**Mathematical Sciences Individual Project Report**

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# Comparative Analysis Of Volatility Models for Oil Price

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**ABSTRACT**

The aim of the project is to investigate the behaviour of Brent oil prices and its volatility. The set of sophisticated volatility models is employed for this purpose. The modelling includes ARCH models of high order, symmetric and asymmetric GARCH models. The models will be compared based on goodness of fit criteria as well as on its forecasting performance. The period of consideration is chosen from the day when price hit the value of 100 US dollars, 1st of February 2011 until 31st January 2014. The results of research should be useful to members of the oil industry, governments and financial market players who need to be able to understand and forecast oil price movements.

## 1. INTRODUCTION

The crude oil price has substantial macroeconomic and microeconomic impact. Large oil price movements increase uncertainty about future prices and thus cause delays in business investments. In addition, oil price volatility plays an important role for pricing various financial instruments. The demand for accurate volatility model is obvious.

1st of February 2011, the Brent oil price became to cost over 100 USD. It is possible to separate the oil price behaviour before this date, when the price was generally on the upward trend and after this date when the price stabilised in the range of 95 – 125 US dollar per barrel. This work considered the latter period as it represents the new macroeconomic and geopolitical state of things. The aim of this work is to find the model which describes the behaviour of the Brent oil price in the most accurate way. The data set consists of daily closing prices over the period from February 1, 2011 to January 31, 2014, resulting to 751 observations (Quandl, 2014).

The adequate model should be able to reflect properties usually observed in real life. Volatility clustering and leverage effect are commonly observed in the crude oil price time series. The volatility clustering can be described as the tendency of large returns to be followed by large returns of either sign, and small returns to be followed by small returns. This property is reflected in positive significant auto-correlations of squared returns, which slowly decay towards zero. The leverage effect refers to an asymmetric response of volatility to negative and positive past returns.

From a practical point of view, volatility models are mostly used not only to understand and describe known behaviour of oil price, but for the purpose of its forecast. The wide range of various statistics is employed so that to assess the forecasting performance of models.

## 2. DATA

### 2.1 Properties of data set

Data values are sequenced in time, hence we deal with time series. In order to ensure that time series is "de-trended", meaning that data are stationary, we replace daily oil prices by daily log returns.

$R_t = \ln(\frac{P_t}{P_{t-1}})$ – log return at time t , where $P_t$ is the price of crude oil at time t.

Thus, the data set to analyse consists of 750 data points $R_t$. As estimated, the mean of sample μ = 0.01% and standard deviation σ = 1.46%.

As seen from the Figure 1 (a), returns behave in non-systematic manor and dynamics of returns gives an evidence of volatility clustering, i.e. periods of high volatility are followed by periods of relatively low volatility.

As to correlogram, Figure 1 (b), returns have no sign of dependencies among observations, as there is only one value of autocorrelation function (lag k =15) outside the critical region, which is agreed with 95% confidence level. In addition, Ljung-Box Test shows that there is no autocorrelation in data set, as Q = 20.45 < Z = 32.67. This implies that returns most likely follow a process similar to a random walk $R_t = \mu + \sigma_t\varepsilon_t$, where $\varepsilon_t$ – independent identically distributed (iid) variable.

The Normality test performed with Minitab shows that returns are not normally distributed, Figure 1 (c).

### 2.2 Data cleaning

This section is devoted to treatment of outliers. The outlier is the observation which has extreme value relative to other observations under the same condition. The presence of outliers affects the performance of volatility models, but its removal can lead to failure of the model to capture the underlying phenomena. Therefore, it is crucial to justify the deletion of extreme data points from a sample.

Taking into account the assumption of a random walk model $R_t = \mu + \sigma_t\varepsilon_t$ for returns and making another assumption about $\varepsilon_t$ to follow unit normal distribution, allow us to suggest that the absolute value of $R_t$ greater than $3.29\sigma_t$ is likely to appear in a sample with the probability of 0.1%. These values are regarded to be outliers for further consideration. Values $\sigma_t$ are estimated by the 20-day volatility window.

There are six return values outside the critical value. Let's look at news to understand the nature of outliers: 12.04.11 (|-4.14| > 3.29x1.12|) – oil price drop is associated with price correction due to insufficient demand from emerging economies and also not a promising economic outlook for China and Japan (The Guardian, 2011); 05.05.2011 (|-8.25| > 3.29x1.73) – weak US employment data, dollar strengthening and demand destruction (The New York Times, 2011); 04.05.12 (|-3.74| > 3.29x1.13) – weak US employment data, the growth of Europe and US economies is slower than expected, weak demand and high inventory levels (Bloomberg, 2012); 01.06.12 (|-5.14| > 3.29x1.44) – worse than expected US job figures, weak Chinese and European manufacturing data (Financial Times,, 2012); 19.09.2012 (|-5.14| > 3.29x1.44) contracting Chinese manufacturing activity and Saudi's commitment to keep oil price low; and 06.11.12 (3.43>3.29x1.07) – US election factor, the potential geopolitical conflict associated with Iran if Romney wins (USA Today, 2012).

Five out of six large price movements are triggered by economic factors which occur systematically and therefore they cannot be removed from the data set. The data point associated with US election factor can be removed since the event happened only once during the period of consideration. Thus, the value of 3.43% is replaced with the mean value of 0.01%.

## 3. FITTING MODEL TO DATA

As shown in Part 2.1, the random walk model $R_t = \mu + \sigma_t \varepsilon_t$ is chosen for oil returns. Since oil markets are characterized by the presence of shock, therefore general conditional heteroskedasticity models are used in order to capture this feature. Also, the asymmetric response of volatility to negative and positive past returns is tested for presence by comparing symmetric and asymmetric models.

For all models considered below, the notation is as follows: $a_t = R_t - \mu$ is mean-corrected log return at time t; $\sigma_t$ – volatility at time t; $\varepsilon_t$ – innovations, independent identically distributed variable (iid), assumed to be unit-normally distributed $\varepsilon_t \sim N(0,1)$; $\alpha_0$ – unconditional mean; $\alpha_1$ , $\beta_1$ – model parameters; ϒ – asymmetry term; θ – power term.

The maximum likelihood method (MLE) is used for the purpose of estimation of model's parameters. The MLE approach requires an innovation $\varepsilon_t$ to be unit-normally distributed. This implies the conditional distribution $a_t \sim N(0,\sigma^2)$ and hence the probability distribution function for a set of parameters P = (α, β, …, θ) is

$f(P,a_t) = \frac{1}{2\pi} e^{-\frac{a_t^2}{2\sigma_t^2}} = L(a_t, P)$ – likelihood, the probability of parameter values given the data set.

Applying this expression to every observation in the series gives the joint probability of all observations for given parameters set P

$L(P; a) = \prod_{t=0}^{T} \frac{1}{2\pi} e^{-\frac{a_t^2}{2\sigma_t^2}}$ , where T is the number of data points.

Further applying the log function to the product gives:

$Log(L(P; a)) = \sum_{t=0}^{T} \log(\frac{1}{2\pi} e^{-\frac{a_t^2}{2\sigma_t^2}}) = constant - \sum_{t=0}^{T} \log(\sigma_t + \frac{a_t^2}{2\sigma_t^2})$, thus the latter term must be maximised. All MLE calculations are done using Excel.

**MODELS**

The ARCH model due to Engel (1982) captures the volatility clustering phenomena, when large return values (shocks) tend to be followed by other large shocks. Due to the model large past squared shocks imply a large conditional variance. ARCH (m) models require a high order to adequately describe the volatility structure of the returns series. For current research, the order is chosen to be m=4,5,6,7. The ARCH model is

$\sigma^2_t = \alpha_0 + \alpha_1 a^2_{t-1} + \alpha_2 a^2_{t-2} + \ldots + \alpha_m a^2_{t-m}$ , where $\alpha_0 > 0$, $\alpha_i \geq 0$.

Forecast:

$\sigma^2_{t+s} = \alpha_0 + \alpha_1 a^2_{t+s-1} + \alpha_2 a^2_{t+s-2} + \ldots + \alpha_m a^2_{t+s-m}$ , where for s > 2, $\sigma^2_t$ are replaced by corresponding forecasted values if needed.

The GARCH model proposed by Bollerslev (1986) extends an ARCH model so that the conditional variance is an Auto Regressive Moving Average process. This model also responds equally to positive and negative returns.

The mode used for later consideration is the GARCH (1,1) model:

$\sigma^2_t = \alpha_0 + \alpha_1 a^2_{t-1} + \beta_1 \sigma^2_{t-1}$, where $\alpha_1, \beta_1 \geq 0$.

Forecast:

$\sigma^2_{t+s} = \alpha_0 + (\alpha_1 + \beta_1) \sigma^2_{t+s-1}$ , for s ≥ 2.

The Nonlinear GARCH (NGARCH) model was introduced by Engle and Ng (1993). Unlike GARCH model NGARCH responds unequally to positive and negative returns.

The mode used for later consideration is the NGARCH(1,1) model:

$\sigma^2_t = \alpha_0 + \alpha_1 (a_{t-1} + \Upsilon \sigma_{t-1})^2 + \beta_1 \sigma^2_{t-1}$, where $\alpha_0, \alpha_1, \beta_1 \geq 0$.

Forecast:

$\sigma^2_{t+s} = \alpha_0 + (\alpha_1 (1 + \Upsilon)^2 + \beta_1) \sigma^2_{t+s-1}$ , for s ≥ 2.

Another asymmetric model is exponential the GARCH (EGARCH) model proposed by Nelson (1991).

The mode used for later consideration is the EGARCH(1,1) model:

$\sigma^2{}_t = e^P$ , where $P = \alpha_0 + \alpha_1(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \Upsilon\varepsilon_{t-1} + \theta \ln\sigma^2{}_{t-1}$,

Forecast:

$$\sigma^2{}_{t+s} = \sigma^{2\theta}{}_{t+s-1} + \exp(\alpha_0 - \alpha_1\sqrt{\frac{2}{\pi}})\ [\exp(\frac{(\square+\alpha 1)^2}{2})\Phi(\Upsilon+\alpha_1) + \exp(\frac{(\square-\alpha 1)^2}{2})\Phi(\Upsilon-\alpha_1)].$$

where $\Phi(x)$ – standard normal cumulative distribution function

The GJR–GARCH model proposed by Glosten, Jagannathan and Runkle (1993):

$$\sigma^2{}_t = \alpha_0 + (\alpha_1 + \Upsilon\, I_{t-1})\ \varepsilon^2{}_{t-1} + \beta_1\sigma^2{}_{t-1}, \quad \text{where } I_{t-1} = \begin{cases} 0, & a_{t-1} \geq 0 \\ 1, & a_{t-1} < 0 \end{cases}$$

Forecast:

$$\sigma^2{}_{t+s} = \alpha_0 + (\alpha_1 + \frac{\Upsilon}{2} + \beta_1)\ \sigma^2{}_{t+s-1}, \text{ for } s \geq 2.$$

The Asymmetric Power ARCH (APARCH) model proposed by Ding, Granger and Engle (1993) allows the possibility to estimate a power coefficient θ, which is assumed to be equal to 2 in all models discussed earlier.

$$\sigma^{\theta}{}_t = \alpha_0 + \alpha_1(|\varepsilon_{t-1}| - \Upsilon\varepsilon_{t-1})^{\theta} + \beta_1\sigma^{\theta}{}_{t-1}$$

Prediction

$$\sigma^{\theta}{}_{t+s} = \alpha_0 + \sigma^{\theta}{}_{t+s-1}\,(\alpha_1\frac{1}{\sqrt{2\pi}}\,[(1+\Upsilon)^{\theta} + (1-\Upsilon)^{\theta}] + 2^{\frac{\theta-1}{2}}\,\Gamma(\frac{\theta+1}{2}) + \beta_1, \text{ for } s \geq 2.$$

where $\Gamma(x)$ – Gamma function.

## 4. COMPARISON OF MODELS

### 4.1 Validation of models

Once the model is fitted to data and volatility is estimated, then the next step is to check how well ARCH effect and autocorrelation have been removed. This can be done by Ljung-Box method (Ljung, Box, 1978) , testing whether residuals and squared residuals series have significant autocorrelation or not.

The Ljung-Box test statistic

$$Q(k) = N(N+2)\sum_{i=1}^{k}\frac{p_i{}^2}{N-i},$$

where k is the number of tested lags, N is the sample size, $\rho_i{}^2$ is the sample autocorrelation at lag k. Null hypothesis is rejected if at 5% significance level if $Q(k) > \chi_{0.95}{}^2{}_{,k}$, where $\chi_{0.95}{}^2{}_{,k}$ – is 5% quantile of chi-square distribution with k degrees of freedom. For given analysis $\chi_{0.95}{}^2{}_{,21} = 32.67$.

Normality test for residuals of each model performed by Minitab shows that residuals are not normally distributed, but according to Ljung-Box test, residuals and squared residuals for each model are not correlated across the series (Q-values in Table 1). This is a necessary condition to conclude that models are valid.

### 4.2 Model selection based on information criteria

AIC (Akaike Information Criteria) and BIC (Bayesian Information Criteria) are used as a measure of the goodness-of-fit of an estimated model. They establish the relationship between how well the model fits the data and complexity of the model.

$AIC = -\frac{2}{N}(\mathrm{Log}(L) - K)$ – proposed by Akaike (1974)

$BIC = -\frac{2}{N}(\mathrm{Log}(L) - K\ln(N))$ – proposed by Gideon and Schwarz (1978)

where L – is the maximised value of the likelihood function for the estimated model, K – the number of parameters in the model, N – the sample size.

### 4.3 Model selection based on forecasting performance.

Four forecasting performance measures are selected for evaluating the performance of volatility forecast from selected models:

$MAPE = \frac{100}{s}\sum_{r=1}^{s}\left|\frac{\sigma(t+r)-ft,r}{\sigma(t+r)}\right|$, where $\sigma_{t+r} \neq 0$ – Mean Absolute Percentage Error

$MAPE_{adj} = \frac{100}{s}\sum_{r=1}^{s}\left|\frac{\sigma(t+r)-ft,r}{\sigma(t+r)+ft,r}\right|$, where $\sigma_{t+r} + f_{t,r} \neq 0$ – Adjusted MAPE

$MSE = \frac{1}{s}\sum_{r=1}^{s}(\sigma_{t+r} - f_{t,r})^2$ – Mean Square Error

$$\text{Theil's U Statistic} = \frac{\sqrt{\sum_{r=1}^{s}\left(\frac{\sigma_{t+r} - f_{t,r}}{\sigma_{t+r}}\right)^2}}{\sqrt{\sum_{r=1}^{s}\left(\frac{\sigma_{t+r} - f^{b}_{t,r}}{\sigma_{t+r}}\right)^2}}, \text{ where } \sigma_{t+r} \neq 0,\ \sigma_{t+r} - f^{b}_{t,r} \neq 0.$$

Where s is the number of days for the forecasting period, $\sigma_{t+r}$ is an actual volatility at time t+r, $f_{t,r}$ – volatility forecast at time t+r, $f^{b}_{t,r}$ – benchmark volatility, which is chosen to be a naïve forecast.

In-sample forecasting method is used to calculate these measures. The data set is divided in two parts, the first one is used to estimate one step ahead volatility predictions, while the remaining part is used to test forecasts. Two time periods are considered, namely 20 and 60 days periods.

## 5. Results and interpretations

ARCH models, even having high orders up to 7, have maximised log-likelihood function values not greater than 2500, which put them aside from the group of GARCH family models. Further model comparison suggests that NGARCH(1,1) and GJR–GARCH(1,1) models, with the value of 2529, fit the data in the best way. Comparison of AIC values also tells that NGARCH(1,1) and GJR–GARCH(1,1) models are the best ones, with the lowest AIC value of –6.722. Comparison of BIC values chooses GARCH(1,1) model as the most effective model, with BIC value of –6.654. This can be explained by the relative simplicity of the model in comparison to its rivals, since BIC penalises the use of more complicated models in greater extend. As a result, based on measures of accuracy and efficacy, NGARCH(1,1) and GJR–GARCH(1,1) models are proved to be the best ones for volatility modelling of Brent oil returns. In case if the difference in accuracy is negligible in comparison to advantages that the simplicity of the model gives, than GARCH(1,1) model has greater priority.

Comparative analysis of short term performance indicators suggests that GARCH(1,1) model gives the most accurate forecast. The values of MAPE = 0.1047, $MAPE_{adj}$ = 0.0489 and MSE = 1.40E-06 are the lowest ones. The Theil's U statistic value is 1.41, which is slightly greater than for GJR–GARCH(1,1) with the value of 1.38. This tells that GJR–GARCH(1,1) model forecasts better than GARCH(1,1) in comparison to the naïve forecast. Considering long term performance, GARCH(1,1) model is still the best accurate based on MAPE, $MAPE_{adj}$ and MSE indicators, with values of 0.1207, 0.0557 and 2.10E-06 respectively, but when it comes to comparison with naïve forecast than GARCH(1,1) model concede in accuracy to GJR-GARCH(1,1) and NGARCH(1,1), the latter shows the best performance with the value of statistic to be 1.11.

The value of parameter $\beta_1 > 0.93$ for all GARCH family models means that shocks are highly persistent on the Brent oil market. The coefficient $\alpha_1$ for the GARCH model is 0.041 (0.05 expected), which proves that new shocks influence volatility. The asymmetry coefficient ϒ for NGARCH is 0.57, which reflects that negative returns increase future volatility by a larger amount than positive returns of the same magnitude. The power coefficient equal to 1.81 suggests that the power of GARCH models may be adjusted.

## 6. Conclusions

The range of widely used models, such as high order ARCH models and symmetric and asymmetric GARCH family models, were employed to model returns of Brent crude oil prices. It is shown that heteroskedasticity of the returns time series was removed successfully by applying selected models. NGARCH and GJR-GARCH models describe the behaviour of oil returns in the best way. However, for forecasting purposes GARCH generally outperforms all other considered models. Results of modelling generally show that shocks have a permanent effect on volatility. Moreover, through NGARCH modelling, it is confirmed that negative returns increase future volatility by larger amount than positive returns do.

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**APPENDIX 1: Pictures**

Figure 1.a – Brent oil returns behaviour

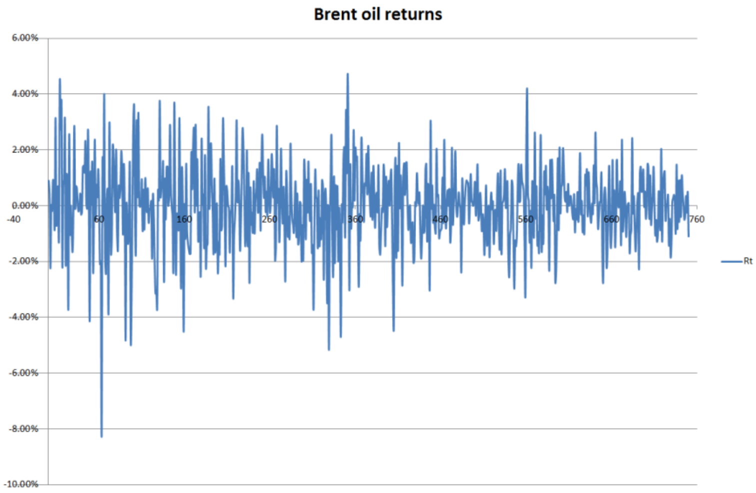



Figure 1.b – Autocorrelation of returns

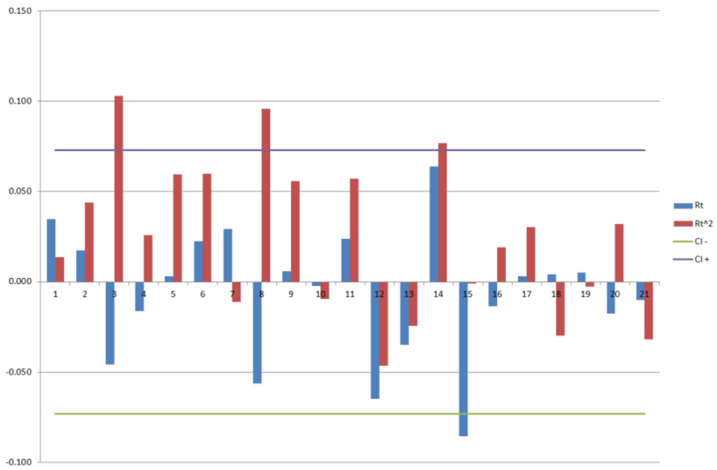
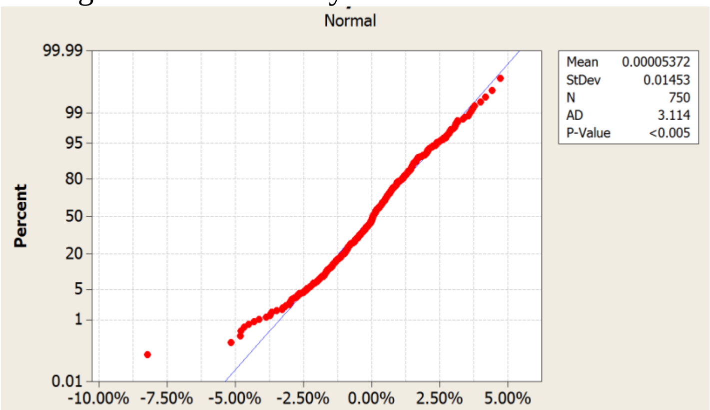



Figure 1.c – Normality test for Brent oil returns



## APPENDIX 2: TABLES OF RESULTS

| | ARCH(4) | ARCH(5) | ARCH(6) | ARCH(7) | NGARCH(1,1) | GARCH(1,1) | GJR-GARCH (1,1) | APARCH (1,1) | EGARCH(1,1) |
|---|---|---|---|---|---|---|---|---|---|
| Coefficients | | | | | | | | | |
| $\theta$ | -- | -- | -- | -- | | -- | -- | 1.815245 | |
| $\Upsilon$ | -- | -- | -- | -- | 0.573188 | -- | 0.040937 | 0.000000 | 0.000000 |
| $\beta$ | -- | -- | -- | -- | 0.934717 | 0.951780 | 0.951050 | 0.950462 | 0.999722 |
| $\alpha_0$ | 0.629861 | 0.561157 | 0.493293 | 0.491788 | 0.007645 | 0.006705 | 0.007680 | 0.000000 | 0.000000 |
| $\alpha_1$ | 0.084247 | 0.085901 | 0.081980 | 0.002500 | 0.043550 | 0.040983 | 0.018684 | 0.000024 | 0.093504 |
| $\alpha_2$ | 0.071959 | 0.086943 | 0.099828 | 0.081609 | -- | -- | -- | -- | -- |
| $\alpha_3$ | 0.149123 | 0.054856 | 0.078425 | 0.099395 | -- | -- | -- | -- | -- |
| $\alpha_4$ | 0.087772 | 0.147576 | 0.047679 | 0.080392 | -- | -- | -- | | -- |
| $\alpha_5$ | -- | 0.091589 | 0.129339 | 0.047051 | -- | -- | -- | -- | -- |
| $\alpha_6$ | -- | -- | 0.099515 | 0.129372 | -- | -- | | | -- |
| $\alpha_7$ | -- | -- | -- | 0.099287 | -- | -- | -- | -- | -- |
| LB Test, z = 32.67 | | | | | | | | | |
| Q_ε[21] | **17.3** | **17.85** | **18.31** | **18.43** | **18.14** | **19.17** | **18.3** | **18.6** | **19.87** |
| Q_ε^2[21] | **30.19** | **21.95** | **16.34** | **16.43** | **15.38** | **14.02** | **15.03** | **14.74** | **15.49** |
| Log(L) | **2500** | **2499** | **2500** | **2496** | **2529** | **2525** | **2529** | **2523** | **2522** |
| Information criteria | | | | | | | | | |
| AIC | **-6.642** | **-6.636** | **-6.636** | **-6.623** | **-6.722** | **-6.714** | **-6.722** | **-6.703** | **-6.703** |
| BIC | **-6.552** | **-6.532** | **-6.517** | **-6.488** | **-6.647** | **-6.654** | **-6.647** | **-6.613** | **-6.628** |
| Forecasting performance | | | | | | | | | |
| 20-day horizon | | | | | | | | | |
| MAPE | **0.1753** | **0.204** | **0.2256** | **0.2154** | **0.1560** | **0.1047** | **0.1177** | **0.1441** | **0.1929** |
| MAPEadj | **0.0795** | **0.0909** | **0.0991** | **0.0949** | **0.0707** | **0.0489** | **0.0546** | **0.0805** | **0.1120** |
| MSE | **5.30E-06** | **6.90E-06** | **8.10E-06** | **7.50E-06** | **3.20E-06** | **1.40E-06** | **1.80E-06** | **3.60E-06** | **5.10E-06** |
| Theil's U | **1.89** | **2.52** | **3.09** | **3.01** | **1.49** | **1.41** | **1.38** | **1.47** | **2.34** |
| 60-day horizon | | | | | | | | | |
| MAPE | **0.14.98** | **0.1831** | **0.2189** | **0.2198** | **0.2050** | **0.1207** | **0.1261** | **0.4909** | **0.5584** |
| MAPEadj | **0.0687** | **0.0825** | **0.0969** | **0.0973** | **0.0912** | **0.0557** | **0.0585** | **0.3528** | **0.4218** |
| MSE | **4.20E-06** | **5.90E-06** | **7.80E-06** | **7.90E-06** | **5.40E-06** | **2.10E-06** | **2.10E-06** | **3.90E-05** | **4.01E-05** |
| Theil's U | **1.85** | **2.46** | **1.33** | **1.34** | **1.11** | **1.38** | **1.14** | **4.9** | **6.63** |