

Serie geométrica

$$S = \sum_{n=p}^{\infty} r^n = r^p + r^{p+1} + r^{p+2} + \dots + r^{p+k} + \dots$$

$$S = \sum_{n=0}^{\infty} r^n : \text{Suma de infinitos términos}$$

S_n : Suma de n términos.

$$S_n = r^p + r^{p+1} + r^{p+2} + \dots + r^{p+n-1} \rightarrow n \text{ términos}$$

$$rS_n = r^{p+1} + r^{p+2} + r^{p+3} + \dots + r^{p+n}$$

$$S_n - rS_n = r^p - r^{p+n} \xleftrightarrow{\text{factor común}} (1-r)S_n = r^p - r^{p+n}$$

$$S_n = \frac{r^p - r^{p+n}}{1-r}, \text{ para } r \neq 1.$$

$$\text{Si } r=1, S = \sum 1^n = \sum 1 \rightarrow S_n = n$$

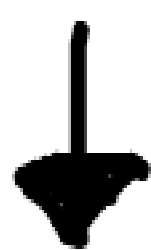
$$S_n = \begin{cases} \frac{r^p - r^{p+n}}{1-r} & \text{Si } r \neq 1 \\ n & \text{Si } r = 1 \end{cases}$$

$$S = \sum_{n=p}^{\infty} r^n = \lim_{n \rightarrow \infty} S_n$$

$$S = \sum_{n=p}^{\infty} r^n = \lim_{m \rightarrow \infty} S_m$$

$$S_n = \begin{cases} \frac{r^n - r^{n+m}}{1-r} & \text{Si } r \neq 1 \\ n & \text{Si } r = 1 \end{cases}$$

$$\lim_{n \rightarrow \infty} r^{n+m} \begin{cases} 0 & \text{si } |r| < 1 \\ \infty & \text{si } |r| > 1 \\ \pm 1 & \text{si } |r| = 1 \end{cases}$$



S es divergente si $|r| > 1$, porque $\lim_{n \rightarrow \infty} r^{n+m}$ sería ∞
exp más grande

Si $|r| < 1$, $S = \lim_{n \rightarrow \infty} S_n = \frac{r^n}{1-r}$, porque $\lim_{n \rightarrow \infty} r^{n+m} = 0$

Si $|r| = 1$, también diverge, porque $\lim_{n \rightarrow \infty} n = \infty$

$$S = \begin{cases} \frac{r^n}{1-r} & \text{Si } |r| < 1 \\ \pm \infty & \text{Si } |r| \geq 1 \end{cases}$$

$$\textcircled{1} \quad S = \sum_{n=1}^{\infty} \left(\frac{5}{4^n} \right) = \sum 5 \cdot \frac{1}{4^n} = \sum 5 \left(\frac{1}{4} \right)^n = 5 \sum \left(\frac{1}{4} \right)^n$$

(geometric)

$$S = \lim S_n = \frac{5 \cdot \frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{5}{4}}{\frac{3}{4}} \cdot 5 = \frac{4}{12} \cdot 5 = \frac{5}{3} \quad r = \frac{1}{4}$$

$$S = \frac{5}{3}$$

$$\textcircled{2} \quad S = \sum_{n=1}^{\infty} \frac{5^{2n+1}}{4^{3n}} = \sum \frac{5^{2n} \cdot 5}{4^{3n}} = 5 \sum \frac{5^{2n}}{4^{3n}} = 5 \sum \frac{25^n}{64^n} = 5 \sum \left(\frac{25}{64} \right)^n$$

$$S = \lim S_n = \frac{\overset{\times 5}{25/64}}{1 - \overset{\times 5}{25/64}} = \frac{\overset{\times 5}{25/64}}{\overset{\times 5}{39/64}} = \frac{25 \cdot \cancel{64}}{39 \cdot \cancel{64}} = \frac{25}{39} \cdot 5$$

$$S = \frac{125}{39}$$

$$\textcircled{3} \quad \sum_{n=1}^{\infty} \left(\frac{1}{4^n} - \frac{1}{5^n} \right) \quad \left\{ \begin{array}{l} S_1 = \sum \left(\frac{1}{4} \right)^n \\ S_2 = 5 \left(\frac{1}{5} \right)^n \end{array} \right.$$