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Adaptive locally weighted learning tracking control for a class of unknown strict-feedback nonlinear systems via differentiable higher order kernels

Yu-Fa Liu [a,b,c], Dong Wang [a,b,c], Zhuo Wang [a,b,c], Ante Su [d], Yong-Hua Liu [a,b,c,*], Chun-Yi Su [a,b,c]

[a] School of Automation, Guangdong University of Technology, Guangzhou, Guangdong, 510006, PR China
[b] Guangdong-Hong Kong Joint Laboratory for Intelligent Decision and Cooperative Control, Guangzhou, Guangdong, 510006, PR China
[c] Guangdong Province Key Laboratory of Intelligent Decision and Cooperative Control, Guangzhou, Guangdong, 510006, PR China
[d] School of Mechanical Engineering, Shandong University, Jinan 250002, PR China

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ABSTRACT

This study addresses the problem of adaptive locally weighted learning (LWL) tracking control for a class of n-order unknown strict-feedback nonlinear systems (SFNSs). Without involving a priori information on the system dynamics, an adaptive tracking control algorithm is designed by fusing LWL and the technique of backstepping, in which the LWL is employed to identify the unknown nonlinear functions. Particularly, by introducing a novel weighting function with sufficient differentiability, the obstacle of the integration of LWL and backstepping to control SFNSs is successfully circumvented. The developed adaptive LWL tracking control scheme ensures that all closed-loop signals are bounded. Finally, simulation results are performed to testify the effectiveness of the proposed LWL tracking control approach.

1. Introduction

During the past thirty year, online approximation-based control has evolved as an effective tool that utilizes cooperative learning methods to identify completely unknown or partially known system dynamics to obtain improved performance in trajectory tracking [1–8]. Many interesting results in this field have been found [9–15] and extensively implemented to various types of electrical and mechanical systems, including wastewater treatment systems [13], bionic robots [14], ship maneuvering motion systems [15].

It is well known that the approximation accuracy of cooperative learning for identifying a parameter vector is determined by the linear combination of all the basis elements. However, the jointly optimal values of all elements of the parameter vector may be affected, while adding extra basis elements. To tackle this problem, LWL control approach, first proposed in [16], was devoted to mitigate the impact of data overfitting and facilitate the concepts of the approximation structure, but the issue of the closed-loop stability was not discussed. Subsequently, several LWL control solutions with stability guarantee have been developed in literature [17–20]. In [21–23], the designs of LWL tracking control were applied to nonlinear systems with relative degree two. The LWL control methodology has also been extended to multiple types of nonlinear systems with higher relative degree [24–28]. In [29,30], the tools of barrier Lyapunov functions (BLFs) and backstepping were incorporated into LWL controller design to

* Corresponding author at: School of Automation, Guangdong University of Technology, Guangzhou, Guangdong, 510006, PR China.
E-mail addresses: yufa.liu@outlook.com (Y.-F. Liu), dongWang.gdut@outlook.com (D. Wang), 248120738@qq.com (Z. Wang), antesu@mail.sdu.edu.cn (A. Su), yonghua.liu@gdut.edu.cn (Y.-H. Liu), cysu@alcor.concordia.ca (C.-Y. Su).

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guarantee the state and control constraints. In [31], by integrating LWL with BLFs, a backstepping-free adaptive control framework was developed for a class of nonaffine uncertain systems.

Although the elegant results mentioned in [21–31] have been proposed, these are applicable only to special nonlinear systems in Brunovsky form, not to more general strict-feedback nonlinear systems (SFNSs). It is noticeable that backstepping control is a commonly used methodology for a class of SFNSs, requiring the sufficient differentiability of each virtual control signal. Unfortunately, the classical kernel used in LWL control is a biquadratic function and only continuously differentiable, which may limit the integration of LWL and backstepping control to solve SFNSs. Recently, command filters were introduced into the framework of adaptive backstepping, relaxing the sufficient differentiability requirement on the virtual control signals, then the adaptive LWL controllers were presented for SFNSs [32]. In [33], through self-organizing approximation, a control design of command filtered backstepping was developed for SFNSs. Nevertheless, command filtered based LWL control suffers from the "explosion of the dynamic order" of the filters adopted and the resulting closed-loop stability highly relies on the selection of the filter parameters, preventing its potential applications.

Motivated by the above-mentioned investigations, the study focuses on LWL tracking control for a class of SFNSs with unknown dynamics, where a novel weighting function with sufficient differentiability is constructed to promote the integration of LWL and backstepping control. The contributions of this study are twofold as outlined below.

(1) A new adaptive LWL tracking control algorithm is designed for a class of unknown SFNSs. The distinctive characteristic of the proposed LWL tracking controller is that a novel weighting function with sufficient differentiability is introduced, which successfully overcomes the obstacle of the integration of LWL and backstepping to control n-order unknown SFNSs without resorting the command filters.

(2) Compared to existing adaptive control solutions using LWL for SFNSs [32,33], the adaptive LWL tracking controller presented in this study does not include any precise knowledge of system dynamics, excluding even the typically required bounding functions.

Notations: The sets of nonnegative real numbers, real numbers and positive real numbers are represented as $R_{\geq 0}$, R and $R_{>0}$, respectively. The R^n represents real n-dimensional space. The Euclidean norm of $x \in R^n$ is represented as $\|x\|$. The $n \times n$ identity matrix is denoted as I_n. The exponential function is denoted as $\exp(\cdot)$.

2. Problem formulation

Consider a class of SFNSs described by

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + x_{i+1}, i = 1, \ldots, n-1, \\ \dot{x}_n &= f_n(\bar{x}_n) + u, \\ y &= x_1, \end{aligned} \tag{1}$$

where $\bar{x}_n = [x_1, \ldots, x_n]^T \in R^n$ is the system state vector, $u \in R$ and $y \in R$ are the system input and output, respectively; $\bar{x}_i = [x_1, \ldots, x_i]^T \in R^i$; $f_i(\bar{x}_i) : R^i \to R$, $i = 1, \ldots, n$ are unknown nonlinear functions.

Remark 1. System (1) represents a class of strict-feedback systems, which has been widely studied in many practical systems, such as ship maneuvering motion systems [15], single-link robotic systems [34], and quadrotor helicopters [35].

The *primary objective* of this study is to develop an adaptive LWL controller, denoted as u, with the *objective* of ensuring that the system output y tracks a desired signal y_d and that all signals of the closed-loop system are bounded. For the purpose, the following standard assumptions are exploited.

Assumption 1 (*[36]*)**.** The unknown nonlinearities $f_i(\bar{x}_i)$ are C^1 functions.

Assumption 2 (*[29]*)**.** The desired signal $y_d \in C^n$ to be tracked is available and there is a known positive constant Y_d such that

$$|y_d| + |\dot{y}_d| + \cdots + |y_d^{(n)}| \leq Y_d. \tag{2}$$

Remark 2. Assumptions 1 and 2 are given to facilitate controller design. A certain degree of conservatism is inevitable. These assumptions are quite standard in most existing work on the tracking control problem of system (1) [29,36].

3. Locally weighted learning

In this section, we focus on a novel weighting function and present the LWL framework, which are utilized in our development of adaptive tracking control.

3.1. Novel weighting functions

Definition 1. For a given compact operational region $\mathcal{D}_i \subset R^n$, a continuous, nonnegative, and locally supported function $\omega_{i,k}(Z_i)$ is said a weighting function if the supporting set $M_{i,k} = \{Z_i \in \mathcal{D}_i \mid \omega_{i,k}(Z_i) \neq 0\}$ is convex and connected.

A classical weighting function that satisfies above conditions is the biquadratic kernel, which is meticulously detailed below [26]

$$\omega_{i,k}(Z_i) = \begin{cases} (1-\psi_{i,k}^2)^2, & \psi_{i,k} \le 1, \\ 0, & \psi_{i,k} > 1. \end{cases} \tag{3}$$

where $\psi_{i,k} = \frac{\|Z_i - c_{i,k}\|}{\mu_{i,k}}$, $c_{i,k}$ represents the center location of the kth weighting function, and $\mu_{i,k}$ denotes the radius of the supporting set $M_{i,k}$. However, it is noted that the classical weighting function (3) is only continuously differentiable [12–33], which does not ensure the sufficient differentiability required for backstepping design. This issue poses a crucial obstacle to integrating LWL and backstepping for controlling SFNSs. To address this, in this study, we introduce a novel type of weighting function as

$$\omega_{i,k}^p(Z_i) = \begin{cases} \left(1-\psi_{i,k}^{2(n+1-p)}\right)^{n+1-p}, & \psi_{i,k} \le 1, \\ 0, & \psi_{i,k} > 1. \end{cases} \tag{4}$$

where p is a positive integer.

Remark 3. The new weighting function (4) introduced in this study is a C^{n-p} function. It meets the differentiability requirements for the backstepping design process, effectively overcoming the key obstacle of integrating LWL and backstepping methods for controlling SFNSs.

The normalized weighting functions are

$$\bar{\omega}_{i,k}^p(Z_i) = \frac{\omega_{i,k}^p(Z_i)}{\sum_{k=1}^{N_i} \omega_{i,k}^p(Z_i)}, k = 1, \dots, N_i, \tag{5}$$

where $\sum_{k=1}^{N_i} \bar{\omega}_{i,k}^p(Z_i) = 1$ for all $Z_i \in \mathcal{D}_i$.

3.2. Local approximators

Consider a continuous function $h_i(Z_i)$ that is identified by LWL as follows:

$$\hat{h}_i(Z_i) = \frac{\sum_{k=1}^{N_i} \omega_{i,k}^p(Z_i)\hat{h}_{i,k}(Z_i)}{\sum_{k=1}^{N_i} \omega_{i,k}^p(Z_i)}, k = 1, \dots, N_i, \tag{6}$$

where Z_i is the input vector, $\omega_{i,k}^p(Z_i)$ are weighting functions, $\hat{h}_{i,k}(Z_i) = \hat{\theta}_{i,k}^T \varphi_{i,k}(Z_i)$ is the kth local approximator of $h_i(Z_i)$, $\hat{\theta}_{i,k}$ is a parameter vector that will be estimated online, $\varphi_{i,k}(Z_i)$ is a prespecified vector of continuous basis functions. (6) can be rewritten in accordance with (5) as follows:

$$\hat{h}_i(Z_i) = \begin{cases} \sum_{k=1}^{N_i} \bar{\omega}_{i,k}^p(Z_i)\hat{h}_{i,k}(Z_i), & Z_i \in \bar{M}_{i,k}. \\ 0, & Z_i \in \mathcal{D}_i - \bar{M}_{i,k}. \end{cases} \tag{7}$$

where $\bar{M}_{i,k}$ denotes the closure of $M_{i,k}$.

For all $Z_i \in \mathcal{D}_i$, the optimal parameter estimates $\theta_{i,k}^*$ and local approximation error $\epsilon_{i,k}(Z_i)$ are defined as

$$\theta_{i,k}^* = \arg\min_{\hat{\theta}_{i,k}} \left(\int_{Z_i \in \mathcal{D}_i} \omega_{i,k}^p(Z_i) |h_i(Z_i) - \hat{h}_{i,k}(Z_i)| dZ_i \right), \tag{8}$$

$$\epsilon_{i,k}(Z_i) = \begin{cases} h_i(Z_i) - \theta_{i,k}^{*T}\varphi_{i,k}(Z_i), & Z_i \in \bar{M}_{i,k}, \\ 0, & Z_i \in \mathcal{D}_i - \bar{M}_{i,k}. \end{cases} \tag{9}$$

Lemma 1 ([26]). *For any $Z_i \in \mathcal{D}_i$, the LWL is utilized to identify the continuous function $h_i(Z_i)$, i.e.,*

$$h_i(Z_i) = \sum_{k=1}^{N_i} \bar{\omega}_{i,k}^p(Z)\theta_{i,k}^{*T}\varphi_{i,k}(Z_i) + \epsilon_{i,k}(Z_i), \tag{10}$$

where $\epsilon_{i,k}(Z_i)$ represents the approximation error, which meets $| \epsilon_{i,k}(Z_i) | \le \bar{\epsilon}_i$ with an unknown positive constant $\bar{\epsilon}_i$.

4. Adaptive locally weighted learning tracking controller design

In this section, the novel weighting functions (4) are embedded into the adaptive LWL tracking controller with rigorous stability guarantees for unknown SFNSs.

4.1. Tracking controller design

To begin with, the change of variables are introduced as follows:

$$z_1 = x_1 - y_d, \tag{11}$$

$$z_i = x_i - \alpha_{i-1}, i = 2, \ldots, n, \tag{12}$$

where at the $(i-1)$-th step, the virtual control signal α_{i-1} can be determined. According to (11) and (12), the adaptive LWL tracking controller, as detailed below.

Step 1*:* The design of the first virtual control signal α_1 and the associated adaptive law $\hat{\theta}_{1,k}$ are formulated as

$$\alpha_1 = -\rho_1 z_1 - \sum_{k=1}^{N_1} \bar{\omega}^1_{1,k} \hat{\theta}^T_{1,k} \varphi_{1,k}(Z_1), \tag{13}$$

$$\dot{\hat{\theta}}_{1,k} = \Gamma_1(\bar{\omega}^1_{1,k} \varphi_{1,k}(Z_1) z_1 - \sigma_1 \hat{\theta}_{1,k}), \tag{14}$$

where $Z_1 = [x_1, \dot{y}_d]^T \in R^2$, $\hat{\theta}_{1,k}$ denotes the estimated value for $\theta^*_{1,k}$, which will be described in detail below. The matrix Γ_1 is confirmed to be positive definite, while ρ_1 and σ_1 serve as positive design constants.

Step i $(i = 2, \ldots, n-1)$*:* The design of the ith virtual control signal α_i and the associated adaptive law $\hat{\theta}_{i,k}$ are formulated as

$$\alpha_i = -\rho_i z_i - \sum_{k=1}^{N_i} \bar{\omega}^i_{i,k} \hat{\theta}^T_{i,k} \varphi_{i,k}(Z_i) - z_{i-1}, \tag{15}$$

$$\dot{\hat{\theta}}_{i,k} = \Gamma_i(\bar{\omega}^i_{i,k} \varphi_{i,k}(Z_i) z_i - \sigma_i \hat{\theta}_{i,k}), \tag{16}$$

where $Z_i = [x_1, \ldots, x_i, \frac{\partial \alpha_1}{\partial x_1}, \ldots, \frac{\partial \alpha_{i-1}}{x_{i-1}}, \phi_{i-1}]^T \in R^{2i}$, the expression for ϕ_{i-1} is provided in (31), $\hat{\theta}_{i,k}$ denotes the estimated value for $\theta^*_{i,k}$, which will be described in detail below. The matrix Γ_i is confirmed to be positive definite, while ρ_i and σ_i serve as positive design constants.

Step n*:* The design of the actual control input u and the associated adaptive law $\hat{\theta}_{n,k}$ are formulated as

$$u = -\rho_n z_n - \sum_{k=1}^{N_n} \bar{\omega}^n_{n,k} \hat{\theta}^T_{n,k} \varphi_{n,k}(Z_n) - z_{n-1}, \tag{17}$$

$$\dot{\hat{\theta}}_{n,k} = \Gamma_n(\bar{\omega}^n_{n,k} \varphi_{n,k}(Z_n) z_n - \sigma_n \hat{\theta}_{n,k}), \tag{18}$$

where $Z_n = [x_1, \ldots, x_n, \frac{\partial \alpha_1}{\partial x_1}, \ldots, \frac{\partial \alpha_{n-1}}{x_{n-1}}, \phi_{n-1}]^T \in R^{2n}$, the expression for ϕ_{n-1} is provided in (42), $\hat{\theta}_{n,k}$ denotes the estimated value for $\theta^*_{n,k}$, which will be described in detail below. The matrix Γ_n is confirmed to be positive definite, while ρ_n and σ_n serve as positive design constants.

Remark 4. As shown in (13)–(18), the developed adaptive LWL controller does not involve any a priori knowledge about the system nonlinearities or their boundary functions, quite differing to the LWL results in [32,33].

The entire block diagram of the resulting LWL control system can be displayed in Fig. 1.

4.2. Analysis of stability

To analyze the closed-loop stability of the resulting LWL system, the following theorem is established.

Theorem 1. *Consider the closed-loop system composing of the SFNSs (1), the actual adaptive LWL tracking controller (17) and the associated adaptive laws designed in (14), (16) and (18) under Assumptions 1 and 2. Then, for any initial states $\bar{x}_n(0)$, all signals in the closed-loop system are semi-globally uniformly ultimately bounded and the output tracking error converges to a residual set that can be made arbitrarily small by suitably tuning the design constants.*

Proof. Following the standard backstepping approach, the proof consists n steps.

Step 1*:* Select the Lyapunov function candidate as follows

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \sum_{k=1}^{N_1} \tilde{\theta}_{1,k} \Gamma_1^{-1} \tilde{\theta}_{1,k}, \tag{19}$$

where $\tilde{\theta}_{1,k} = \theta^*_{1,k} - \hat{\theta}_{1,k}$.

Recalling (1) and (11), the time derivative of V_1 is calculated as

$$\dot{V}_1 = z_1(f_1(\bar{x}_1) + z_2 + \alpha_1 - \dot{y}_d) - \sum_{k=1}^{N_1} \tilde{\theta}_{1,k} \Gamma_1^{-1} \dot{\hat{\theta}}_{1,k}. \tag{20}$$

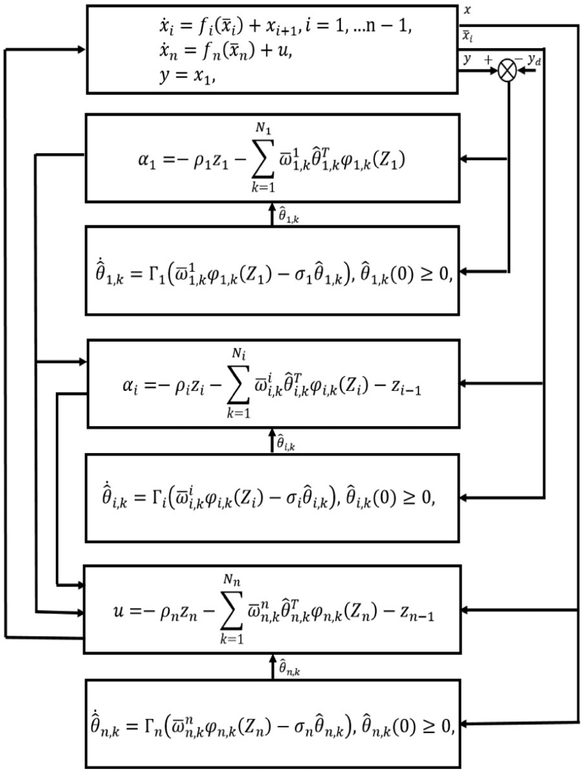

Fig. 1. Entire block diagram of the LWL control system.

Let

$$h_1(Z_1) = f_1(\bar{x}_1) - \dot{y}_d \tag{21}$$

with $Z_1 = [x_1, \dot{y}_d]^T$. The continuity of $h_1(Z_1)$ is apparent. Via Lemma 1, $h_1(Z_1)$ is identified as

$$h_1(Z_1) = \sum_{k=1}^{N_1} \bar{\omega}^1_{1,k} \theta^{*T}_{1,k} \varphi_{1,k}(Z_1) + \epsilon_{1,k}(Z_1), \tag{22}$$

where $\epsilon_{1,k}(Z_1)$ represents the approximation error, which meets $| \epsilon_{1,k}(Z_1) | \leq \bar{\epsilon}_1$ with an unknown positive constant $\bar{\epsilon}_1$.

Substituting (22) into (20) yields

$$\dot{V}_1 = z_1\Big(\sum_{k=1}^{N_1} \bar{\omega}^1_{1,k} \theta^{*T}_{1,k} \varphi_{1,k}(Z_1) + \epsilon_{1,k}(Z_1) + z_2 + \alpha_1\Big) - \sum_{k=1}^{N_1} \tilde{\theta}_{1,k} \Gamma_1^{-1} \dot{\hat{\theta}}_{1,k}. \tag{23}$$

Invoking (13) and (14) into (23), we have

$$\begin{aligned}\dot{V}_1 &= -\rho_1 z_1^2 + z_1 z_2 + z_1 \epsilon_{1,k}(Z_1) - \sum_{k=1}^{N_1} \sigma_1 \tilde{\theta}_{1,k} \hat{\theta}_{1,k} \\ &= -\rho_{1,1} z_1^2 - \rho_{1,2} z_1^2 + z_1 z_2 + z_1 \epsilon_{1,k}(Z_1) - \sum_{k=1}^{N_1} \sigma_1 \tilde{\theta}_{1,k} \hat{\theta}_{1,k},\end{aligned} \tag{24}$$

where $\rho_1 = \rho_{1,1} + \rho_{1,2}$ with positive constants $\rho_{1,1}$, $\rho_{1,2}$.

Adopting Young's inequality generates to

$$z_1 \epsilon_{1,k}(Z_1) \leq \rho_{1,2} z_1^2 + \frac{\bar{\epsilon}_1^2}{4\rho_{1,2}}, \tag{25}$$

$$-\sigma_1 \tilde{\theta}_{1,k} \hat{\theta}_{1,k} \leq \frac{\sigma_1}{2} \|\theta^*_{1,k}\|^2 - \frac{\sigma_1}{2} \|\tilde{\theta}_{1,k}\|^2. \tag{26}$$

Then, one gets from (24)

$$\dot{V}_1 \leq -\rho_{1,1} z_1^2 - \sum_{k=1}^{N_1} \frac{\sigma_1}{2} \|\tilde{\theta}_{1,k}\|^2 + z_1 z_2 + \sum_{k=1}^{N_1} \frac{\sigma_1}{2} \|\theta^*_{1,k}\|^2 + \frac{\bar{\epsilon}_1^2}{4\rho_{1,2}}. \tag{27}$$

Step i $(i = 2, \dots, n-1)$*:* Select the ith Lyapunov function candidate as follows

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2} \sum_{k=1}^{N_i} \tilde{\theta}_{i,k} \Gamma_i^{-1} \tilde{\theta}_{i,k}, \tag{28}$$

where $\tilde{\theta}_{i,k} = \theta^*_{i,k} - \hat{\theta}_{i,k}$.

Invoking (1) and (12), we have

$$\dot{V}_i = \dot{V}_{i-1} + z_i(f_i(\bar{x}_i) + z_{i+1} - \dot{\alpha}_{i-1} + \alpha_i) - \sum_{k=1}^{N_i} \tilde{\theta}_{i,k} \Gamma_i^{-1} \dot{\hat{\theta}}_{i,k}, \tag{29}$$

where

$$\dot{\alpha}_{i-1} = \sum_{j=1}^{i} \frac{\partial \alpha_{i-1}}{\partial x_j}(f_j(\bar{x}_j) + x_{j+1}) + \phi_{i-1} \tag{30}$$

with

$$\phi_{i-1} = \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{j+1} + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_{j,k}} \dot{\hat{\theta}}_{j,k}. \tag{31}$$

Let

$$h_i(Z_i) = f_i(\bar{x}_i) - \dot{\alpha}_{i-1} \tag{32}$$

with $Z_i = [x_1, \dots, x_i, \frac{\partial \alpha_1}{\partial x_1}, \dots, \frac{\partial \alpha_{i-1}}{\partial x_{i-1}}, \phi_{i-1}]^T$. Apparently the function $h_i(Z_i)$ is continuous, which can be identified via Lemma 1 as

$$h_i(Z_i) = \sum_{k=1}^{N_i} \bar{\omega}^i_{i,k} \theta^{*T}_{i,k} \varphi_{i,k}(Z_i) + \epsilon_{i,k}(Z_i), \tag{33}$$

where $\epsilon_{i,k}(Z_i)$ represents the approximation error, which meets $|\epsilon_{i,k}(Z_i)| \le \bar{\epsilon}_i$ with an unknown positive constant $\bar{\epsilon}_i$.

Replacing (33) with (29), results in

$$\dot{V}_i = \dot{V}_{i-1} + z_i(\sum_{k=1}^{N_i} \bar{\omega}^i_{i,k} \theta^{*T}_{i,k} \varphi_{i,k}(Z_i) + \epsilon_{i,k}(Z_i) + z_{i+1} + \alpha_i) - \sum_{k=1}^{N_i} \tilde{\theta}_{i,k} \Gamma_i^{-1} \dot{\hat{\theta}}_{i,k}. \tag{34}$$

Recalling (15) and (16), it yields

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i-1} - \rho_i z_i^2 + z_i z_{i+1} + z_i \epsilon_{i,k}(Z_i) - \sum_{k=1}^{N_i} \sigma_i \tilde{\theta}_{i,k} \hat{\theta}_{i,k} \\ &= \dot{V}_{i-1} - \rho_{i,1} z_i^2 - \rho_{i,2} z_i^2 + z_i z_{i+1} + z_i \epsilon_{i,k}(Z_i) - \sum_{k=1}^{N_i} \sigma_i \tilde{\theta}_{i,k} \hat{\theta}_{i,k}, \end{aligned} \tag{35}$$

where $\rho_i = \rho_{i,1} + \rho_{i,2}$ with positive constants $\rho_{i,1}$, $\rho_{i,2}$.

By Young's inequality, it has

$$z_i \epsilon_{i,k}(Z_i) \le \rho_{i,2} z_i^2 + \frac{\bar{\epsilon}_i^2}{4\rho_{i,2}}, \tag{36}$$

$$-\sigma_i \tilde{\theta}_{i,k} \hat{\theta}_{i,k} \le -\frac{\sigma_i}{2} \|\tilde{\theta}_{i,k}\|^2 + \frac{\sigma_i}{2} \|\theta^*_{i,k}\|^2. \tag{37}$$

Combining the inequalities (36) and (37), one gets

$$\dot{V}_i \le -\sum_{j=1}^{i} \rho_{j,1} z_j^2 - \sum_{j=1}^{i} \sum_{k=1}^{N_j} \frac{\sigma_j}{2} \|\tilde{\theta}_{j,k}\|^2 + \sum_{j=1}^{i} \sum_{k=1}^{N_j} \frac{\sigma_i}{2} \|\theta^*_{j,k}\|^2 + \sum_{j=1}^{i} \frac{\bar{\epsilon}_j^2}{4\rho_{j,2}} + z_i z_{i+1}. \tag{38}$$

Step n: Select the nth Lyapunov function candidate as follows

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} \sum_{k=1}^{N_n} \tilde{\theta}_{n,k} \Gamma_n^{-1} \tilde{\theta}_{n,k}, \tag{39}$$

where $\tilde{\theta}_{n,k} = \theta^*_{n,k} - \hat{\theta}_{n,k}$.

Along with (1) and (12), the time derivative of V_n is obtained

$$\dot{V}_n = \dot{V}_{n-1} + z_n(f_n(\bar{x}_n) + u - \dot{\alpha}_{n-1}) - \sum_{k=1}^{N_n} \tilde{\theta}_{n,k} \Gamma_n^{-1} \dot{\hat{\theta}}_{n,k}, \tag{40}$$

where

$$\dot{\alpha}_{n-1} = \sum_{j=1}^{n} \frac{\partial \alpha_{n-1}}{\partial x_j}(f_j(\bar{x}_j) + x_{j+1}) + \phi_{n-1} \tag{41}$$

with

$$\phi_{n-1} = \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{j+1} + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_{j,k}} \dot{\hat{\theta}}_{j,k}. \tag{42}$$

Let

$$h_n(Z_n) = f_n(\bar{x}_n) - \dot{\alpha}_{n-1} \tag{43}$$

with $Z_n = [x_1, \ldots, x_n, \frac{\partial \alpha_1}{\partial x_1}, \ldots, \frac{\partial \alpha_{n-1}}{\partial x_{n-1}}, \phi_{n-1}]^T$. Obviously, $h_n(Z_n)$ is a continuous function. With the aid of Lemma 1, $h_n(Z_n)$ is identified as

$$h_n(Z_n) = \sum_{k=1}^{N_n} \bar{\omega}_{n,k}^n \theta_{n,k}^{*T} \varphi_{n,k}(Z_n) + \epsilon_{n,k}(Z_n), \tag{44}$$

where $\epsilon_{n,k}(Z_n)$ represents the approximation error, which meets $| \epsilon_{n,k}(Z_n) | \le \bar{\epsilon}_n$ with an unknown positive constant $\bar{\epsilon}_n$.

Thanks to (44), (40) transforms into

$$\dot{V}_n = \dot{V}_{n-1} + z_n\Big(\sum_{k=1}^{N_n} \bar{\omega}_{n,k}^n \theta_{n,k}^{*T} \varphi_{n,k}(Z_n) + \epsilon_{n,k}(Z_n) + u\Big) - \sum_{k=1}^{N_n} \tilde{\theta}_{n,k} \Gamma_n^{-1} \dot{\hat{\theta}}_{n,k}. \tag{45}$$

Utilizing (17) and (18) yields the following result:

$$\begin{aligned} \dot{V}_n &= \dot{V}_{n-1} - \rho_n z_n^2 + z_n z_{n-1} + z_n \epsilon_{n,k}(Z_n) - \sum_{k=1}^{N_n} \sigma_n \tilde{\theta}_{n,k} \hat{\theta}_{n,k} \\ &= \dot{V}_{n-1} - \rho_{n,1} z_n^2 - \rho_{n,2} z_n^2 + z_n z_{n-1} + z_n \epsilon_{n,k}(Z_n) - \sum_{k=1}^{N_n} \sigma_n \tilde{\theta}_{n,k} \hat{\theta}_{n,k}, \end{aligned} \tag{46}$$

where $\rho_n = \rho_{n,1} + \rho_{n,2}$ with positive constants $\rho_{n,1}$, $\rho_{n,2}$.

Similar to (36) and (37), the following hold

$$z_n \epsilon_{n,k}(Z_n) \le \rho_{n,2} z_n^2 + \frac{\bar{\epsilon}_n^2}{4\rho_{n,2}}, \tag{47}$$

$$-\sigma_n \tilde{\theta}_{n,k} \hat{\theta}_{n,k} \le -\frac{\sigma_n}{2} \|\tilde{\theta}_{n,k}\|^2 + \frac{\sigma_n}{2} \|\theta_{n,k}^*\|^2. \tag{48}$$

Inserting (47) and (48) into (46), it yields

$$\begin{aligned} \dot{V}_n &\le -\sum_{j=1}^{n} \rho_{j,1} z_j^2 - \sum_{j=1}^{n} \sum_{k=1}^{N_j} \frac{\sigma_j}{2} \|\tilde{\theta}_{j,k}\|^2 + \sum_{j=1}^{n} \sum_{k=1}^{N_j} \frac{\sigma_j}{2} \|\theta_{j,k}^*\|^2 + \sum_{j=1}^{n} \frac{\bar{\epsilon}_j^2}{4\rho_{j,2}} \\ &\le -\kappa V_n + \chi, \end{aligned} \tag{49}$$

where

$$\begin{aligned} \kappa &= \min\Big\{2\rho_{j,1}, \frac{\sigma_j}{\lambda_{\max}(\Gamma_j^{-1})}\Big\}, \\ \chi &= \sum_{j=1}^{n} \sum_{k=1}^{N_j} \frac{\sigma_j}{2} \|\theta_{j,k}^*\|^2 + \sum_{j=1}^{n} \frac{\bar{\epsilon}_j^2}{4\rho_{j,2}}. \end{aligned}$$

From (49), it is concluded that

$$\frac{1}{2} \sum_{k=1}^{n} z_i^2 \le V_n \le \varpi, \tag{50}$$

$$\frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{N_j} \tilde{\theta}_{j,k} \Gamma_j^{-1} \tilde{\theta}_{j,k} \le V_n \le \varpi, \tag{51}$$

where $\varpi = \max\{V_n(0), \frac{\chi}{\kappa}\}$. Then, it is deduced that $z_1, \ldots, z_n, \hat{\theta}_{1,1}^T, \ldots, \hat{\theta}_{1,N_1}^T, \ldots, \hat{\theta}_{i,1}^T, \ldots, \hat{\theta}_{i,N_i}^T, \ldots, \hat{\theta}_{n,1}^T, \ldots, \hat{\theta}_{n,N_n}^T$ are bounded. Furthermore, together with Assumptions 1 and 2, it is deduced that $x_1, \ldots, x_n$ are bounded. Furthermore, (49) gives that

$$\lim_{t\to\infty} V_n \le \frac{\chi}{\kappa}, \tag{52}$$

which implies that by increasing the value of κ, the tracking error z_1 can converge to an arbitrarily small neighborhood of the origin. □

Remark 5. It is well-established that the sufficient differentiability of the virtual control signals is a fundamental requirement in the backstepping procedure. However, the biquadratic kernel (3) adapted in LWL control is only continuously differentiable [12–33], which makes the integration of LWL and backstepping for unknown SFNSs challenging. In this study, we introduce a novel weighting function (4) with sufficient differentiability, as revealed in (4), to address this challenge. Based on this weighting function, a new adaptive LWL tracking control algorithm is designed for SFNSs.

Remark 6. Comparing to existing LWL controllers for SFNSs [32,33], the proposed LWL tracking control strategy does not adapt any extra command filters, avoiding the issues of the “explosion of the dynamic order” of the filters and the selection of the filter parameters.

5. Simulation study

In this section, we present three simulation studies that verify the effectiveness and universality of the proposed LWL tracking control strategy, according to which the same controller (with respect to structure and control parameters) is capable of solving the adaptive LWL tracking control problem for systems that belong to the class described by (1), with different nonlinearities. Specifically, Example 1 are two general SFNSs model with different nonlinearities. Example 2 is illustrated through a practical physical model.

Example 1. Two general SFNSs with different nonlinearities are considered as follows:

$$\Sigma_1 : \begin{cases} \dot{x}_1 = f_{1,\Sigma_1} + x_2, \\ \dot{x}_2 = f_{2,\Sigma_1} + u, \\ y = x_1, \end{cases} \tag{53}$$

$$\Sigma_2 : \begin{cases} \dot{x}_1 = f_{1,\Sigma_2} + x_2, \\ \dot{x}_2 = f_{2,\Sigma_2} + u, \\ y = x_1, \end{cases} \tag{54}$$

where x_1 and x_2 are the states with the initial condition $[x_1(0), x_2(0)]^T = [0.5, 0.5]^T$; $f_{1,\Sigma_1} = 0.1x_1\exp(-0.5x_1)$, $f_{2,\Sigma_1} = \cos(x_1^2x_2)\exp(-0.5\sin(x_2)) + \cos(x_1x_2^3)\sin(x_1^2 + x_2)$, $f_{1,\Sigma_2} = x_1^2\sin(x_1)$ and $f_{2,\Sigma_2} = \cos(x_1)\exp(2x_2\sin(x_1)) + x_1x_2$. The *control objective* is to compel the output y tracks the desired signal $y_d = \frac{1}{2}(\sin(2t) + \cos(t))$. To meet the this objective, the adaptive LWL controller with the adaptive laws is implemented on the basis of (13)–(18), as detailed below.

Step 1*:* The virtual control signal α_1 and the adaptive laws $\hat{\theta}_{1,k}$ are constructed as

$$\begin{aligned} \alpha_1 &= -\rho_1 z_1 - \sum_{k=1}^{N_1} \bar{\omega}_{1,k}^1 \hat{\theta}_{1,k}^T \varphi_{1,k}(Z_1), \\ \dot{\hat{\theta}}_{1,k} &= \Gamma_1(\bar{\omega}_{1,k}^1 \varphi_{1,k}(Z_1)z_1 - \sigma_1\hat{\theta}_{1,k}), \hat{\theta}_{1,k}(0) = 0, \end{aligned}$$

where $Z_1 = [x_1, \dot{y}_d]^T$. In the first LWL, $N_1 = 100$. $c_{1,k}$ is the center location of the kth weighting function evenly spaced on $[-5,5] \times [-1,1]$, and $\mu_{1,k}$ the radius of the supporting set $M_{1,k}$ is chosen as $\mu_{1,k} = 2$. Select the following design parameters as $\Gamma_1 = 5I_1$, $\rho_1 = 5$ and $\sigma_1 = 3$.

Step 2*:* The actual control signal u and the adaptive laws $\hat{\theta}_{2,k}$ are constructed as

$$\begin{aligned} u &= -\rho_2 z_2 - \sum_{k=1}^{N_2} \bar{\omega}_{2,k}^2 \hat{\theta}_{2,k}^T \varphi_{2,k}(Z_2) - z_1, \\ \dot{\hat{\theta}}_{2,k} &= \Gamma_2(\bar{\omega}_{2,k}^2 \varphi_{2,k}(Z_2)z_2 - \sigma_2\hat{\theta}_{2,k}), \hat{\theta}_{2,k}(0) = 0, \end{aligned}$$

where $Z_2 = [x_1, x_2, \frac{\partial\alpha_1}{\partial x_1}, \phi_1]^T$, $\phi_1 = \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d + \frac{\partial\alpha_1}{\partial \dot{y}_d}\ddot{y}_d + \frac{\partial\alpha_1}{\partial\hat{\theta}_{1,k}}\dot{\hat{\theta}}_{1,k}$. In the second LWL, $N_2 = 625$. $c_{2,k}$ is the center location of kth weighting function evenly spaced on $[-5,5]\times[-5,5]\times[-20,20]\times[-20,20]$, and the radius is selected as $\mu_{2,k} = 6$. Select the following design parameters as $\Gamma_2 = 6I_1$, $\rho_2 = 5$ and $\sigma_2 = 3$.

According to (4), the weighting function with sufficient differentiability is set as

$$\omega_{i,k}^i(Z_i) = \begin{cases} (1 - \psi_{i,k}^{2(3-i)})^{(3-i)}, & \psi_{i,k} \le 1, \\ 0, & \psi_{i,k} > 1. \end{cases} , i = 1, 2,$$

where $\psi_{i,k} = \|\frac{Z_i - c_{i,k}}{\bar{\mu}_{i,k}}\|_\infty$, and $\bar{\mu}_{i,k} = [\mu_{1,k}, \dots, \mu_{i,k}]$, $i = 1, 2$. The basis functions are chosen as $\varphi_{1,k}(Z_1) = [\varepsilon_{1,k}, 1]^T$, $\varphi_{2,k}(Z_2) = [\varepsilon_{2,k}, 1]^T$, in which $\varepsilon_{1,k} = \iota_1\exp(-\frac{\|Z_1 - c_{1,k}\|^2}{v_{1,k}^2})$, $\varepsilon_{2,k} = \iota_2\exp(-\frac{\|Z_2 - c_{2,k}\|^2}{v_{2,k}^2})$, $v_{1,k} = 2$, $v_{2,k} = 1$, $\iota_1 = 2$ and $\iota_2 = 1$.

The simulation results are illustrated in Figs. 2–6. Fig. 2 shows the output tracking performance, while Fig. 3 presents the control input u. Fig. 4 displays the trajectories of the adaptive parameters $\|\hat{\theta}_1\|$ and $\|\hat{\theta}_2\|$, where $\|\hat{\theta}_1\| = \sqrt{\sum_{j=1}^{100}\hat{\theta}_{1,j}^2}$ and $\|\hat{\theta}_2\| = \sqrt{\sum_{j=1}^{625}\hat{\theta}_{2,j}^2}$. As shown in Figs. 2–4, the proposed adaptive LWL controller effectively guides the system output to track the desired trajectory, maintaining the boundedness of all closed-loop signals as predicted by the theoretical analysis, even in the presence of unknown system nonlinearities. Figs. 5 and 6 compare the learned functions with the ground truth, demonstrating the strong learning capability of the LWL method.

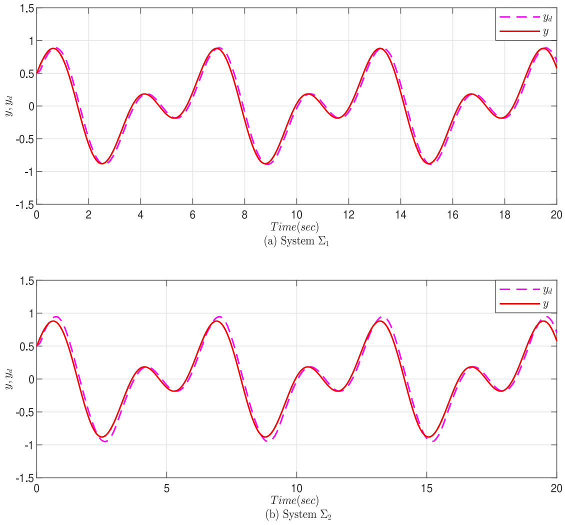

Fig. 2. Output tracking performance of systems Σ_1 and Σ_2.

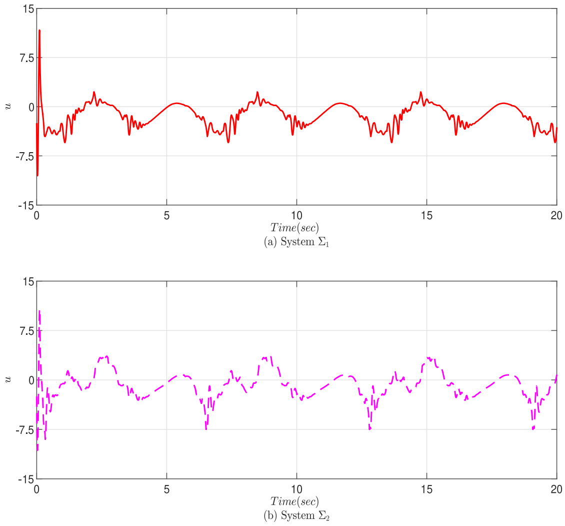

Fig. 3. Control signals u of systems Σ_1 and Σ_2.

Example 2. To illustrate the practicality of the proposed control scheme, an inverted pendulum system is taken into consideration. The dynamic equations of this system are described as [37]

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{g\sin(x_1) - \frac{\mathcal{M}_p \mathcal{L} x_2^2 \cos(x_1)\sin(x_1)}{\mathcal{M}_c + \mathcal{M}_p}}{\mathcal{L}\left(\frac{4}{3} - \frac{\mathcal{M}_p \cos^2(x_1)}{\mathcal{M}_c + \mathcal{M}_p}\right)} + \frac{\frac{\cos(x_1)}{\mathcal{M}_c + \mathcal{M}_p}}{\mathcal{L}\left(\frac{4}{3} - \frac{\mathcal{M}_p \cos^2(x_1)}{\mathcal{M}_c + \mathcal{M}_p}\right)} u, \\ y &= x_1, \end{aligned} \tag{55}$$

where x_1 and x_2 represent the angle and angular velocity of the pendulum, respectively. $\mathcal{M}_c$ is the mass of a cart, $g = 9.8\ \mathrm{m/s^2}$ represents the gravitational acceleration. $\mathcal{L}$ denotes the half length of a pole, $\mathcal{M}_p$ represents the mass of the pole. u represents the demanded force.

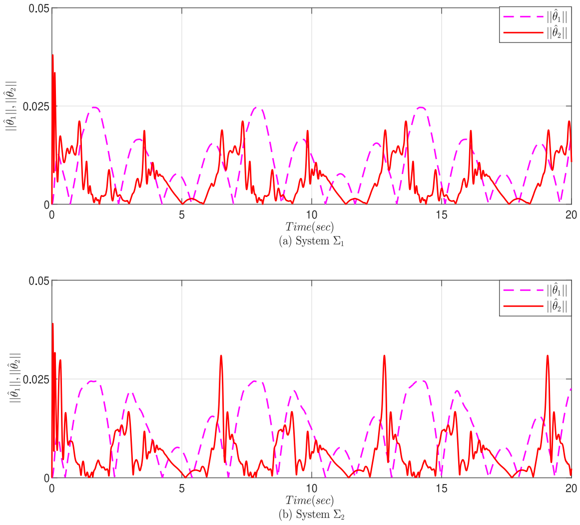

Fig. 4. Adaptive parameters $\|\hat{\theta}_1\|$ and $\|\hat{\theta}_2\|$ of systems Σ_1 and Σ_2.

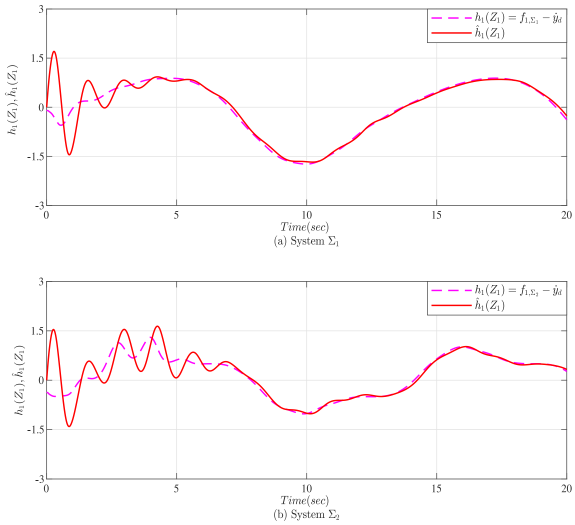

Fig. 5. The function $h_1(Z_1)$ and its approximation $\hat{h}_1(Z_1)$.

In the simulation, the system parameters are set as $\mathcal{M}_c = 1$ kg, $\mathcal{M}_p = 0.1$ kg, $\mathcal{L} = 0.5$ m. $[x_1(0), x_2(0)]^T = [0.1, 0]^T$ is the initial condition of (55). The desired signal $y_d = \frac{1}{2}(\sin(2t) + \cos(t))$.

Highlighting the universal applicability and effectiveness of the adaptive LWL tracking control scheme, We employ the controller described in Example 1 with the same design parameters to the plant (55). The results of the simulations can be observed in Figs. 7–9. Fig. 7 displays the performance of output tracking. Fig. 8 depicts the control input u. The trajectories of the adaptive parameters $\|\hat{\theta}_1\|$ and $\|\hat{\theta}_2\|$ are presented in Fig. 9, in which $\|\hat{\theta}_1\| = \sqrt{\sum_{j=1}^{100} \hat{\theta}_{1,j}^2}$, $\|\hat{\theta}_2\| = \sqrt{\sum_{j=1}^{625} \hat{\theta}_{2,j}^2}$. Obviously, all closed-loop signals are bounded by observing the results in Figs. 7–9, confirming the effectiveness of the adaptive LWL tracking controller.

To showcase the superiority of the proposed adaptive LWL method, a comparative experiment with the command filter in [32] is conducted. The design parameters as $\rho_1 = \rho_2 = 5$, $\Gamma_1 = 5I_1$, $\Gamma_2 = 6I_1$ and $\sigma_1 = \sigma_2 = 2$. The simulation results are shown in Figs. 10–11. It can be seen from Figs. 10–11. Compared with the command filter-based control method in [32], the adaptive LWL

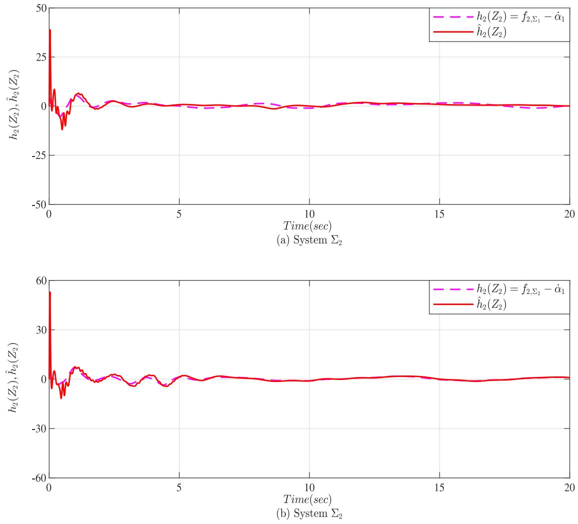

Fig. 6. The function $h_2(Z_2)$ and its approximation $\hat{h}_2(Z_2)$.

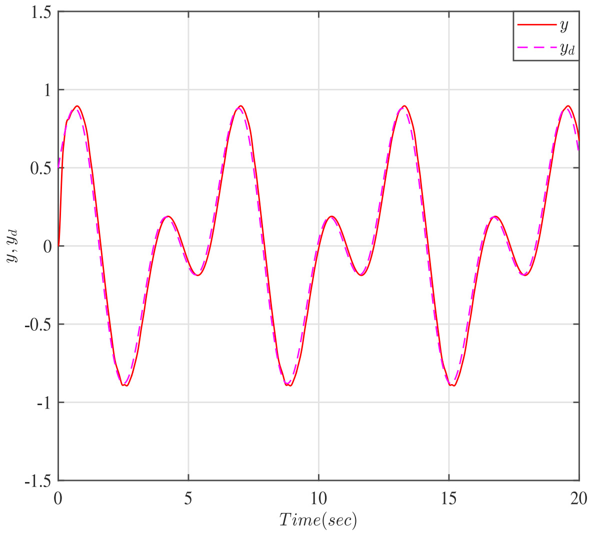

Fig. 7. Trajectories of y and y_d.

tracking control method proposed in this paper can obtain better tracking performance with less control cost, and has significant advantages.

6. Conclusion

This study investigates the problem of tracking control for a class of n-order unknown SFNSs. An adaptive LWL tracking control approach is developed without including *a priori* knowledge about the system functions, even their bounding functions. The key contribution of this method is that a novel weighting function with sufficient differentiability is constructed, evading the obstacle of the integration of LWL and backstepping for SFNSs. The simulation confirms the validity and practicality of our method.

Several interesting issues remain worth exploring in this direction. For instance, from an implementation and computational perspective, the proposed controller designed in (13)–(18) necessitates the repeated calculation of virtual control variable derivatives. Consequently, the controller's complexity increases significantly with the system's order. To mitigate this issue of "explosion of complexity", one available method is the use of dynamic surface control techniques, which will be addressed in future research. Another interesting topic for investigation is whether the proposed method can be applied to multi-agent networks and their applications [38–40].

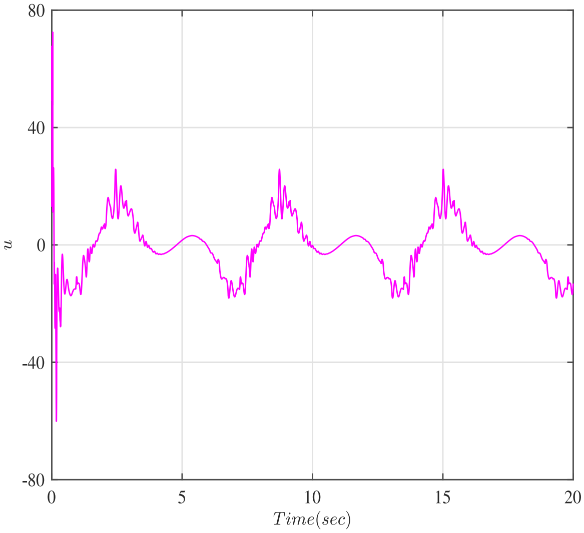

Fig. 8. Trajectories of the control input u.

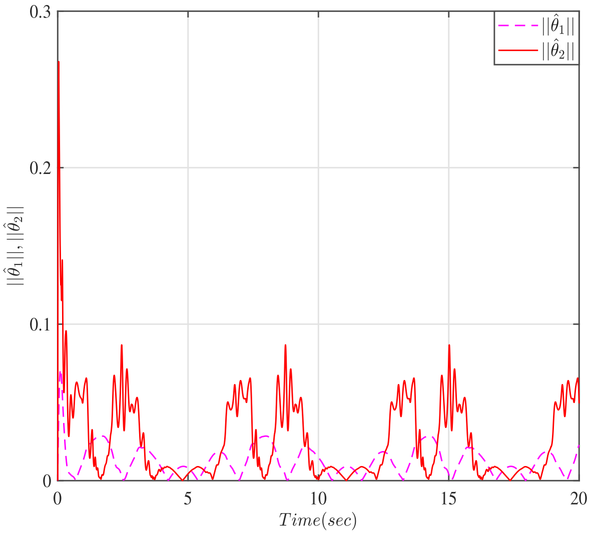

Fig. 9. Trajectories of $\|\hat{\theta}_1\|$ and $\|\hat{\theta}_2\|$.

CRediT authorship contribution statement

Yu-Fa Liu: Writing – review & editing, Writing – original draft, Formal analysis. **Dong Wang:** Methodology, Investigation, Formal analysis. **Zhuo Wang:** Investigation, Data curation. **Ante Su:** Writing – review & editing. **Yong-Hua Liu:** Writing – review & editing, Data curation, Conceptualization. **Chun-Yi Su:** Writing – original draft, Investigation, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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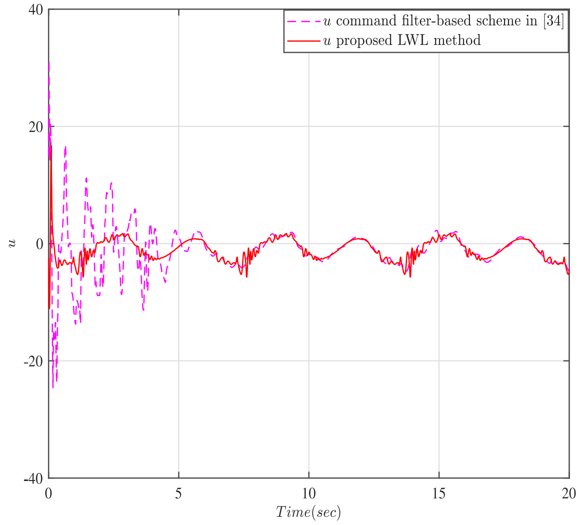

Fig. 10. Control signal u with proposed LWL method and command filter-based scheme in [34].

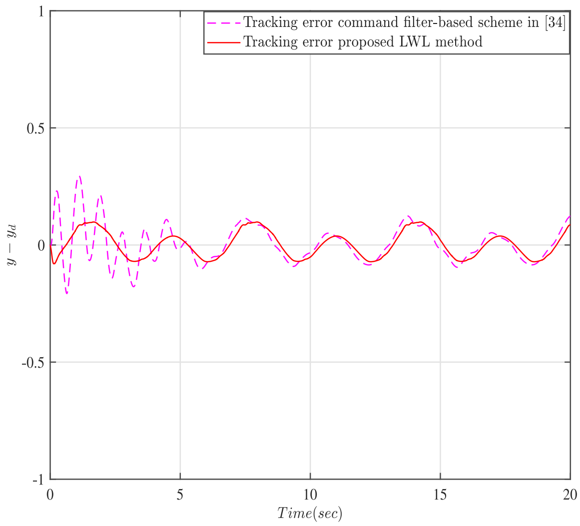

Fig. 11. Tracking error with proposed LWL method and command filter-based scheme in [34].

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