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MARIE SKŁODOWSKA-CURIE ACTIONS

Individual Fellowships (IF)
Call: H2020-MSCA-IF-2016

PART B

“HSPLS”

This proposal is to be evaluated as:

[**Standard EF**]

Contents

List of Participating Organisations

Participating organisations	Legal Entity Short Name	Academic (tick)	Non-academic (tick)	Country	Dept./ Division/ Laboratory	Supervisor
Beneficiary						
Imperial College of Science, Technology and Medicine (Imperial College London)	Imperial	✓		United Kingdom	Dept. of Mathematics	Prof. Martin Liebeck

There are no partner organisations in this project.

1. Excellence

1.1. **Quality and credibility of the research/innovation action (level of novelty, appropriate consideration of inter/multidisciplinary and gender aspects).**

Introduction. One of the most mysterious aspects of mathematics is the possibility of enumerating certain symmetric structures. An illustration of this phenomenon is the enumeration of the finite **linear spaces** with a **2-transitive** automorphism group. A linear space consists of a set of points and a set of subsets of points called lines for which each pair of distinct points lies on **exactly** one line, and each line contains at least two points; its automorphism group is 2-transitive when any pair of distinct points can be mapped to any other. For example, both the projective space $\mathrm{PG}(d, q)$ and the affine space $\mathrm{AG}(d, q)$ have these properties. On the face of it, the assumptions of finite 2-transitive linear spaces do not appear to be especially restrictive, and yet, it turns out that besides the projective and affine spaces, there are only two more infinite families and four sporadic examples.[1] Not only is this list extremely short, but it also reveals how the very general conditions "finite linear space" and "2-transitive" encapsulate far more detailed structures; symmetry is an excellent tool for compressing information.

Another occurrence of this phenomenon is the classification of the finite **homogeneous graphs**. These (simple undirected) graphs are characterised by the property that any isomorphism between induced subgraphs can be lifted to an automorphism of the entire graph. Once again, the list of examples is very short:[2–4] it consists of the disjoint unions of same-sized complete graphs; their complements, the complete multipartite graphs; the 3×3 grid graph; and the pentagon C_5. In some sense, this list is too short. We should be able to capture other highly symmetric graphs, such as the $n \times n$ grid graph or the cycle C_n. And we can, provided we relax the condition of homogeneity so that only isomorphisms of **connected** induced subgraphs lift, in which case we obtain both of these families, as well as the famous Petersen graph, and not much more.[5] In fact, the concept of homogeneity is far too strong, for if we insist that only isomorphisms of induced graphs with **at most 5** vertices lift, then the resulting list still comprises only the homogeneous graphs![6]

This behaviour leads to the central question behind the candidate's research: to what extent can we weaken certain symmetry hypotheses so that we can (perhaps) find extra examples but still enumerate them?

There are several possible avenues for answering this question. The first is to weaken homogeneity to ***X*-homogeneity** for some class of graphs X, where only isomorphisms of induced subgraphs in X lift to automorphisms. For instance, if we take X to be the class of paths, then the X-homogeneous graphs are precisely the distance-transitive graphs, which have been studied extensively and whose classification is well under way.[7,8] If we instead take X to be the class of graphs with at most 2 vertices, then the automorphism group of a non-complete X-homogeneous graph with at least one edge has exactly 3 orbits on ordered pairs of vertices; in group theoretic terms, the automorphism group is transitive of **rank 3**, and using the classification of the **primitive** permutation groups of rank 3,[9] the X-homogeneous graphs can be completely enumerated. Thus another avenue for weakening homogeneity is to study graphs admitting automorphism groups of **low rank**. A third avenue is to extend the concepts of X-homogeneity and low rank to a class of relational structures that generalises both graphs and linear spaces: **partial linear spaces**. These consist of a set of points and a set of subsets of points called lines for which each pair of points lies on **at most** one line, and each line contains at least two points. As with graphs, there are very few homogeneous

[1] Kantor, W. M. Homogeneous designs and geometric lattices. *J. Combin. Theory, Ser. A* 38 (1985), 66–74.
[2] Sheehan, J. Smoothly embeddable subgraphs. *J. London Math. Soc.* 2 (1974), 212–218.
[3] Gardiner, A. Homogeneous graphs. *J. Combin. Theory, Ser. B* 20 (1976), 94–102.
[4] Gol'fand, Y. and Klin, M. On k-homogeneous graphs. *Algorithmic Studies in Combinatorics* (1978), 76–85.
[5] Gardiner, A. Homogeneity conditions in graphs. *J. Combin. Theory Ser. B* 24 (1978), 301–310.
[6] Cameron, P. J. 6-transitive graphs. *J. Combin. Theory, Ser. B* 28 (1980), 168–179.
[7] Brouwer, A. E., Cohen, A. M. and Neumaier, A. *Distance-regular graphs.* Springer-Verlag, Berlin, 1989.
[8] van Bon, J. Finite primitive distance-transitive graphs. *Eur. J. Combin.* 28 (2007), 517–532.
[9] Liebeck, M. W. and Saxl, J. The finite primitive permutation groups of rank three. *Bull. London Math. Soc.* 18 (1986), 165–172.

partial linear spaces, and if we take X to be the class of partial linear spaces with at most 6 points, then the X-homogeneous partial linear spaces are all homogeneous.[10]

The candidate has two long-term research goals. The first is to understand X-homogeneous partial linear spaces $\mathcal{S}$ where X is the class of connected partial linear spaces (possibly with bounded size). This leads to two main research objectives. **Objective 1** is to focus on the case when $\mathcal{S}$ is not a graph or a linear space, where nothing is yet known. **Objective 2** is to continue her work on the case when $\mathcal{S}$ is a graph and the members of X have bounded size.[11] The second long-term goal of the candidate is to understand partial linear spaces admitting automorphism groups of low rank, which again leads to two main research objectives. **Objective 3** is to complete the classification of the primitive permutation groups of rank at most 5. **Objective 4** is to use this classification to investigate certain partial linear spaces with rank 4 automorphism groups.

State-of-the-art. The study of primitive permutation groups of **low rank** goes back to Burnside,[12] who proved in 1897 that a finite 2-transitive (or rank 2) permutation group has a unique minimal normal subgroup that is either non-abelian and simple, or abelian and regular. The former condition characterises **almost simple** groups, while the latter characterises **affine** groups. With the advent of the classification of the finite simple groups,[13] the list of 2-transitive permutation groups could then be determined.[14–17] More generally, the O'Nan-Scott Theorem implies that a classification of the finite primitive groups of rank at most 5 essentially reduces to a consideration of the almost simple and affine cases.[18,19] The primitive almost simple groups of rank 3, 4 or 5 were classified in a series of papers,[9,19–22] while the primitive affine groups of rank 3 were classified by Foulser and the supervisor Liebeck.[23,24] The case of primitive affine groups with rank 4 or 5 remains open.

The study of **homogeneous** (or ultrahomogeneous) relational structures began with the theory of Fraïssé,[25] and this theory was then used to characterise the countable homogeneous graphs.[26] We have already discussed some of the impressive research on homogeneous graphs, including the result that every finite 2-homogeneous graph is known by the classification of the primitive groups of rank 3. (A partial linear space is ***k*-homogeneous** if it is X-homogeneous where X is the class of partial linear spaces with at most k points.) Before this classification was completed, Buczak proved that only two finite graphs are 4- but not 5-homogeneous,[27] and Cameron and Macpherson determined the finite 3-homogeneous graphs.[28] Thus finite k-homogeneous graphs are well understood.

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[10] Devillers, A. Ultrahomogeneous semilinear spaces. *Proc. London Math. Soc.* 84 (2002), 35–58.

[11] Devillers, A., Fawcett, J. B., Li, C. H., Praeger, C. E. and Zhou, J.-X. k-connected-homogeneous graphs. In prep.

[12] Burnside, W. *Theory of groups of finite order.* CUP, Cambridge, 1911.

[13] Gorenstein, D. *Finite simple groups.* Plenum Press, New York and London, 1982.

[14] Maillet, E. Sur les isomorphes holédriques et transitifs des groupes symétriques ou alternés. *J. Math. Pures et Appl.* 1 (1895), 5–34.

[15] Huppert, B. Zweifach transitive, auflösbare Permutationsgruppen. *Math. Z.* 68 (1957), 126–150.

[16] Curtis, C. W., Kantor, W. M. and Seitz, G. M. The 2-transitive permutation representations of the finite Chevalley groups. *Trans. Amer. Math. Soc.* (1976), 1–59.

[17] Hering, C. Transitive linear groups and linear groups which contain irreducible subgroups of prime order, II. *J. Algebra* 93 (1985), 151–164.

[18] Liebeck, M. W., Praeger, C. E. and Saxl, J. On the O'Nan-Scott theorem for finite primitive permutation groups. *J. Austral. Math. Soc.* 44 (1988), 389–396.

[19] Cuypers, H. Low rank permutation representations of the finite simple groups of Lie type, Part 1 of: Geometries and permutation groups of small rank. *Doctoral thesis, Rijksuniversiteit, Utrecht* (1989).

[20] Bannai, E. Maximal subgroups of low rank of finite symmetric and alternating groups. *J. Fac. Sci. Univ. Tokyo Sect. IA* 18 (1972), 475–486.

[21] Kantor, W. M. and Liebler, R. A. The rank 3 permutation representations of the finite classical groups. *Trans. Amer. Math. Soc.* 271 (1982), 1–71.

[22] Praeger, C. E. and Soicher, L. H. *Low rank representations and graphs for sporadic groups.* CUP, Cambridge, 1997.

[23] Foulser, D. A. Solvable primitive permutation groups of low rank. *Trans. Amer. Math. Soc.* 143 (1969), 1–54.

[24] Liebeck, M. W. The affine permutation groups of rank three. *Proc. London Math. Soc.* 54 (1987), 477–516.

[25] Fraïssé, R. Sur certaines relations qui généralisent l'ordre des nombres rationnels. *C. R. Acad. Sci.* 237 (1953), 540–542.

[26] Lachlan, A. H. and Woodrow, R. E. Countable ultrahomogeneous undirected graphs. *Trans. Amer. Math. Soc.* 262 (1980), 51–94.

[27] Buczak, J. M. J. Finite group theory. *D.Phil. thesis, Oxford University* (1980).

[28] Cameron, P. J. and Macpherson, H. D. Rank three permutation groups with rank three subconstituents. *J. Combin. Theory, Ser. B* 39 (1985), 1–16.

The finite k-homogeneous linear spaces are also well understood, for they can be determined directly from the classification of the 2-transitive linear spaces;[1] in particular, all finite 6-homogeneous linear spaces are homogeneous. In fact, Devillers proved that this holds for all finite partial linear spaces.[10] Devillers also investigated **proper** partial linear spaces—those that are not graphs or linear spaces—and proved the remarkable result that a finite proper 3-homogeneous partial linear space must be one of three well-studied classes: a polar space, a copolar space or a partial geometry. Later, in her Ph.D. thesis, Devillers classified the finite proper 4- and 5-homogeneous partial linear spaces.[29] Finally, the finite proper 2-homogeneous partial linear spaces are precisely those whose automorphism groups are transitive of rank 3, which Devillers classified in the primitive non-affine cases.[30,31] The candidate has nearly completed the primitive affine case.[32]

One of the candidate's long-term goals is to understand X-homogeneous partial linear spaces where X is the class of connected partial linear spaces with at most k points; these are ***k*-connected-homogeneous** (***k*-CH**), or **connected-homogeneous** (**CH**) when k is the number of points in the partial linear space. The finite and countable CH graphs have been determined,[5,33] as have certain infinite 3-CH graphs,[34] but little is known about finite k-CH graphs for small k. This problem is intractable for $k \leqslant 2$ and extremely difficult for $k = 3$. The candidate is currently investigating 4-CH graphs;[11] together with her collaborators, she has classified the finite 4-CH graphs whose local graphs are connected and obtained partial results in the disconnected case. Nothing is known about k-CH proper partial linear spaces for any k. (Note that a k-CH linear space is trivially k-homogeneous.)

Objectives and overview of the action. We now expand on the research objectives provided in the introduction.

Objective 1. The candidate will initiate a study of the finite proper k-CH partial linear spaces. She will classify those partial linear spaces that are CH and determine whether there exists an absolute constant c for which every c-CH partial linear space is CH. She will also develop a programme for classifying well-behaved families of k-CH partial linear spaces for small k.

Objective 2. The candidate will investigate finite k-CH graphs with girth 4. When $k = 7$, such graphs are CH,[11] so she wishes to determine the minimal constant c for which c-CH graphs with girth 4 are CH. This constant is at least 5 since the n-cube ($n \geqslant 6$) is a graph with girth 4 that is 4-CH but not 5-CH, and she conjectures that c is in fact equal to 5.

Objective 3. The candidate will complete the classification of the finite primitive permutation groups of rank at most 5 by determining those primitive permutation groups of affine type with rank 4 or 5. Through this process, the candidate will develop her knowledge of Aschbacher reduction and the irreducible projective representations of the almost simple groups of Lie type.

Objective 4. The candidate will commence an analysis of the finite proper partial linear spaces $\mathcal{S}$ admitting a primitive automorphism group G of rank 4 by focusing on the case when $\mathcal{S}$ is a partial geometry and G is an affine group acting transitively on pairs of collinear points.

Research methodology and approach.

Approach to Objective 1. The candidate expects that the methods for enumerating the finite proper CH partial linear spaces will be primarily combinatorial. Some of the methods from the classifications of the homogeneous partial linear spaces and CH graphs will apply,[5,10] but she will also need to develop new techniques to deal with the complication that, unlike a graph, the partial

[29] Devillers, A. Classification of some homogeneous and ultrahomogeneous structures. *Ph.D. thesis, Université Libre de Bruxelles* (2002).

[30] Devillers, A. A classification of finite partial linear spaces with a primitive rank 3 automorphism group of almost simple type. *Innov. Incid. Geom.* 2 (2005), 131–177.

[31] Devillers, A. A classification of finite partial linear spaces with a primitive rank 3 automorphism group of grid type. *Eur. J. Combin.* 29 (2008), 268–272.

[32] Bamberg, J., Devillers, A., Fawcett, J. B. and Praeger, C. E. Partial linear spaces with a primitive rank 3 automorphism group of affine type. In prep.

[33] Gray, R. and Macpherson, D. Countable connected-homogeneous graphs. *J. Combin. Theory Ser. B* 100 (2010), 97–118.

[34] Gray, R. k-CS-transitive infinite graphs. *J. Combin. Theory Ser. B* 99 (2009), 378–398.

linear space $\mathcal{S}_p$ induced by the set of points collinear to a point p need not be homogeneous. In fact, $\mathcal{S}_p$ is either a disjoint union of lines, or admits an imprimitive permutation group of rank 4 that is X-homogeneous where X is the class of partial linear spaces with no lines. Determining whether there exists an absolute constant c for which c-CH partial linear spaces are CH will likely require the classification of the finite simple groups, together with combinatorial techniques.

Approach to Objective 2. Let Γ be a 5-CH graph with girth 4. The candidate conjectures that Γ is CH. For a vertex u, the stabiliser $\mathrm{Aut}(\Gamma)_u$ acts 4-transitively on the neighbours of u. Thus the permutation group induced by $\mathrm{Aut}(\Gamma)_u$ is either the symmetric group, the alternating group, or a Mathieu group M_n acting on n points where $n \in \{11, 12, 23, 34\}$. By a result of Cameron, only the third case is possible if Γ is not CH.[35] Combinatorial arguments then force $n \in \{12, 24\}$. By developing the arguments of Cameron or using the amalgam method,[36,37] the candidate aims to prove that no such graph exists. If successful, the candidate will use similar ideas to study 4-CH graphs with girth 4. Regardless, she hopes to obtain a classification of the 5-CH graphs with girth 4 and therefore determine the minimal constant c for which all c-CH graphs with girth 4 are CH.

Approach to Objective 3. A primitive permutation group of affine type G is a subgroup of the general affine group $\mathrm{AGL}(V)$ for some $\mathbb{F}_p$-vector space V and prime p with the form TG_0 where T is the group of translations of V, and G_0, the stabiliser of the 0 vector, is an irreducible subgroup of the general linear group $\mathrm{GL}(V)$. The group G has rank r on V if and only if G_0 has $r-1$ orbits on the non-zero vectors of V. Using a theorem of Aschbacher concerning the maximal subgroups of the finite classical groups,[38] the candidate will reduce the classification of such G with rank 4 or 5 to the case where the quotient of G_0 by its centre is almost simple with minimal normal subgroup L, and the projective representation of L on V is absolutely irreducible. She will then perform a case-by-case analysis of such groups using methods developed by the supervisor.[24,39]

Approach to Objective 4. Let $\mathcal{S}$ be a proper partial linear space with a primitive affine automorphism group G of rank 4 (where G may not be the full automorphism group). The collinearity graph Γ of $\mathcal{S}$ has vertex set the points of $\mathcal{S}$ with two points adjacent whenever they are collinear in $\mathcal{S}$. Since G has rank 4, the diameter of Γ is at most 3. If it is 3, then Γ is distance-transitive and therefore known,[40] so determining $\mathcal{S}$ should be straightforward. Otherwise, Γ has diameter 2 and is not distance-transitive. One well-known family of partial linear spaces with diameter 2 are the partial geometries, for which there exists a positive constant α such that for each anti-flag (p, ℓ)—that is, for a point p not on a line ℓ—the point p is collinear with exactly α points on ℓ. We assume that $\mathcal{S}$ is a partial geometry. Further, G has one or two orbits on pairs of collinear points, so we assume that it has one, in which case $\mathcal{S}$ is flag-transitive. From the candidate's classification in Objective 3, the group G and its orbits will be known, and, using techniques developed previously by the candidate,[32] she will determine which orbits (if any) can be the set of points collinear with a given point in a partial geometry.

Originality and innovative aspects of the research programme. The candidate is the first to investigate k-CH partial linear spaces (Objective 1). Finite partial linear spaces are a sizeable and relevant class of structures, and those that are **flag-transitive** are of special interest, as the substantial effort to classify the flag-transitive linear spaces demonstrates.[41] Moreover, CH partial linear spaces have a far richer structure than homogeneous ones, for 2-homogeneity alone implies that the collinearity

[35] Cameron, P. J. Suborbits in transitive permutation groups. In: *Combinatorics: Proceedings of the NATO Advanced Study Institute held at Nijenrode Castle, Breukelen, The Netherlands, 8-20 July 1974.* Springer, 1975, 419–450.

[36] Cameron, P. J. Permutation groups with multiply transitive suborbits. *Proc. London Math. Soc* 3 (1972), 427–440.

[37] Ivanov, A. A. and Shpectorov, S. V. *Geometry of Sporadic Groups II, Representations and Amalgams.* CUP, Cambridge, 2002.

[38] Aschbacher, M. On the maximal subgroups of the finite classical groups. *Invent. Math.* 76 (1984), 469–514.

[39] Liebeck, M. W. On the orders of maximal subgroups of the finite classical groups. *Proc. London Math. Soc.* 50 (1985), 426–446.

[40] van Bon, J., Cohen, A. M. and Cuypers, H. Affine distance-transitive graphs and classical groups. *J. Combin. Theory, Ser. A* 110 (2005), 291–335.

[41] Buekenhout, F., Delandtsheer, A., Doyen, J., Kleidman, P. B., Liebeck, M. W. and Saxl, J. Linear spaces with flag-transitive automophism groups. *Geom. Dedicata* 36 (1990), 89–94.

graph has diameter at most 2. Consequently, the candidate expects that several infinite families of non-homogeneous CH partial linear spaces will exist; one such example is the $n \times n$ grid for $n \geqslant 4$. This research will require the use of computer software such as MAGMA and GAP.[42,43] Leonard Soicher (Queen Mary, University of London) is the chair of the GAP council, and working with this expert will lead to a new collaboration for the host institution, Imperial College London.

Graphs with girth 4 that are 3-CH (Objective 2) are **2-arc-transitive**, and such graphs are of great interest to algebraic graph theorists.[44] The candidate's recent work on k-CH graphs leaves several open problems in the difficult case where the local graph is complete multipartite,[11] and Objective 2 is the most tractable of these, though still challenging. One method of attack is to develop the techniques of Peter Cameron (University of St Andrews),[36] which will result in a new collaboration for the host institution. Alternatively, the candidate anticipates that the amalgam method will be fruitful, and although she is unfamiliar with this method, Sasha Ivanov (Imperial College London) is an expert. Another expert is Chris Parker (University of Birmingham), and the candidate will seek to build a new collaboration with this researcher.

Given the extensive study of primitive permutation groups of low rank, it is surprising that this classification was never completed (Objective 3). Classifying the primitive affine groups of rank 4 and 5 will therefore be a significant achievement for the candidate. Further, her knowledge of the techniques involved is limited, while the supervisor is an expert; indeed, he classified the primitive affine groups of rank 3.[24] Through this work, the candidate hopes to substantially improve her understanding of Aschbacher reduction and representations of almost simple groups, as these topics frequently arise in her field of research. In addition, this classification will be vital for Objective 4.

The candidate is working on a project to classify the finite 2-homogeneous proper partial linear spaces;[30–32] these have an automorphism group of rank 3. Only several infinite families arise in the primitive affine case (including one family of partial geometries), and she believes that weakening the assumptions of 2-homogeneity to allow two orbits on pairs of non-collinear points—in which case the automorphism group has rank 4—will result in new and interesting families of partial linear spaces. Partial geometries are of particular interest, for their collinearity graphs are **strongly regular**. The supervisor's classification of the flag-transitive affine linear spaces puts him in a unique position to aid the candidate with this research.[45] This objective may also lead to a future collaboration between the host institution and Hans Cuypers (Technische Universiteit Eindhoven), who is an expert on primitive almost simple groups of rank 4.[19]

These four objectives, especially the broader goals of Objectives 1 and 4, will lay the foundations for a future programme of study for the candidate, and they will greatly improve her chances of finding a permanent academic position in the UK.

There are no gender issues in the proposed research. The research is inherently interdisciplinary, as it relates two fields of mathematics: combinatorics and group theory.

1.2. **Quality and appropriateness of the training and of the two way transfer of knowledge between the researcher and the host.** The candidate's research training objectives are to become competent in the following three theories.

(TO1) Algebraic groups and projective representations of finite almost simple groups of Lie type.
(TO2) Aschbacher reduction.
(TO3) The amalgam method.

Aschbacher reduction (TO2) is a fundamental tool for permutation group theorists.[38] This procedure essentially divides the study of primitive affine groups into nine classes: the first eight constitute the **geometric classes**, while the ninth is the **almost simple class**, in which case the theory of (TO1) is applied. Though the candidate has used many results that employ Aschbacher reduction, she has never performed one herself. Moreover, though the candidate has a thorough understanding of the

[42] BOSMA, W., CANNON, J. AND PLAYOUST, C. The Magma algebra system. I. The user language. *J. Symbolic Comput.* 24 (1997), 235–265.
[43] GAP – *Groups, Algorithms, and Programming, Version 4.6.3.* `http://www.gap-system.org`. The GAP Group. 2013.
[44] SERESS, Á. Toward the classification of s-arc transitive graphs. In: *Groups St Andrews 2005.* CUP, Cambridge, 2007, 401–414.
[45] LIEBECK, M. W. The classification of finite linear spaces with flag-transitive automorphism groups of affine type. *J. Combin. Theory, Ser. A* 84 (1998), 196–235.

representations of symmetric and alternating groups, she has little knowledge of the representations of almost simple groups of Lie type, and in order to understand these, it is also necessary to learn about algebraic groups, another weakness for the candidate. It is vital for the candidate to improve her skills in these areas. In her experience, the best way to learn is by doing, and this is a key benefit of pursuing Objective 3, where she will directly apply (TO1) and (TO2) under the supervisor's guidance, who is a world expert in these techniques. She will supplement this hands-on approach by attending a graduate course on Lie Algebras. Learning about the very active field of algebraic groups will also broaden the candidate's research interests. Finally, the amalgam method (TO3) is a powerful tool in group theory for studying graphs using information about their local actions. The candidate will try this method for the first time through Objective 2. Sasha Ivanov from the host institution is an expert and will be an invaluable resource.[37]

The Imperial **Postdoc Development Centre** aims to maximise the potential of postdoctoral fellows at Imperial through a range of courses, workshops and other forms of support. In order to fulfil her goal of acquiring a permanent academic position in the UK, the candidate will take the following courses: *Interviewing for an Academic Career*, and *Lectureship Applications: What You Need to Know to be Successful*. She will also take the course *Science Communication: Reaching a Wider Audience* in order to learn how to promote her work, including to the general public; she will then apply the skills gained to her proposed research.

The candidate has held a postdoctoral position in the Centre for the Mathematics of Symmetry and Computation (CMSC) at The University of Western Australia for three years. The Centre is one of the foremost groups in the world for algebraic combinatorics, with researchers in group theory, graph theory and finite geometry. Through seminars, reading groups and numerous collaborations, the candidate has been exposed to a wide range of techniques for studying symmetric partial linear spaces. Though the host institution is well represented in permutation group theory and algebraic graph theory, none of its researchers have worked with proper partial linear spaces. Thus the candidate has the opportunity to bring such expertise to the host.

The candidate will communicate her expertise through informal discussion, collaboration and seminars. She gained experience with this type of knowledge transfer through her collaboration with Jin-Xin Zhou (Beijing Jiaotong University). The candidate developed some relevant combinatorial techniques, while Jin-Xin Zhou developed some relevant group theoretic ones, and they were able to teach these techniques to one another and work together, resulting in the classification of various types of 4-CH graphs.[11] Moreover, while at the CMSC, the candidate was heavily involved with the Outreach Committee, which runs mathematical activities for primary and secondary students designed to foster an interest in mathematics. She chaired the committee in 2016 and was a member of the committee from 2013–2015; through this commitment, she learned various activities that are highly effective in convincing young students that mathematics is fun and worth studying. She is keen to contribute this knowledge to the outreach programme at Imperial by helping out with departmental events and volunteering for the Imperial *Outreach Team* as a *STEM activity leader*.

1.3. **Quality of the supervision and of the integration in the team/institution.** Prof. Martin Liebeck received a D.Phil. in mathematics from Oxford University in 1979 under the supervision of Peter Neumann. He has held a permanent position at Imperial College London since 1986 and been Head of the Pure Mathematics Section since 1997. He is an ISI Highly Cited Researcher and was awarded the 2010 Imperial Rector's Medal for excellence in teaching. He has supervised 20 Ph.D. students, many of whom now have permanent academic positions, including Jonathan Brundan (University of Oregon), Eugenia O'Reilly-Regueiro (Universidad Nacional Autónoma de México) and Tim Burness (University of Bristol). He has obtained 10 EPSRC grants and recently supervised a Marie Curie Intra-European Fellow, Jeroen Schillewaert. He is an editor for the Journal of Algebra and has been for the London Mathematical Society journals. He has helped organise nearly a dozen conferences and workshops, including several at the Mathematical Research Institute of Oberwolfach and the Banff International Research Station. He is frequently invited to speak at conferences worldwide and has held visiting positions at The University of Western Australia, California Institute of Technology, University of Amsterdam and The University of Auckland, to name a few. He is internationally recognised as an expert in permutation groups, algebraic groups, finite simple groups and algebraic combinatorics, each of which is relevant to the proposed research. His extensive knowledge on algebraic groups and the representation theory of finite simple groups will be vital to Objectives 3 and 4, and his more general knowledge of algebraic combinatorics will be very useful for Objectives 1 and 2. He has well over 100 publications, many of them relevant to this proposal,[9,18,24,39,41,45–47] and he has published in Annals of Mathematics, Inventiones Mathematicae, Duke Mathematical Journal, and Crelle. His collaborators include Eamonn O'Brien (The University of Auckland), Robert Guralnick (University of Southern California), Cheryl Praeger (The University of Western Australia), Jan Saxl (University of Cambridge), Gary Seitz (University of Oregon), Donna Testerman (École Polytechnique Fédérale de Lausanne) and Rob Wilson (Queen Mary, University of London).

Imperial College London is routinely ranked among the top 10 universities in the world; for example, it is 9th in the 2016-17 QS World University Rankings. According to the 2014 Research Excellence Framework (REF), which assesses the quality of research in UK universities, Imperial College has the greatest concentration of high impact research of any major UK university. Further, the Department of Mathematics at Imperial was joint 3rd for its proportion of world-leading research. The Pure Section has 25 permanent academic members, 28 postdoctoral fellows and 39 Ph.D. students. There are numerous weekly seminars covering a broad range of topics, and 10 graduate level courses on algebra and discrete mathematics alone. The Algebra Group has 5 permanent members: Prof. David Evans, Prof. Sasha Ivanov, Prof. Martin Liebeck, Dr Oliver Pretzel and Dr Travis Schedler, as well as a teaching fellow, Dr John Britnell. From October 2016, the Algebra Group will have 10 Ph.D. students. Thus the candidate will be part of a vibrant and active research community.

The research expertise of Martin Liebeck and Sasha Ivanov will be crucial to the success of the candidate's proposal. She will regularly attend the London Algebra Colloquium, a joint seminar series held between Imperial College, Queen Mary and City University. As part of her career development plan, where possible, the candidate will teach and supervise M.Sc. student projects. She will be provided with extensive information on computers, phones, safety, and security, as well as HR matters through the department's dedicated contact. The candidate will attend *Imperial insights*, where she will be welcomed to Imperial by senior staff and given an overview of the organisation. She will have access to both the *Learning and Development Centre*, through which she will refine her career development plan, and the *Postdoc Development Centre*, through which she will improve her capabilities on interviewing and writing lectureship applications in order to secure her first permanent academic position.

1.4. **Capacity of the researcher to reach or re-enforce a position of professional maturity/independence.** For the candidate, reaching a position of professional maturity means finding a permanent academic position, preferably in the UK; this is a key step in her career development plan. This fellowship will contribute significantly towards her plan, as she will achieve the research training objectives (TO1), (TO2) and (TO3) laid out in Section 1.2 and be part of a high-quality research environment with access to the extensive research network of the supervisor and host institution. By regularly attending the London Algebra Colloquium, the candidate will also have frequent access to the wider research environment provided by Queen Mary and City University. The former has a strong focus in computational group theory, and the latter has a strong focus in representation theory; both are relevant fields for the candidate's research. Moreover, this fellowship will be the candidate's second major postdoctoral appointment, having held a three year appointment at The University of Western Australia, so she will be in a strong position to apply for lectureships at its completion. This position will be made even stronger by attending courses at the *Postdoc Development Centre*, as outlined in Section 1.2.

As a Research Associate in the Centre for the Mathematics of Symmetry and Computation (CMSC) at The University of Western Australia—one of the foremost groups in the world for algebraic combinatorics—the candidate took significant steps towards reaching professional maturity. She acquired excellent time management skills working on multiple concurrent projects, as demonstrated by a strong track record of publication while there: she published five papers, submitted two

[46]Kleidman, P. and Liebeck, M. *The subgroup structure of the finite classical groups.* CUP, Cambridge, 1990.

[47]Liebeck, M. W. and Seitz, G. M. The maximal subgroups of positive dimension in exceptional algebraic groups. *Memoirs Amer. Math. Soc.* 169, 802 (2004), 1–227.

others, and is in the process of writing three more. She was awarded CMSC Paper Prizes in 2015 and 2016 for the best paper submitted by an early career researcher affiliated with the Centre. Through her research projects there, she forged many contacts with whom she will continue to collaborate. In fact, she branched out to work with a group of applied mathematicians on a problem at the intersection of graph theory and statistical physics, recently published in the prestigious *Proceedings of the National Academy of Sciences (PNAS)*. She gained valuable experience both supervising an honours student—which resulted in a paper submitted for publication—and chairing and running activities for the Outreach Committee for three years, through which she endeavoured to convince young students that mathematics is interesting and worth studying. Further, for two years as a doctoral student at the University of Cambridge, she co-organised a weekly seminar series giving internal and external graduate students the opportunity to speak about their research.

2. Impact

2.1. **Enhancing the potential and future career prospects of the researcher.** The candidate's long-term goal is to obtain a permanent academic position at a UK institution and develop a group focused on studying symmetry. Through the proposed research and with the supervisor's guidance, the candidate will gain specific skills in the representation theory of finite almost simple groups of Lie type (TO1), as well as Aschbacher reduction (TO2); these are fundamentally important for her intended career as a permutation group theorist. She also hopes to gain new competence in the theory of amalgams (TO3), another important field in group theory. Through the *Learning and Development Centre*, she will refine her career development plan, and by attending courses at the *Postdoc Development Centre* (see Section 1.2), she will improve her interviewing skills and learn about applying for lectureships. She also hopes to gain more teaching experience, which will greatly improve her chances of finding a lectureship at the end of the fellowship. Lastly, as a result of her four research objectives, she will commence a much broader programme of research into symmetric partial linear spaces, which are rather poorly understood. This will lead to further international recognition and independence for the candidate, thereby resulting in excellent career opportunities.

2.2. **Quality of the proposed measures to exploit and disseminate the action results.** The candidate will have day-to-day discussions with the supervisor and other researchers at the host institution. She will attend the *London Algebra Colloquium* (**LAC**), which is a weekly seminar series run jointly by Imperial, Queen Mary and City University; this will give her regular contact with other researchers in her field. She will speak at seminars, conferences and workshops throughout the UK and worldwide at various stages of her research; this supports the Horizon 2020 objective of Open Science. She will attend *Combinatorics 2018* (**C18**), a biannual conference that is focused on finite geometry and usually held in Italy, as well as the *British Combinatorial Conference 2019* (**BCC**). She has been invited to the *Tutte Centenary Retreat* (**Tutte**) in late November 2017; this is a workshop on symmetric graphs at Matrix@Melbourne, the new Australian facility for mathematical research programmes. She will visit Peter Cameron at the University of St Andrews, hopefully during the first three months of Work Package 2 (see Section 3.1). The candidate aims to publish five papers on her research in a range of high-quality research journals such as the Bulletin or Journal of the London Mathematical Society, Transactions of the American Mathematical Society, the Journal of Algebraic Combinatorics, the Journal of Combinatorial Theory and the Journal of Algebra. She will also post her preprints on the arXiv, further supporting the Horizon 2020 objective of Open Science. This project will not generate a large amount of data, and there is no immediate potential for commercial exploitation; nevertheless, the candidate's research will enhance the expertise of the scientific community at large.

2.3. **Quality of the proposed measures to communicate the action activities to different target audiences.** The candidate will take the course *Science Communication: Reaching a Wider Audience* through Imperial's *Postdoc Development Centre*, in order to learn techniques for promoting her research. Whenever the opportunity arises, she will give talks on her research that are pitched at a wide range of mathematicians. The candidate will also volunteer for the *Imperial Festival* (**IF**), an annual event put on by the host institution to share its research with the public through interactive activities, performances and workshops. The 2016 festival, which was attended by 15,000 people, was described by the *Guardian* as "a fascinating event". While a research associate at The University of Western Australia, the candidate was heavily involved with the Outreach Committee, through which she ran activities for primary and secondary students designed to encourage their interest in mathematics. She intends to continue this work by helping out with outreach events run by the host department, as well as volunteering as a *STEM activity leader* for the *Outreach Team* at the host institution, which delivers a wide range of exciting and inspiring activities for students from all backgrounds and ages, with a focus on science, technology, engineering and mathematics (STEM).

3. Quality and Efficiency of the Implementation

3.1. **Coherence and effectiveness of the work plan.** In what follows, "Work Package x" corresponds to "Objective x" from page 6 of Section 1.1. A "deliverable" is a paper submitted for publication to a high-quality international peer-reviewed journal. The Gantt chart is designed under the assumption that the candidate will commence the fellowship in October 2017.

Work package 1. (*22 months*) First the candidate will classify the finite proper CH partial linear spaces (*10 months*); completing this task is **Milestone 1.1**, as this enables her to carry on with the other tasks set out in Objective 1. She will then prepare **Deliverable 1.1** on this research (*2 months*). Next she will consider whether there exists an absolute constant c such that every c-CH partial linear space is CH (*4 months*), develop a programme for understanding k-CH partial linear spaces for small k (*4 months*), and prepare **Deliverable 1.2** (*2 months*).

Work package 2. (*11 months*) The candidate has two different approaches for proving that every 5-CH graph with girth 4 is CH. She will first develop the techniques of Cameron (*3 months*),[36] and if successful, she will study 4-CH graphs with girth 4 using the same techniques or the amalgam method (*6 months*). Otherwise, she will study 5-CH graphs using the amalgam method (*6 months*). In either eventuality, she will prepare **Deliverable 2.1** (*2 months*).

Work package 3. (*16 months*) To classify the primitive affine groups of rank 4, the candidate will reduce to the almost simple case (*3 months*), consider almost simple groups of Lie type in characteristic p (*6 months*), consider almost simple groups of Lie type in characteristic different from p (*2 months*), and finish with the alternating and sporadic groups (*2 months*). Completing this classification is **Milestone 3.1**, as this enables her to start WP4. Lastly, she will prepare **Deliverable 3.1** (*3 months*).

Work package 4. (*11 months*) To classify the finite proper partial geometries with a primitive automorphism group of rank 4 that is transitive on pairs of collinear points, the candidate will first investigate the restrictions imposed by the structure of a partial geometry (*1 month*). Then, using her classification from WP3, she will analyse the geometric classes (*4 months*), analyse the almost simple classes (*4 months*), and prepare **Deliverable 4.1** (*2 months*).

Month	1	2	3	4-7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Work Package 1																					
Work Package 2																					
Work Package 3																					
Work Package 4																					
Deliverable									1.1				3.1		2.1				1.2		4.1
Milestone							1.1			3.1											
Workshop		Tutte																			
Conference						C18													BCC		
Seminar											LAC										
Public Engagement					IF												IF				

3.2. **Appropriateness of the allocation of tasks and resources.** WP1 has been allocated the most time because its success depends on the development of new techniques. The first 6 months of WP3 will involve heavy reading as the candidate begins work on training objectives (TO1) and (TO2) (see Section 1.2), so she has deliberately delayed the start of WP2 and WP4; WP1 is more exploratory in nature and will complement WP3 well. Similarly, learning about the amalgam method (TO3) will require extensive reading and is therefore scheduled to start after the most intensive periods for (TO1) and (TO2). WP4 is scheduled to start near the end of WP3, as WP4 is dependent on Milestone 3.1. The candidate expects that the research of WP4 will be particularly intense near the end as she deals with new almost simple classes not encountered in her previous work,[32] so she has matched this with WP1, which, again, is more exploratory in nature. The candidate finds that, despite writing up results as she goes, the final weeks before the submission of a paper are especially busy, so she has allocated 2–3 months for preparing deliverables, ensuring that there is no overlap between the 5 deliverables. The candidate does not anticipate that any workshops, conferences, seminars or public engagement will interfere with the mathematical schedule.

3.3. **Appropriateness of the management structure and procedures, including risk management.** The candidate will meet with the supervisor and other members of the department on a day-to-day basis. She will assess her progress with the supervisor every 6 months. The host institution has a detailed policy for dealing with research misconduct, and both the supervisor and the candidate take issues of research integrity very seriously. Research in pure mathematics is inherently open since the validity of a claim can be assessed directly from its proof; for additional transparency, when the candidate uses a computer programme such as Magma to aid in the proof of a claim,[42] she will make the code and results available, either on her website or upon request. She will always endeavour to give credit where it is due, through citations, acknowledgements and authorships. The *Research Service Team* at Imperial will assist with financial reporting, while the *EU Team* will aid with grant negotiations. There is a risk that classifying the CH partial linear spaces (WP1) will take too long or be too difficult to finish, but nothing is known about these structures, so even partial progress towards Objective 1 will result in at least one paper. There is a risk that neither of the contingencies for WP2 will succeed, but using the extensive literature on 2-arc-transitive graphs, the candidate will find an alternative approach. Another risk is that WP3 will take too long and delay the start of WP4, but WP4 can be started regardless of whether WP3 has finished since both work packages involve a case-by-case analysis. There is also a risk that many new almost simple cases will arise in WP3, so that WP4 cannot be finished by the end of the 24 months; however, given the supervisor's expertise with the groups involved and the candidate's expertise with the techniques involved, this work will be completed soon after. Despite the unpredictable nature of research in pure mathematics, the candidate is confident that she will have the necessary resources to meet her four objectives.

3.4. **Appropriateness of the institutional environment (infrastructure).** According to the 2014 Research Excellence Framework (REF), which assesses the quality of research in UK higher education institutions, Imperial College London has the greatest concentration of high impact research of any major UK university, and the Department of Mathematics at Imperial was joint 3rd for its proportion of world-leading research. The host is fully dedicated to Horizon 2020 with over 50 Marie Skłodowska-Curie Individual Fellowships; in the past, it was fully dedicated to Framework Programme 7 with 170 Marie Curie Individual Fellowships. In fact, Imperial is the perfect fit for the candidate's proposal. The supervisor is committed to working with the candidate, and both he and Sasha Ivanov have the precise expertise required for her research, as training objectives (TO1), (TO2) and (TO3) demonstrate in Section 1.2.[9,18,24,37,39,41,45–48] Both researchers have an extensive network of collaborators, and visits from these mathematicians, including those listed in Section 1.3 as well as Sergey Shpectorov (University of Birmingham), will be advantageous to the candidate. Further, the Algebra Group's strong connection to Queen Mary and City University via the London Algebra Colloquium provides special access to a highly beneficial wider research environment. The host will supply the candidate with a personal desk, access to computing facilities including the algebra software Magma,[42] administrative support and excellent library facilities, including access to online journals. Lastly, the *Learning and Development Centre* and *Postdoc Development Centre* will help the candidate acquire the necessary skills for a career in academia. Thus all of the candidate's research and training needs are well met by Imperial.

[48]Ivanov, A. A. and Praeger, C. E. On finite affine 2-arc transitive graphs. *Eur. J. Combin.* 14 (1993), 421–444.