1 Excellence

1.1 Quality and pertinence of the project's research and innovation objectives (and the extent to which they are ambitious, and go beyond the state of the art)

Introduction, historical remarks and state-of-the-art.

Motivation

In a very basic and general approach, most sciences and scientific disciplines follow a standard scientific method. Very generally speaking, this method consists of a series of clearly defined steps that can be summarised as: observation, measurement, experimentation, and formulation, testing and modification of hypotheses.

The development of theoretical mathematical knowledge requires the formulation and formal proof of hypotheses. For this reason, it is almost impossible to follow the standard scientific steps mentioned above. Every day work of a mathematician usually starts with an idea that eventually becomes a hypothesis based on merely intuition and sometimes a handful of examples. More often than not the hypothesis turns to be incorrect and this only shows when failing to give a formal and complete proof. Usually the hypotheses needs to be modified and the process must start over again, without being able to report any knowledge from those previous attempts. Moreover, this need not to be the worst case scenario, it could be that a hypothesis is perfectly correct but the tools needed to develop a formal proof are not developed yet. Reference to say, Fermat's last theorem?

What is OurP?

Clearly, the two scenarios presented above show that there is a need to improve and speed the way mathematical knowledge is developed. Unfortunately, there is not much to do for the latter situation; however we could try to improve the quotidian labor of a theoretical mathematician. This is precisely where OurP fits. It is not uncommon that examples of mathematical objects are easy to construct in a theoretical context but hard to visualize in a concrete way. This usually comes from the very nature of the mathematical objects. However, discrete and combinatorial objects often possess a finite and natural way of save them as computational objects. Of course, this natural setting is usually limited by computational power or by the lack of efficient representation of such objects. OurP's main objective is to develop data sets and computational tools of highly symmetric maps, abstract polytopes and maniplexes.

The nature of OurP involves a two-way flow of knowledge: With the aim of generating new datasets of highly symmetric polytopes and related objects, we will develop new theoretical results and methods, enabling us to devise new algorithms and combinatorial representation for constructing highly symmetric abstract poltytopes. The algorithms will then be carefully implemented and executed. The obtained datasets will then be analysed with the aim of finding interesting patterns, suggesting new conjectures and proposing new directions for further research.

Datasets of mathematical objects

Enumeration and classification of mathematical objects is a natual way of conducting research. The discrete nature of combinatorial objects turn them into natural candidates to not only classify families of interesting objects but to enumerate and explicitly list the elements of such families. This research approach has resulted in the development of interesting data sets of combinatorial objects. Highly symmetric graphs is arguably the most studied family of combinatorial objects from the approach of building datasets of objects. It is believed that empirical study of symmetric graphs of small valence started in 1930s, when R.M. Foster began collecting examples of interesting graphs that could serve as models for electrical networks[1].

Of course, this area of research has taken advantage of the development and improvement of computational power but the theoretical research goes back to Tutte and his work on classifying 3-valent arc-transitive graphs[2,3]. It is believed that empirical study of symmetric graphs of small valence started in 1930s, when R.M. Foster began collecting examples of interesting graphs that could serve as models. His work was published in a book which now carries the name Foster's census[4]. The development of the

[1] R. M. Foster. "Geometrical Circuits of Electrical Networks". In: *Transactions of the American Institute of Electrical Engineers* 51.2 (June 1932), pp. 309–317.

[2] W. T. Tutte. "A family of cubical graphs". In: *Proc. Cambridge Philos. Soc.* 43 (1947), pp. 459–474.

[3] W. T. Tutte. "On the symmetry of cubic graphs". In: *Canadian J Math* 11 (1959), pp. 621–624.

[4] R. Foster. "A census of trivalent symmetrical graphs". In: *Conference on Graph Theory and Combinatorial Analysis, University of Waterloo, Ontario.* 1966.

theory, together with more powerful computers, resulted in a breakthrough of datasets of highly symmetric graphs constructions. Using the classification of automorphism groups of 3-valent arc-transitive[5] and bitransitive[6] graphs, together with new methods for finding normal subgroups of finite index in a finitely presented group allowed a construction of complete list of all 3-valent arc-transitive graphs[7 8] of order up to 10000 vertices, and a list of all 3-valent bitransitive graph on up to 768 vertices[9]. Based on their deep theoretical result on the order of automorphism groups[10] , Spiga, Verret and Potočnik compiled a complete list[11] of all trivalent vertex-transitive graphs of order at most 1280. Very recently, using the database of vertex-transitive groups of small degree, Conder and Verret have compiled a complete list of all edge-transitive graphs (of arbitrary valence) up to order 63[12] , while Holt and Royle have extended their census of all vertex-transitive graphs up to order 48[13] .

The classification and enumeration of groups has been also an intriguing problem since the beginning of theory. In 1854 Cayley[14] introduced the axiomatic definition of a group and enumerated the groups of order up to 6. Of course this is just the first step in what became an active research in both, theoretical mathematics[15] , as well as a motivation to develop computation tools such as the library `SmallGrp`[16] of small groups of GAP. In fact, one of the principal motivators on the study of symmetries of discrete objects the the *classification of Finite Simple Groups.* It turns out that many of the so-called sporadic simple groups can be understood as symmetry groups of discrete objects. This classification eventually derived in the construction of the ATLAS of Finite Groups[17] .

Once upon a time of polytopes

The enumerations and classification of the five Platonic Solids is one the most antique classification problems. There is archaeological evidence of stone balls representing what we now know as the symmetry groups of the five Platonic Solids. This evidence was discovered in Scotland and dates from the first half of the third millennia BCE. It is also known that the Egyptians and Babillonials were aware of the existence of such object but undoubtedly the Greeks have the credit of studying them form a purely mathematical interest. In fact, the thirteenth book of Euclid's Elements is devoted to the classification of the five Platonic Solids. A classification problem often relates to the definition of the objects that are being classified. In a paint from 1420 by Paolo Uccello and an engraving from 1568 by Wenzel Jamnitzer appear the oldest representation of what we know as *regular stellated polyhedra.* These are objects that share many properties with platonic solids but have the special characteristic of having stellated polygons as faces. These polyhedra were rediscovered first by Kepler in the late 1500's and then by Poinsot in 1809 who also discovered their duals; both authors described them with a mathematical approach. Soon later, in 1811 Cauchy show that the four objects descibed by Poinsot were the only possible regular stellated polyhedra. In the second half of the 19th century that Schäfli formally studied the symmetries of Platonic Solids and their higher dimensional analogous, those that we today know as regular convex

[5] D. Ž. Djoković and G. L. Miller. "Regular groups of automorphisms of cubic graphs". In: *J. Combin. Theory Ser. B* 29.2 (1980), pp. 195–230.

[6] D. M. Goldschmidt. "Automorphisms of trivalent graphs". In: *Ann. of Math. (2)* 111.2 (1980), pp. 377–406.

[7] M. Conder and P. Dobcsányi. "Trivalent symmetric graphs on up to 768 vertices". In: *J. Combin. Math. Combin. Comput.* 40 (2002), pp. 41–63.

[8] M. Conder. *Trivalent (cubic) symmetric graphs on up to 10000 vertices.* `https://www.math.auckland.ac.nz/~conder/symmcubic10000list.txt`, Accessed online January 7th 2022.

[9] M. Conder et al. "A census of semisymmetric cubic graphs on up to 768 vertices". In: *J. Algebraic Combin.* 23.3 (2006), pp. 255–294.

[10] P. Potočnik, P. Spiga, and G. Verret. "Bounding the order of the vertex-stabiliser in 3-valent vertex-transitive and 4-valent arc-transitive graphs". en. In: *J. Combin. Theory Ser. B* 111 (Mar. 2015), pp. 148–180.

[11] P. Potočnik, P. Spiga, and G. Verret. "A census of 4-valent half-arc-transitive graphs and arc-transitive digraphs of valence two". In: *Ars Math. Contemp.* 8.1 (2015), pp. 133–148.

[12] M. D. E. Conder and G. Verret. "Edge-transitive graphs of small order and the answer to a 1967 question by Folkman". In: *Algebr. Comb.* 2.6 (2019), pp. 1275–1284.

[13] D. Holt and G. Royle. "A census of small transitive groups and vertex-transitive graphs". In: *J. Symbolic Comput.* 101 (2020), pp. 51–60.

[14] A. Cayley. "On the theory of groups, as depending on the symbolic equation $\theta^n = 1$". In: *Philos Mag* 7.42 (Jan. 1854), pp. 40–47.

[15] S. R. Blackburn, P. M. Neumann, and G. Venkataraman. *Enumeration of Finite Groups.* Cambridge Tracts in Mathematics. Cambridge: Cambridge University Press, 2007.

[16] H. U. Besche, B. Eick, and E. A. O'Brien. "The groups of order at most 2000". In: *Electronic Research Announcements of the American Mathematical Society* 7 (2001), pp. 1–4.

[17] J. H. Conway. *Atlas of Finite Groups. Maximal Subgroups and Ordinary Characters for Simple Groups.* Oxford University Press, USA, 1986, p. 284.

polytopes. To this point it is important to remark that geometrical properties of convex polytopes allow us to fully classify them combinatorially by a sequence of numbers, the Schläfli symbol (explained in detail below), and hence their reconstruction from a computational viewpoint is extremely simple.

The theory of polyhedral-like structures took a complete new breath with the contributions of H.S.M. Coxeter. Those contributions are extremely numerous to list in here and spread all along the 20th century. Coxeter's monograph[18] on regular polytopes is most likely its most influential publication, but some of his remarkable contributions date as early as 1937 when together with J.F. Petrie described the *regular skew polyhedra*[19] as infinite analogues of Platonic solids. Coxeter is also attributed to classify the groups generated by hyperplane reflections, leading to what we today know as *Coxeter groups*. Coxeter groups have an influential role in several branches of mathematics. They of course, appear as the symmetry groups of regular polytopes and tessellations of the Euclidean and Hyperbolic spaces[20], but they have made their way to Tits geometries[21], computational Lie group theory, Hecke algebras[22], just to mention some.

Coxeter's work serve as inspiration for many mathematicians, one of them being Branko Grünbaum who in the 70's introduced[23] the notion of *polystroma*, which is an ancestor of what we today call *abstract polytopes*. Grünbaum is also responsible of first treating symmetric polyhedra from a combinatorial viewpoint. By relaxing the definition of a regular polyhedron he presented[24] a list of 47 regular polyhedra which included the Platonic Solids, Stellated polyhedra as well as Petrie-Coxeter skew polyhedra. Soon after A. Dress describes[25] another polyhedron and proves[26] that the list of 48 regular polyhedra is complete.

Almost at the same time as Grünbaum was describing his families of regular polyhedra on the Euclidean space, G. Jones and D. Singerman published his classical manuscript[27] which settle the necessary theory to identify maps on orientable surfaces with what in modern terminology we called its *monodromy group*. The ideas behind this paper show important equivalences between topological maps (embedding of graphs on orientable surfaces), certain quotients triangular groups (Coxeter groups of rank 3), maps on Riemmann surfaces and certain permutations on the darts of the map. These equivalences are a combinatorial/discrete version of the classical Uniformization theorem[28] for Riemann surfaces, since in particular show that every map can be obtained as a quotient of a regular tessellation of the sphere, the Euclidean plane or the hyperbolic plane. The work of Jones and Singerman was an important contribution on the theory of discrete group actions on Riemann surfaces and it was eventually connected the theory Grothendieck's *Dessins d'enfant*.[29]

Some combinatorial equivalences of maps on surfaces were also explored by Tutte, who gave an axiomatic combinatorial definition of a map.[30] Vince extended[31] these equivalences and introduced a very general definition of combinatorial maps. A slightly more restrictive definition is that of a *maniplex* in-

[18] H. S. M. Coxeter. *Regular polytopes.* Third. Dover Publications, Inc., New York, 1973, pp. xiv+321.
[19] H. S. M. Coxeter. "Regular Skew Polyhedra in Three and Four Dimension, and their Topological Analogues". In: *Proc. London Math. Soc.* S2-43.1 (1937), p. 33.
[20] J. E. Humphreys. *Reflection Groups and Coxeter Groups.* Vol. 29. Cambridge Studies in Advanced Mathematics. Cambridge University Press, June 1990, pp. xii+204.
[21] J. Tits. *Buildings of spherical type and finite BN-pairs.* Lecture Notes in Mathematics, Vol. 386. Springer-Verlag, Berlin-New York, 1974, pp. x+299.
[22] A. M. Cohen. "Coxeter Groups and three Related Topics". en. In: ed. by A. Barlotti et al. NATO ASI Series. Dordrecht: Springer Netherlands, 1991, pp. 235–278.
[23] B. Grünbaum. "Regularity of graphs, complexes and designs". In: *Problèmes combinatoires et théorie des graphes (Colloq. Internat. CNRS, Univ. Orsay, Orsay, 1976).* Vol. 260. Colloq. Internat. CNRS. CNRS, Paris, 1978, pp. 191–197.
[24] B. Grünbaum. "Regular polyhedra—old and new". In: *Aequationes Math.* 16.1-2 (1977), pp. 1–20.
[25] A. W. M. Dress. "A combinatorial theory of Grünbaum's new regular polyhedra. I. Grünbaum's new regular polyhedra and their automorphism group". In: *Aequationes Math.* 23.2-3 (1981), pp. 252–265.
[26] A. W. M. Dress. "A combinatorial theory of Grünbaum's new regular polyhedra. II. Complete enumeration". In: *Aequationes Math.* 29.2-3 (1985), pp. 222–243.
[27] G. A. Jones and D. Singerman. "Theory of maps on orientable surfaces". In: *Proc. London Math. Soc. (3)* 37.2 (1978), pp. 273–307.
[28] W. Abikoff. "The Uniformization Theorem". In: *The American Mathematical Monthly* 88.8 (Oct. 1981), pp. 574–592.
[29] G. A. Jones and J. Wolfart. *Dessins d'Enfants on Riemann Surfaces.* Springer London, Limited, 2016.
[30] W. T. Tutte. "What is a map?" In: *New directions in the theory of graphs (Proc. Third Ann Arbor Conf., Univ. Michigan, Ann Arbor, Mich., 1971).* 1973, pp. 309–325.
[31] A. Vince. "Combinatorial maps". In: *Journal of Combinatorial Theory. Series B* 34.1 (1983), pp. 1–21.

Reference (?)

troduced by Wilson[32] more than twenty years after Vince's manuscript. We shall describe this notion slightly more detail in Section ??

Highly symmetric polyhedra have captivated humankind for almost as long as history itself. The period succeeding the Greeks goes to the Roman and eventually the Byzantine empire, whose attitude to mathematics (and to other sciences as well) was ambiguous going form encouragement to suppression. However they must be credited by preserving the Greek's mathematical knowledge. The Arabs did many contributions to mathematics in this period, however it seems that geometry and hence symmetric objects was not of their scientific interest. However, they did have a strong empirical knowledge of symmetry. It is possible to find patterns in the Alhambra that exemplify many of what we know today as the planar crystallographic groups, which are closely related to the symmetry groups of highly symmetric polyhedra. This takes us to the middle ages where as in other sciences, mathematical knowledge was not of heavy interest. However again the notion of symmetry did developed through artistic pieces. Notably, the oldest appearances of what we today know as stellated polyhedra appeared in a paint from 1420 by Paolo Uccello and an engraving from 1568 by Wenzel Jamnitzer. These objects were rediscovered first by Kepler in the late 1500's and then by Poinsot in 1809; both of which described them with a mathematical approach. Soon later, in 1811 these objects where classified by Cauchy. Although Platonic solids are as old as mathematics themselves, their theoretical relevance did not develop the same way as some other aspects of mathematics. It was not until the second half of the 19th century that Schäfli formally studied the symmetries of Platonic Solids and their higher dimensional analogous, those that we today know as regular convex polytopes. To this point it is important to remark that geometrical properties of convex polytopes allow us to fully classify them combinatorially by a sequence of numbers, the Schläfli symbol (explained in detail below), and hence their reconstruction from a computational viewpoint is extremely simple. The theory of polyhedral-like structures took a complete new breath with the contributions of H.S.M Coxeter. Those contributions are extremely numerous to list in here and spread al along the 20th century. However, we should mention that Coxeter formally brought together the interplay between symmetry (group theory and geometry) and combinatorial objects. One of his most remarkable contributions is that of classifying symmetry groups generated by reflections (known today as Coxeter groups). The impact of Coxeter's contributions has served not only as reference but also as inspiration of many distinguished mathematicians and impacted not only to the community working with highly symmetric discrete objects but also to a huge extend of subareas of modern mathematics, just to mention some of them we refer to the work by Grünbaum and Dress who settle the first steps to what we today call abstract polytopes, and to the work by Tits on buildings and its further generalisation by Buekenhout as diagram geometries, both of which are related to the understanding of some of the so called sporadic almost simple groups.

Maps on surfaces. A map M on a surface S can be though as an embedding of a graph G on a surface such that the faces (connected components S) are homeomorphic to discs. The group Aut(M) of automorphisms of the map can be understood as the group of automorphism of G that extend to homeomorphisms of S. This allows us to understand the surface in a combinatorial way. Moreover, the baricentric subdivision of the map induces a triangulation of the surface into flags (figure?) so that the the action of Aut(M) on the set of flags is free. This implies that we can understand Aut(M) as a fixed-point free permutation group on the set of flags. Whenever this action is transitive the map is called regular (reflexible). This is consistent with the notion of regularity of polyhedra introduced by [REF]. In a regular map M, all the faces have the same amount p of vertices at its boundary and each vertex have the same degree q. In this situation we say that M is of (Schläfli) type p,q. Informally speaking, a regular map is one that admits fully reflectional symmetry. A regular map of type p,q is fully determined by its automorphism group and moreover, this automorphism group is a quotient of the Coxeter group [p,q]. This fact has been heavily exploited to build data sets of regular maps. Currently, there are several data sets of regular maps available. The approach taken on the development of each of them differ slightly one from another. We list the available data below. In [C2006ROM101] Conder lists all the regular maps on ori-

[32] S. Wilson. "Maniplexes: Part 1: maps, polytopes, symmetry and operators". In: *Symmetry* 4.2 (2012), pp. 265–275.

entable surfaces of genus 2 to 101 (3378 entries); this census was later improved in [C2011 ROMg301] to list all regular orientable maps of genus 2 to 301. The census [C2011ROMg301] has 15824 entries. Every non-orientable regular maps admit an regular orientable double cover, as a consequences the previous cenci automatically have their non-orientable analogous: [C2006RNOMg202] and [C2011RNOMg602] which list the regular non-orientable maps of genus 2 to 202 (862 entries) and of genus 2 to 602 (3260). The data sets described above were computed using the LowIndexNormalSubgroups of MAGMA. Further uses of this routine lead to the data sets [C2012RMe1000] of regular (orientable and non-orientable) maps with at most 1000 edges. A regular map on an orientable surface admits an index-two subgroup of automorphisms inducing full rotational symmetry. The maps (regular or not) admitting this high degree of rotational symmetry are often call rotary maps and such maps can divided in two classes: reflexible (what we before called regular) and irreflexible or chiral. Rotary maps are often regarded as the most symmetric maps, since many authors are only interested in orientation-preserving automorphisms. The rotation group of a rotary map of type p,q is a quotient of$[p, q]+$, the even subgroup of the Coxeter Group [p,q] by a normal subgroup M of [p,q]+. If M is normal not only in $[p, q]+$ but also in [p,q], then the associated rotary map is regular, otherwise is chiral. This fact has been used to build families of rotary maps and some data sets of such have been built as well. Notably, in [C2012RotM1000e] Conder lists all the rotary maps with up to 1000 edges while in [P2014RotM3000e] Potočnik impoves this dataset by listing all rotary maps with up to 3000 edges (255,980 entries). Both datasets include a version where only regular or only chiral maps are listed. In [C2006ChiM101g] Conder presents a census of chiral maps on orientable surfaces of genus from 2 to 101 (594 entries). This census was later improved by Conder and in [C20014ChiM301g] he presents the list of the 3870 chiral maps on orientable surfaces of genus from 2 to 301.

Abstract polytopes. An abstract polytope is a partially ordered set that shares many properties with the face-lattice of a convex polytopes. Examples of such are, of course (the face lattices of), all convex polytopes but also tillings of Euclidean and hyperbolic spaces, maps on surfaces as well as many objects with in principle, not necessarily nice geometrical representation. In a bit more detailed way, an abstract polytope (n-polytope, for short) is a combinatorial generalisation of all such geometrical objects. Most maps on surfaces are 3-polytopes and every 3-polytope can be regarded as a map on a not necessarily compact surface. The notion of flags extend in a natural from maps to n-polytopes as maximal chains of the poset containing exactly one face of each of the n possible ranks (dimensions). Each flag F has a unique i-adjacent flag Fi that shares all the faces but the one of rank i. This notion of adjacency of flags turns the set of flags into a n-edge coloured graph, the flag graph of the polytope. The group of automorphism of a polytope is the group of colour-preserving automorphisms of the flag graph. This is in fact isomorphic to the automorphism group of P as poset, namely, the set of order-preserving bijections and also coincide with the topological definition of automorphisms for maps. The degree of symmetry of a polytope can be measured with the number of falg-orbits of the automorphism group. As with maps, a polytope is regular if it is transitive on flags; regular polytopes are the most symmetric ones. The automorphism group of a regular polytope is a quotient of a Coxeter group with string diagram. As before, this result has been extensively used to build regular polytopes from a theoretical approach. However, the amount of datasets available regular n-polytopes, with n >= 4 is significantly small compared to that for maps. In [H2006RP2000f] Hartley lists all the possible regular polytopes with at most 2000 flags, excetp those with 1024 or 1536 flags). He used the library of small groups of GAP, that lists contains all possible groups with at most 2000 elements. The census [C2012RP2000f] containing essentially the same information was computed by Conder. There are some datasets containing all regular polyopes whose automorphism group belongs to a relevant family of groups. Notably Hartley's [HRPSSG] contains the regular polytopes whose automorphism group is one of the sporadic symple groups of order smaller thant that of the Held group (aprox. 4X109). While in [LRPASG] Leemans keeps an on going census of regular polytopes whose automorphism group is an almost simple group. Both data sets offer several thousands of examples of regular polytopes but most of them (over 90In a very similar ways as with maps, there is a combinatorial definition of rotary n-polytopes. Rotary but non-regular polytopes are called chiral and besides regular polytopes. Chiral polytopes have 2-flag orbits and the class of chiral polytopes is the second most studied symmetry family of abstract polytopes. However, the availability of data sets or even explicit examples of chiral polytopes of rank higher than 4 is almost non-existent. Chiral polytopes of rank 3 (chiral maps) are a classical part of the theory and the definition naturally extends to higher ranks. Chiral polytopes were formally introduced by Schulte and Weiss in beginning of the 90's [REF!] and for many years it was hard to theoretically find examples of chiral polytopes of rank 4 or 5. Moreover, in the attempt of finding such examples many non-existence results were established. It was until 2010 that Pellicer proved that chiral n-polytopes exists for any n [REF]. Pellicer's proof is constructive but trying to compute the smallest of his examples of rank larger than 10 is not practical with the current computational power; the size of such polytopes grows as a tower of exponential functions whose length grows with n. As mentioned before the existence of data sets of chiral polytopes is very limited. Every chiral polytope admits a minimal regular cover, which implies that we can explore the data sets of regular polytopes and check which of them are regular covers of chiral polytopes. Using this approach, Hartley collected in [HSCP] all the chiral polytopes whose minimal regular cover appears in [H2012RP2000f]. This census of chiral polytopes contains only 56 polytopes (compared to the over 20 000 entries in the census of regular polytopes used to build it). Moreover, only 8 of such polytopes are of rank 4, the remaining 48 are chiral maps. Using LowIndexNormalSubgroups routine for MAGMA , Conder constructed the census [CCP2000f] which contains all chiral polytopes with at most 2000 flags. To no one surprise, this atlas contains mostly examples in rank 3, a few examples in rank 4 and just one example in rank 5. As with regular polytopes. Leemans et al. Keep in [LCPASG] a on going census of chiral polytopes whose automorphism group is one of the almost simple groups. This census contains just a few thousands of entries and as with the other cenci of chiral polytopes, the vast majority of them are or rank 3, whit not a single example of rank larger than 5. Chiral polytopes are just one of (2n) -1 possible symmetry type class of 2-orbit polytopes. Classical examples of some of the other classes of 2-orbit polytopes are known. However, the general problem of determining if for every pair (n,T) with n >3 and T a symmetry type of 2-orbit polytopes exists an n-polytope of symmetry type T remains mostly open. Of course, collecting and classifying known examples and constructions is of the interest of the community and it might help to solve existence problems as the one just presented. Constructing and classifying n-polytopes with k-orbits and given symmetry type is an fairly recent and active research area and just as with more symmetric polytopes, trying to identify patterns and constructing new examples from previously known examples could be seriously improved by the construction of data sets of polytopes. Some of the possible uses will be discussed below.

Maniplexes. We very briefly mention the notion of an n-maniplex. Roughly speaking, a maniplex is a combinatorial generalization of both maps and (the flag graph of) polytopes. They were introduced by Wilson in [REF] but have been gaining popularity recently. In particular, the usage of maniplexes has allowed to use graph-theoretical techniques to build polytopes [REF! PrimozMicaelDanie, Tero, Elias, EliasIsaTero, Gabe]. Highly symmetric maniplexes that are not polytopes seem to be very degenerate. Moreover, there is a characterisation of maniplexes that are polytopes [REF Isa]. There is no current data set explicitly dedicated to maniplexes (and not to maps or polytopes) but recent research shows that it might be worthy to start treating polytopes as a particular class fo maniplexes and leave polytopality as an attribute of such.

Problem identification and Research and Innovation objectives.

he general objective of OURPROJECT is to build a environment of data sets and computational tools for maps, maniplexes and polytopes. We expect this environment to be not only of the interest but more importantly, useful to the community doing research on this area. We explain in the following paragraphs how we split this general objective into several particular and very concrete objectives. The existent datasets of highly symmetric maps and polytopes have not been exploded to its full capacity. One of the reasons behind it is that the information in most of these data sets is not very user-friendly. Even the small amount of existing data sets have been developed by several people, mostly in an independent way, using different notation and different computer algebra systems. This usually stops a researcher to use a given data set just because he or she might not be familiar with the notation or programming language. Moreover, many of these cenci exists only as raw text which is not always easy to consult. However, we could use the fact the the amount of data is not huge in our advantage. All the facts mentioned above outline our first objective:

Objective 1. Collect, unify and make the existing data sets user-friendly and in a FAIR-ly way so that OURPROJECT eventually becomes the standard to-go when consulting or publishing new data on highly symmetric polytopes by i. Create and develop a unique data set from the existent ones that unifies notation and identification of the data and make it available so that other researches could experiment and eventually contribute to OURPROJECT. ii. Build a web-based interface to our data set for easy and quick consultation. iii. Develop this data set in a way that can be easily implemented in some of the standards computer algebra systems such as MAGMA, GAP and SageMath. iv. Write appropriate documentation so that OURPROJECT eventually becomes available for others to contribute.

Objective 1 is very concrete and offers and very easy to verify the success of OURPROJECT while at the same time sets a good start point to our ambitious general objective. The current state-of-theart allow us to start with Objective 1 from the very beginning of OURPROJECT

Many of the first data sets of abstract polytopes are based on the (small) size of the objects. Either by taking advantage of previously computed (such as the library of small groups) or by using computational routines that, because of their own nature are limited by the size of the input (such as the LowIndex-Subgroup). However, it has been shown that the size of the smallest regular polytope of rank n grows exponentially with n [REF] while the size of the smallest chiral polytope of rank n is at least of factorial growth with respect to n [REF]. This explains why de amount of examples of higher rank polytopes drops dramatically on the current data sets available. On the other hand, there are interesting infinite families (eg, the Toroids) or constructions (eg. generalized cubes) that of course will not come as a result of an exhaustive exploration but that are not too demanding from a computational approach. Motivated from the previous discussion we propose our second objective.

Objective 2. Explore the literature and implement appropriate routines to construct new examples of polytopes from previously known ones.

Of course the previous very practical objective comes alongside with the next one which, just by its theoretical relevance, presents a very ambitious part of OURPROJECT.

Objective 3. Develop new constructions of highly symmetric polytopes. In particular, focus on the constructions of abstract regular polytopes with given symmetry type to eventually build data sets of given symmetry type (besides regular and chiral).

Objective 3 by itself represents an extremely ambitious and it goes beyond any two year project. Any theoretical contribution is already of great interest to the community. However, we strongly believe that in order to really cause an impact on the workflow of research on abstract polytopes all theoretical contributions should be accompanied by its computational analogue.

Of course, OURPROJECT is aimed to become a long term and eventually permanent resource for the community doing research on symmetries of maps and abstract polytopes. This will not be achievable without the involvement of such community.

Objective 4. Encourage and motivate both well-established and young reseachers to use and contribute to OURPROJECT so that it eventually becomes the standard way to explore, experiment and publish data sets, routines and computational tools for the development on the research of abstract polytopes.

Of course, Objective 4 is very ambitious and we acknowledge that it depends on the community more that on ourselves, but we strongly believe that our approach and the the current status of the could fit together to fill a gap that has been present for many year now. It is important to remark that the community has faced a similar scenario before. Many of the early research on abstract polytopes was collected on the comprehensive manuscript [REF! ARP] and nowadays it serves as the natural and standard theoretical reference. Our expectations is that OURPROJECT eventually becomes the computational analogue for our community.

1.2 Soundness of the proposed methodology (including interdisciplinary approaches, consideration of the gender dimension and other diversity aspects if relevant for the research project, and the quality of open science practices, including sharing and management of research outputs and engagement of citizens, civil society and end users, where appropriate)

The nature of our project involves a two way flow of knowledge between from discrete mathematics and group theory to the development of computational tools and data management. In order to build new data sets of abstract polytopes we need computational-efitient ways to compute and storage property of such objects and the other way around, if we want to OURPROJECT to eventually becomes a useful tool on theoretical research we need to develop ways to access and present the computed information in a user friendly way. We describe our proposed metodology from this two complementary approaches.

Mathematical representation of highly symmetric abstract polytopes.

Abrstract polytopes as posets. The original definition of an abstract polytope is on the form of a poset [REF]. It makes sense from an hystorical view point. They are intendet to be combinatorial generalisation of the (geometric) convex polytopes. This generalisation was obtained by taking some of the properties of the face lattice of a convex polytope and use them as defining properties of an abstract polytope. Unfortunatelly, the computational cost and the combinatorial problem of storaging a poset seems to be very inneficient. Moreover, modern computer algebra systems do not have particullarly efficient tools to deal with posets. We should try to avoid representing abstract polytopes as parially ordered sets.

Abstract polytopes from their automorphism group. Most likely this is the most exploided representation of an abstract polytopes. When an abstract polytope has a high degree of symmetry its automorphism group contains many combinatorial information of the poltyope. In particular, regular polytopes are incorrespondence with string C-groups [REF!], which are smoot quotient of Coxeter groups satisfying certain interection property. This fact has been strongly used to build the existing datasets mentioned in Section 1.1. The census [CONDER] was built by computing all possible normal subgroups of index at most 2000 of the universal string Coxeter group. This approach is computational expensive but it has the advantage that the computations have to be done only once. It might be worthy to try an push

1.3 Quality of supervision, training, and knowledge transfer

2 Impact

2.1 Credibility of the measures to enhance the career perspectives and employability of the researcher and contribution to his/her skills development

2.2 Suitability and quality of the measures to maximise expected outcomes and impacts, as set out in the dissemination and exploitation plan, including communication activities

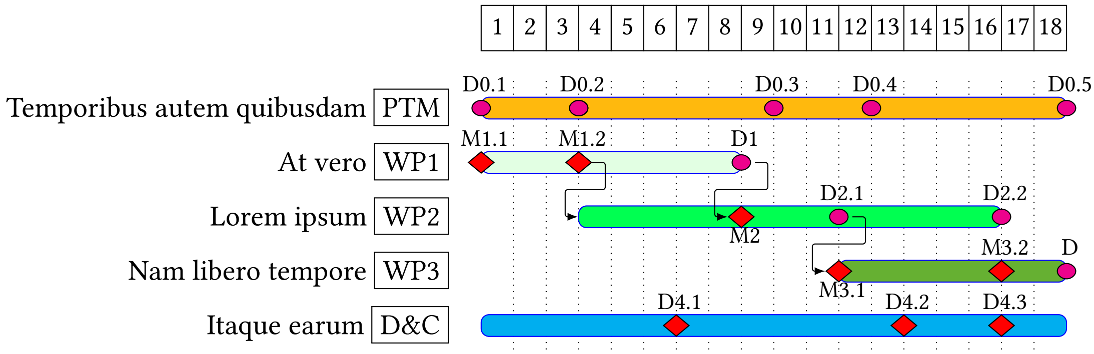
At a minimum, address the following aspects: Plan for the dissemination and exploitation activities, including communication activities:1 Describe the planned measures to maximize the impact of your project by providing a first version of your 'plan for the dissemination and exploitation including communication activities'. Describe the dissemination, exploitation measures that are planned, and the target group(s) addressed (e.g. scientific community, end users, financial actors, public at large). Regarding communication measures and public engagement strategy, the aim is to inform and reach out to society and show the activities performed, and the use and the benefits the project will have for citizens. Activities must be strategically planned, with clear objectives, start at the outset and continue through the lifetime of the project. The description of the communication activities needs to state the main messages as well as the tools and channels that will be used to reach out to each of the chosen target groups. • Strategy for the management of intellectual property, foreseen protection measures: if relevant, discuss the strategy for the management of intellectual property, foreseen protection measures, such as patents, design rights, copyright, trade secrets, etc., and how these would be used to support exploitation.

• All measures should be proportionate to the scale of the project, and should contain concrete actions to be implemented both during and after the end of the project.

2.3 The magnitude and importance of the project's contribution to the expected scientific, societal and economic impacts

3 Quality and Efficiency of the Implementation

3.1 Quality and effectiveness of the work plan, assessment of risks and appropriateness of the effort assigned to work packages



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Risk management

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3.2 Quality and capacity of the host institutions and participating organisations, including hosting arrangements