Modeling Complex Systems (CS/CSYS 302), Fall 2020

Assignment #2 of 3
Done in teams of 2 or 3. Due on Blackboard by midnight on Friday, October 23rd. Your write-up should contain everything but the codes, such as any necessary figures as well as your answers to the questions. A brief justification should accompany your answers to each question. Codes should be submitted separately.

Cellular Automata (CA)

1. Come up with a research question about discrete state dynamics. These can be general question: (1) How can we model the role of controlled burns in controlling forest fires? Or more specific questions: (2) With limited resources, how should we design an awareness campaign before a flu season? (3) Can adding lanes to a highway slow traffic?

2. Come up with a cellular automaton to describe your system. (1) This could be a 2 state system — sites are either trees or burned — with fires starting randomly in trees and spreading to neighboring tree sites. (2) This could be a 3 state system — sites are either susceptible, vaccinated, or infectious — with outbreaks starting in the susceptible population and spreading to susceptible or vaccinated neighbors with different probabilities. (3) This could be a 2 state-system — sites are either cars or empty space — with cars advancing if they can, or switching lanes, or staying still with no options.

3. Try to answer your question with your CA. Make sure to run your CA on a variety of initial conditions and (if the CA is stochastic) with enough trials.

4. Discuss your methods and results. This includes important assumptions you made, and how they relate to your model system (e.g. Do you use synchronous or asynchronous updates? How did you choose your initial conditions?). Provide figures or tables that help answer your research question.

5. Prepare a short (5 minutes) presentation of your question, models and results.

Diffusion-Limited Aggregation (DLA)

6. DLA is a model of particles undergoing a random walk (usually in 2D) and aggregating together. The classic model starts with a seed somewhere in space, which grows whenever a new random walker comes in contact with it (by being in an adjacent space). Implement your own version of DLA. You can try to speed it up by drawing the length of steps in the random walk from a power-law distribution (often called a Lévy flight model), or by adding bias (e.g. gravity) in the random walk to steer it towards the aggregated cluster.

7. Produce a figure of the structure created by your DLA code.

8. Bonus question: Implement the box-counting method and measure the dimension of your DLA structure. Report your findings with a log-log plot of number of boxes needed to cover the structure as a function of box size. What does the measured dimension tell you about the DLA structure? Is it fractal?

Gambler's ruin

9. Suppose you and your friend are playing the following game: in each round, your friend flips a biased coin, which is heads with probability p. If the coin is heads, you gain a dollar, and if tails, you lose a dollar. The game ends when you run out of money or you reach a fixed winning value W , which you get to keep if you win. Suppose you have an initial $0 < z < W$ dollars to begin the game with. Let S_n denote your money in round n and $T = \min\{n : S_n = 0, S_n = W\}$ be the time it takes for the game to end. What is the probability that you eventually win? Prove your conclusion with rigorous arguments.

10. Prove $\Pr(T = \infty) = 0$. Together with your answer to the previous question, what does this say about your "friend"?