Series geométricas

$$S = \sum_{n \geq p} \overset{\text{razón}}{r}{}^{n} = r^{p} + r^{p+1} + r^{p+2} + \dots + r^{p+k} + \dots$$

$S = \sum_{n \geq 0} r^{n}$: Suma de infinitos términos

S_n : Suma de n términos.

$$S_n = r^{p} + \cancel{r^{p+1}} + \cancel{r^{p+2}} + \cancel{\dots} + r^{p+n-1} \rightarrow n \text{ términos}$$

$$rS_n = \cancel{r^{p+1}} + \cancel{r^{p+2}} + \cancel{r^{p+3}} + \cancel{\dots} + r^{p+n}$$

$$S_n - rS_n = r^{p} - r^{p+n} \xleftrightarrow{\text{f. común}} (1-r)S_n = r^{p} - r^{p+n}$$

$$S_n = \frac{r^{p} - r^{p+n}}{1-r}, \text{ para } r \neq 1.$$

Si $r = 1$, $S = \sum 1^{n} = \sum 1 \rightarrow S_n = n$

$$S = \sum_{n \geq p} r^{n} = \lim_{n \to \infty} S_n$$

$$S_n = \begin{cases} \dfrac{r^{p} - r^{p+n}}{1-r} & \text{Si } r \neq 1 \\ n & \text{Si } r = 1 \end{cases}$$

$$S = \sum_{n \geq p} r^n = \lim_{n \to \infty} S_n$$

$$S_n = \begin{cases} \dfrac{r^p - r^{n+1}}{1-r} & \text{Si } r \neq 1 \\ n & \text{Si } r = 1 \end{cases}$$

$$\lim r^{n+1} \begin{cases} 0 & \text{si } |r| < 1 \\ \infty & \text{si } |r| > 1 \\ \pm 1 & \text{si } |r| = 1 \end{cases}$$

S es divergente si $|r|>1$, porque $\underbrace{\lim r^{n+1}}_{\text{exp más grande}}$ sería ∞

Si $|r|<1$, $S = \lim S_n = \dfrac{r^p}{1-r}$, porque $\lim r^{n+1} = 0$

Si $|r| = 1$, también diverge, porque $\lim n = \infty$

$$S = \begin{cases} \dfrac{r^p}{1-r} & \text{Si } |r| < 1 \\ \pm\infty & \text{Si } |r| \geq 1 \end{cases}$$

① $S = \sum_{n \geq 1} \left(\frac{5}{4^n} \right) = \sum 5 \cdot \frac{1}{4^n} = \sum 5 \left(\frac{1}{4} \right)^n = 5 \sum \left(\frac{1}{4} \right)^n$

(geométrica) $r = \frac{1}{4}$

$S = \lim S_n = 5 \frac{1/4}{1 - 1/4} = \frac{1/4}{3/4} \cdot 5 = \frac{4}{12} \cdot 5 = 5/3$

$S = 5/3$

② $S = \sum_{n \geq 1} \frac{5^{2n+1}}{4^{3n}} = \sum \frac{5^{2n} \cdot 5}{4^{3n}} = 5 \sum \frac{5^{2n}}{4^{2n}} = 5 \sum \frac{25^n}{64^n} = 5 \sum \left(\frac{25}{64} \right)^n$

$S = \lim S_n = \overset{\times 5}{\downarrow} \frac{25/64}{1 - 25/64} = \overset{\times 5}{\downarrow} \frac{25/64}{39/64} = \overset{\times 5}{\downarrow} \frac{25 \cdot \cancel{64}}{39 \cdot \cancel{64}} = \frac{25}{39} \cdot 5$

$S = \frac{125}{39}$

③ $S = \sum_{n \geq 1} \left(\frac{1}{4^n} - \frac{1}{5^n} \right) \begin{cases} S_1 = \sum \left(\frac{1}{4} \right)^n \\ S = \sum \left(\frac{1}{\ } \right)^n \end{cases}$