$$f(x) = \sum a_n x^n$$

$$\int f(x)\,dx = c + \int a_1 x^1\,dx + \int a_2 x^2\,dx + \dots + \int a_n x^n\,dx =$$

$$c + \frac{1}{2} a_1 x^2 + \frac{1}{3} a x^3 + \dots + \frac{1}{n+1} a_n x^{n+1} = \sum \frac{1}{n+1} a x^{n+1}$$

$$f(x) = \sum \frac{x^n}{n} \qquad g(x) = \sum_{n \geq 0} x^n = \frac{1}{1-x}$$

$$\int g(x) = \int \sum_{n \geq 0} x^n = \int \frac{1}{1-x}$$

$$\int g(x) = \sum_{n \geq 0} \frac{x^{n+1}}{n+1} = \ln\left(\frac{1}{1-x}\right) + c$$

$$\int g(0) = \ln(1) + c = c$$

$$\int g(0) = \sum \frac{0}{n+1} = 0$$

$$\to c = 0$$

$$\int g(x) = \sum_{n \geq 0} \frac{x^{n+1}}{n} = \ln\left(\frac{1}{1-x}\right)$$

$$\int g(x) = \sum_{n \geq 0} \frac{x^{n+1}}{n+1} = \sum_{n \geq 1} \frac{x^n}{n} = \ln\left(\frac{1}{1-x}\right)$$

$$f(x) = \sum \frac{x^n}{3^{n+1}} = \frac{1}{3} \sum \left(\frac{x}{3}\right)^n$$

$$f'(x) = \frac{1}{3} \sum \frac{n x^{n-1}}{3^n} = \sum \frac{n x^{n-1}}{3^{n+1}}$$

$$\int f(x) = \sum \frac{1}{3^{n+1}} \int x^n dx = \sum \left(\frac{x^{n+1}}{(n+1)\, 3^{n+1}}\right) + C$$

$$f(x) = \sum \frac{x^n}{n}$$

$$f'(x) = \sum \frac{1}{\cancel{n}} \cancel{n} x^{n-1} = \sum x^{n-1}$$

$$\int f(x) = \sum \frac{1}{n} \int x^n dx = C + \sum \frac{x^{n+1}}{(n+1) n}$$