18. Feladat. Legyen $g\colon \mathbf{R} \to \mathbf{R}$ egy adott folytonos függvény, x_0 egy rögzített valós szám és

$$f\colon \mathbf{R} \to \mathbf{R}, \qquad f(x) := g(x)|x - x_0|.$$

Adjunk szükséges és elégséges feltételt arra, hogy az f függvény differenciálható legyen az x_0 pontban!

19. Feladat. Adja meg a következő függvények deriváltját!

1. $f(x) = 4x^5 + 2x^3 - 8$,
2. $f(x) = 3x^6 - 2x^3 + x$,
3. $f(x) = x^2\sqrt{x^3\sqrt{x^3}}$,
4. $f(x) = \dfrac{1}{x^2\sqrt[3]{x^2}}$,
5. $f(x) = 4x^5 + \dfrac{1}{x^3} + \dfrac{3}{\sqrt[3]{x}}$,
6. $f(x) = \dfrac{2x^3 + 3x^2}{\sqrt[3]{x^4}}$,
7. $f(x) = 2\sin\dfrac{\pi}{4} - 4\cos x$,
8. $f(x) = \operatorname{ctg} x - \operatorname{tg} x + e^{\pi+2}$,
9. $f(x) = \left(x^3 + 5\right)\ln x$,
10. $f(x) = xe^x + x$,
11. $f(x) = \sqrt[4]{x}\operatorname{tg} x - \operatorname{arcth} 1$,
12. $f(x) = (3 \cdot 2^x + 2^3)\sin x$,
13. $f(x) = \sqrt{x}\log_2 x + 3$,
14. $f(x) = e^x \arcsin x$,
15. $f(x) = \sin x + x^2\cos x$,
16. $f(x) = 3\cos x + \sqrt[4]{x}\arccos x$,
17. $f(x) = \left(x^4 - \dfrac{3}{x}\right)2^x$,
18. $f(x) = (\sqrt[3]{x^2} + 2x)\cos x$,
19. $f(x) = \dfrac{x+1}{x-1}$,
20. $f(x) = \dfrac{\ln x}{\cos x}$,
21. $f(x) = \dfrac{\sin x - \cos x}{\sin x + \cos x}$,
22. $f(x) = \dfrac{e^x + x}{e^x + 1}$,
23. $f(x) = \dfrac{x^3 + 2x - 1}{\operatorname{ctg} x}$,
24. $f(x) = \dfrac{\operatorname{tg} x + x}{\sqrt{2}}$,
25. $f(x) = \dfrac{x\ln x}{\sqrt{x} + 1}$,
26. $f(x) = \dfrac{x^2\sin x}{\cos x + 1}$,
27. $f(x) = \dfrac{x^3 - 1}{xe^x}$,
28. $f(x) = \dfrac{x + \sin x}{2 - x + \cos x}$,

29. $f(x) = \dfrac{x \arcsin x + \cos x}{\sin x - 1}$,

30. $f(x) = \dfrac{\log_3 x + x^2}{e^x + x + 1}$,

31. $f(x) = \left(8 - \dfrac{1}{x^2}\right)^5$,

32. $f(x) = \dfrac{1}{(\operatorname{tg} x + x - 2)}$,

33. $f(x) = \sqrt[3]{x^4 + 4x^2 - \dfrac{1}{x}}$,

34. $f(x) = \dfrac{x - x^2}{\sqrt{x^2 + 1}}$,

35. $f(x) = (3x - 2)^6 \operatorname{tg} x$,

36. $f(x) = \sin^2 x + \sin x$,

37. $f(x) = \sqrt{\sin x^3 + x \cos x}$,

38. $f(x) = e^{x^2+1}$,

39. $f(x) = x^2 e^{\cos x}$,

40. $f(x) = \dfrac{e^{2x} + x}{e^{\frac{x}{2}}}$,

41. $f(x) = xe^{x^2} + x^2 e^x$,

42. $f(x) = \ln(\cos x)$,

43. $f(x) = \ln\left(\dfrac{x+2}{x-3}\right)$,

44. $f(x) = \ln(x^2 \sin x)$,

45. $f(x) = \ln\left(2^x + 2\right)$,

46. $f(x) = 2^{\sin x + 1}$,

47. $f(x) = \log_2\left(\dfrac{x + x^2}{\sqrt{x} + x}\right)$,

48. $f(x) = \dfrac{\log_3(x^2 - x) + x}{3 + \operatorname{tg} x}$,

49. $f(x) = \sin x^2 + x \cos x^4$,

50. $f(x) = \sin\sqrt{2^x + x^2}$,

51. $f(x) = \operatorname{tg}\dfrac{1}{x} \cdot e^{\operatorname{ctg} x}$,

52. $f(x) = \operatorname{arctg}\dfrac{x}{2} + \ln^2\dfrac{x}{2}$,

53. $f(x) = x \arcsin\sqrt{\dfrac{x}{x+1}}$,

54. $f(x) = \dfrac{1}{(e^{\operatorname{arctg} x} + 2)^4}$,

55. $f(x) = \ln^3\sqrt{\dfrac{\sin x + x}{\cos x - x}}$,

56. $f(x) = x \arcsin\sqrt{x} + \sqrt{3}$,

57. $f(x) = 10^{\ln^2 x + xe^{2x}}$,

58. $f(x) = x^2 \operatorname{arctg}(\sqrt{x} + 1)$,

59. $f(x) = \sin^3(x^2 + 1)\sqrt{x + 1}$,

60. $f(x) = 2^{\frac{x}{x+1}} \sin x^2$,

61. $f(x) = \left(\sqrt{x + 2}\cos x\right)^{-\frac{3}{2}}$,

62. $f(x) = \log_\pi \dfrac{1}{\operatorname{ctg} x + x}$,

63. $f(x) = \dfrac{\cos(\ln 5x)}{x^2 \ln x}$,

64. $f(x) = e^{\sin^2 x - \pi}(x + e^2)$,

65. $f(x) = \arcsin e^{x^2}$,

66. $f(x) = \ln\left(\ln^2(x^3 + 1)\right)$,

67. $f(x) = \sqrt{\operatorname{tg}\sqrt{x2^{x+2}}}$,

68. $f(x) = \pi^{\log_2(\sqrt{x} + \frac{1}{x} - 1)}$,

69. $f(x) = \dfrac{\cos x^4 \operatorname{tg}(x+2)}{\cos^4 x + 2}$,

70. $f(x) = \dfrac{\operatorname{ctg}(x + \ln 2x)}{3\sqrt{x} + \pi^2}$.

20. Feladat. Adja meg a következő függvények deriváltját!

1. $f(x) = \operatorname{tg}\left(\cos x + \sqrt{\sin\dfrac{1}{x^3}}\right)\ln\left(x - \sqrt[3]{\cos\dfrac{1}{x}}\right)^2$,
2. $f(x) = \dfrac{x \operatorname{tg} x e^x + \arcsin^2(2\sin x + \pi) - \frac{1}{x}}{\sqrt{x + \pi^2} e^{\frac{x}{2}}}$,
3. $f(x) = \sqrt[3]{x^2} 2^{(\operatorname{tg}^2 x^3 + \sin(x+\pi)^2)(\log_2^2 x^3 + \sqrt{x^2+1})}$,
4. $f(x) = \sqrt[3]{\dfrac{3^{x^2+1}\sin^3 x + \ln^2(x - e)}{\sin x\left(3 - \frac{1}{x^2}\right)}} \cdot e^{x \operatorname{arctg} x^2 + 8}$.

21. Feladat. Adja meg a következő függvények deriváltját!

1. $f(x) = x^x \qquad (x > 0)$,
2. $f(x) = x^{\sqrt{x}} \qquad (x > 0)$,
3. $f(x) = (\sin x)^{\cos x} \qquad \left(0 < x < \dfrac{\pi}{2}\right)$,
4. $f(x) = (1 + \operatorname{tg} x)^{\operatorname{ctg} x} \qquad \left(0 < x < \dfrac{\pi}{2}\right)$,
5. $f(x) = (\operatorname{arctg} x)^{x^2+1} \qquad (x > 0)$,
6. $f(x) = (x^3 + x)^{\ln x} \qquad (x > 1)$,
7. $f(x) = x^{x^x} \qquad (x > 0)$.