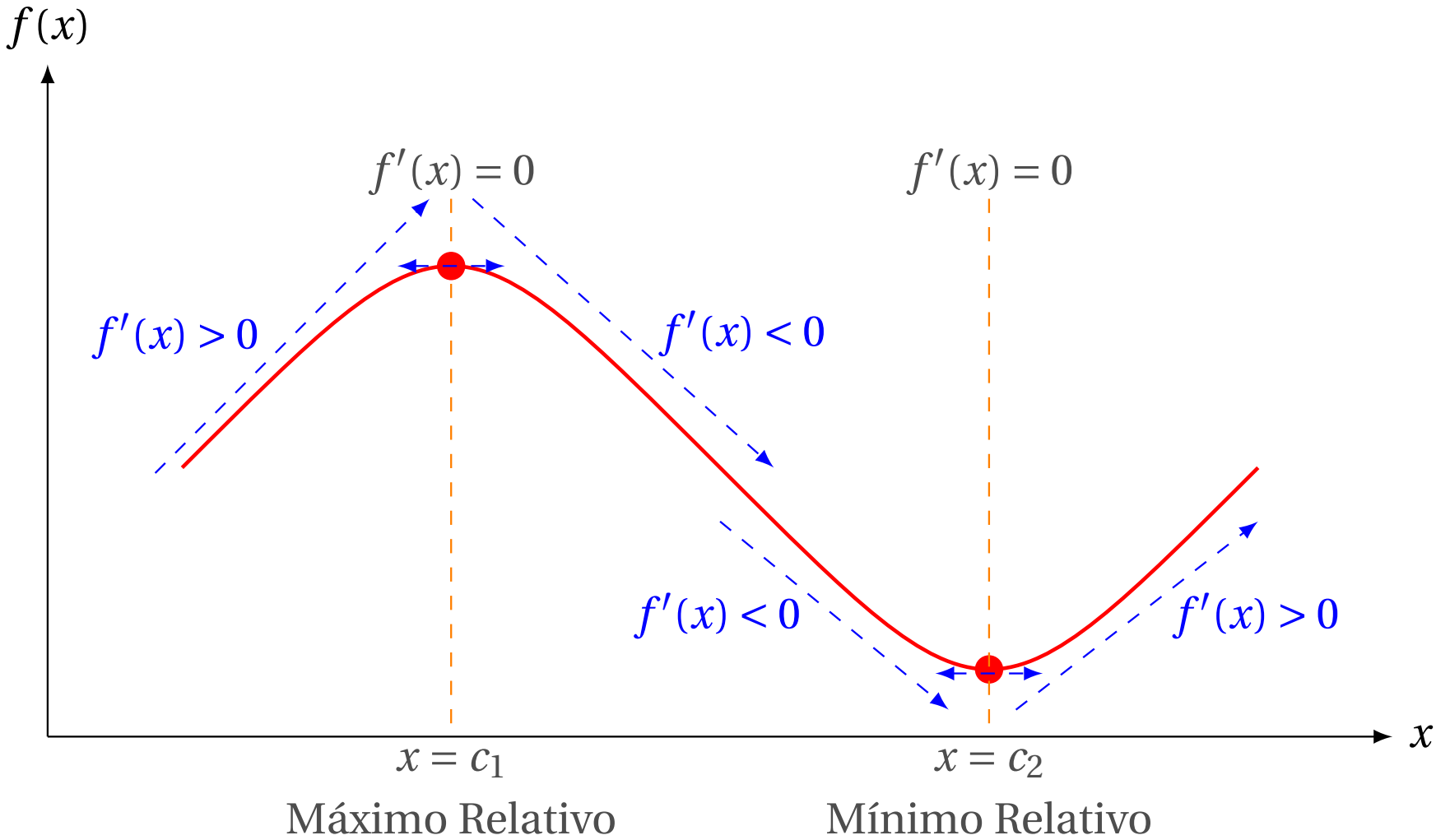
$$\sin 3 = \frac{1}{3}$$

$$\cos 3 = \sqrt{1 - (\frac{1}{3})^2} = \sqrt{\frac{8}{9}} = \frac{2}{3}\sqrt{2}$$

95 ./images-tikz/extremos.pgf

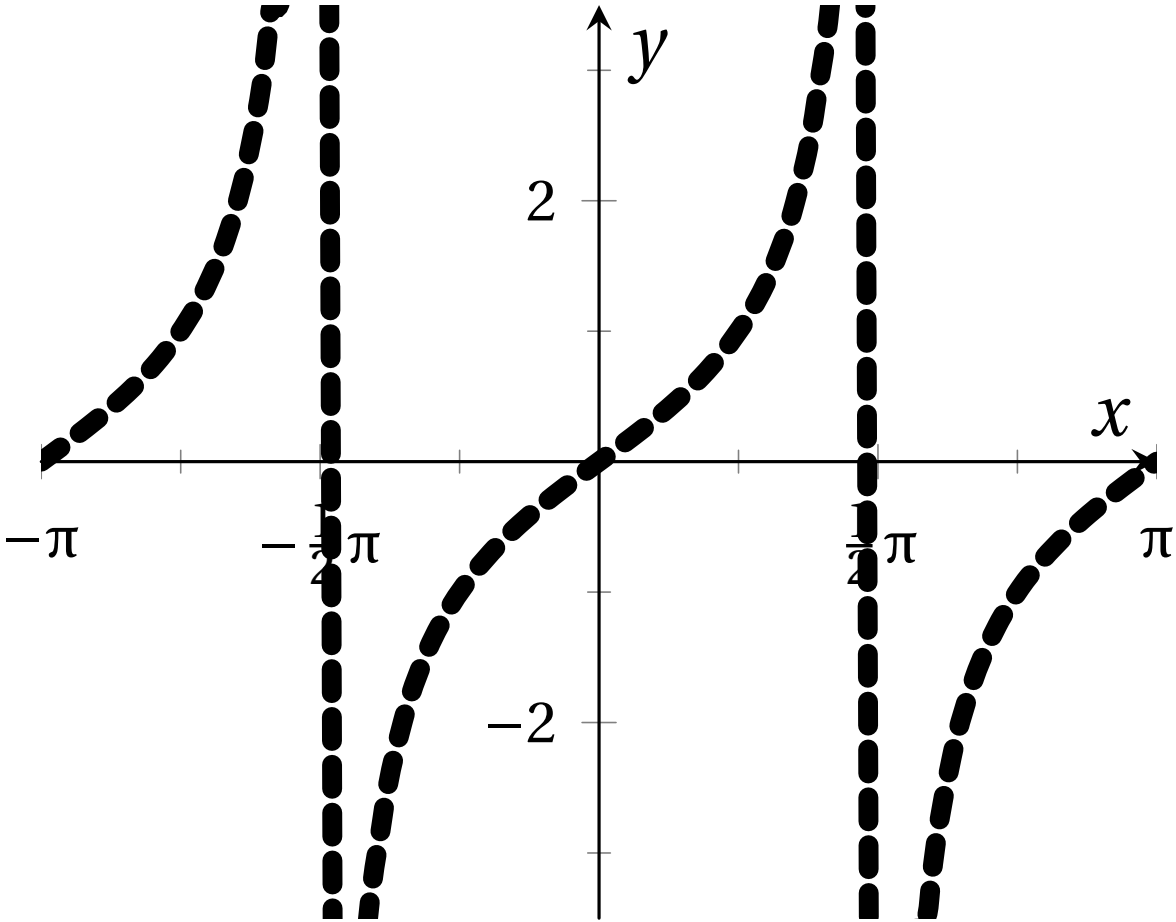

$f(x)$
$f'(x) < 0$
$f'(x) > 0$
$x = c$
x

./images-tikz/extremos-b.pgf

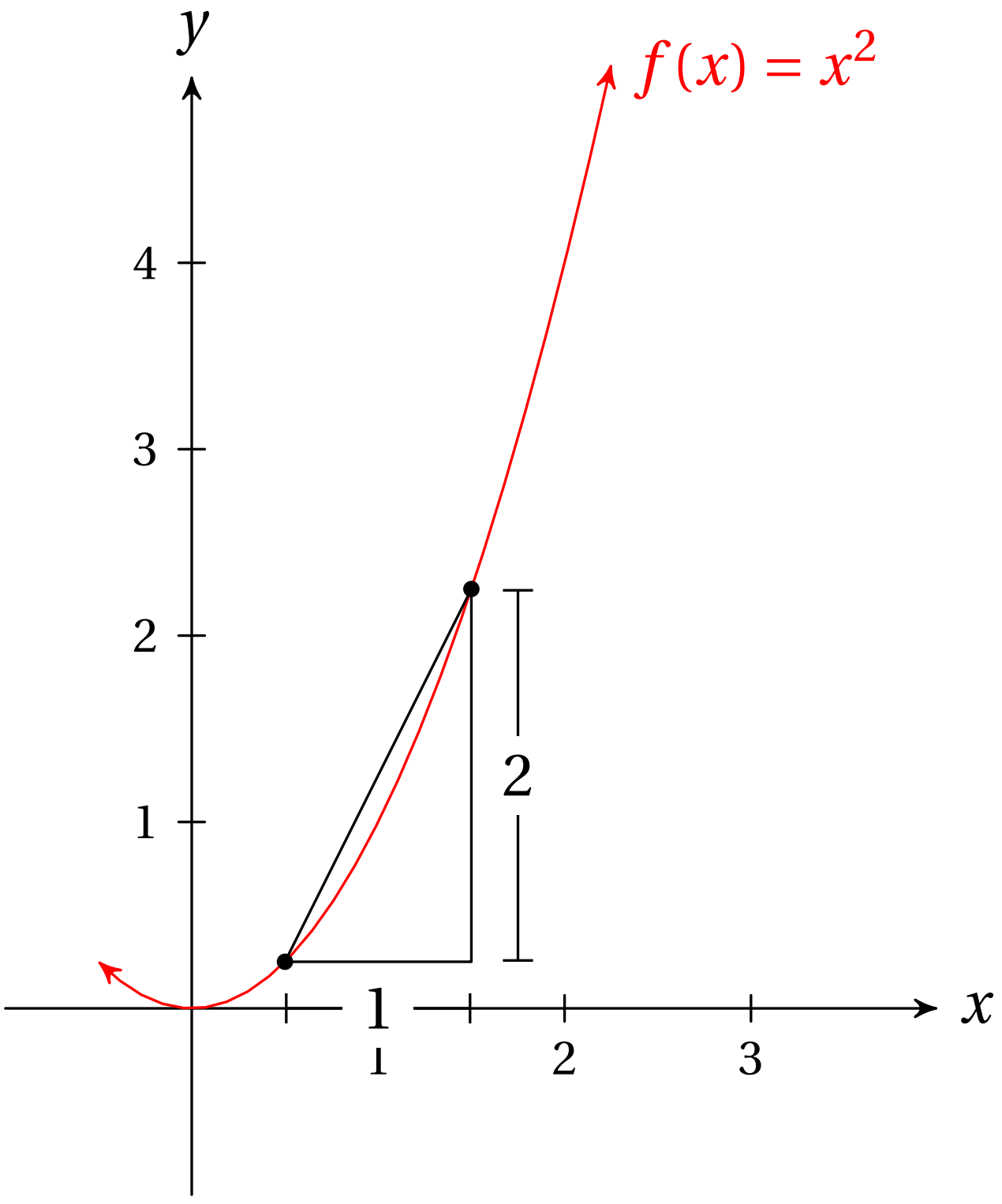

$f(x)$
$f'(x) = 0$
$f'(x) = 0$
$f'(x) > 0$
$f'(x) < 0$
$f'(x) < 0$
$f'(x) > 0$
x
$x = c_1$
$x = c_2$
Máximo Relativo
Mínimo Relativo

97 ./images-tikz/trig3.pgf

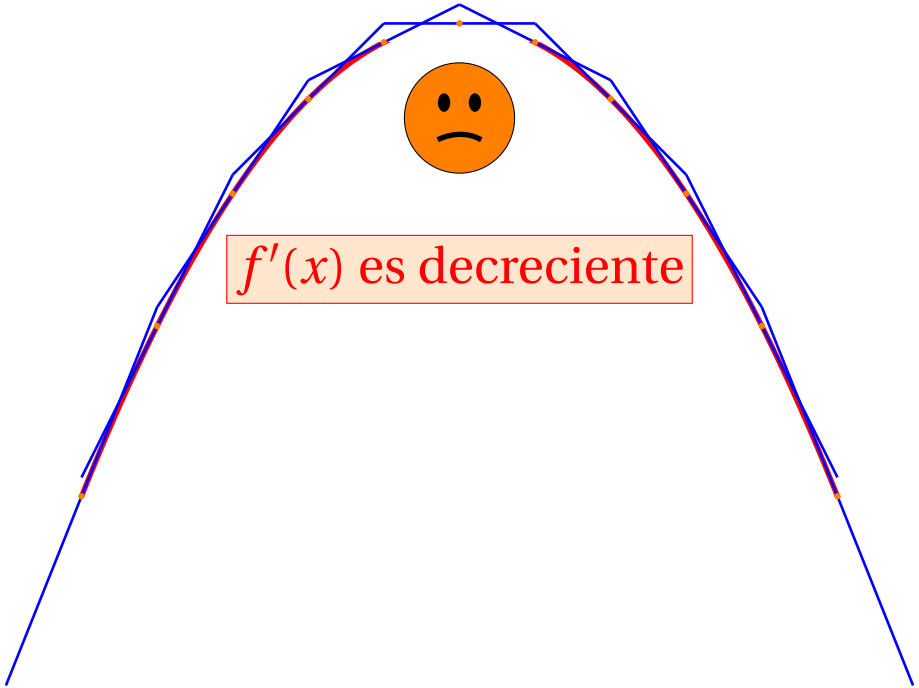
Trimmed bounding boxes



98 **./images-tikz/image$_3$6.pgf**

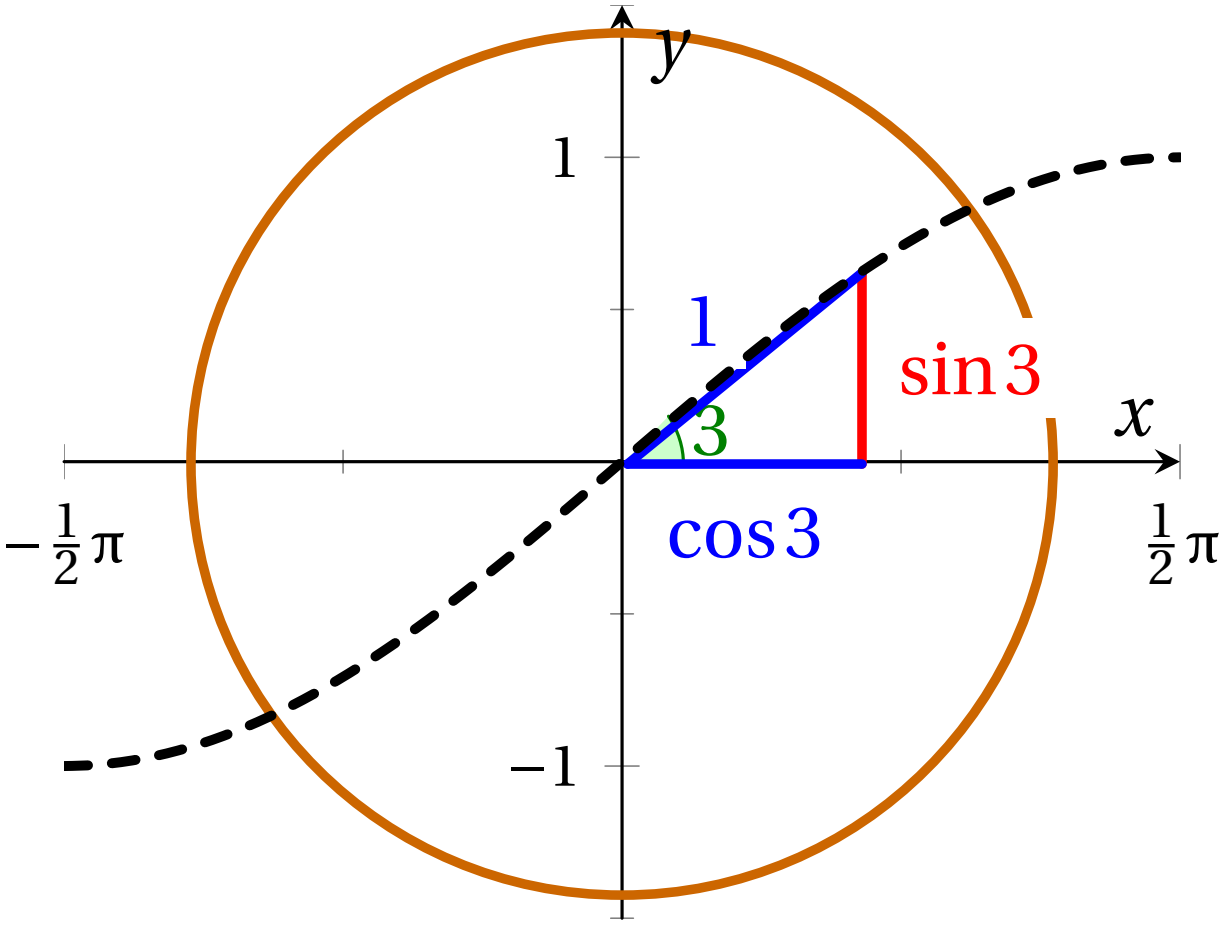

y
x
f(x) = x^2
4
3
2
1
2
1
1
2
3

99 ./images-tikz/concavidad-b.pgf

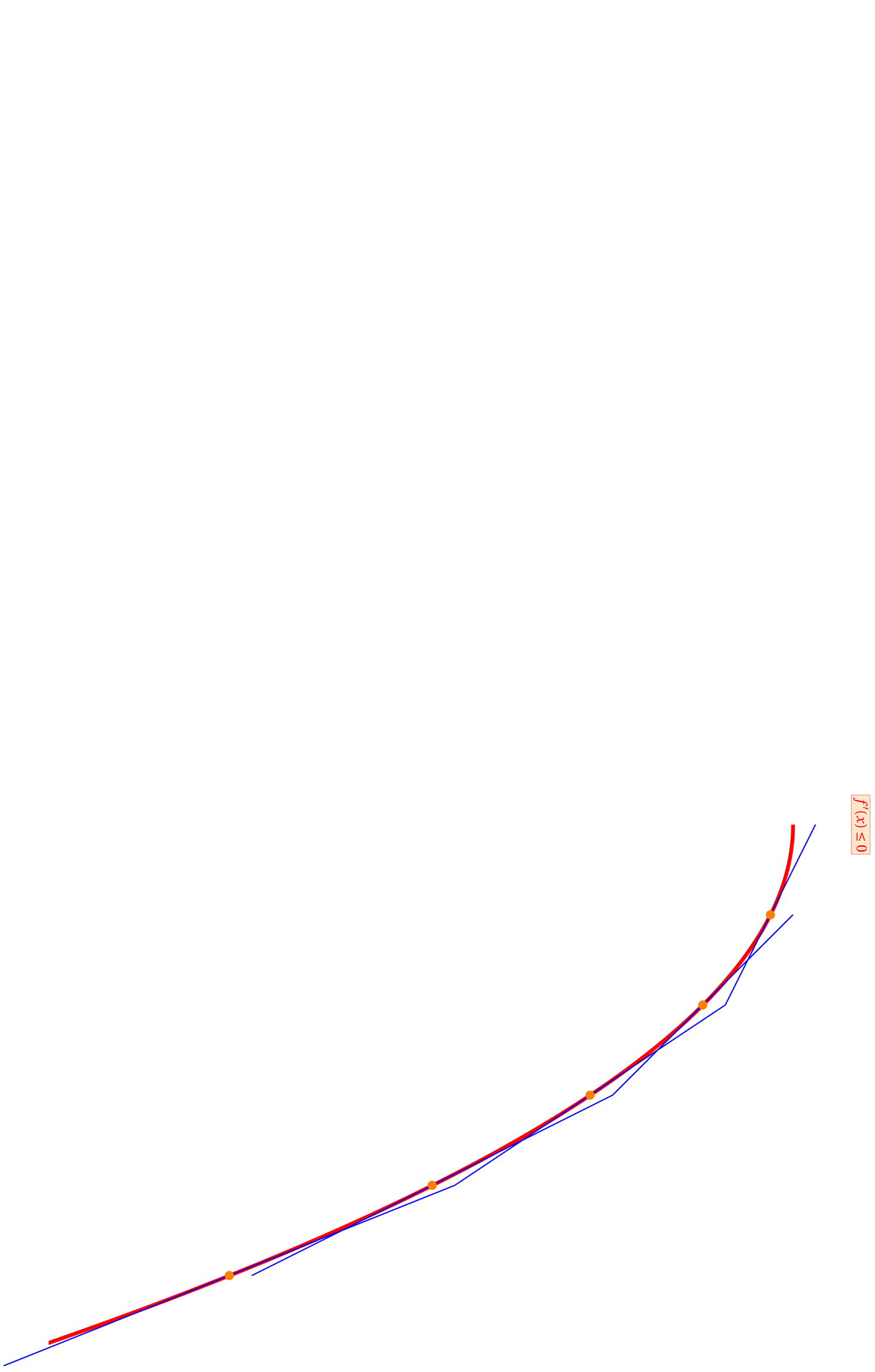

$f'(x)$ es decreciente

100 ./images-tikz/trig.pgf

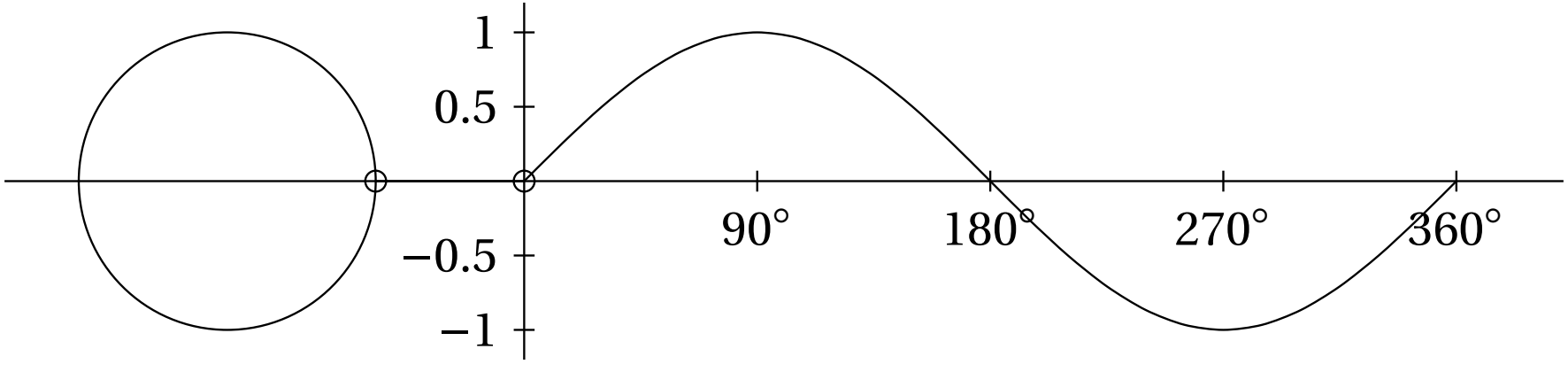
Trimmed bounding boxes

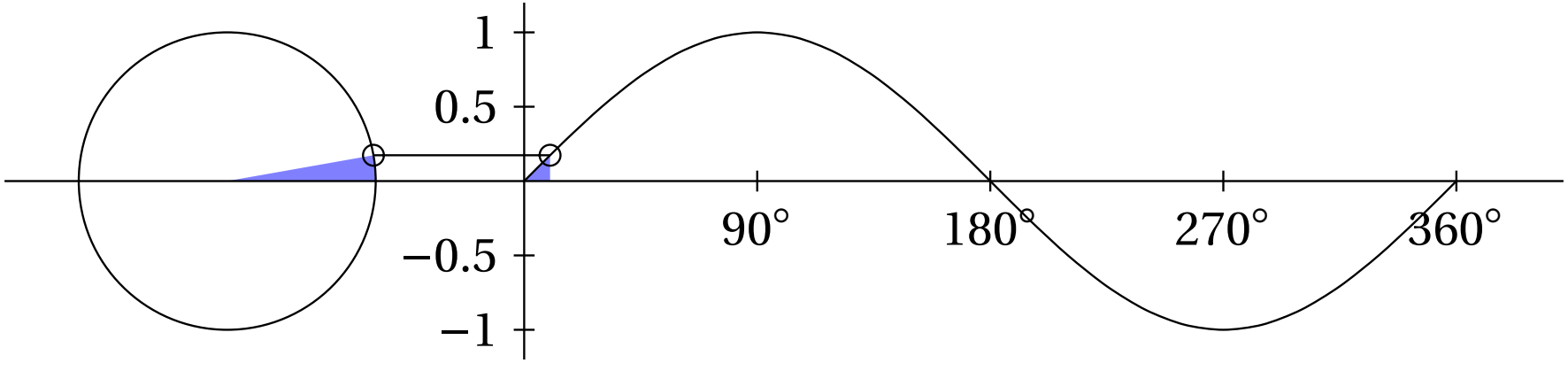


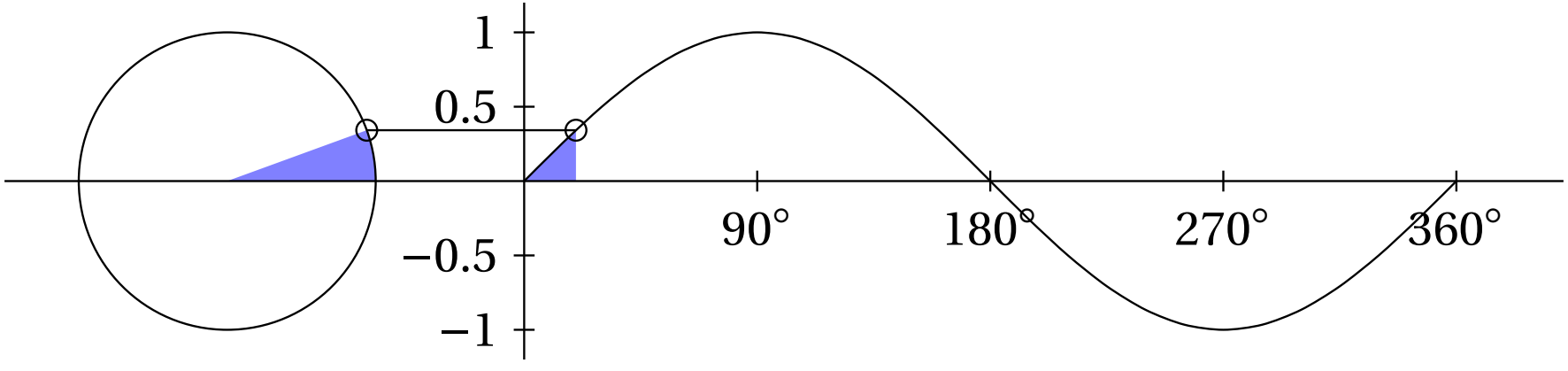
101 ./images-tikz/creciente-b.pgf

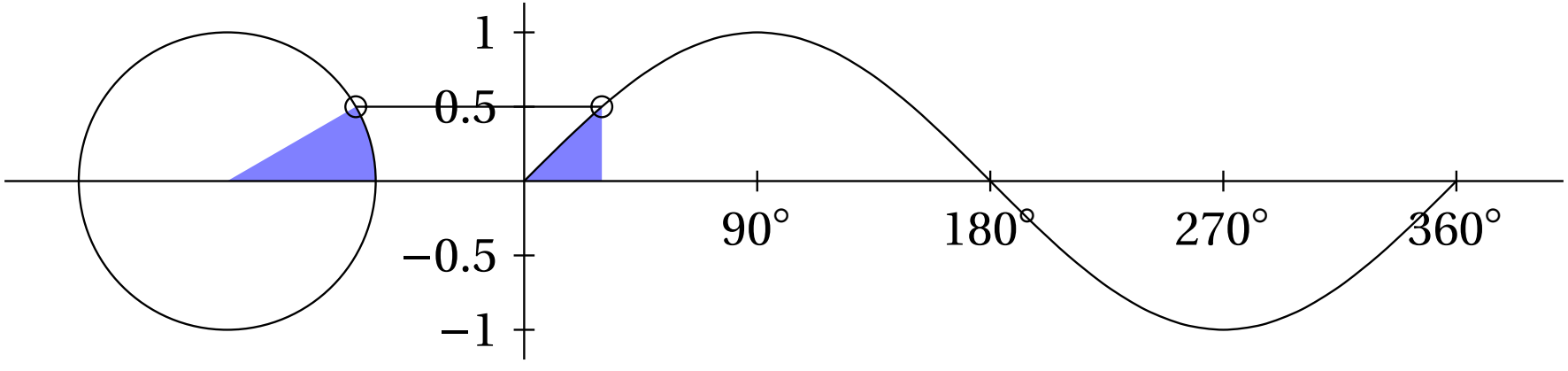

$f'(x) \leq 0$

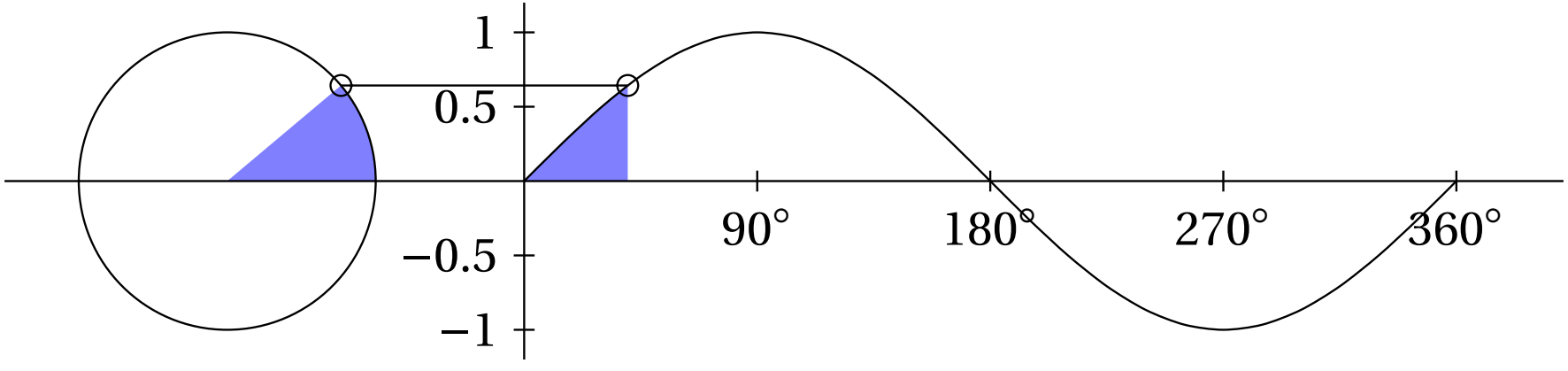
102 ./images-tikz/sin.pgf

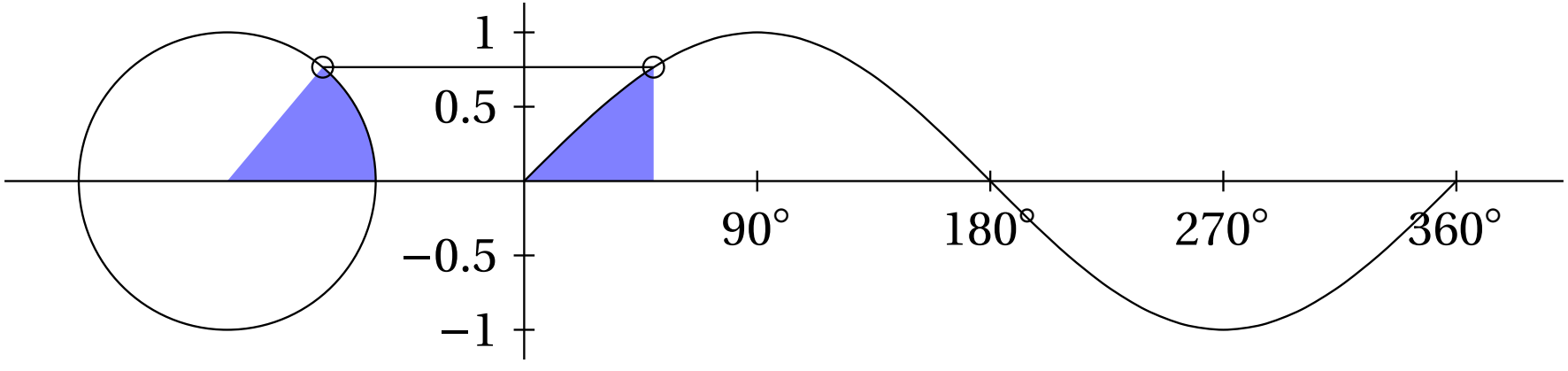

1
0.5
−0.5
−1
90°
180°
270°
360°

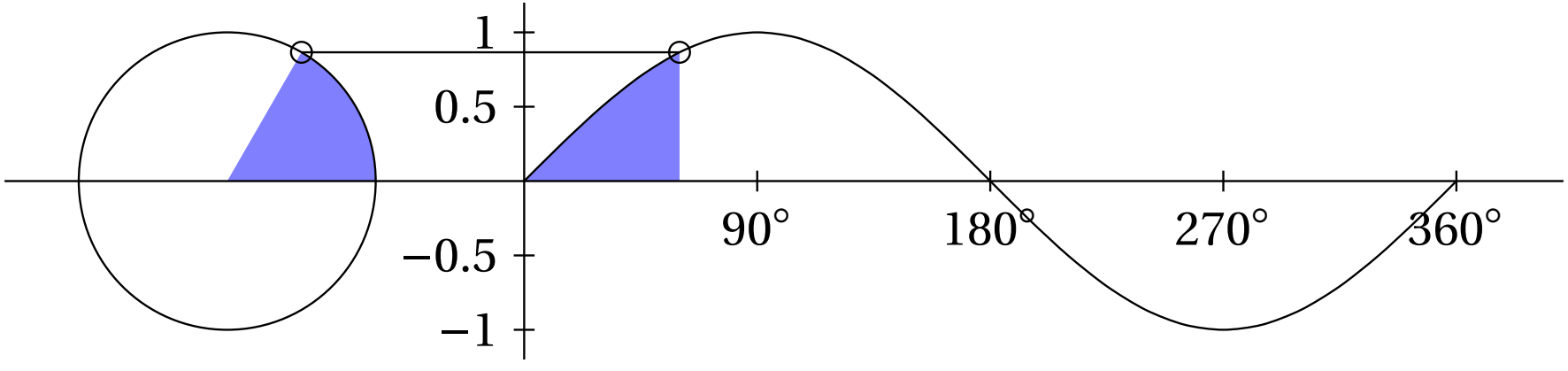

1
0.5
−0.5
−1
90°
180°
270°
360°

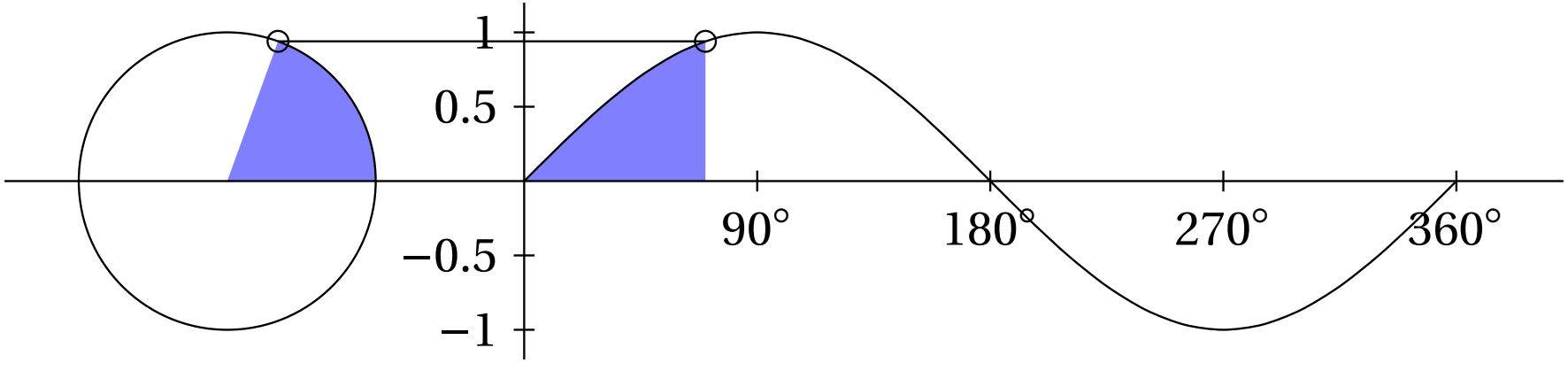

1
0.5
−0.5
−1
90°
180°
270°
360°

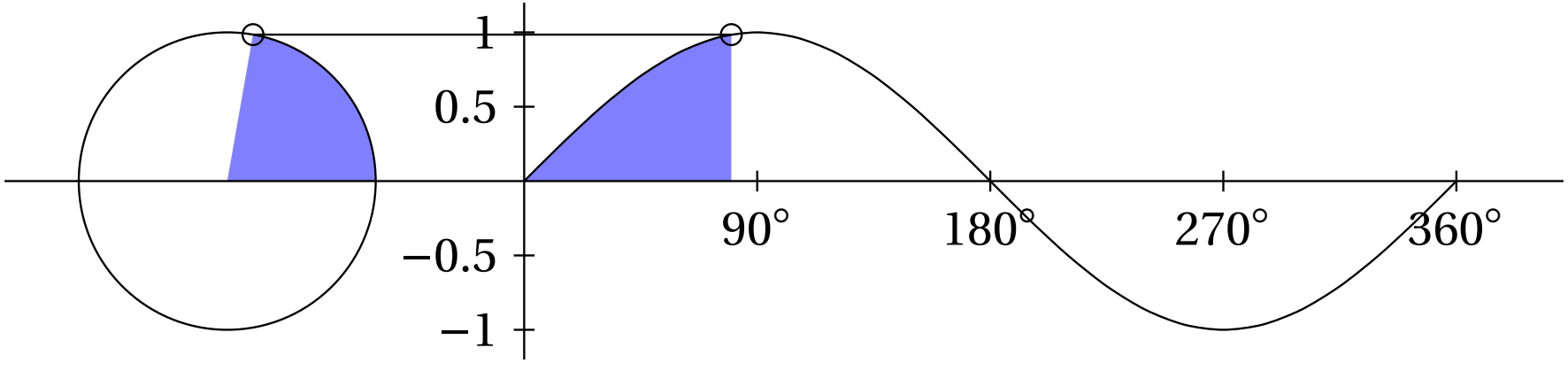

1
0.5
−0.5
−1
90°
180°
270°
360°


1
0.5
−0.5
−1
90°
180°
270°
360°


1
0.5
−0.5
−1
90°
180°
270°
360°


1
0.5
−0.5
−1
90°
180°
270°
360°


1
0.5
−0.5
−1
90°
180°
270°
360°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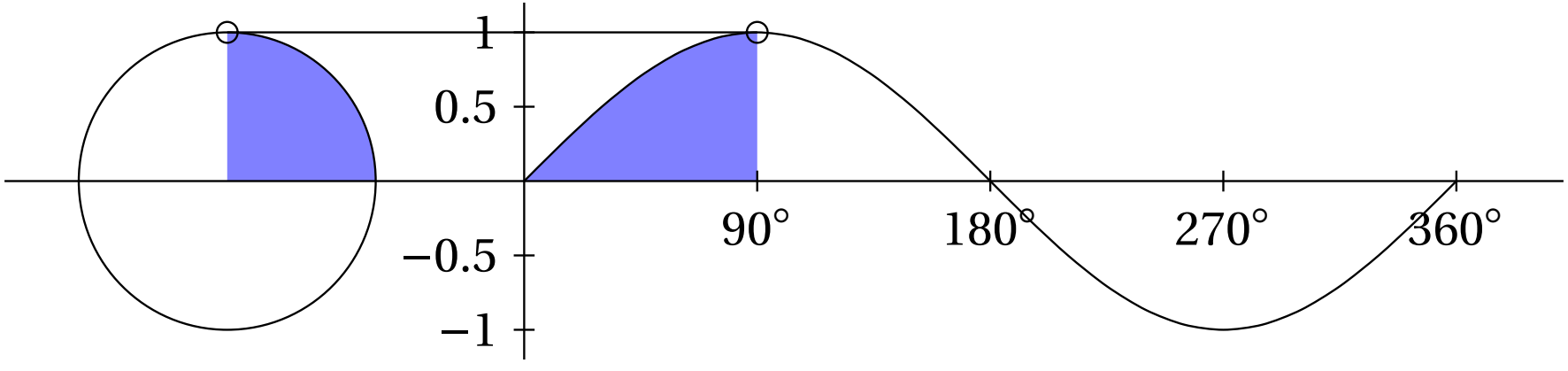
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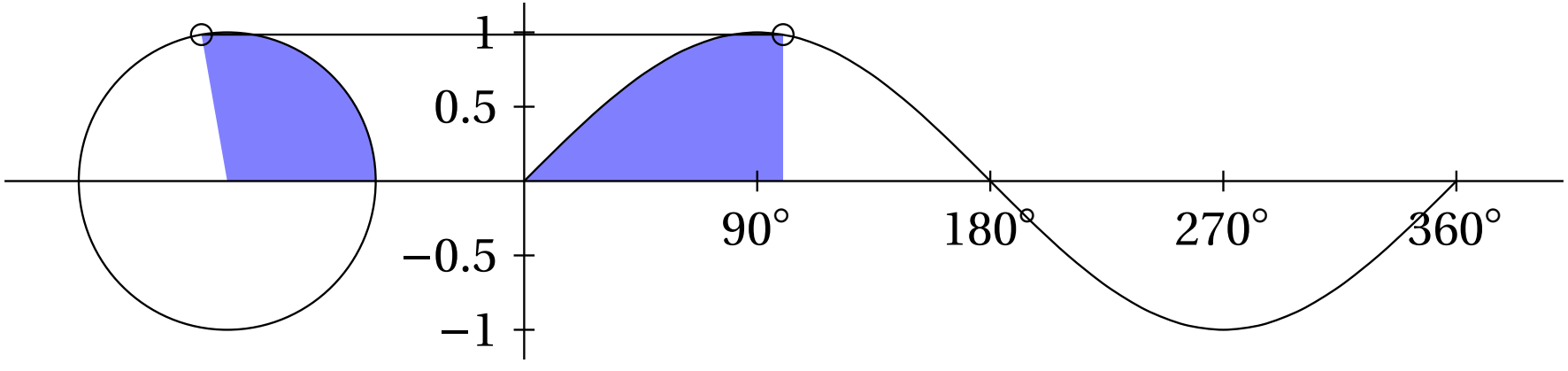
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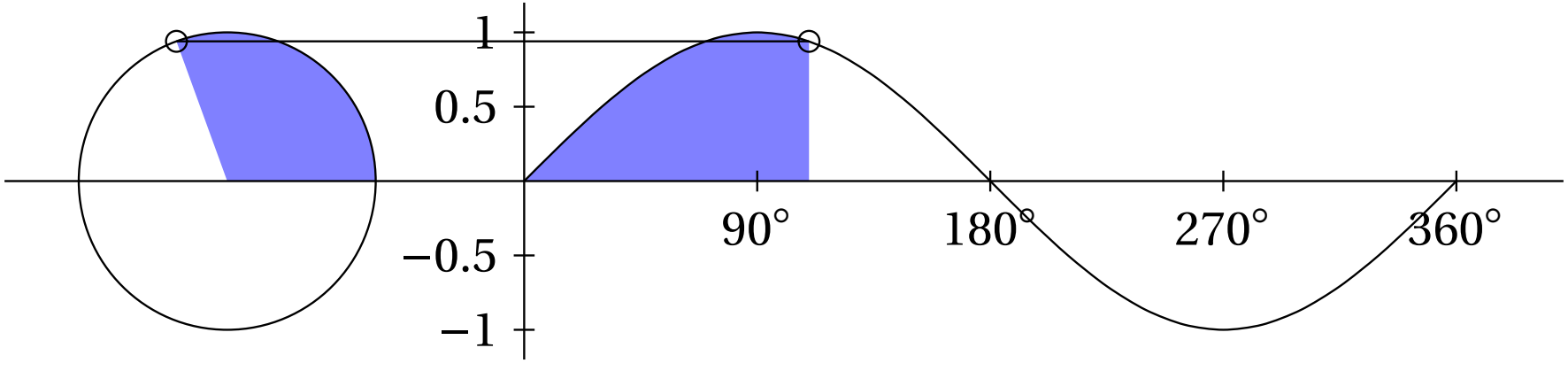
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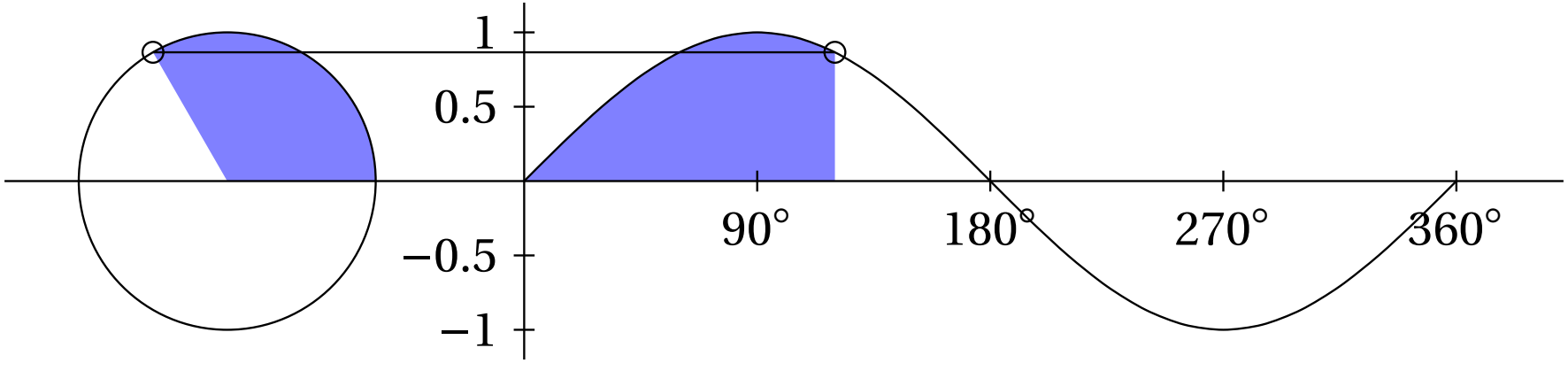
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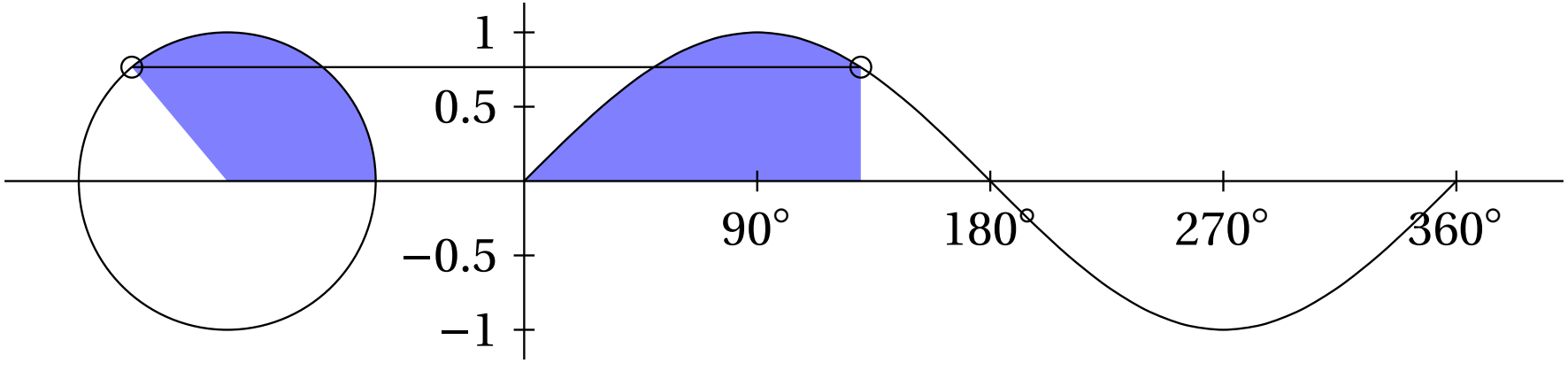
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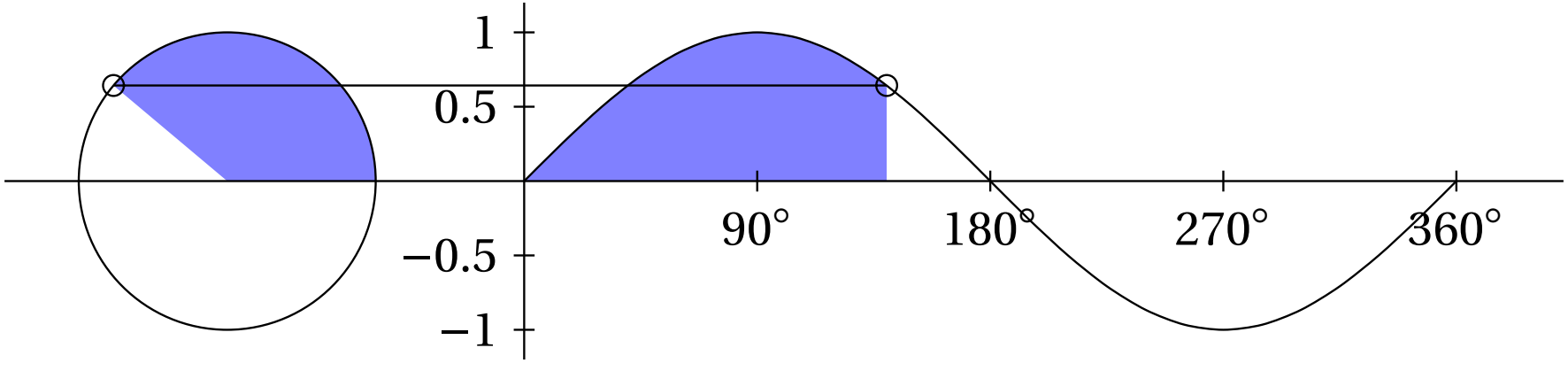
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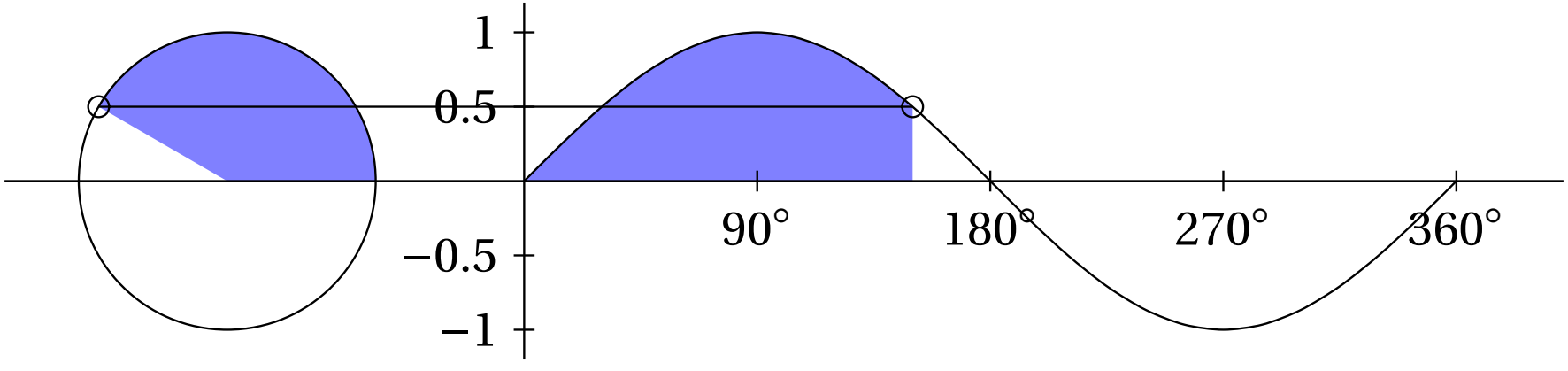
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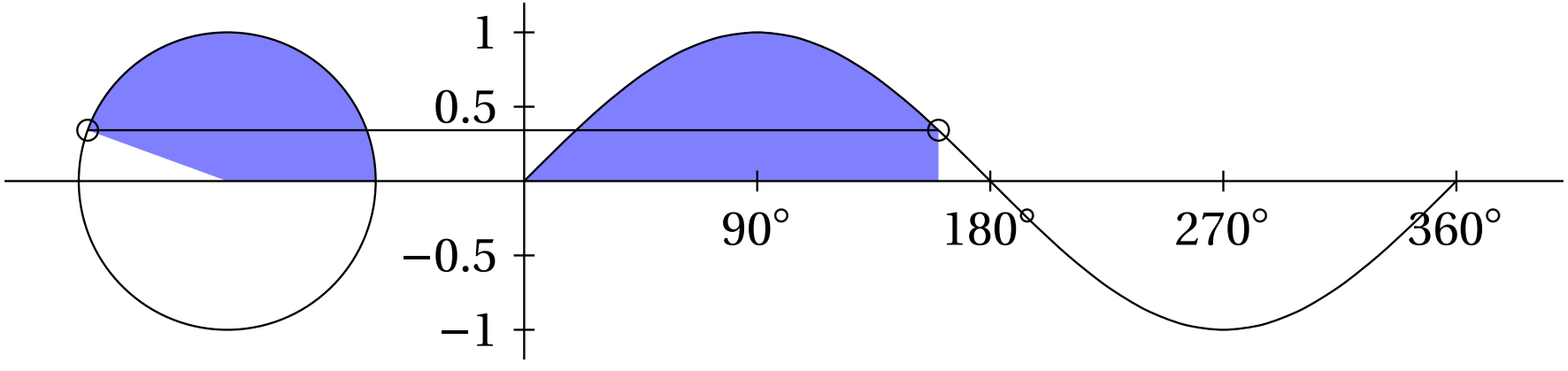
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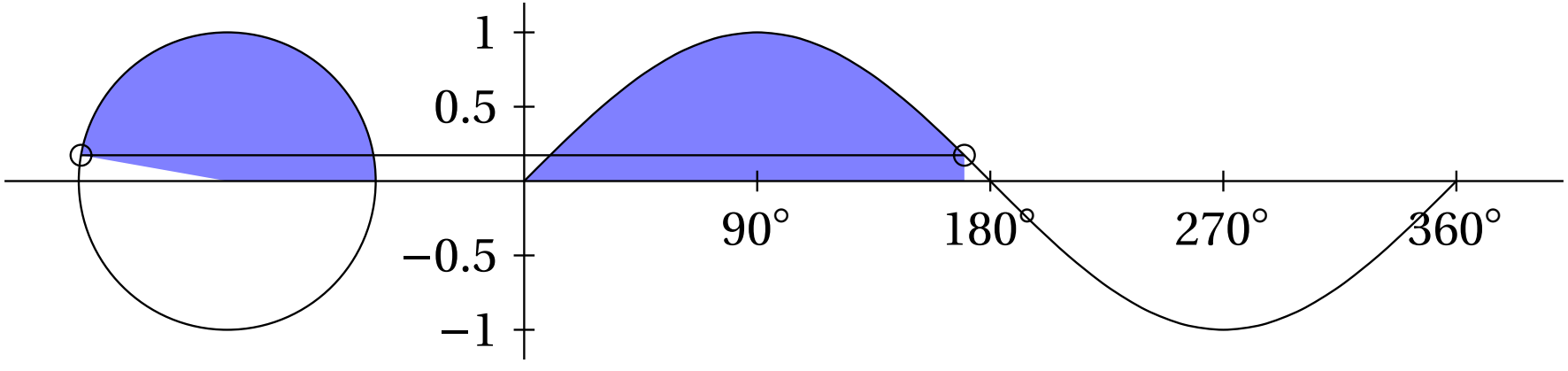
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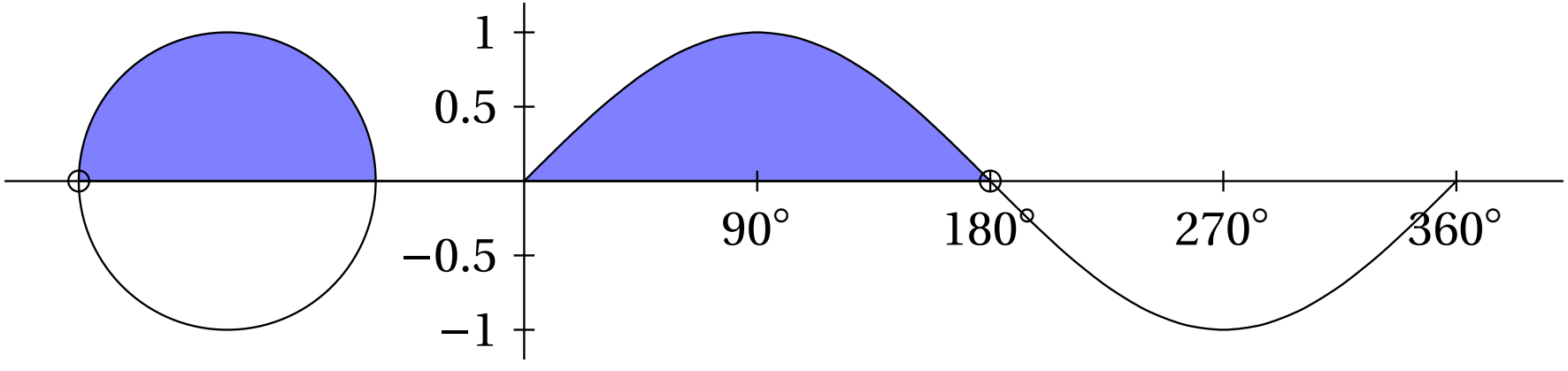
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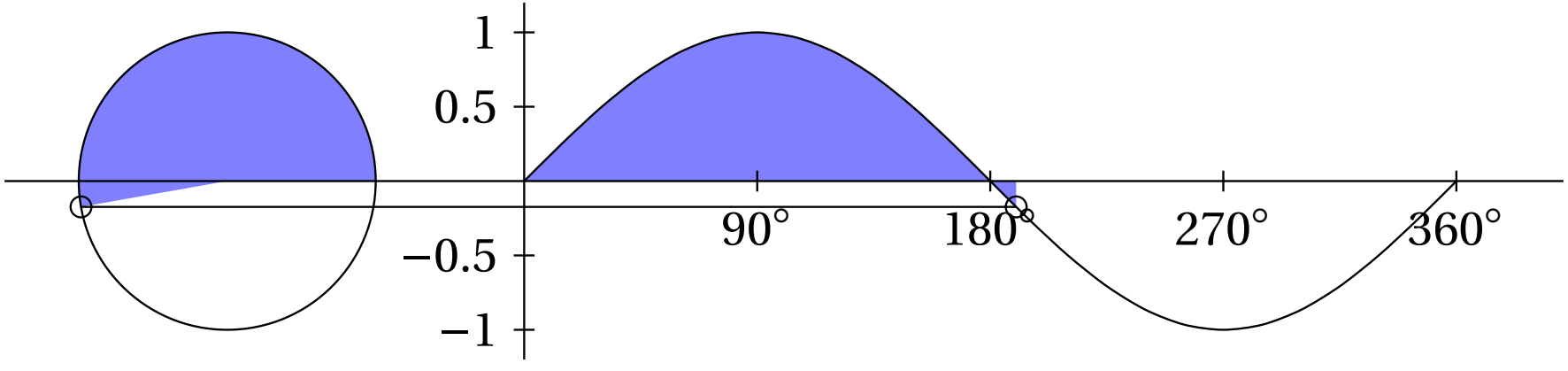
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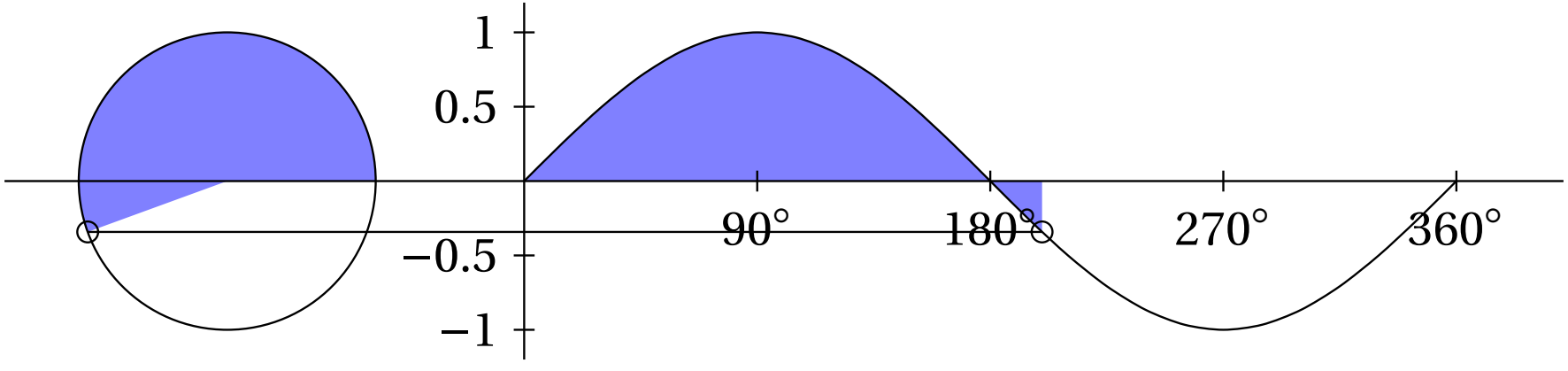
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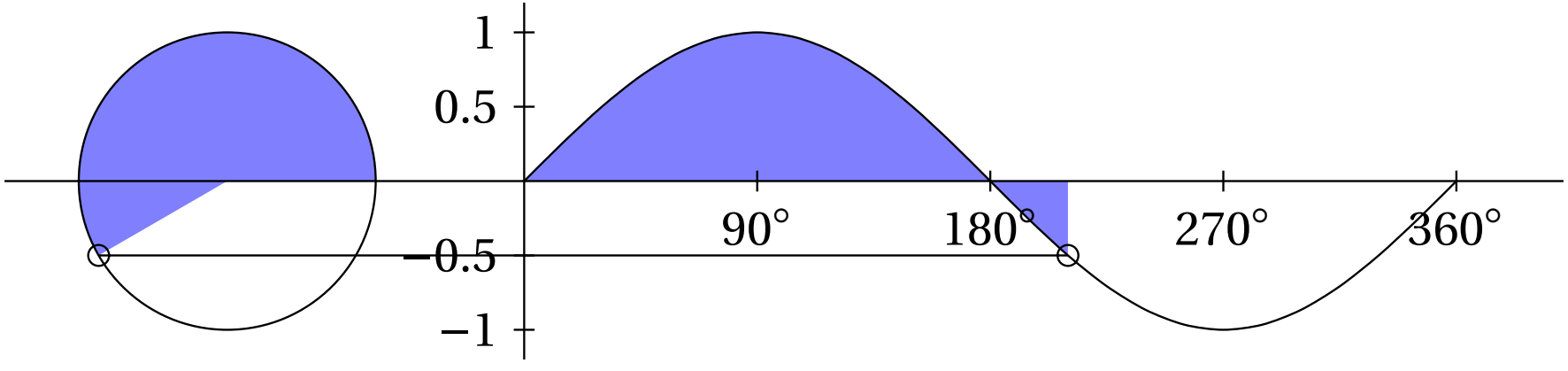
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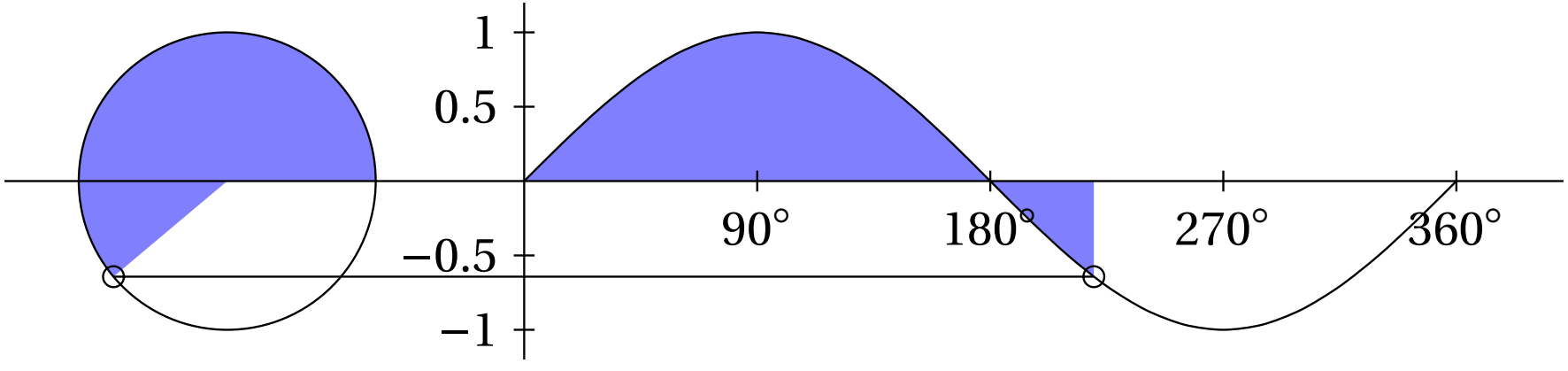
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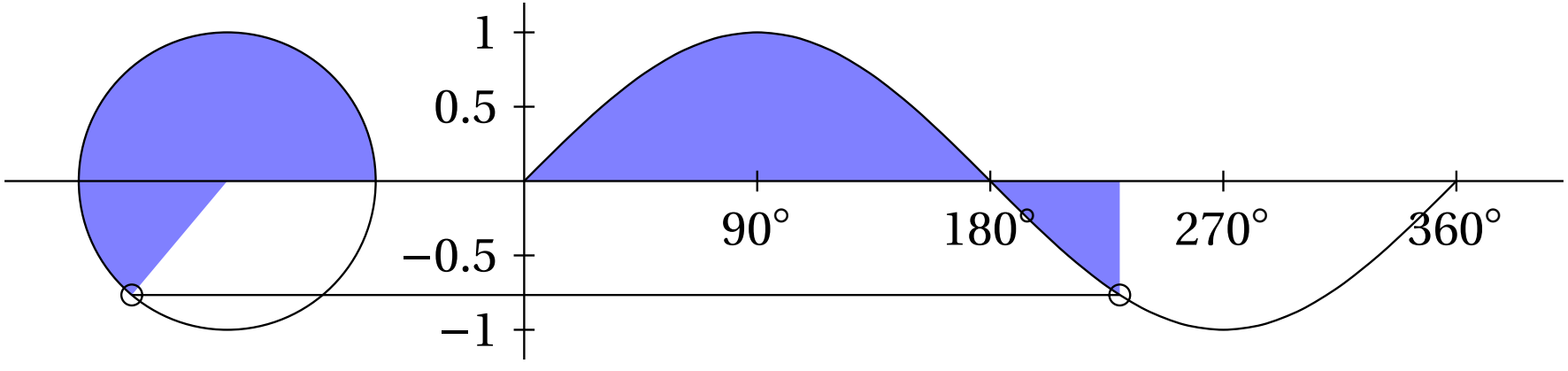
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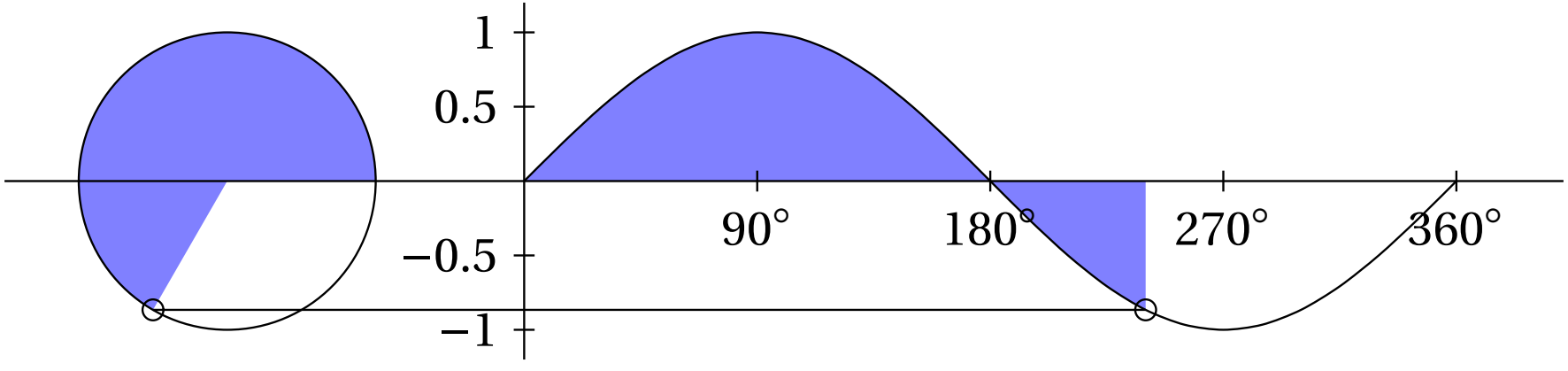
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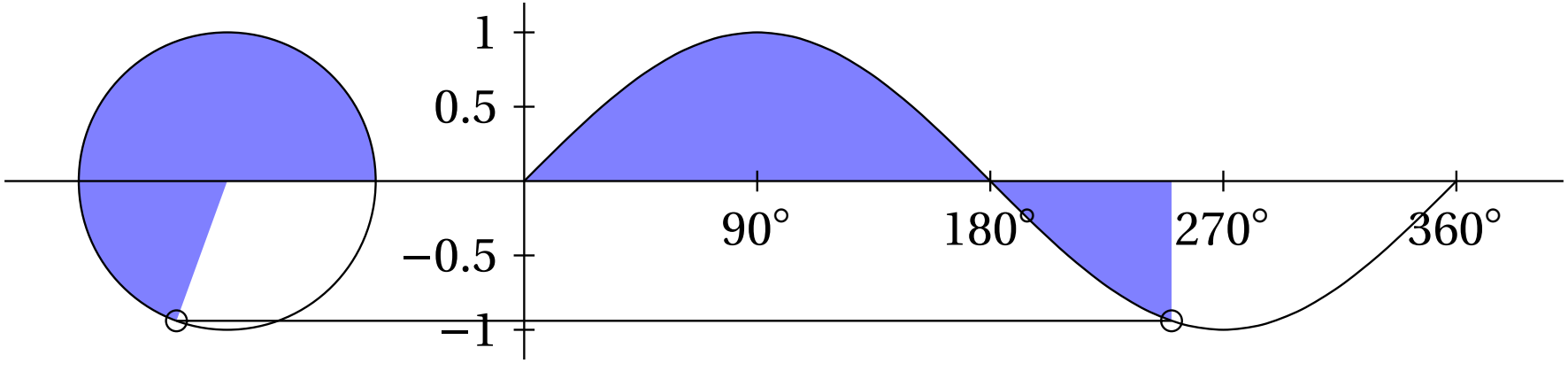
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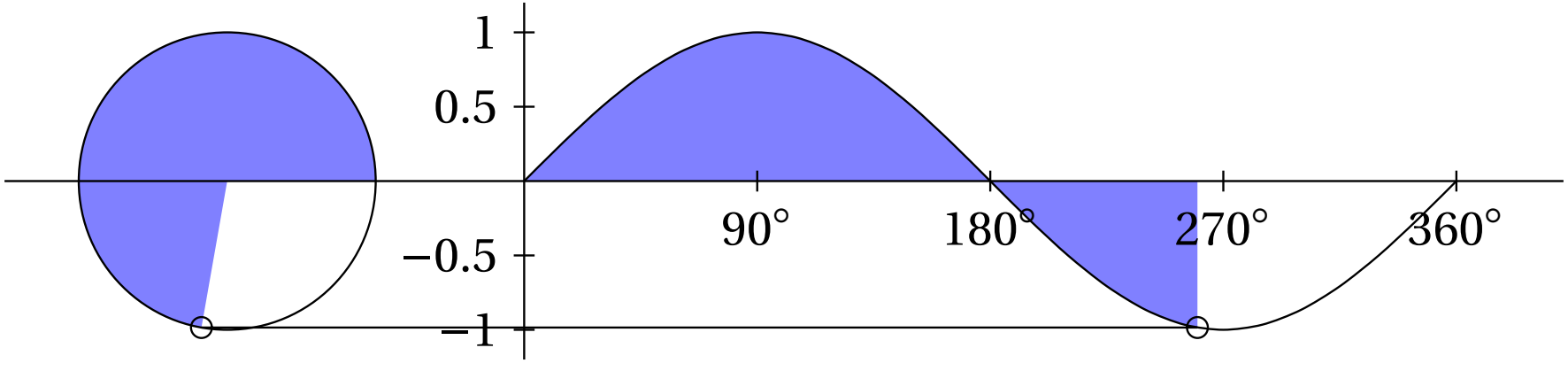
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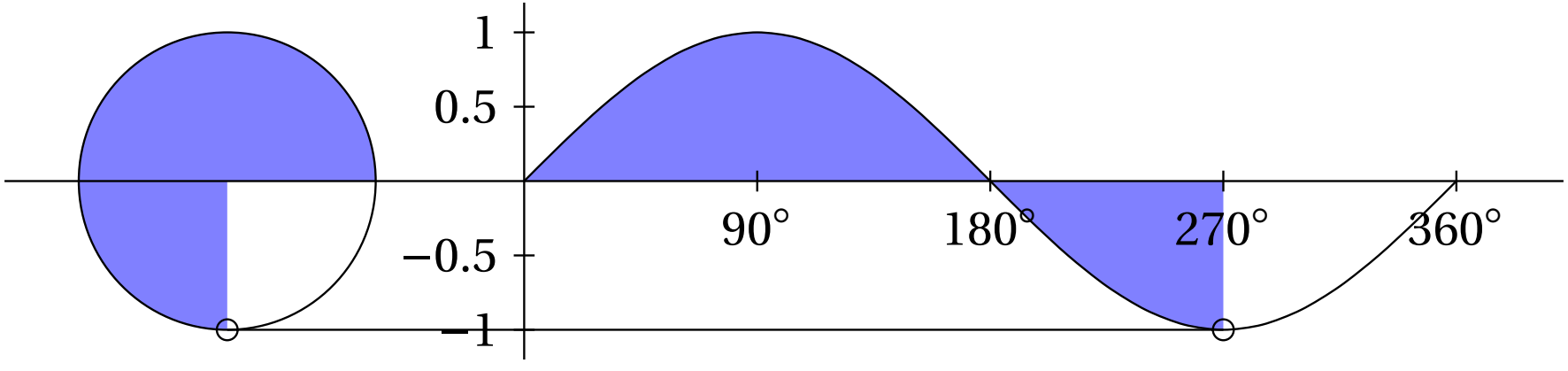
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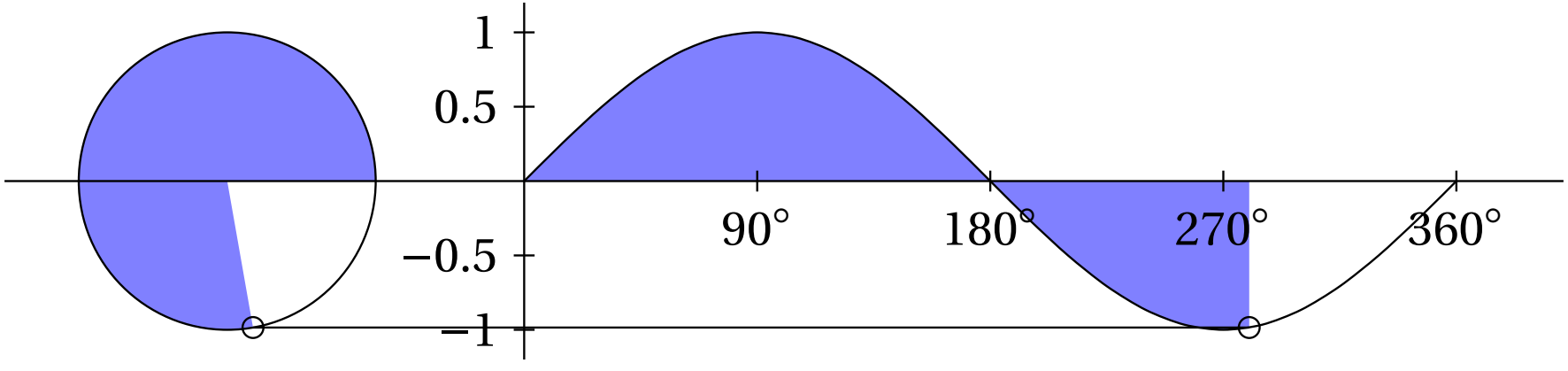
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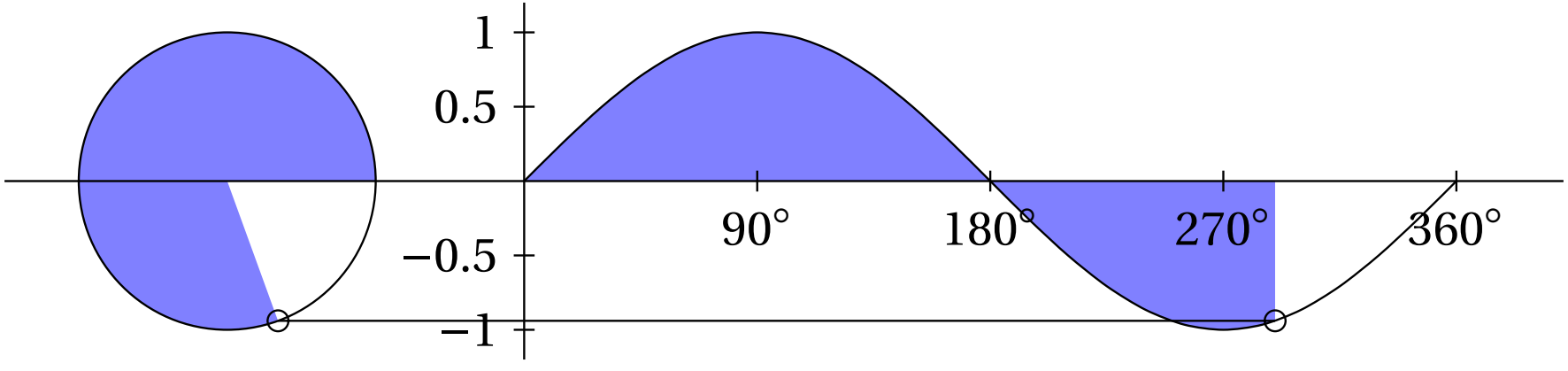
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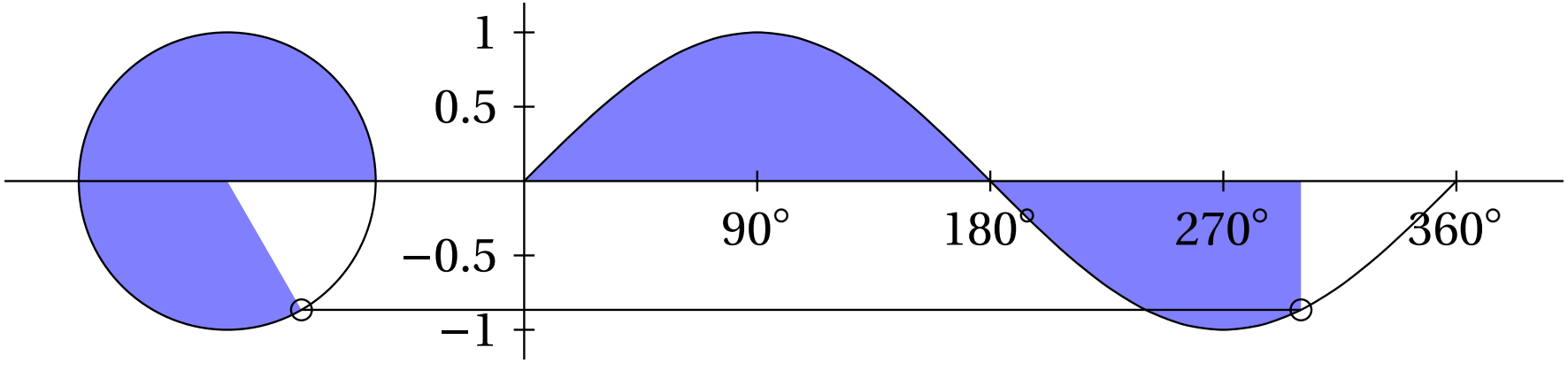
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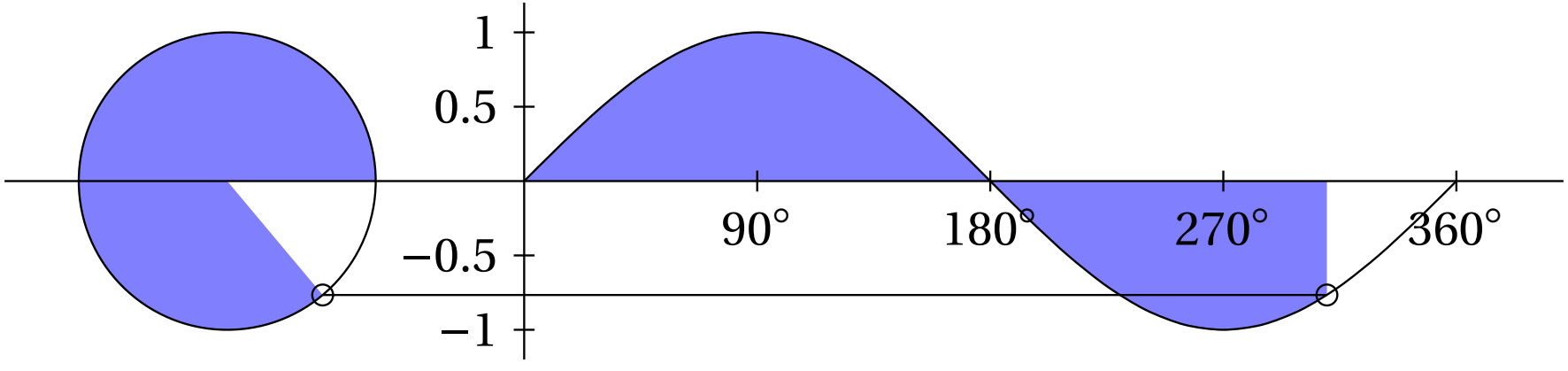
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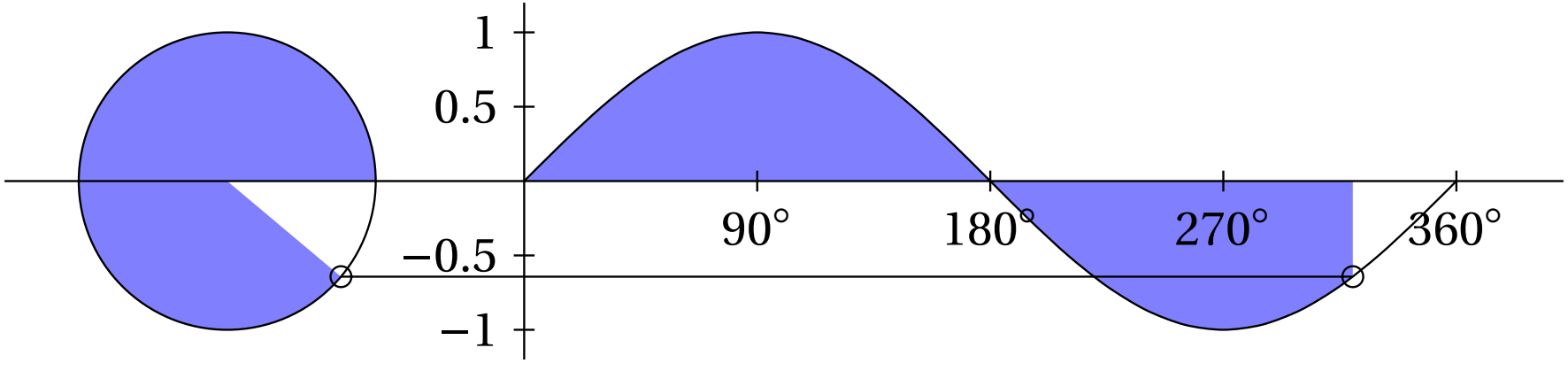
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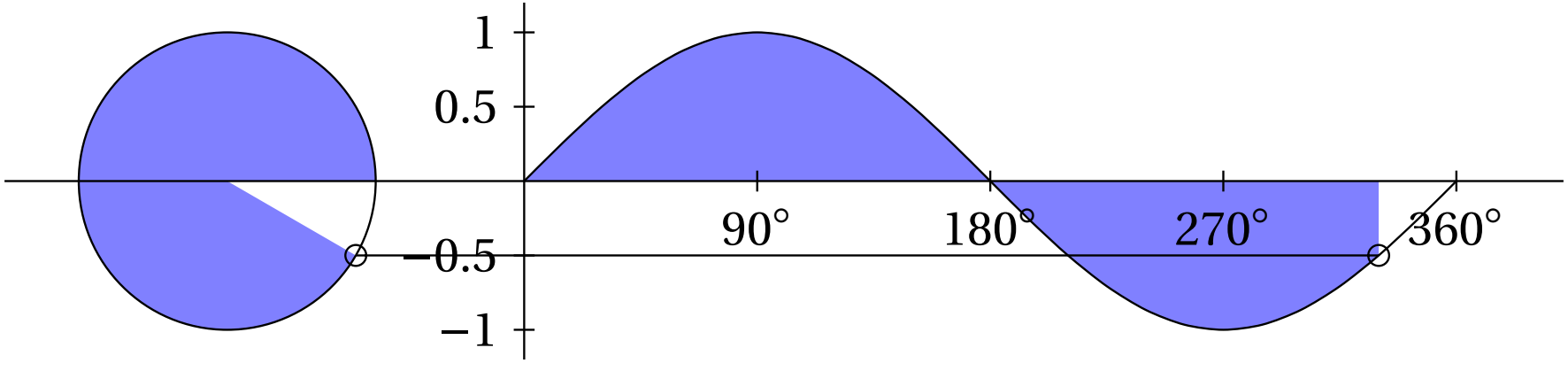
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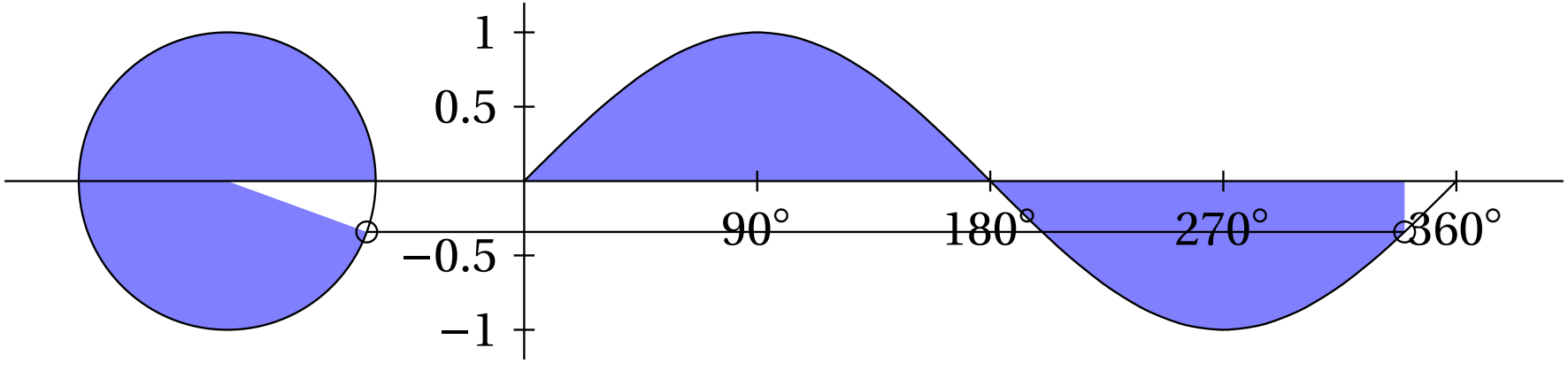
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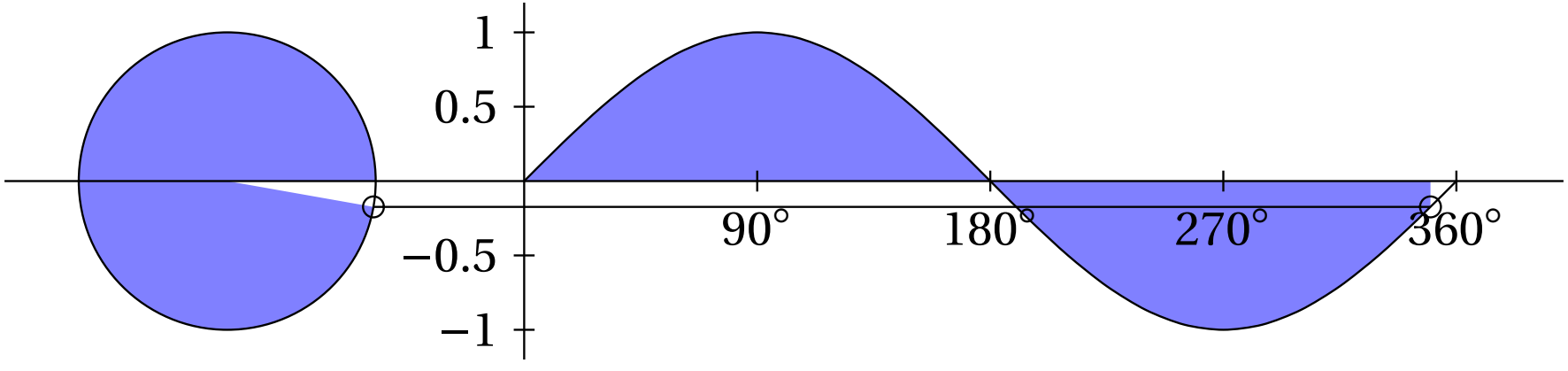
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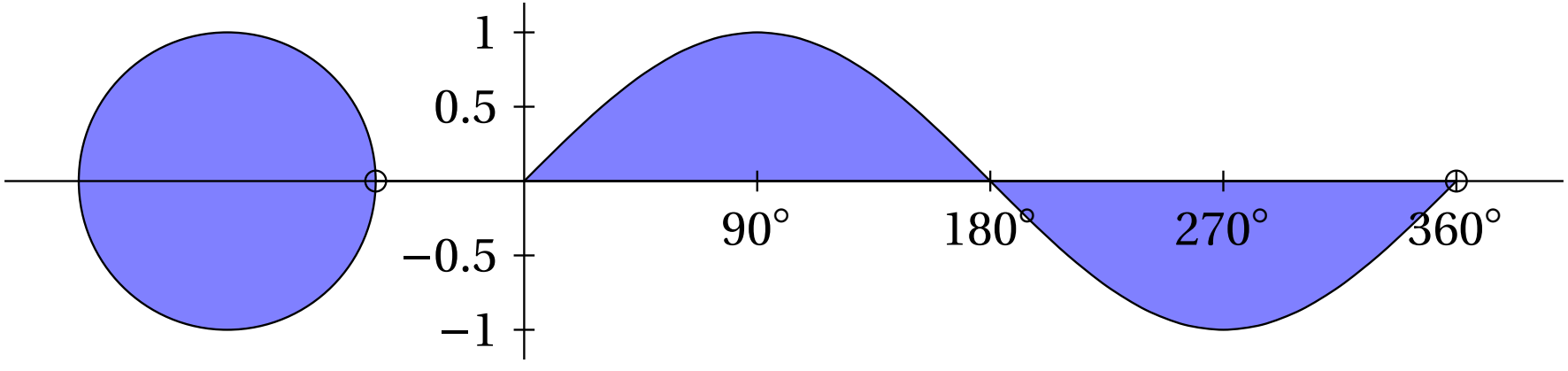
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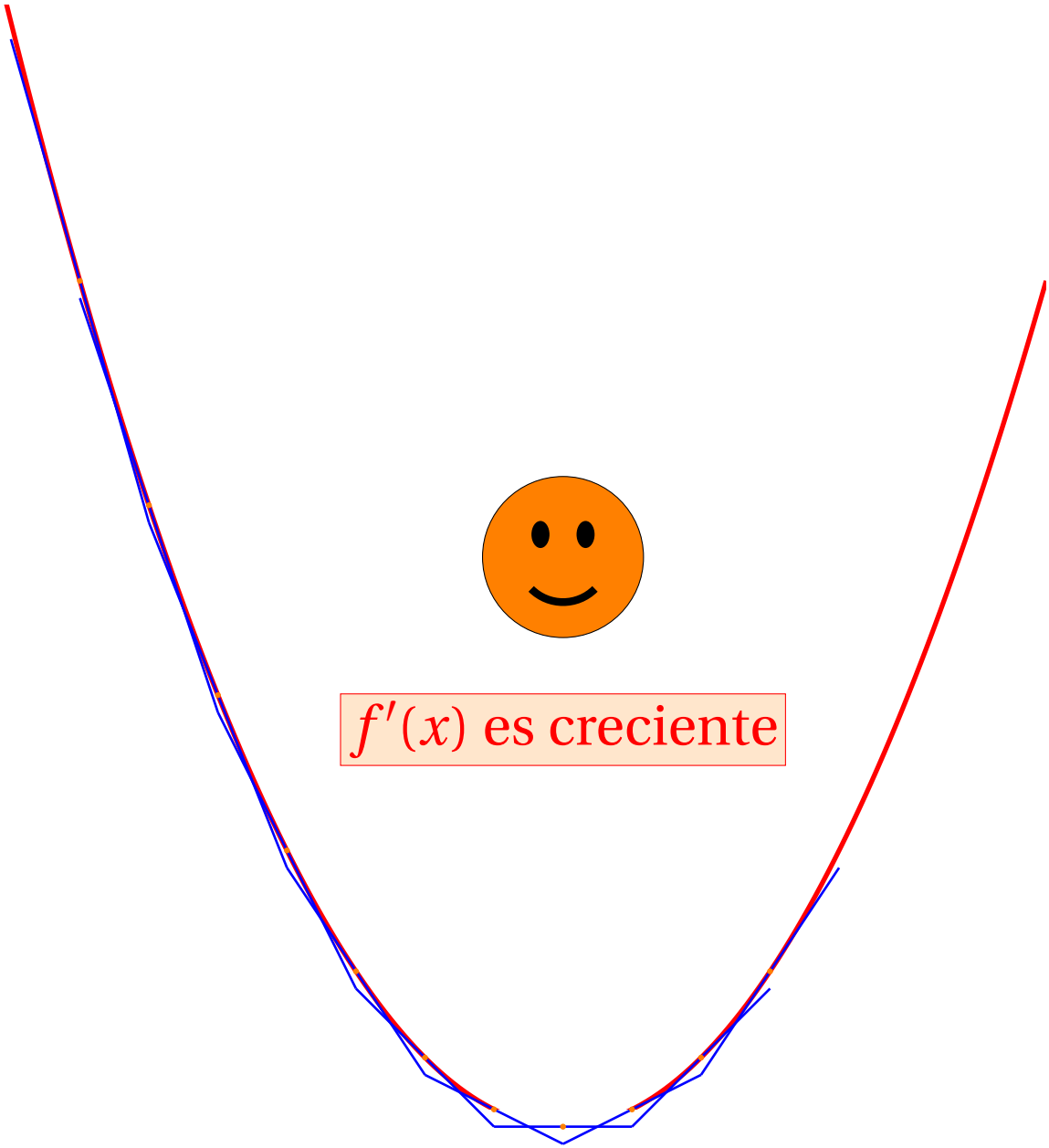
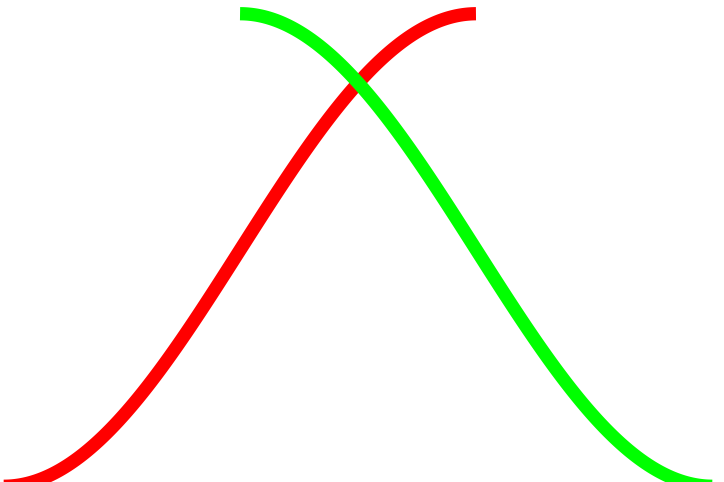
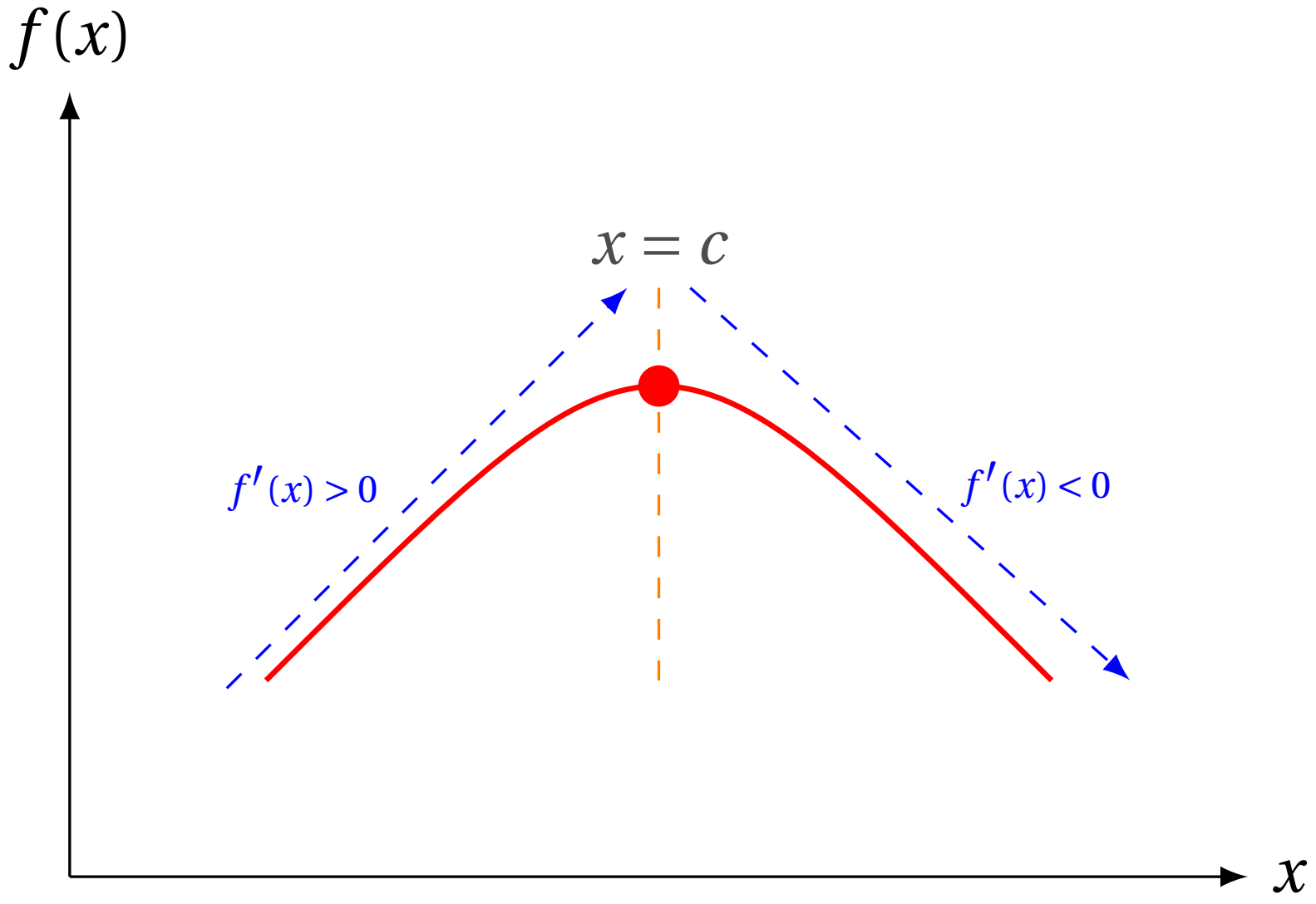
103 ./images-tikz/concavidad-a.pgf

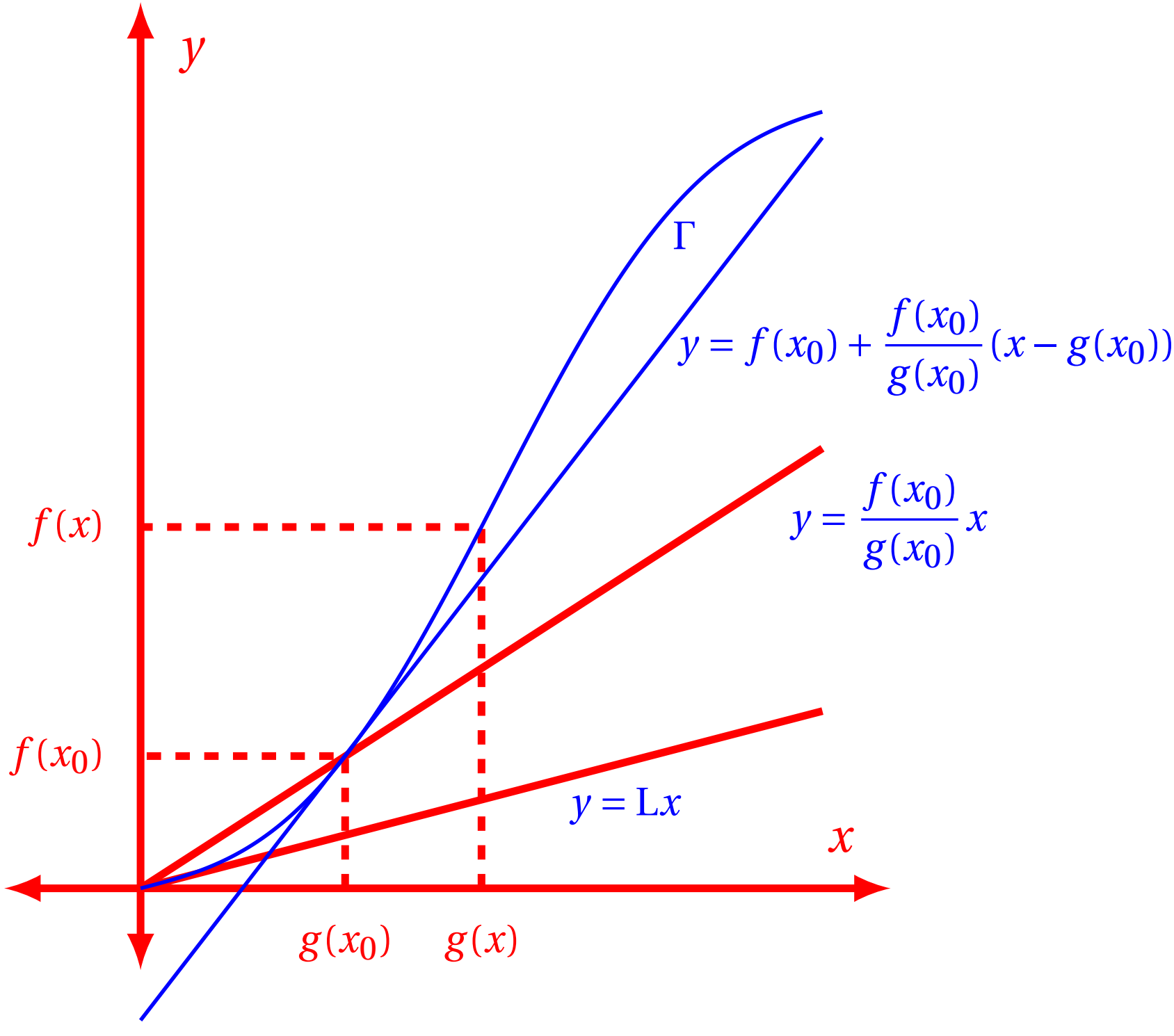

$f'(x)$ es creciente



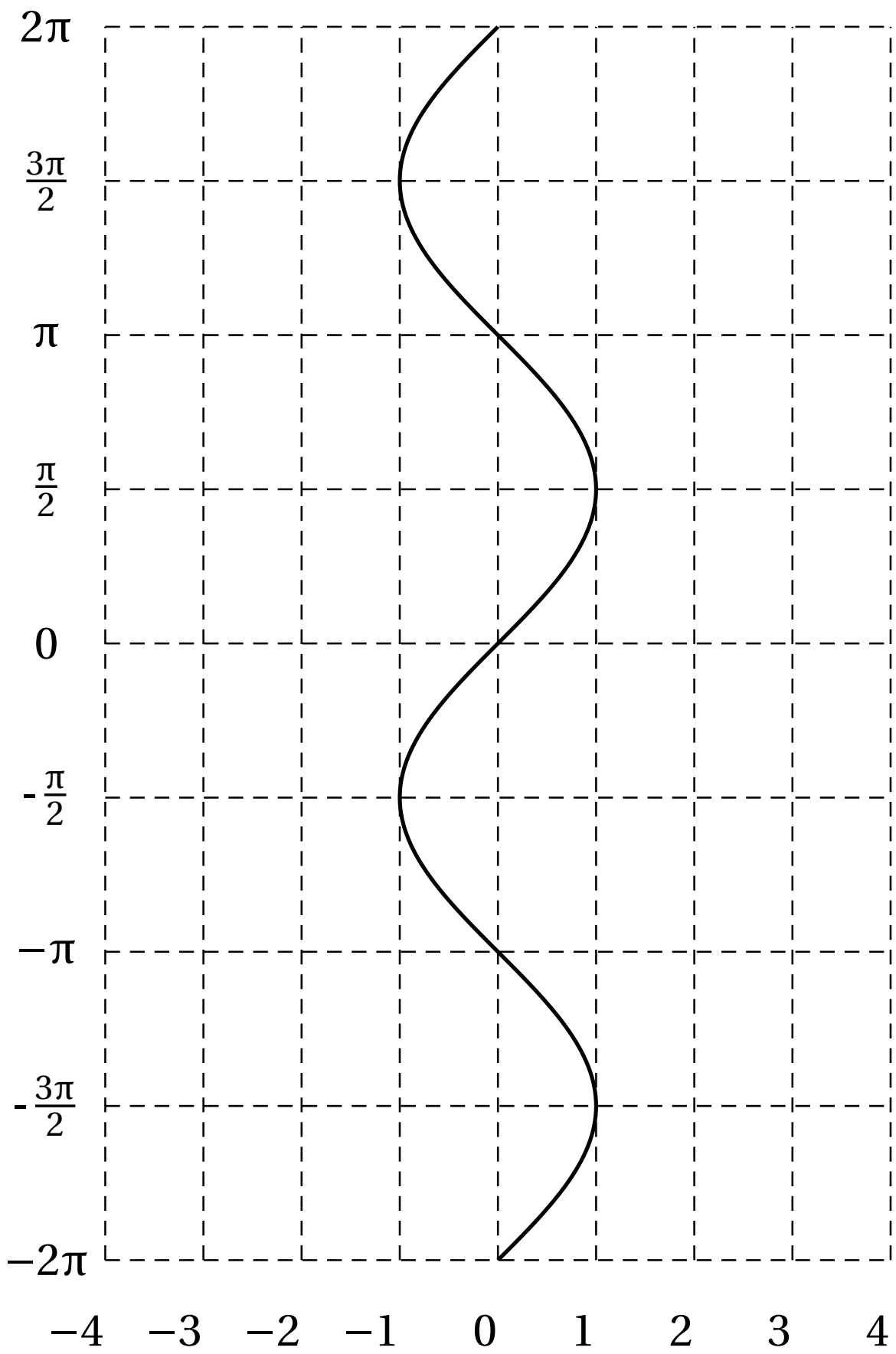
105 ./images-tikz/extremos-a.pgf


$f(x)$
$x = c$
$f'(x) > 0$
$f'(x) < 0$
x

106 ./images-tikz/lhopital.pgf


y
Γ
$y = f(x_0) + \frac{f(x_0)}{g(x_0)}(x - g(x_0))$
$y = \frac{f(x_0)}{g(x_0)}x$
$f(x)$
$f(x_0)$
$y = \mathrm{L}x$
x
$g(x_0)$
$g(x)$

107 ./images-tikz/tikznew4.pgf


2π
3π/2
π
π/2
0
-π/2
−π
-3π/2
−2π
−4
−3
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−1
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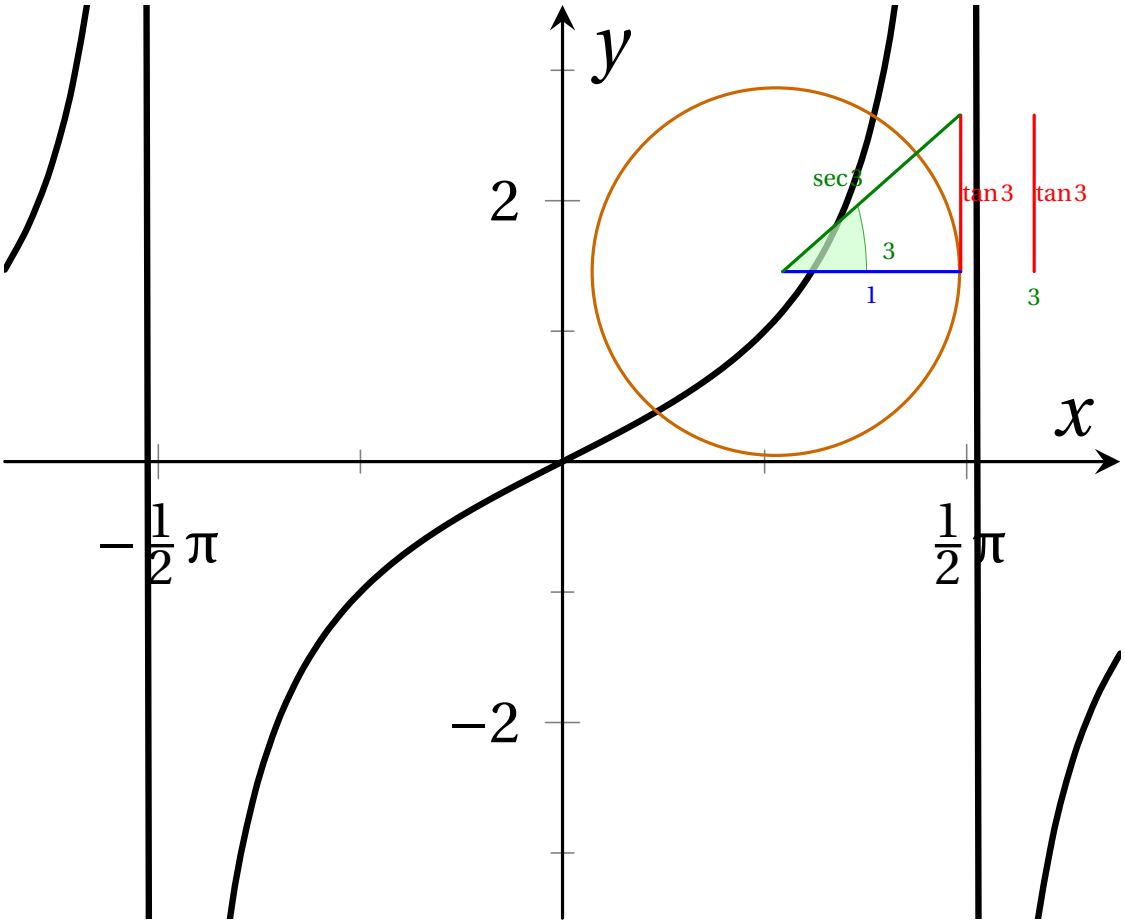
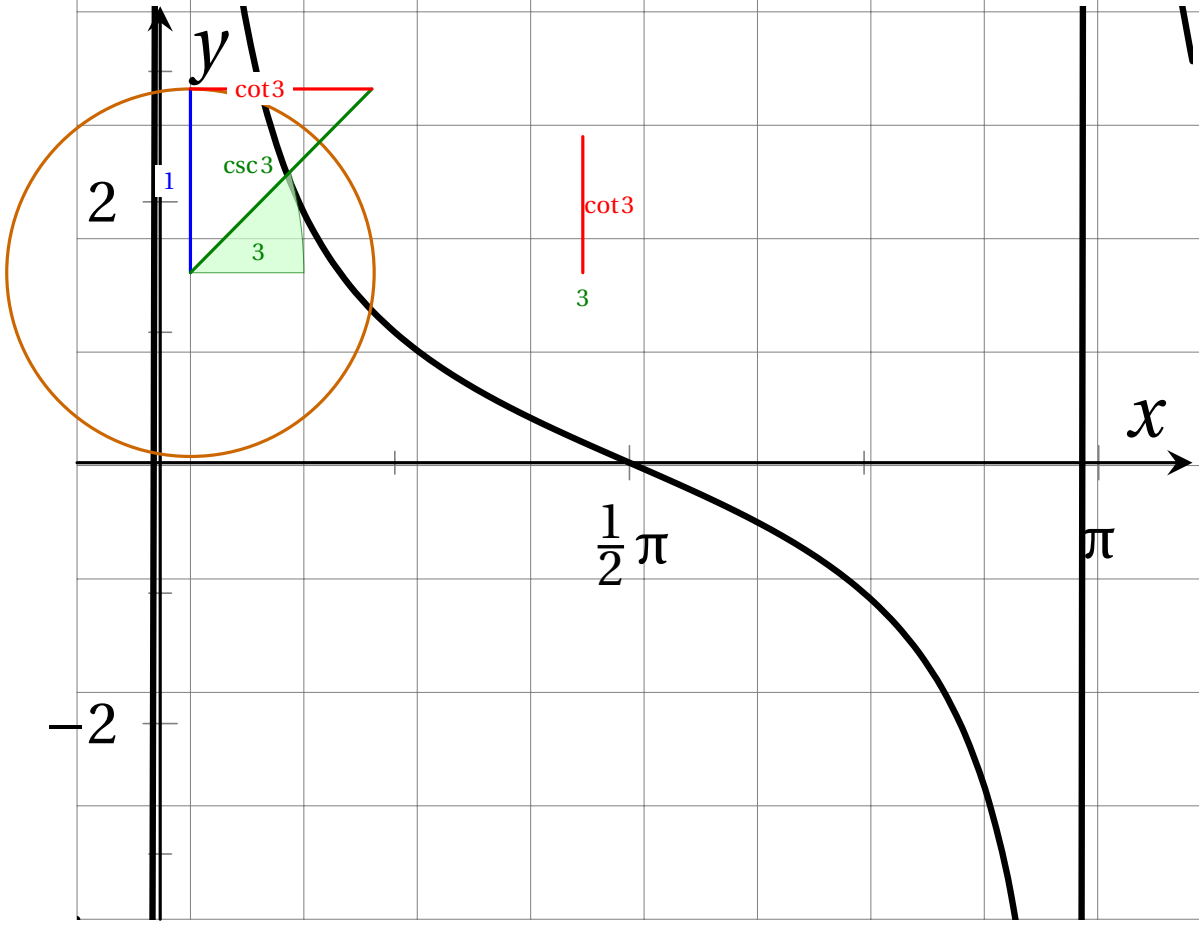
4

Trimmed bounding boxes

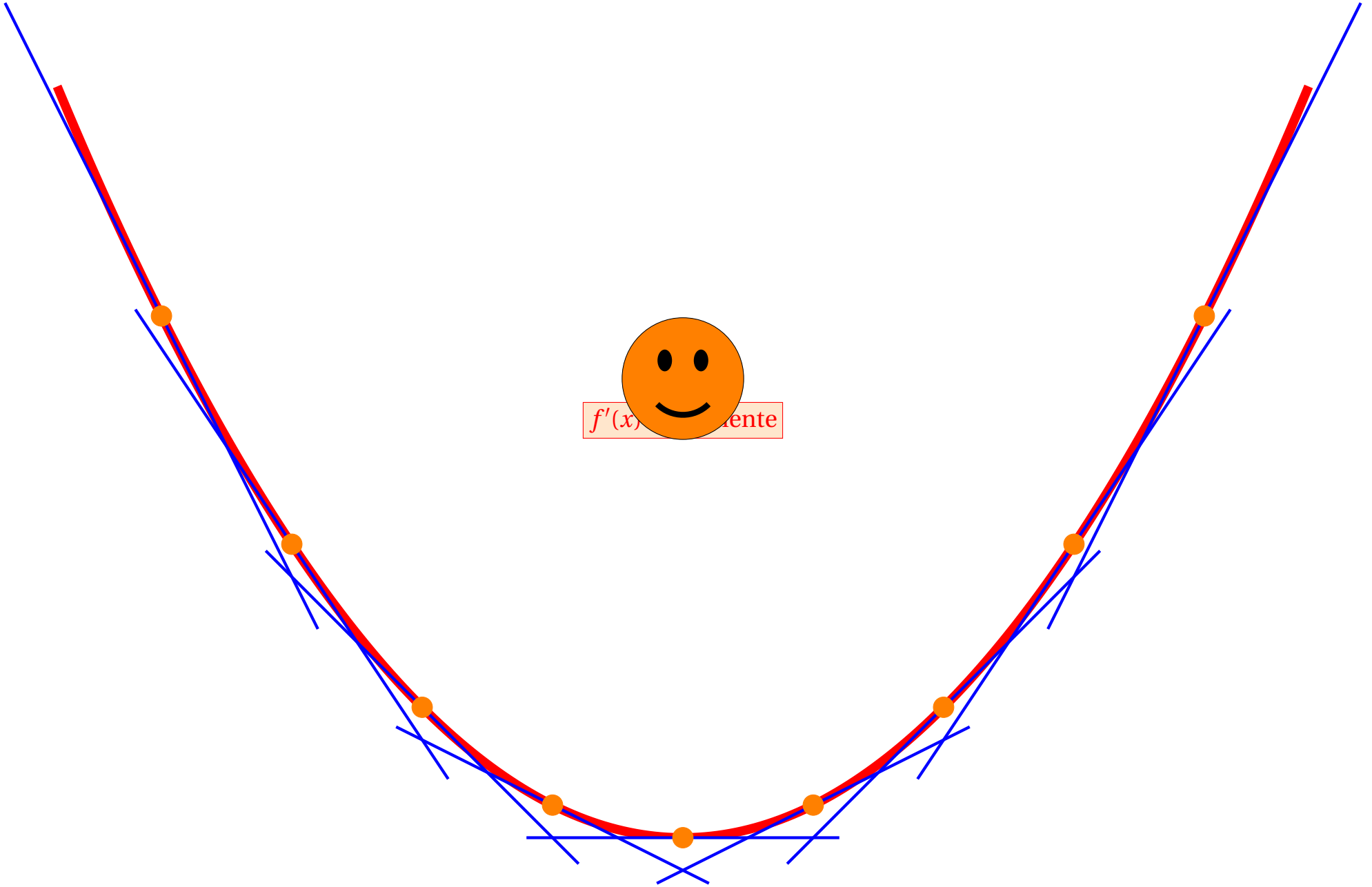


109 ./images-tikz/trig3c.pgf

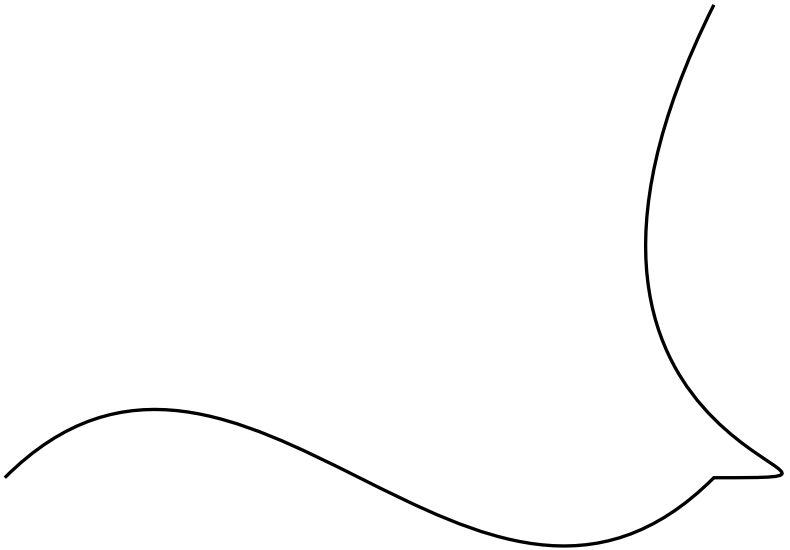
Trimmed bounding boxes



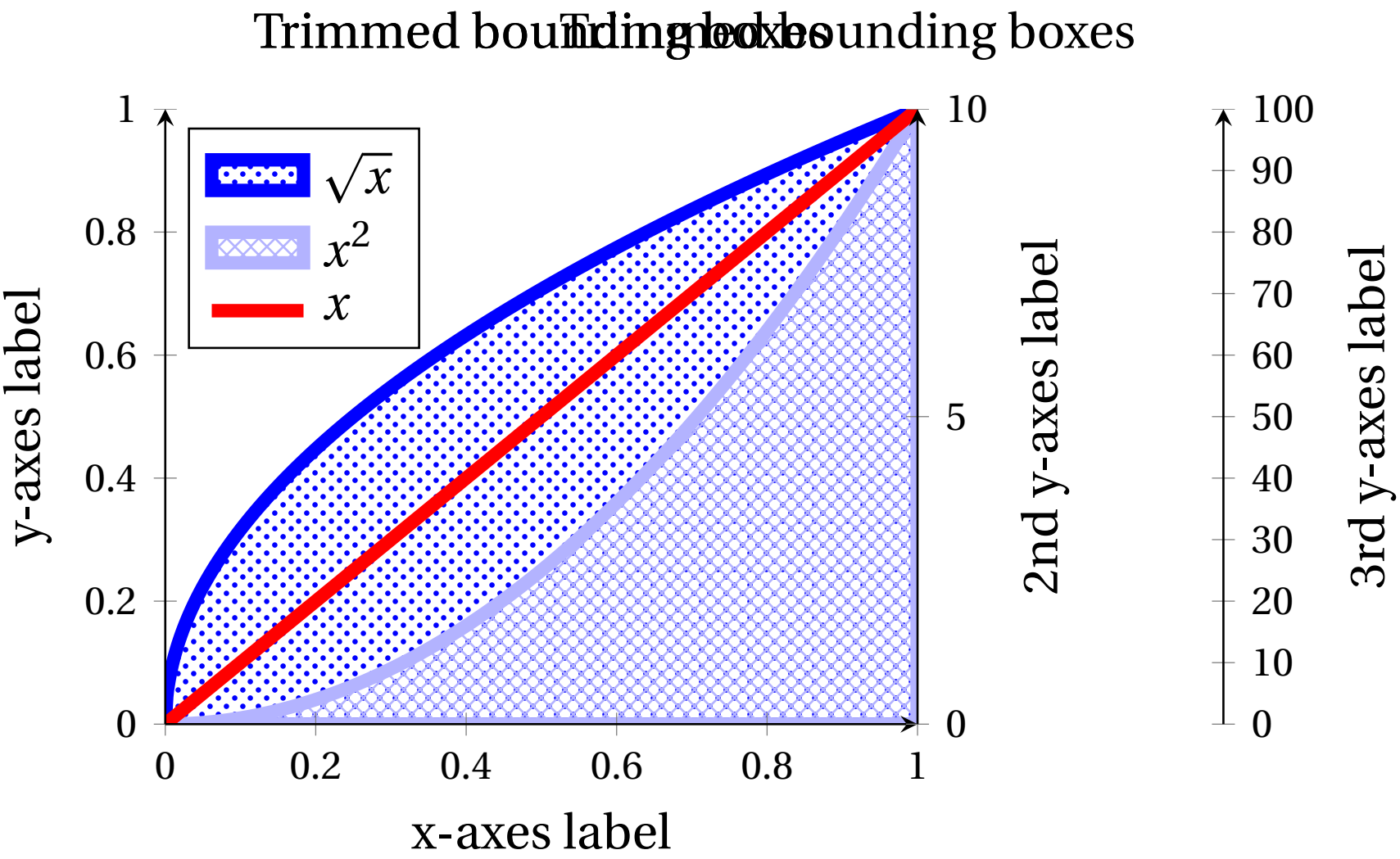
110 ./images-tikz/concavidad.pgf


$f'(x)$ ente

111 ./images-tikz/bezier.pgf



112 ./images-tikz/multiaxes.pgf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$\sqrt{x}$
x^2
x
y-axes label
x-axes label
2nd y-axes label
3rd y-axes label
0
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0.4
0.6
0.8
1
0
5
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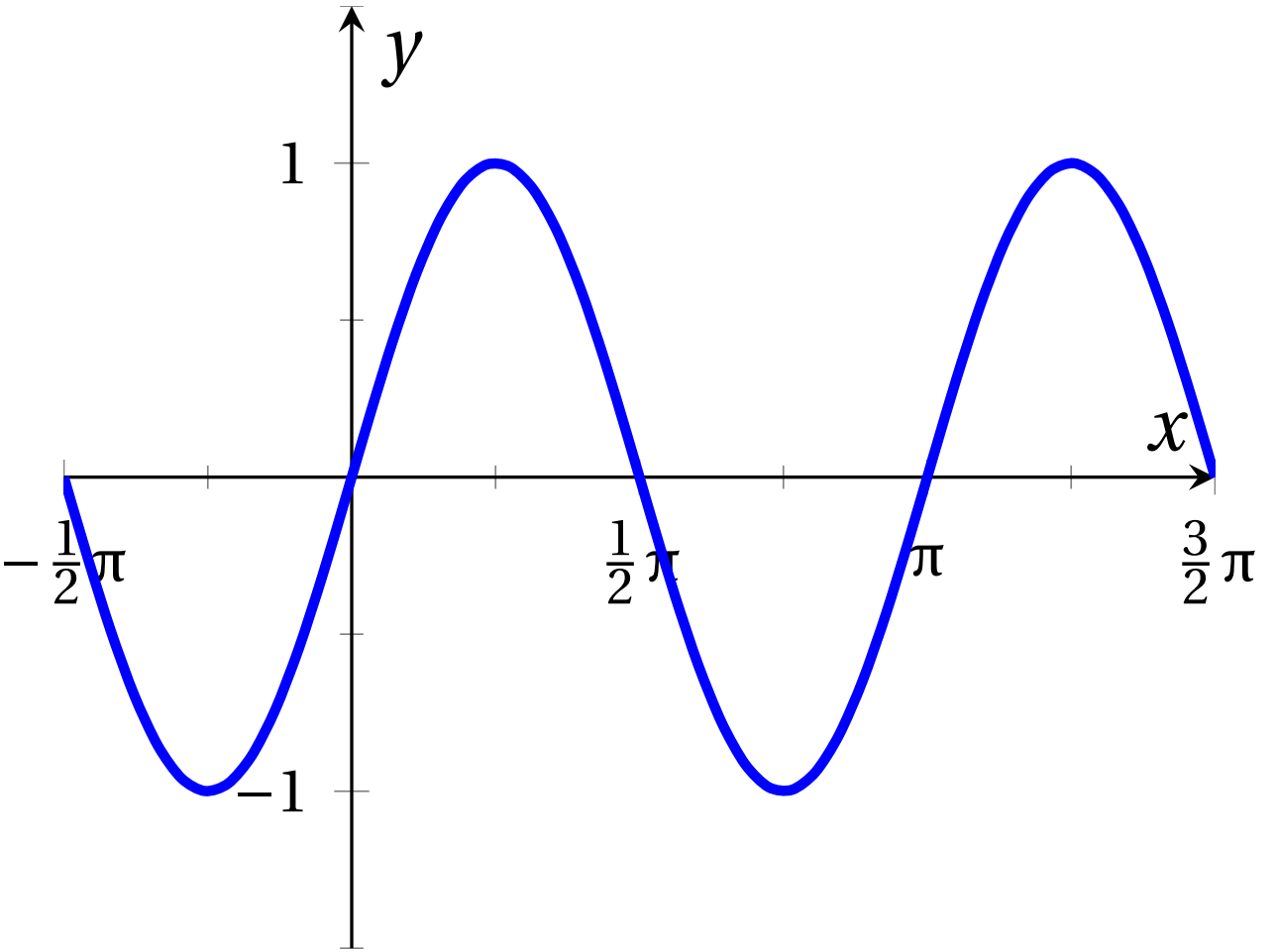
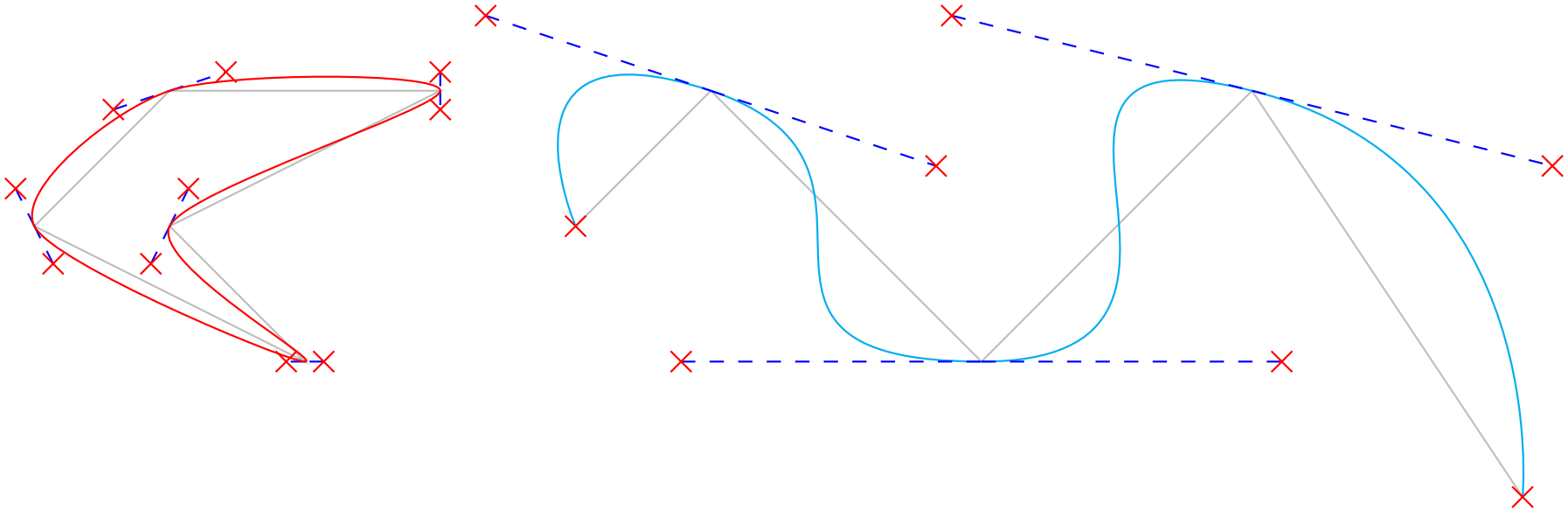
0
10
20
30
40
50
60
70
80
90
100

113 ./images-tikz/senx.pgf

Trimmed bounding boxes

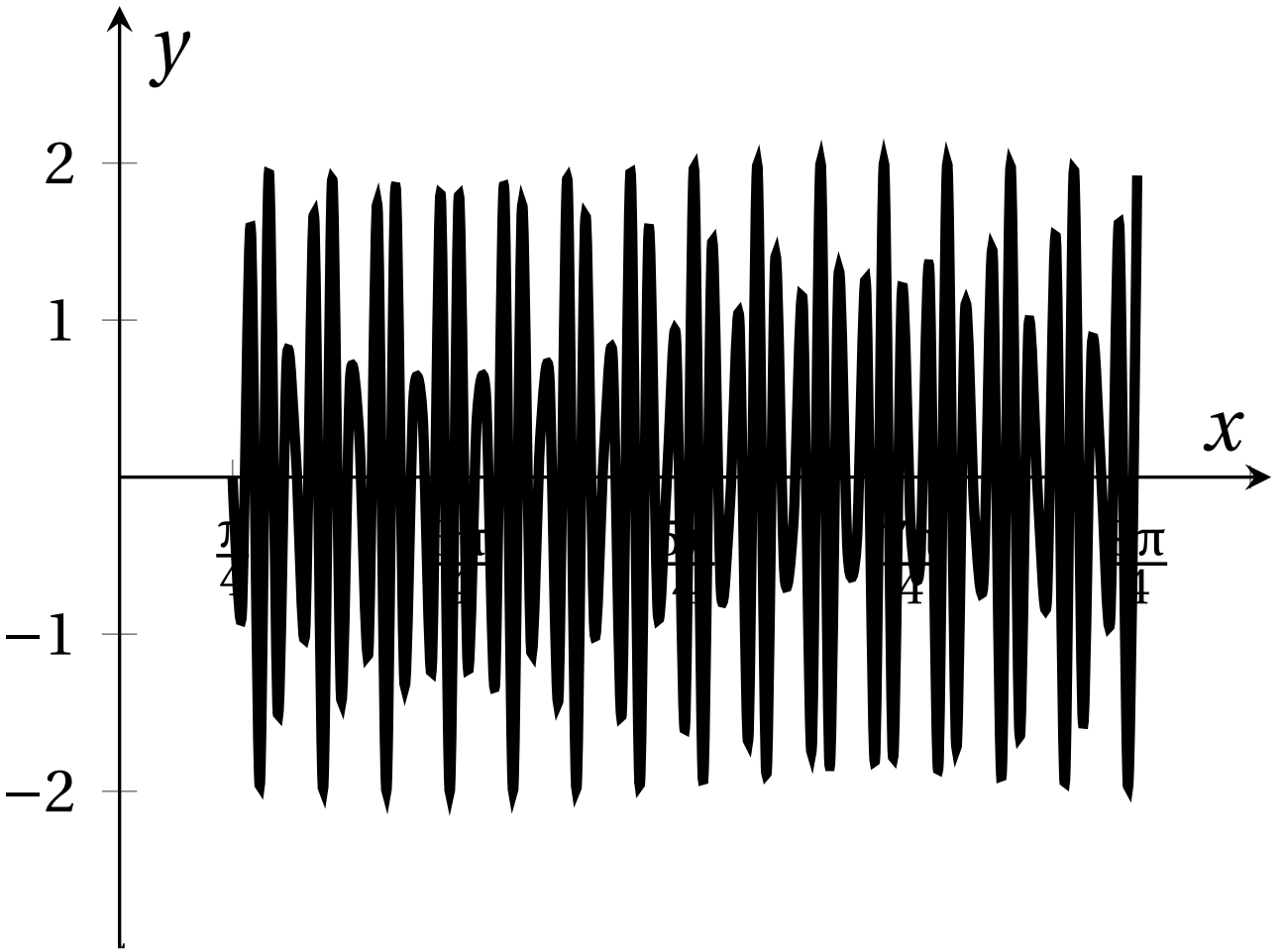


114 ./images-tikz/image$_1$7.pgf

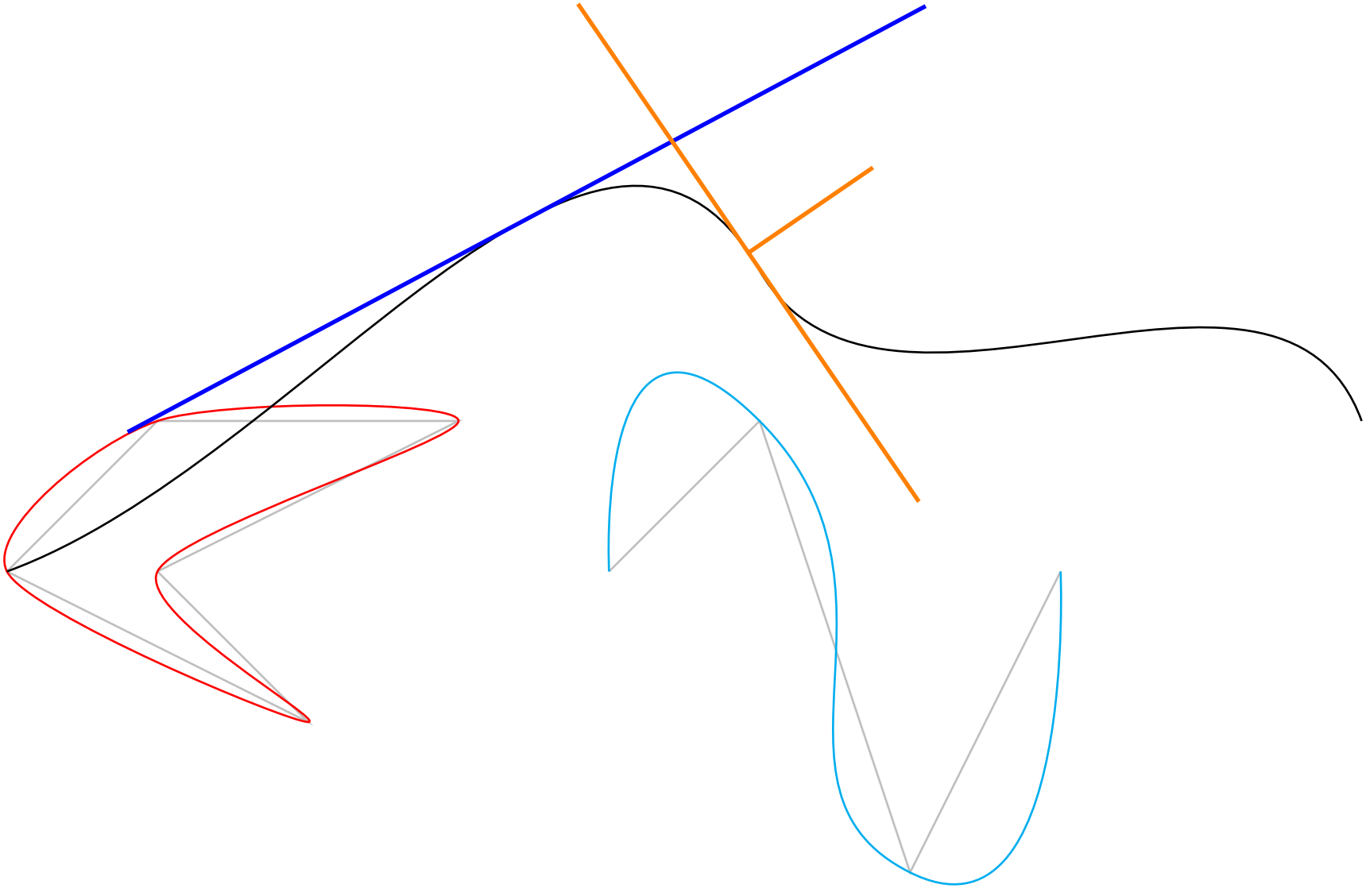


115 ./images-tikz/senx2.pgf

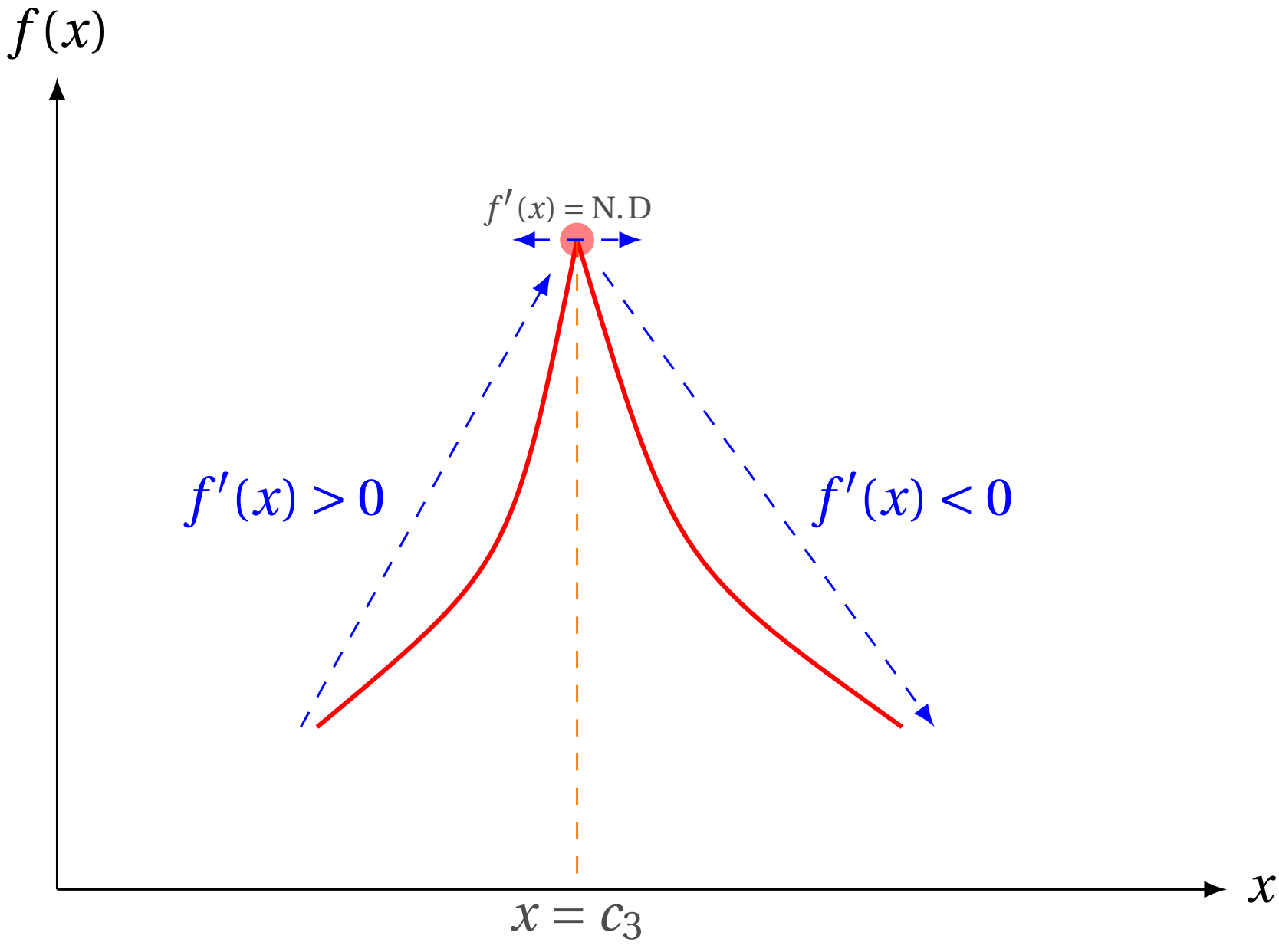
Trimmed bounding boxes



116 ./images-tikz/image$_1$6.pgf

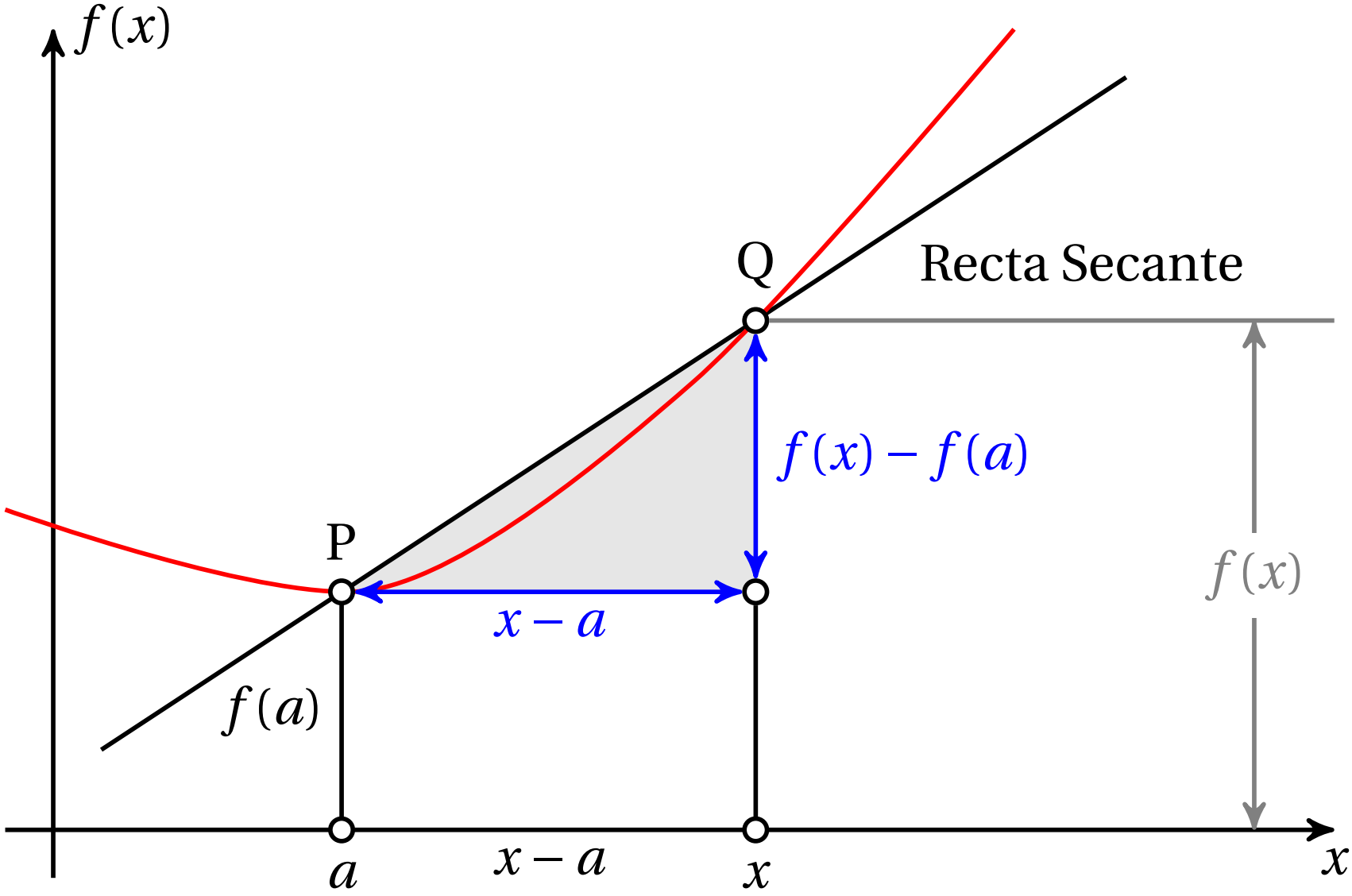


117 ./images-tikz/extremos-c.pgf



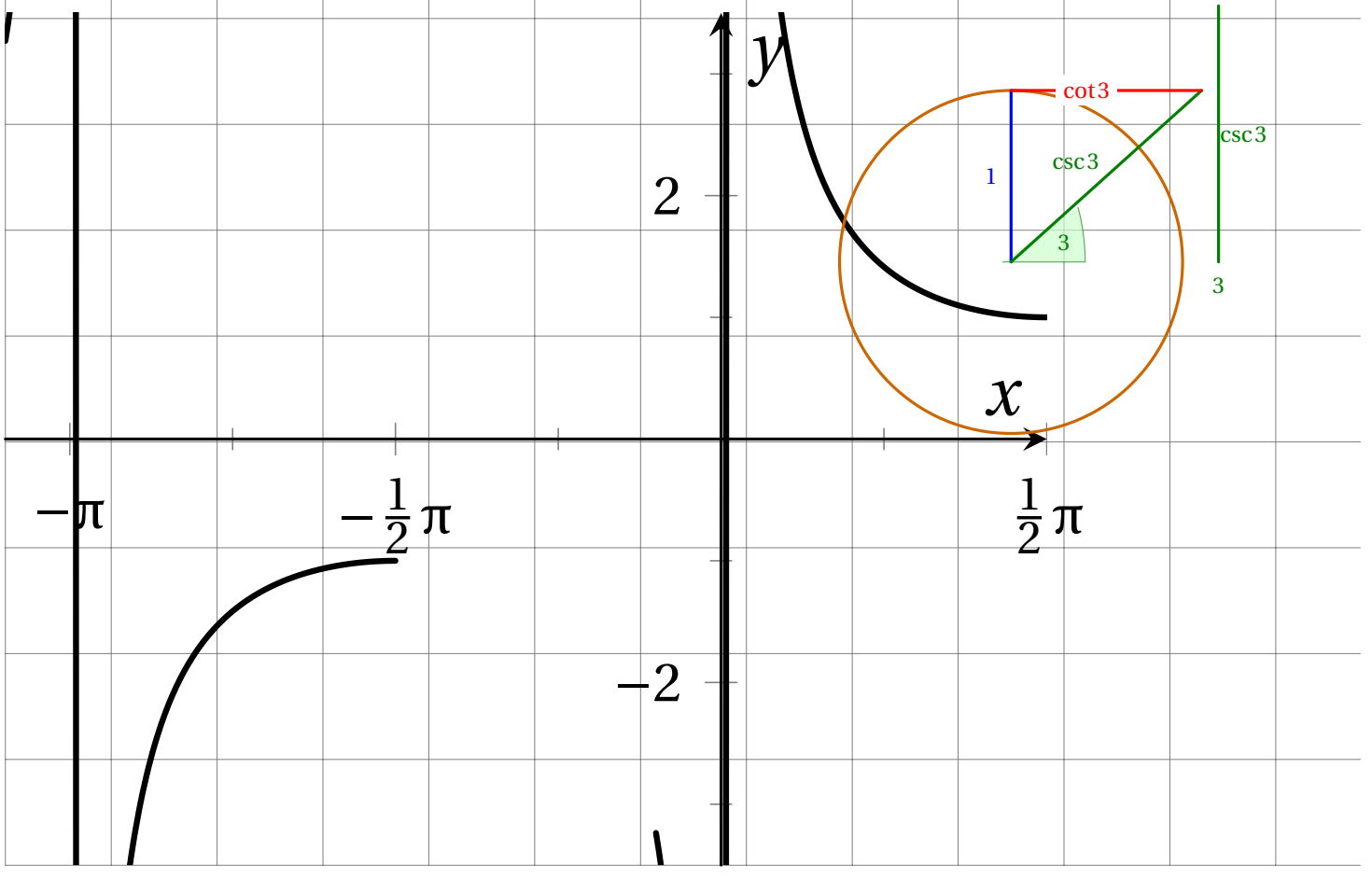
Máximo Relativo

118 ./images-tikz/derivada-1.pgf

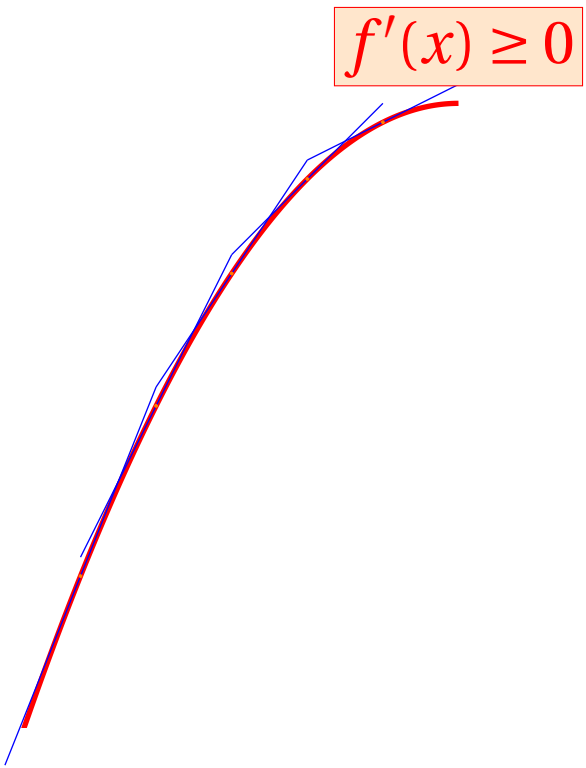

$f(x)$
Q
Recta Secante
$f(x) - f(a)$
P
$f(x)$
$x - a$
$f(a)$
a
$x - a$
x
x

119 ./images-tikz/trig3d.pgf

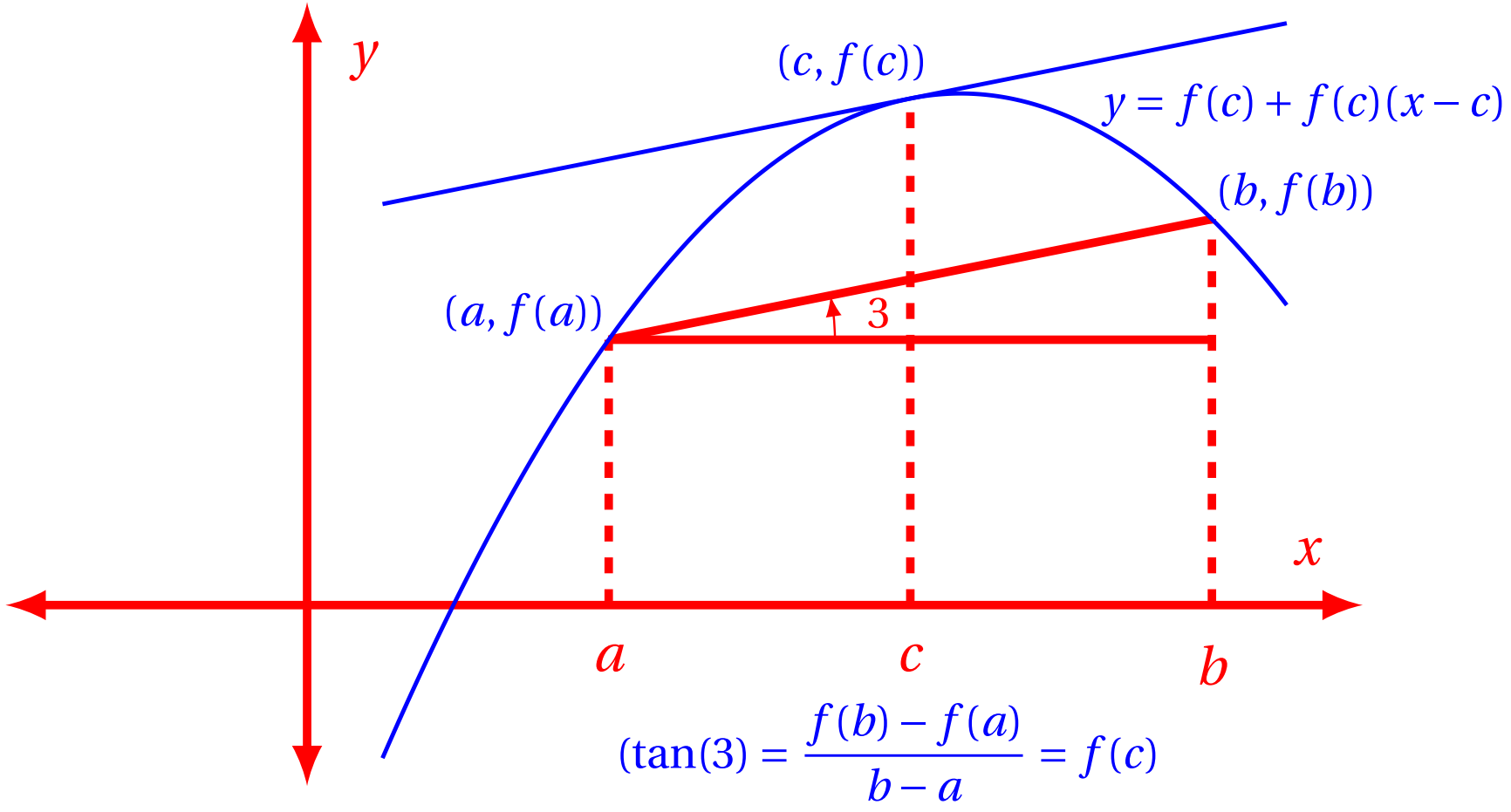
Trimmed bounding boxes



120 ./images-tikz/creciente-a.pgf

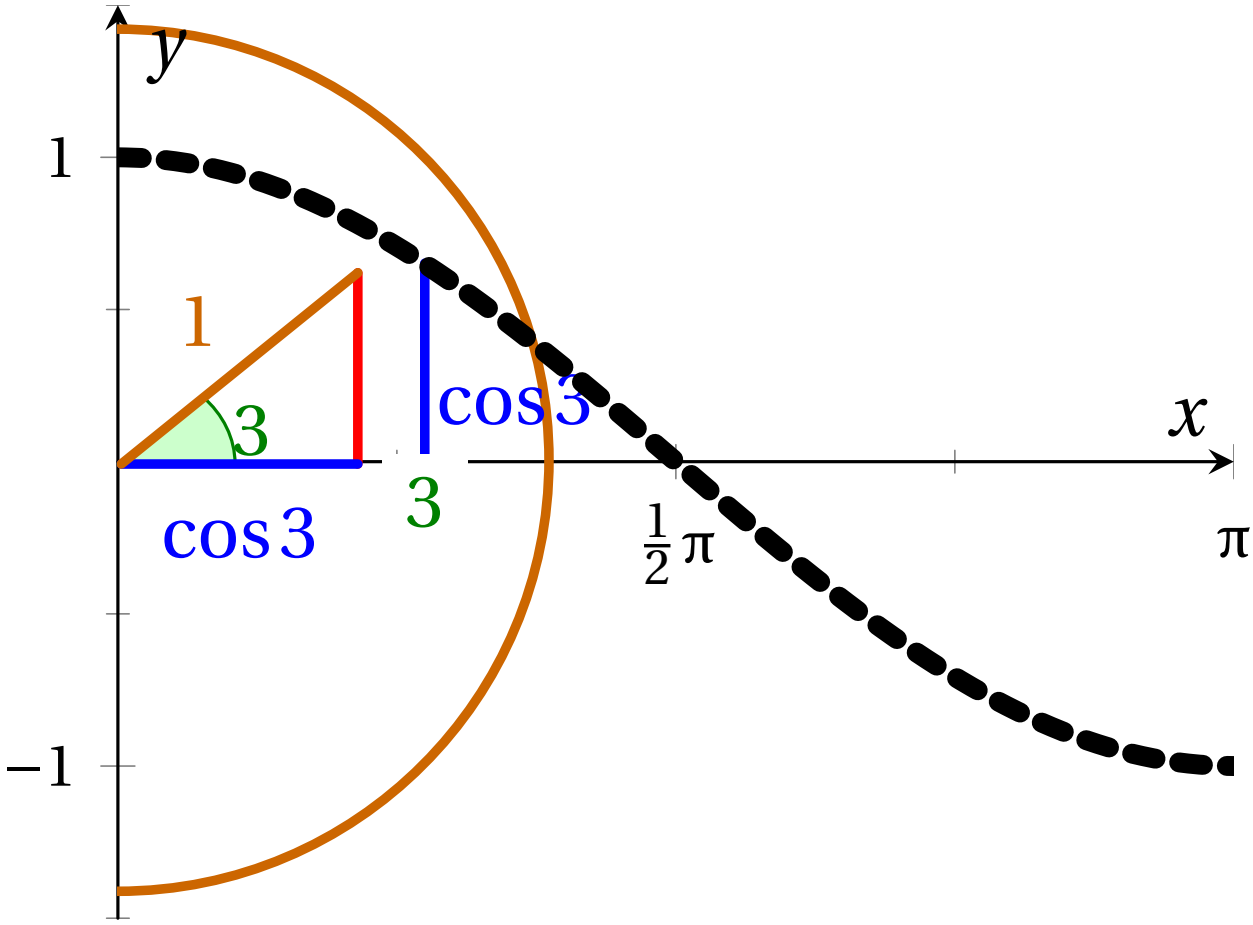

$f'(x) \geq 0$

121 ./images-tikz/lagrange.pgf

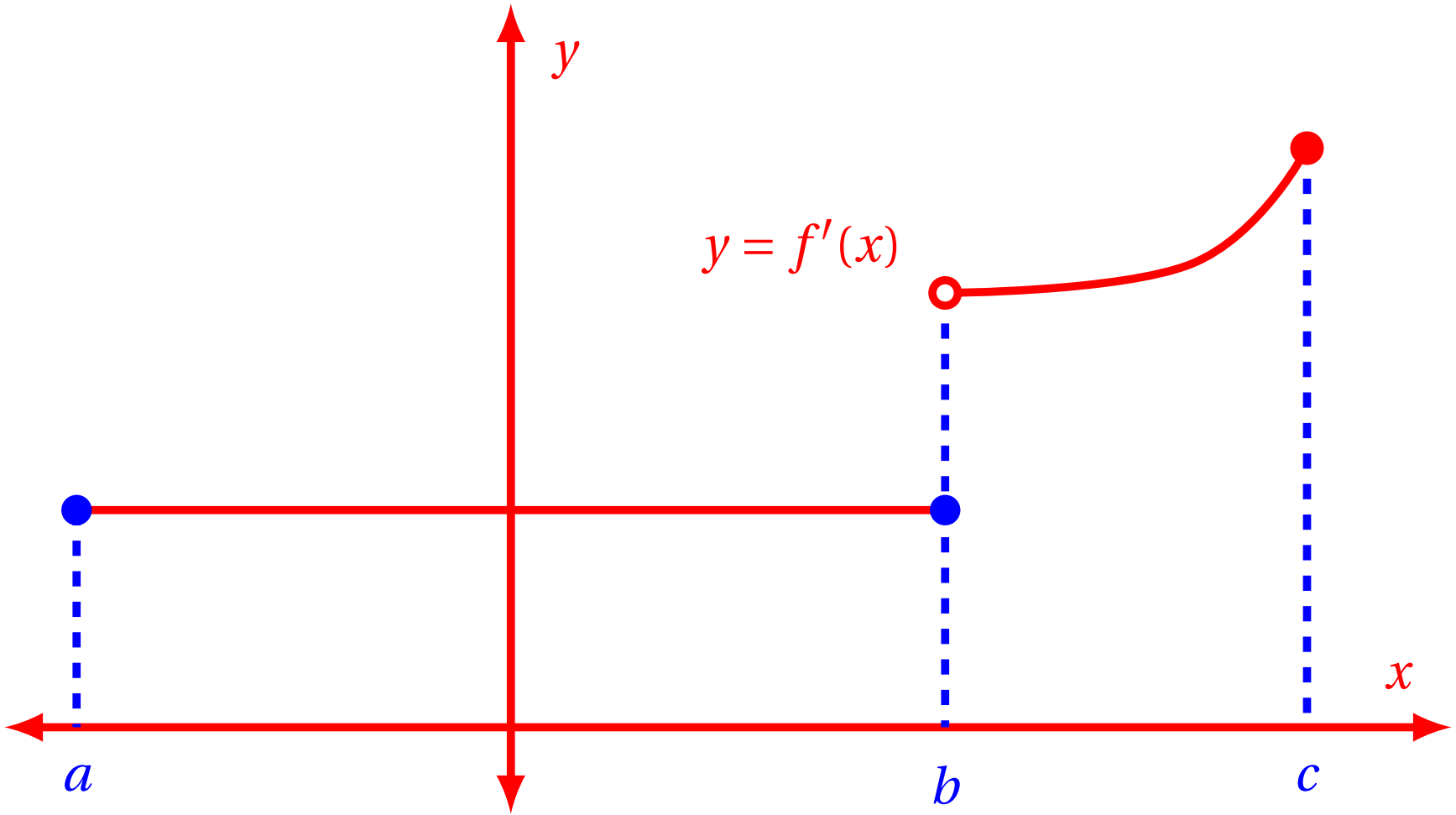

y
(c, f(c))
y = f(c) + f(c)(x − c)
(b, f(b))
(a, f(a))
3
x
a
c
b
(tan(3) = f(b) − f(a) / b − a = f(c)

122 ./images-tikz/trig2.pgf

Trimmed bounding boxes



123 ./images-tikz/ejsegde.pgf


y
y = f′(x)
x
a
b
c