Discrete Maths Notes

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Contents

1 Logic

1.1 Propositional Logic

1.1.1 Basics

A **proposition** is a statement that is either true or false.
Prepositions will be represented mathematically with capital letters A, B, C...
These prepositions are then are connected into more complex compound prepositions using *connectives*. Connectives are statements like "and, implies, if-then" and are represented mathematically with the symbols below.

It's not always easy to determine if they're true/false.

Connectives			
Symbol	Name	English Term(s)	Reading
$\wedge$	AND	And, But, Also	A and B
$\vee$	OR	-	A or B
$\implies$	IMPLICATION	If A, Then B If A, then B A implies B A, therefore B A only if B B follows from A A is a sufficient condition for B B is a necessary condition for A	A implies B
$\iff$	BICONDITIONAL	If & only if A is necessary and sufficient for B	A if and only if B
$\neg$	NEGATION	Not...	Not A

A Bicondtional can also be thought of $(A \implies B) \wedge (B \implies A)$
Negation may sometimes be represented as A' *or* $\overline{A}$

1.1.2 Terminology

$A \wedge B$ - conjuction of conjuncts A and B
$A \vee B$ - disjunction of disjuncts A and B
$A \implies B$ - A is the hypothesis$_{\text{/antecedant}}$ and B is the conclusion$_{\text{/consequence}}$

1.1.3 Examples

1 Compound Proposition

If all humans are mortal$_{\text{prp } A}$ and all Greeks are human$_{\text{prp } B}$
then all Greeks are moral$_{\text{prp } C}$ can be represented as $A \land B \implies C$

2 Negation

Chocolate is sweet $\rightarrow$ Chocolate is not sweet

Peter is tall and thin $\rightarrow$ Peter is short or fat

Short and fat would be incorrect!

The river is shallow or polluted $\rightarrow$ The river is deep and polluted.

Not shallow or not polluted would be incorrect!

3 Implication: hypothesis and conclusion

If the rain continues then the river will flood

A sufficient condition for a network failure is that the central switch goes down

The avocados are ripe only if they are dark and soft

A good diet is a necessary condition for a healthy cat

1.1.4 Satifiability, Tautology, Contradiction

A proposition is satisfiable if it is true for *at least one* combination of boolean values.

You don't need a whole truth table for this, just look for one!

A Boolean Satisfiablity Problem (SAT) is checking for satifiability in a propositional logic formula.

A Tautology is a proposition that is always true
ex $A \lor \neg A$

A Contradiction is a proposition that is always false.
ex $A \land \neg A$

1.2 Truth Tables

1.2.1 Basics

Truth Tables are used for determining all the possible outputs of a complex compound propostion.

The Columns Are for the prepositions, intermediate compound prepositions and the whole compound preposition.

The intrmt' prepositions are optional steps to make solving easier, use as needed.

The Rows Are to contian the different sets of possible truth values for each proposition. You will have 2^p rows where p is the number of propositions (then +1 for the header).

The connectives in a compound propositional logic problem follow an order of precedence (the PEMDAS of logic) in the following order;
$\neg$, $\wedge$, $\vee$, $\implies$, $\iff$

1.2.2 Connective Outputs

Negation

A	$\neg A$
T	F
F	T

And

A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

Or

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

Implication

A	B	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

An implication is true when the hypothesis is false or the conclusion is true.

Bicondtional

A	B	$A \iff B$
T	T	T
T	F	F
F	T	F
F	F	T

A Bicondtional is true when the two prepositions have the same value.

1.2.3 Examples

$A \implies B \iff B \implies A$

A	B	$A \implies B$	$B \implies A$	——
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$A \wedge \neg B \implies \neg C$

A	B	C	$A \wedge \neg B$	——
T	T	T	F	T
T	T	F	F	T
T	F	T	T	F
T	F	F	T	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

ℹ *Remember, columns like $A \implies B$ are optional in-between steps to help solve each problem.*