A Constructive Approach for Neural Network Approximation Sets in Adaptive Control of Strict-Feedback Systems

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***Abstract*—Determining the neural network (NN) approximation sets for adaptive control of strict-feedback uncertain systems has posed a persistent challenge. This article proposes a novel and constructive solution that incorporates signal substitution technique, barrier functions (BFs), and backstepping approach. By applying the signal substitution technique, all system states are transformed into state error variables, facilitating the approximation of unknown system functions through NNs. The use of BFs subsequently allows for the restriction of state errors, enabling the calculation of exact bounds for the NN weight estimators. This process reveals the determination of the approximation sets of NN in advance. Illustrative examples are conducted to validate the effectiveness of the proposed approach.**

***Index Terms*—Approximation sets, barrier functions (BFs), neural network (NN), signal substitution technique, strict-feedback systems.**

I. INTRODUCTION

OVER the last few decades, neural network (NN) control has become a prominent method for addressing the challenges of controlling highly uncertain nonlinear systems, thanks to its capacity to learn complex input-output mappings. Numerous significant advancements have been developed in [1], [2], [3], [4], [5], [6], [7], [8], [9], and [10], and successfully applied to various industrial systems, including robots [11], near-space morphing vehicles [12], power systems [13], etc.

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It is well known that during the implementation of adaptive NN controllers, the NN input signals must remain within a specific approximation set to enable effective approximation of the unknown nonlinear dynamics [14]. However, determining such an approximation set in advance is challenging and often overlooked. To address this issue, an adaptive tracking control framework was introduced for a class of uncertain nonlinear dynamic systems [15], where the switching mechanisms were employed to drive system states into predefined compact sets for NN approximation. Building on this idea, Fabri and Kadirkamanathan [16] proposed an adaptive control scheme for affine nonlinear systems, employing a sliding mode approach that utilizes dynamically structured Gaussian radial basis functions. Nevertheless, the controllers proposed in [15] and [16] are prone to chattering due to the use of nondifferentiable switching functions. To mitigate this issue, several adaptive NN controllers utilizing differentiable switching functions have been developed for strict-feedback systems, as seen in [17], [18], [19], [20], [21], [22], and [23]. Alternatively, adaptive NN backstepping control methods have been suggested for different types of uncertain nonlinear systems [24], [25], [26], where signal substitution techniques were employed to transform the variables of nonlinear terms into desired reference signals, thereby clearly defining the NN approximation set.

Despite notable results obtained in [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], and [26] for determining NN approximation sets, the developed control algorithms typically impose strict restrictions on nonlinear functions. Specifically, these approaches assume that the bounding functions of the system nonlinearities are known, which can be a significant limitation. To address this issue, approximation-based adaptive control approaches with predefined approximation sets have been studied for unknown nonlinear systems [27], [28], [29]. In [28], a backstepping adaptive control framework was proposed for a class of strict-feedback uncertain systems by integrating NN with barrier Lyapunov functions (BLFs). The BLFs were employed to determine the existence of approximation sets in advance, ensuring the validity of the NN approximation. Similarly, a neuro-adaptive backstepping control approach, as discussed in [29], was developed for unknown pure-feedback nonaffine systems by integrating BLFs with the mean value theorem. Recently, Yu and Chen [30] examined adaptive NN control method for strict-feedback systems with mismatched uncertainties. They proposed a novel iterative design method that effectively solves the existence issue of NN approximation sets. Nevertheless, while works, such as [27], [28], [29], and [30], guarantee the existence of these sets, they do not provide methods to accurately determine them, which may affect the approximation accuracy of the unknown nonlinearities in the system. To accurately determine NN approximation sets, Zou et al. [31] introduced a constructive approach for NN control of first-order and second-order unknown nonlinear systems by imposing constraints on system states. Additionally, in [32], an adaptive consensus protocol with known NN approximation sets was developed for a class of heterogeneous nonlinear multiagent systems, incorporating both adaptive and nonsmooth control strategies. However, the results in [31] and [32] are exclusively applicable to uncertain nonlinear systems with relative degrees of one or two.

Building on the aforementioned studies, in this article, we investigate the problem of determining the NN approximation sets for adaptive control of strict-feedback uncertain systems. The main contributions are summarized as follows.

1) We develop a novel, constructive solution for determining NN approximation sets in adaptive control of strict-feedback uncertain systems. In contrast to classical NN control approaches, which struggle with predetermining approximation sets, our method integrates signal substitution, barrier functions (BFs), and backstepping techniques. Specifically, the signal substitution technique transforms the original system states into state error variables, enabling NNs to approximate unknown system functions. By constraining these state errors using BFs, we derive precise bounds for the NN weight estimators, allowing the accurate determination of NN approximation sets in advance.
2) Unlike existing adaptive NN control schemes [27], [28], [29], [30], which can only establish the existence of the approximation sets without determining them precisely, our method ensures that the approximation sets of the NN can be obtained accurately.
3) In contrast to existing NN control results with known approximation sets [31], [32], which are typically restricted to systems with relative degrees of one or two, the presented innovative constructive control approach with approximation sets in advance can be applied to uncertain nonlinear systems with arbitrarily high-relative degree.

Notations: The real numbers, nonnegative real numbers, and positive real numbers are represented by $\mathbf{R}$, $\mathbf{R}_{\geq 0}$ and $\mathbf{R}_{>0}$, respectively. The real n-dimensional space is denoted by $\mathbf{R}^n$, while the $n \times n$ identity matrix is given by I_n. The exponential function is denoted as $\exp(\cdot)$.

II. Preliminaries

A. Neural Network Approximation

In this article, we define a radial basis function NN (RBFNN) as

$$H_{\mathrm{NN}}(X) = \zeta^T \Phi(X) \tag{1}$$

to approximate an unknown continuous function, where the NN input vector is $X \in \mathbf{R}^m$. The vector $\zeta = [\zeta_1, \ldots, \zeta_l]^T$ and $\Phi(X) = [\Phi_1(X), \ldots, \Phi_l(X)]^T$ represent the weights and the RBF, respectively, with each $\Phi_i(X) = \exp(-[(X-\delta_i)^T(X-\delta_i)/\sigma_i^2])$ for $i = 1, \ldots, l$, where $l > 1$ represents the number of NN nodes. The parameter $\sigma_i > 0$ is the Gaussian function width, while $\delta_i = [\delta_{i1}, \ldots, \delta_{il}]^T$ represents the center of the receptive field.

Lemma 1 [28]: For any given compact set $\Omega_{\mathrm{NN}} \subset \mathbf{R}^m$, any continuous function $H(X) : \mathbf{R}^m \to \mathbf{R}$ can be approximated by the RBFNN in (1) as

$$H(X) = \zeta^T \Phi(X) + \epsilon(X), X \in \Omega_{\mathrm{NN}} \tag{2}$$

where the approximation error $\epsilon(X)$ meets the condition $|\epsilon(X)| \leq \varepsilon$ with ε being a positive constant.

B. Stability of Nonautonomous Systems

A nonautonomous system of the following form is considered:

$$\dot{\xi}_0 = h(t, \xi_0),\ \ \xi_0(t_0) \in \bar{\Omega}_0 \tag{3}$$

with $h : \mathbf{R}_{\geq 0} \times \bar{\Omega}_0 \to \mathbf{R}^m$, which is piecewise continuous in t and locally Lipschitz within $\bar{\Omega}_0$, and $\bar{\Omega}_0$ is a domain that contains the origin.

Lemma 2 [33]: Let ξ be a solution of (3) on a maximal interval $[t_0, t_s)$ with $t_s < +\infty$. Let $\bar{\Omega}_\xi$ be any compact subset of Ω_ξ. Then, there exists some $t \in [t_0, t_s)$ with $\xi(t) \notin \bar{\Omega}_\xi$.

III. Problem Statement

We focus on a class of strict-feedback systems that can be expressed in the form given below

$$\begin{aligned} \dot{\psi}_i &= \psi_{i+1} + \varphi_i(\bar{\psi}_i), i = 1, \ldots, n-1 \\ \dot{\psi}_n &= u + \varphi_n(\bar{\psi}_n) \\ y &= \psi_1 \end{aligned} \tag{4}$$

where $\psi_1, \ldots, \psi_n$ represent the state variables, control signal $u \in \mathbf{R}$, and output $y \in \mathbf{R}$. Let $\bar{\psi}_i = [\psi_1, \ldots, \psi_i]^T \in \mathbf{R}^i$ for $i = 1, \ldots, n$, with the initial values $\bar{\psi}_n(0) = [\psi_1(0), \ldots, \psi_n(0)]^T$. The unknown functions $\varphi_i(\bar{\psi}_i) : \mathbf{R}^i \to \mathbf{R}$, are locally Lipschitz in $\bar{\psi}_i$ for $i = 1, \ldots, n$.

Remark 1: Strict-feedback systems (4) can accurately model various industrial processes, including offshore surface vehicles [28], continuous stirred tank reactors [36], DC motor systems [37], hydraulic actuators [38], etc. However, parameter identification for these mathematical models is often highly challenging. Therefore, designing adaptive NN controller for strict-feedback systems with unknown dynamics is a valuable and meaningful pursuit.

The *primary objective* is to construct an adaptive NN controller u that defines the NN approximation sets for the adaptive control of strict-feedback uncertain systems (4), ensuring that the output y converges to a neighborhood of zero while maintaining uniform boundedness of all closed-loop system signals.

IV. Adaptive Neural Network Control Design

In this section, we develop a novel adaptive NN controller by integrating the signal substitution technique, BF, NN, and backstepping to determine NN approximation sets for the adaptive control of strict-feedback uncertain systems (4), despite the presence of unknown nonlinearities. Additionally, a detailed stability analysis is carefully performed.

A. Controller Design

To begin with, we define the following coordinate transformations:

$$z_1 = \psi_1 \tag{5}$$

$$z_i = \psi_i - \alpha_{i-1}, i = 2, \ldots, n \tag{6}$$

where α_{i-1}, $i = 2, \ldots, n$ is the virtual control law at the $(i-1)$th step.

Following (5) and (6), the construction of the proposed adaptive NN controller is as follows.

Step 1: Select the design constant ρ_1 such that $|z_1(0)| < \rho_1$. The first virtual control law α_1 and the corresponding adaptive weight estimator $\hat{\zeta}_1$ can then be designed as

$$\alpha_1 = -c_1 z_1 - \hat{\zeta}_1^T \Phi_1(X_1) - \frac{k_1 z_1}{\rho_1^2 - z_1^2} \tag{7}$$

$$\dot{\hat{\zeta}}_1 = \Gamma_1\Big(\Phi_1(X_1) z_1 - \eta_1 \hat{\zeta}_1\Big) \tag{8}$$

where $X_1 = z_1 \in \mathbf{R}$, $\hat{\zeta}_1 = [\hat{\zeta}_{1,1}, \ldots, \hat{\zeta}_{1,l_1}]^T$ are the weight vector, and $\Phi_1(X_1)$ is RBF vector of the first approximator as detailed in (29). Here, l_1 is the number of the NN nodes, $\hat{\zeta}_1$ is the estimate of ζ_1, which will be specified in the sequel; c_1, k_1 and η_1 are positive design constants. The matrix $\Gamma_1 = \mathrm{diag}(\gamma_{1,1}, \ldots, \gamma_{1,l_1})$ with $\gamma_{1,j} > 0$ for $j = 1, \ldots, l_1$.

Step i ($i = 2, \ldots, n-1$): Select the design constant ρ_i such that $|z_i(0)| < \rho_i$. The ith virtual control law α_i and the corresponding adaptive weight estimator $\hat{\zeta}_i$ can then be designed as

$$\alpha_i = -c_i z_i - \hat{\zeta}_i^T \Phi_i(X_i) - \frac{k_i z_i}{\rho_i^2 - z_i^2} \tag{9}$$

$$\dot{\hat{\zeta}}_i = \Gamma_i\Big(\Phi_i(X_i) z_i - \eta_i \hat{\zeta}_i\Big) \tag{10}$$

where $X_i = [z_1, \ldots, z_i, \hat{\zeta}_1, \ldots, \hat{\zeta}_{i-1}]^T \in \mathbf{R}^{2i-1}$, $\hat{\zeta}_i = [\hat{\zeta}_{i,1}, \ldots, \hat{\zeta}_{i,l_i}]^T$ are the weight vector, and $\Phi_i(X_i) = [\Phi_{i,1}, \ldots, \Phi_{i,l_i}]^T$ are the weight vector and RBF vector of the ith approximator as detailed in (39). Here, l_i represents the number of nodes in the ith layer of the NN, $\hat{\zeta}_i$ is the estimate of ζ_i, which will be specified later; c_i, k_i and η_i are positive design constants. The matrix $\Gamma_i = \mathrm{diag}(\gamma_{i,1}, \ldots, \gamma_{i,l_j})$ with $\gamma_{i,l_j} > 0$ for $j = 1, .., l_i$.

Step n: Select the design constant ρ_n such that $|z_n(0)| < \rho_n$. The actual control law u and the corresponding adaptive weight estimator $\hat{\zeta}_n$ can then be designed as

$$u = -c_n z_n - \hat{\zeta}_n^T \Phi_n(X_n) - \frac{k_n z_n}{\rho_n^2 - z_n^2} \tag{11}$$

$$\dot{\hat{\zeta}}_n = \Gamma_n\Big(\Phi_n(X_n) z_n - \eta_n \hat{\zeta}_n\Big) \tag{12}$$

where $X_n = [z_1, \ldots, z_n, \hat{\zeta}_1, \ldots, \hat{\zeta}_{n-1}]^T \in \mathbf{R}^{2n-1}$, $\hat{\zeta}_n = [\hat{\zeta}_{n,1}, \ldots, \hat{\zeta}_{n,l_n}]^T$ are the weight vector, and $\Phi_n(X_n) = [\Phi_{n,1}, \ldots, \Phi_{n,l_n}]^T$ are the weight vector and RBF vector of the nth approximator as detailed in (49). l_n represents the number of nodes in the nth layer of the NN, $\hat{\zeta}_n$ is the estimate of ζ_n, which will be specified in the sequel; c_n, k_n and η_n are positive design constants. The matrix $\Gamma_n = \mathrm{diag}(\gamma_{n,1}, \ldots, \gamma_{n,l_j})$ with $\gamma_{n,l_j} > 0$ for $j = 1, .., l_n$.

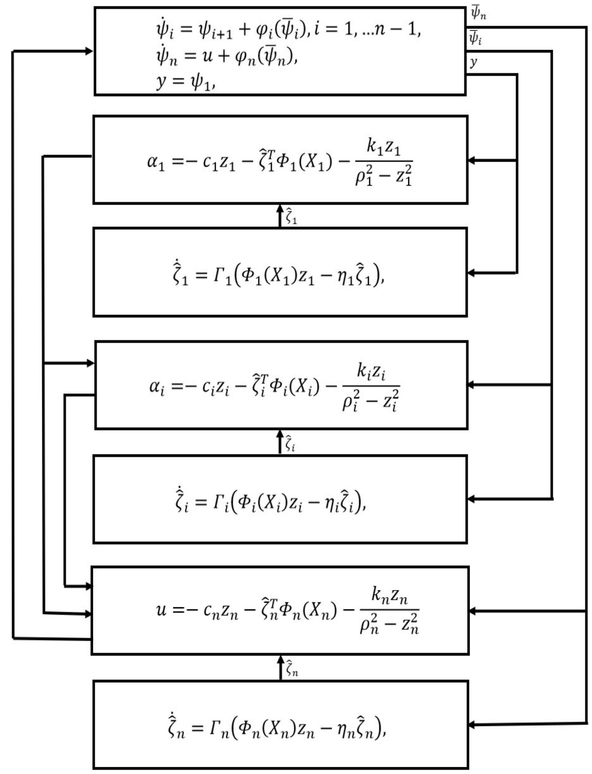

Fig. 1. Overall block diagram of the control system.

The design procedures of the proposed adaptive NN controller (7)–(12) are visualized as the block diagram shown in Fig. 1.

B. Stability Analysis

According to (5)–(12), by applying the signal substitution technique, the virtual control laws $\alpha_1, \ldots, \alpha_{n-1}$ and the actual control law u can be reformulated as

$$\begin{aligned}\alpha_i &= -c_i z_i - \hat{\zeta}_i^T \Phi_i(X_i) - \frac{k_i z_i}{\rho_i^2 - z_i^2} \\ &:= \breve{\alpha}_i\Big(z_1, \ldots, z_i, \hat{\zeta}_1, \ldots, \hat{\zeta}_i\Big), i = 1, \ldots, n-1\end{aligned} \tag{13}$$

$$\begin{aligned}u &= -c_n z_n - \hat{\zeta}_n^T \Phi_n(X_n) - \frac{k_n z_n}{\rho_n^2 - z_n^2} \\ &:= \breve{u}\Big(z_1, \ldots, z_n, \hat{\zeta}_1, \ldots, \hat{\zeta}_n\Big).\end{aligned} \tag{14}$$

Similarly, the states $\psi_1, \ldots, \psi_n$ are transformed into the state error variables $z_1, \ldots, z_n$ and the adaptive weight estimators $\hat{\zeta}_1, \ldots, \hat{\zeta}_n$, as follows:

$$\begin{aligned}\psi_1 &= z_1 \\ &:= \breve{\psi}_1(z_1)\end{aligned} \tag{15}$$

$$\begin{aligned}\psi_i &= z_i + \alpha_{i-1} \\ &:= \breve{\psi}_i\Big(z_1, \ldots, z_i, \hat{\zeta}_1, \ldots, \hat{\zeta}_{i-1}\Big), i = 2, \ldots, n.\end{aligned} \tag{16}$$

To advance the analysis, we present the following two propositions.

Proposition 1: For any initial conditions $\hat{\zeta}_{i,j}(0)$, $i = 1, \ldots, n$ and $j = 1, \ldots, l_i$, if $|z_i| < \rho_i$, the following inequalities hold.

1) $\hat{\zeta}_{i,j} \leq \upsilon_{i,j} = |\hat{\zeta}_{i,j}(0)| + (\rho_i/\eta_i)$, where $\upsilon_{i,j} > 0$.
2) $|\tilde{\zeta}_i| \leq \omega_i$, where $\omega_i > 0$.

Proof: 1) For any initial conditions $\hat{\zeta}_{i,j}(0)$, where $i = 1, \ldots, n$ and $j = 1, \ldots, l_i$, the solutions to the adaptive weight estimators in (8), (10), and (12) can be calculated as

$$\begin{aligned}\hat{\zeta}_{i,j} &= \exp(-\gamma_{i,j}\eta_i t)\Big(\hat{\zeta}_{i,j}(0) \\ &\quad + \gamma_{i,j}\phi_{i,j}z_i \int_0^t \exp(\gamma_{i,j}\eta_i\tau)d\tau\Big) \\ &= \exp(-\gamma_{i,j}\eta_i t)\hat{\zeta}_{i,j}(0) + \frac{\phi_{i,j}z_i}{\eta_i}. \end{aligned} \tag{17}$$

From the definition of $\Phi_i(X_i)$ in (1), it follows that $0 < \Phi_i(X_i)^T\Phi_i(X_i) = \|\Phi_i(X_i)\|^2 \leq 1$. This implies that $|\phi_{i,j}| \leq 1$. Utilizing the fact that $|z_i| < \rho_i$, we can deduce that $|\phi_{i,j}z_i| < \rho_i$. Thus, (13) can be scaled as

$$\begin{aligned}|\hat{\zeta}_{i,j}| &\leq |\hat{\zeta}_{i,j}(0)| + \frac{|\phi_{i,j}z_i|}{\eta_i} \\ &\leq |\hat{\zeta}_{i,j}(0)| + \frac{\rho_i}{\eta_i} \\ &= \upsilon_{i,j}\end{aligned} \tag{18}$$

where $\upsilon_{i,j}$ are known positive constants.

2) Note that $\hat{\zeta}_i = [\hat{\zeta}_{i,1}, \ldots, \hat{\zeta}_{i,l_i}]^T$, together with $|\hat{\zeta}_{i,j}| \leq \upsilon_{i,j}$ in (14), for $i = 1, \ldots, n$, it follows straightforwardly that:

$$|\hat{\zeta}_i| = \sqrt{\sum_{j=1}^{l_i} |\hat{\zeta}_{i,j}|^2} \leq \sqrt{\sum_{j=1}^{l_i} |\upsilon_{i,j}|^2} \tag{19}$$

$$|\tilde{\zeta}_i| \leq |\hat{\zeta}_i| + \zeta_i = \omega_i. \tag{20}$$

■

Remark 2: As demonstrated in (13)–(16), the exact bounds of the adaptive NN estimators can be explicitly and accurately determined. This represents a significant improvement over classical adaptive NN control (CANNC) methods, such as those in [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [31], and [32], where such precise bounds are not readily available.

Proposition 2: Let the vector $\phi(t) = [z_1(t), \ldots, z_n(t), \hat{\zeta}_1(t), \ldots, \hat{\zeta}_n(t)]^T$ be defined, along with the open set $\Omega = \underbrace{(-\rho_1, \rho_1) \times \cdots \times (-\rho_n, \rho_n)}_{n} \times \underbrace{\mathbf{R}^{l_1} \times \cdots \times \mathbf{R}^{l_n}}_{n}$. For the closed-loop system described by (4) and (7)–(12), there exists a unique maximal solution $\phi(t) \in \Omega$ on the time interval $[0, \tau_f)$, i.e., $|z_i| < \rho_i$ for all $t \in [0, \tau_f)$, with $i = 1, \ldots, n$.

Proof: From (5)–(16), the system (4) in closed-loop with (5)–(16) is obtained as follows:

$$\begin{aligned}\dot{z}_1 &= \varphi_1\left(\breve{\psi}_1\right) + \breve{\psi}_2 \\ &:= \kappa_1\left(t, z_1, z_2, \hat{\zeta}_1\right)\end{aligned} \tag{21}$$

$$\begin{aligned}\dot{z}_i &= \varphi_i\left(\underline{\breve{\psi}}_i\right) + \breve{\psi}_{i+1} - \frac{\partial\breve{\alpha}_{i-1}}{\partial t} \\ &\quad - \sum_{j=1}^{i-1}\left(\frac{\partial\breve{\alpha}_{i-1}}{\partial z_j}\kappa_j + \frac{\partial\breve{\alpha}_{i-1}}{\partial\hat{\zeta}_j}\kappa_{n+j}\right) \\ &:= \kappa_i\left(t, z_1, \ldots, z_{i+1}, \hat{\zeta}_1, \ldots, \hat{\zeta}_i\right), i = 2, \ldots, n-1\end{aligned} \tag{22}$$

$$\begin{aligned}\dot{z}_n &= f_n\left(\underline{\breve{\psi}}_n\right) + \breve{u} - \frac{\partial\breve{\alpha}_{n-1}}{\partial t} \\ &\quad - \sum_{j=1}^{n-1}\left(\frac{\partial\breve{\alpha}_{n-1}}{\partial z_j}\kappa_j + \frac{\partial\breve{\alpha}_{n-1}}{\partial\hat{\zeta}_j}\kappa_{n+j}\right) \\ &:= \kappa_n\left(t, z_1, \ldots, z_n, \hat{\zeta}_1, \ldots, \hat{\zeta}_n\right)\end{aligned} \tag{23}$$

$$\begin{aligned}\dot{\hat{\zeta}}_i &= \Gamma_i\left(\Phi_i(X_i)z_i - \eta_i\hat{\zeta}_i\right) \\ &:= \kappa_{n+i}\left(t, z_1, \ldots, z_i, \hat{\zeta}_1, \ldots, \hat{\zeta}_i\right), i = 1, \ldots, n.\end{aligned} \tag{24}$$

Here, $\underline{\breve{\psi}}_n = [\breve{\psi}_1, \ldots, \breve{\psi}_n]^T$.

The closed-loop system in (21)–(24) is also compactly represented as

$$\dot{\phi} = \kappa(t, \phi) = \begin{bmatrix} \kappa_1\left(t, z_1, z_2, \hat{\zeta}_1\right) \\ \vdots \\ \kappa_{2n}\left(t, z_1, \ldots, z_n, \hat{\zeta}_1, \ldots, \hat{\zeta}_n\right)\end{bmatrix}. \tag{25}$$

In the analysis of the closed-loop system (25), it is initially observed that for $i = 1, \ldots, n$, the condition $|z_i(0)| < \rho_i$ holds, implying that $\phi(0) = [z_1(0), \ldots, z_n(0), \hat{\zeta}_1(0), \ldots, \hat{\zeta}_n(0)]^T \in \Omega$ holds. Furthermore, owing to the $\psi_i(\bar{\psi}_i)$, $i = 1, \ldots, n$, are locally Lipschitz in $\bar{\psi}_i$, it follows that $\kappa : \mathbf{R}_{\geq 0} \times \Omega \to \mathbf{R}^{2n}$ is piecewise continuous in t and locally Lipschitz in ϕ. By applying Theorem 54 from [35], we conclude that the system (25) has a unique maximal solution $\phi(t) \in \Omega$ on the time interval $[0, \tau_f)$, i.e., $|z_i| < \rho_i$, for all $t \in [0, \tau_f)$, with $i = 1, \ldots, n$. ■

The primary results of this article are summarized in the theorem below.

Theorem 1: Consider the strict-feedback uncertain systems (4), with the actual control law u designed as in (11) and the adaptive weight estimators designed as in (8), (10), and (12). The following statements hold for any initial condition $\bar{\psi}_n(0)$.

1) The NN approximation sets $\Omega_{\mathrm{NN}_i} \subset \mathbf{R}^{i+\sum_{j=1}^{m} l_j}$, $m = 1, \ldots, i-1$, $i = 1, \ldots, n$ are determined in advance as follows: $\Omega_{\mathrm{NN}_i} = [-\rho_1, \rho_1] \times \cdots \times [-\rho_i, \rho_i] \times [-\upsilon_{1,1}, \upsilon_{1,1}] \times \cdots \times [-\upsilon_{1,l_1}, \upsilon_{1,l_1}] \times [-\upsilon_{i-1,1}, \upsilon_{i-1,1}] \times \cdots \times [-\upsilon_{i-1,l_{i-1}}, \upsilon_{i-1,l_{i-1}}]$, where ρ_i and $\upsilon_{i-1,j} = \hat{\zeta}_{i-1,j}(0) + ([\rho_{i-1}]/[\eta_{i-1}])$ are known positive constants.
2) All signals of the closed-loop systems are uniformly bounded.
3) The output y converges to a neighborhood around zero, which can be made arbitrarily small by appropriately selecting the design parameters.

Proof: The proof consists of two parts. In *Part I*, we use a contradiction method to establish that $\tau_f = +\infty$. Following this, *Part II* is dedicated to demonstrating the achievement of the control objective.

Part I: In this part, we employ a contradiction strategy to substantiate the claim that $\tau_f = +\infty$. To this end, we initially assume the contrary, positing that τ_f is finite, i.e., $\tau_f < +\infty$.

Consider the candidate Lyapunov function given by

$$V_i = \frac{1}{2}z_i^2, i = 1, \ldots, n. \tag{26}$$

Step 1: By combining (4), (6), (21), and (26), we derive the time derivative of V_1 as follows:

$$\dot{V}_1 = z_1\left(\varphi_1(\breve{\psi}_1) + z_2 + \alpha_1\right), t \in [0, \tau_f). \tag{27}$$

Let

$$H_1(X_1) = \varphi_1\left(\breve{\psi}_1\right) \tag{28}$$

with $X_1 = z_1$. Furthermore, note that $|z_1| < \rho_1 \quad \forall t \in [0, t_f)$, as per Proposition 2. Then, we can define the NN approximation set as

$$\Omega_{\mathrm{NN}_1} = [-\rho_1, \rho_1].$$

By Lemma 1, the continuous function $H_1(X_1)$ within the NN approximation set Ω_{NN_1} can be approximated by RBFNN, that is

$$H_1(X_1) = \zeta_1^T\Phi_1(X_1) + \epsilon_1(X_1), X_1 \in \Omega_{\mathrm{NN}_1} \tag{29}$$

where $\epsilon_1(X_1)$ represents the approximation error, which meets $|\epsilon_1(X_1)| \le \varepsilon_1$ with an unknown positive constant ε_1.

Incorporating (7), (28), and (29) into (27) yields

$$\begin{aligned}\dot{V}_1 = &-c_1z_1^2 + \tilde{\zeta}_1^T\Phi_1(X_1)z_1 + \epsilon_1(X_1)z_1\\ &+ z_1z_2 - \frac{k_1z_1^2}{\rho_1^2 - z_1^2} \quad \forall t \in [0, \tau_f).\end{aligned} \tag{30}$$

By leveraging the facts that $|\tilde{\zeta}_1| \le \omega_1$ and $|z_2| < \rho_2$ for all $t \in [0, \tau_f)$, as established in Propositions 1 and 2, respectively, along with facts that $0 < \Phi_1(X_1)^T\Phi_1(X_1) = \|\Phi_1(X_1)\|^2 \le 1$ and $|\epsilon_1| \le \varepsilon_1$, we can apply Young's inequality to deduce that

$$\tilde{\zeta}_1^T\Phi_1(X_1)z_1 \le \frac{c_1z_1^2}{4} + \frac{\omega_1^2}{c_1} \tag{31}$$

$$\begin{aligned}\epsilon_1(X_1)z_1 &\le \varepsilon_1|z_1|\\ &\le \frac{c_1z_1^2}{4} + \frac{\varepsilon_1^2}{c_1}\end{aligned} \tag{32}$$

$$z_1z_2 \le \frac{c_1z_1^2}{4} + \frac{\rho_2^2}{c_1}. \tag{33}$$

Employing (31)–(33), it follows that a deduction can be made as follows:

$$\begin{aligned}\dot{V}_1 &\le -\frac{c_1z_1^2}{4} - \frac{k_1z_1^2}{\rho_1^2 - z_1^2} + \frac{\omega_1^2 + \varepsilon_1^2 + \rho_2^2}{c_1}\\ &\le -\frac{k_1z_1^2}{\rho_1^2 - z_1^2} + \chi_1 \quad \forall t \in [0, \tau_f)\end{aligned} \tag{34}$$

where $\chi_1 = [(\omega_1^2 + \varepsilon_1^2 + \rho_2^2)/c_1]$.

From (34), it follows that there exists a constant $\underline{z}_1$, satisfying $0 < \underline{z}_1 < \rho_1$, such that when $\underline{z}_1 \le |z_1| < \rho_1$, $\dot{V}_1 \le 0$. Furthermore, based on (34), it can also be deduced that $\underline{z}_1 \le \lambda_1\rho_1$ with $\lambda_1 = \sqrt{[\chi_1/k_1 + \chi_1]}$. Thus, for all $t \in [0, \tau_f)$, we can conclude that

$$\begin{aligned}|z_1| &\le \max\left\{|z_1(0)|, \underline{z}_1\right\}\\ &\le \max\left\{|z_1(0)|, \rho_1\sqrt{\frac{\chi_1}{k_1 + \chi_1}}\right\} = \bar{\rho}_1 < \rho_1.\end{aligned} \tag{35}$$

Step i ($i = 2, \ldots, n-1$): Invoking (4), (6), (22), and (26), the time derivative of V_i is derived as

$$\dot{V}_i = z_i\left(\varphi_i(\underline{\breve{\psi}}_i) - \dot{\alpha}_{i-1} + z_{i+1} + \alpha_i\right) \quad \forall t \in [0, \tau_f). \tag{36}$$

By invoking (22) and (24) and signal substitution techniques, $\dot{\alpha}_{i-1}$ can be rewritten as

$$\begin{aligned}\dot{\alpha}_{i-1} &= \sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial z_j}\dot{z}_j + \frac{\partial\alpha_{i-1}}{\partial\hat{\zeta}_j}\dot{\hat{\zeta}}_j\right)\\ &:= \dot{\breve{\alpha}}_{i-1}(X_i).\end{aligned} \tag{37}$$

Let

$$H_i(X_i) = \varphi_i\left(\underline{\breve{\psi}}_i\right) - \dot{\breve{\alpha}}_{i-1}(X_i). \tag{38}$$

From (38), it follows that the NN input as $X_i = [z_1, \ldots, z_i, \hat{\zeta}_1, \ldots, \hat{\zeta}_{i-1}]^T = [z_1, \ldots, z_i, \hat{\zeta}_{1,1}, \ldots, \hat{\zeta}_{1,l_1}, \hat{\zeta}_{2,1}, \ldots, \hat{\zeta}_{2,l_2}, \ldots, \hat{\zeta}_{i-1,1}, \ldots, \hat{\zeta}_{i-1,l_{i-1}}]^T$. By utilizing the facts that $|z_j| < \rho_j$, $j = 1, \ldots, i$ for all $[0, t_f)$, as stated in Proposition 2, and the boundedness of $\hat{\zeta}_1, \ldots, \hat{\zeta}_{i-1}$ as affirmed by Proposition 1, i.e., $|\hat{\zeta}_{i-1}| = \sqrt{\sum_{j=1}^{l_{i-1}}|\hat{\zeta}_{i-1,j}|^2}$, where $|\hat{\zeta}_{i-1,j}| \le \upsilon_{i-1,j}$ and $\upsilon_{i-1,j} = \hat{\zeta}_{i-1,j}(0) + ([\rho_{i-1}]/[\eta_{i-1}])$ are known positive constants, $i = 2, \ldots, n-1$. Subsequently, we define the NN approximation set as

$$\begin{aligned}\Omega_{\mathrm{NN}_i} = &[-\rho_1, \rho_1] \times \cdots \times [-\rho_i, \rho_i]\\ &\times\left[-\upsilon_{1,1}, \upsilon_{1,1}\right] \times \cdots \times \left[-\upsilon_{1,l_1}, \upsilon_{1,l_1}\right]\\ &\times\left[-\upsilon_{i-1,1}, \upsilon_{i-1,1}\right] \times \cdots \times \left[-\upsilon_{i-1,l_{i-1}}, \upsilon_{i-1,l_{i-1}}\right].\end{aligned}$$

Using Lemma 1, the continuous functions $H_i(X_i)$ can be approximated by an RBFNN within the NN approximation set Ω_{NN_i}, as follows:

$$H_i(X_i) = \zeta_i^T\Phi_i(X_i) + \epsilon_i(X_i), X_i \in \Omega_{\mathrm{NN}_i} \tag{39}$$

where $\epsilon_i(X_i)$ represents the approximation error, which meets $|\epsilon_i(X_i)| \le \varepsilon_i$ with an unknown positive constant ε_i.

Inserting (9), (38), and (39) into (36), leads to

$$\begin{aligned}\dot{V}_i = &-c_iz_i^2 + \tilde{\zeta}_i^T\Phi_i(X_i)z_i + \epsilon_i(X_i)z_i\\ &+ z_iz_{i+1} - \frac{k_iz_i^2}{\rho_i^2 - z_i^2} \quad \forall t \in [0, \tau_f).\end{aligned} \tag{40}$$

By applying Young's inequality and the established facts that $|\tilde{\zeta}_i| \le \omega_i$ and $|z_{i+1}| < \rho_{i+1}$ as stated in Propositions 1 and 2, respectively, along with $0 < \Phi_i(X_i)^T\Phi_i(X_i) = \|\Phi_i(X_i)\|^2 \le 1$ and $|\epsilon_i| \le \varepsilon_i$, it follows that:

$$\tilde{\zeta}_i^T\Phi_i(X_i)z_i \le \frac{c_iz_i^2}{4} + \frac{\omega_i^2}{c_i} \tag{41}$$

$$\begin{aligned}\epsilon_i(X_i)z_i &\le \varepsilon_i|z_i|\\ &\le \frac{c_iz_i^2}{4} + \frac{\varepsilon_i^2}{c_i}\end{aligned} \tag{42}$$

$$z_iz_{i+1} \le \frac{c_iz_i^2}{4} + \frac{\rho_{i+1}^2}{c_i}. \tag{43}$$

In view of (41)–(43), it follows that:

$$\dot{V}_i \le -\frac{c_i z_i^2}{4} - \frac{k_i z_i^2}{\rho_i^2 - z_i^2} + \frac{\omega_i^2 + \varepsilon_i^2 + \rho_{i+1}^2}{c_i}$$
$$\le -\frac{k_i z_i^2}{\rho_i^2 - z_i^2} + \chi_i \quad \forall t \in [0, \tau_f) \tag{44}$$

where $\chi_i = ([\omega_i^2 + \varepsilon_i^2 + \rho_{i+1}^2]/c_i)$.

From (44), it can be deduced that there exists a constant $\underline{z}_i$, satisfying $0 < \underline{z}_i < \rho_i$ such that when $\underline{z}_i \le |z_i| < \rho_i$, $\dot{V}_i \le 0$. Furthermore, in view of (44), it can also be deduced that $\underline{z}_i \le \lambda_i \rho_i$ with $\lambda_i = \sqrt{(\chi_i/[k_i + \chi_i])}$, which implies that for all $t \in [0, \tau_f)$

$$|z_i| \le \max\left\{|z_i(0)|, \underline{z}_i\right\}$$
$$\le \max\left\{|z_i(0)|, \rho_i\sqrt{\frac{\chi_i}{k_i + \chi_i}}\right\} = \bar{\rho}_i < \rho_i. \tag{45}$$

Step n: Along with (4), (6), (23), and (26), the time derivative of V_n is derived as

$$\dot{V}_n = z_n\left(\varphi_n(\underline{\breve{\psi}}_n) - \dot{\alpha}_{n-1} + u\right) \quad \forall t \in [0, \tau_f). \tag{46}$$

By using (23) and (24), along with signal substitution techniques, $\dot{\alpha}_{n-1}$ becomes as

$$\dot{\alpha}_{n-1} = \sum_{j=1}^{n-1}\left(\frac{\partial \alpha_{n-1}}{\partial z_j}\dot{z}_j + \frac{\partial \alpha_{n-1}}{\partial \hat{\zeta}_j}\dot{\hat{\zeta}}_j\right)$$
$$:= \dot{\bar{\alpha}}_{n-1}(X_n). \tag{47}$$

Let

$$H_n(X_n) = \varphi_n\left(\underline{\breve{\psi}}_n\right) - \dot{\bar{\alpha}}_{n-1}(X_n). \tag{48}$$

From (48), it can be noted that the NN input as $X_n = [z_1, \ldots, z_n, \hat{\zeta}_1, \ldots, \hat{\zeta}_{n-1}]^T = [z_1, \ldots, z_n, \hat{\zeta}_{1,1}, \ldots, \hat{\zeta}_{1,l_1}, \hat{\zeta}_{2,1}, \ldots, \hat{\zeta}_{2,l_2}, \ldots, \hat{\zeta}_{n-1,1}, \ldots, \hat{\zeta}_{n-1,l_{n-1}}]^T$. By utilizing the facts that, for all $t \in [0, \tau_f)$, $|z_j| < \rho_j$, $j = 1, \ldots, n$, as stated in Proposition 2, and the boundedness of $\hat{\zeta}_1, \ldots, \hat{\zeta}_{n-1}$ as confirmed by Proposition 1, i.e., $|\hat{\zeta}_{n-1}| = \sqrt{\sum_{j=1}^{l_{n-1}} |\hat{\zeta}_{n-1,j}|^2}$, where $|\hat{\zeta}_{n-1,j}| \le \upsilon_{n-1,j}$ and $\upsilon_{n-1,j} = \hat{\zeta}_{n-1,j}(0) + ([\rho_{n-1}]/[\eta_{n-1}])$ are known positive constants. Then, the NN approximation set can be defined as

$$\Omega_{\mathrm{NN}_n} = [-\rho_1, \rho_1] \times \cdots \times [-\rho_n, \rho_n]$$
$$\times\left[-\upsilon_{1,1}, \upsilon_{1,1}\right] \times \cdots \times \left[-\upsilon_{1,l_1}, \upsilon_{1,l_1}\right]$$
$$\times\left[-\upsilon_{n-1,1}, \upsilon_{n-1,1}\right] \times \cdots \times \left[-\upsilon_{n-1,l_{n-1}}, \upsilon_{n-1,l_{n-1}}\right].$$

By Lemma 1, the continuous function $H_n(X_n)$ within the NN approximation set Ω_{NN_n} can be approximated by RBFNN, that is

$$H_n(X_n) = \zeta_n^T \Phi_n(X_n) + \epsilon_n(X_n), X_n \in \Omega_{\mathrm{NN}_n} \tag{49}$$

where $\epsilon_n(X_n)$ represents the approximation error, which meets $|\epsilon_n(X_n)| \le \varepsilon_n$ with an unknown positive constant ε_n.

Combining (11), (48), and (49), we have

$$\dot{V}_n = -c_n z_n^2 + \tilde{\zeta}_n^T \Phi_n(X_n) z_n + \epsilon_n(X_n) z_n$$
$$- \frac{k_n z_n^2}{\rho_n^2 - z_n^2} \quad \forall t \in [0, \tau_f). \tag{50}$$

Similar to (41) and (42), the following inequalities hold:

$$\tilde{\zeta}_n^T \Phi_n(X_n) z_n \le \frac{c_n z_n^2}{4} + \frac{\omega_n^2}{c_n} \tag{51}$$
$$\epsilon_n(X_n) z_n \le \frac{c_n z_n^2}{4} + \frac{\varepsilon_n^2}{c_n}. \tag{52}$$

Integrating (51) with (52), one arrives at

$$\dot{V}_n \le -\frac{c_n z_n^2}{2} - \frac{k_n z_n^2}{\rho_n^2 - z_n^2} + \frac{\omega_n^2 + \varepsilon_n^2}{c_n}$$
$$\le -\frac{k_n z_n^2}{\rho_n^2 - z_n^2} + \chi_n \quad \forall t \in [0, \tau_f) \tag{53}$$

where $\chi_n = [(\omega_n^2 + \varepsilon_n^2)/c_n]$.

Based on (53), it can be deduced that there exists a constant $\underline{z}_n$, satisfying $0 < \underline{z}_n < \rho_n$, such that when $\underline{z}_n \le |z_n| < \rho_n$, $\dot{V}_n \le 0$. Moreover, it follows from (53) that $\underline{z}_n \le \lambda_n \rho_n$, where$\lambda_n = \sqrt{[\chi_n/(k_n + \chi_n)]}$. Therefore, for all $t \in [0, \tau_f)$, we can conclude that

$$|z_n| \le \max\left\{|z_n(0)|, \underline{z}_n\right\}$$
$$\le \max\left\{|z_n(0)|, \rho_n\sqrt{\frac{\chi_n}{k_n + \chi_n}}\right\} = \bar{\rho}_n < \rho_n. \tag{54}$$

From (35), (45), and (54) and Proposition 1, it is deduced that $z_1, \ldots, z_n$, $||\hat{\zeta}_1||, \ldots, ||\hat{\zeta}_n||$ and u are bounded for all $t \in [0, \tau_f)$.

In summary, a compact subset $\bar{\Omega} \subset \Omega$ exists such that the maximal solution of (25) satisfies $\phi(t) \in \bar{\Omega}$, ensuring that the unique solution of the system (25) satisfies $\phi(t) \in \bar{\Omega}$ on the interval $[0, \tau_f)$. By employing Lemma 2, this scenario leads us, via a contradiction, to the conclusive assertion that $\tau_f = +\infty$.

Part II: By systematically applying the analytical methodologies delineated from step 1 to step n in Part I, we progressively establish that $\psi_1, \ldots, \psi_n$, $||\hat{\zeta}_1||, \ldots, ||\hat{\zeta}_n||$, $\alpha_1, \ldots, \alpha_{n-1}$ and u are bounded for all $t \in [0, +\infty)$. Moreover, by utilizing (34) and the inequality $-(k_1 z_1^2/[\rho_1^2 - z_1^2]) \le -(k_1 z_1^2/\rho_1^2)$, it is obtained that

$$\dot{V}_1 \le -\beta_1 V_1 + \chi_1 \tag{55}$$

where $\beta_1 = (k_1/\rho_1^2)$.

Owing to $\dot{V}_1 < 0$ for all $V_1 \ge (\chi_1/\beta_1)$, (55) leads to

$$0 \le V_1 \le \frac{\chi_1}{\beta_1} + \left(V(0) - \frac{\chi_1}{\beta_1}\right)\exp(-\beta_1 t) \tag{56}$$

which indicates that by increasing the value of β_1, the output y can be made to converge to an arbitrarily small neighborhood around zero. ■

Remark 3: In this article, based on Proposition 1, and given initial conditions $\hat{\zeta}_{i,j}(0)$, and the design constants ρ_i, $i = 1, \ldots, n$, the constants $\upsilon_{i-1,l_{i-1}}$, $i = 2, \ldots, n$, can be precisely calculated. Consequently, the NN approximation set $\Omega_{\mathrm{NN}_i} = [-\rho_1, \rho_1] \times \cdots \times [-\rho_i, \rho_i] \times [-\upsilon_{1,1}, \upsilon_{1,1}] \times \cdots \times [-\upsilon_{1,l_1}, \upsilon_{1,l_1}] \times [-\upsilon_{i-1,1}, \upsilon_{i-1,1}] \times \cdots \times [-\upsilon_{i-1,l_{i-1}}, \upsilon_{i-1,l_{i-1}}]$ can be clearly determined. The key benefit of this approach is that neuron nodes can be selected from the approximation set Ω_{NN_i} to more accurately approximate unknown nonlinearities, significantly improving the precision of the NN approximation. This represents a fundamental distinction between the method

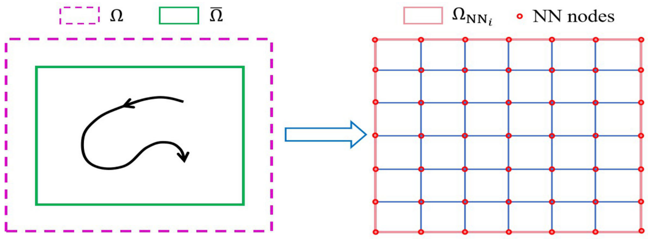

Fig. 2. Relationship between the open set Ω, the NN approximation set Ω_{NN_i}, and the compact subset $\bar{\Omega}$.

proposed in this article and existing adaptive NN control techniques [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30].

Remark 4: It is well known that in implementing an adaptive NN controller, the effectiveness of approximating unknown nonlinearities depends on ensuring that the NN input signals remain within a predefined approximation set [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14]. However, this crucial requirement is often neglected. In this article, we introduce a novel method to determine the NN approximation sets in advance. This method integrates the signal substitution technique, BF, and the backstepping approach. First, the states of the original system are transformed into the state error variables using the signal substitution technique. Next, the open set Ω is defined using BF to constrain the state error variables. Then, at each step of the backstepping design, the NN approximation set Ω_{NN_i} is constructed to ensure the unknown system nonlinearities are accurately approximated within this set. The relationship between the open set Ω, the compact subset $\bar{\Omega}$, and the NN approximation set Ω_{NN_i} is illustrated in Fig. 2.

Remark 5: Compared to existing adaptive NN control schemes with known approximation sets [31], [32], which directly constrain the system states and are therefore restricted to systems with a relative degree of one or two, the design method proposed in this article offers greater flexibility. Our constructive approach is applicable to uncertain nonlinear systems with arbitrarily high-relative degrees, overcoming the limitations of traditional schemes and expanding the applicability to more complex systems.

V. Simulation Study

In this section, we present simulations to validate the effectiveness and robustness of the proposed control algorithm. Specifically, while keeping the control parameters unchanged, we conduct experimental evaluations on two strict-feedback systems with distinct nonlinearities in Example 1 and an actual physical model in Example 2.

Example 1: Two strict-feedback uncertain systems with different nonlinearities are considered as follows:

$$\Sigma_1\colon \begin{cases} \dot{\psi}_1 = \varphi_{1,\Sigma_1}\big(\bar{\psi}_1\big) + \psi_2 \\ \dot{\psi}_2 = \varphi_{2,\Sigma_1}\big(\bar{\psi}_2\big) + u \\ y = \psi_1 \end{cases} \tag{57}$$

$$\Sigma_2\colon \begin{cases} \dot{\psi}_1 = \varphi_{1,\Sigma_2}\big(\bar{\psi}_1\big) + \psi_2 \\ \dot{\psi}_2 = \varphi_{2,\Sigma_2}\big(\bar{\psi}_2\big) + u \\ y = \psi_1 \end{cases} \tag{58}$$

where $\varphi_{1,\Sigma_1}(\bar{\psi}_1) = 3\psi_1^2 + 4\psi_1^3 + 5\psi_1^4 + 2\psi_1^5 + \sin(\psi_1^2)$, $\varphi_{2,\Sigma_1}(\bar{\psi}_2) = \psi_1^2\psi_2^3 + \psi_2^2 + \psi_1^3\psi_2^4 + \sin(\psi_1^3\psi_2^5)\cos(0.5\psi_1\psi_2)$, $\varphi_{1,\Sigma_2}(\bar{\psi}_1) = \psi_1^3 + 3\psi_1^4 + \psi_1^5 + 2\psi_1^7$, and $\varphi_{2,\Sigma_2}(\bar{\psi}_2) = \psi_1^3 + \psi_2^2\sin(\psi_1 + \psi_2) + \psi_1^3\psi_2^5 + \psi_1^4\psi_2^6\sin(\psi_2)\cos(\psi_1\psi_2) + \psi_1^6\cos(\psi_1 + \psi_2)$. The initial condition for the systems (57) and (58) are $[\psi_1(0), \psi_2(0)]^T = [0.1, -0.3]^T$. The adaptive NN controller, along with the adaptive estimators, are implemented in (8)–(13) as described below:

Step 1: The virtual control law α_1 and the adaptive weight estimator $\hat{\zeta}_1$ are constructed as

$$\alpha_1 = -c_1 z_1 - \hat{\zeta}_1^T \Phi_1(X_1) - \frac{k_1 z_1}{\rho_1^2 - z_1^2}$$
$$\dot{\hat{\zeta}}_1 = \Gamma_1\Big(\Phi_1(X_1)z_1 - \eta_1\hat{\zeta}_1\Big)$$

where $X_1 = z_1$.

In the first step, the following design parameters are selected as $c_1 = 5$, $k_1 = 5$, $\rho_1 = 5 > |z_1(0)| = 0.5$, $\Gamma_1 = \mathrm{diag}(3, \ldots, 3)$, $\eta_1 = 3$, and $l_1 = 3$. The initial estimates are given by $\hat{\zeta}_1(0) = [\hat{\zeta}_{1,1}(0), \ldots, \hat{\zeta}_{1,l_1}(0)]^T$, where $\hat{\zeta}_{1,j}(0) = 0$ for $j = 1, 2, \ldots, l_1$. With these parameters, the approximation set for the first step is precisely determined as $\Omega_{\mathrm{NN}_1} = [-\rho_1, \rho_1] = [-5,\ 5]$. The parameter δ_1 represents the center of the RBF vector function, which is uniformly distributed across Ω_{NN_1}. The width of the Gaussian function is selected as $\sigma_1 = 1$. In this approximation set Ω_{NN_1}, the unknown nonlinearity $H_1(X_1) = \varphi_1(\bar{\psi}_1)$ in the system is approximated by the first RBF NN.

Step 2: The actual control law u and the adaptive weight estimator $\hat{\zeta}_2$ are constructed as

$$u = -c_2 z_2 - \hat{\zeta}_2^T \Phi_2(X_2) - \frac{k_2 z_2}{\rho_2^2 - z_2^2}$$
$$\dot{\hat{\zeta}}_2 = \Gamma_2\Big(\Phi_2(X_2)z_2 - \eta_2\hat{\zeta}_2\Big)$$

where $X_2 = [z_1, z_2, \hat{\zeta}_1]^T = [z_1, z_2, \hat{\zeta}_{1,1}, \hat{\zeta}_{1,2}, \hat{\zeta}_{1,3}]^T$.

In the second step, the state errors z_1 and z_2, along with $\hat{\zeta}_1$, are used as the NN input signals. According to Proposition 2, the conditions $|z_j| < \rho_j$, for $j = 1, 2$ hold. Additionally, from Proposition 1, we know that $\hat{\zeta}_{1,j} \le \upsilon_{1,j}$ for $j = 1, 2, 3$, where $\upsilon_{1,j}$ is given by $\upsilon_{1,j} = \hat{\zeta}_{1,j}(0) + (\rho_1/\eta_1)$. In the first step, the design parameters $\rho_1 = 5$ and $\eta_1 = 3$ were selected, with the initial values of the adaptive weight estimators set as $\hat{\zeta}_1(0) = [0, 0, 0]^T$. Using Proposition 1, we can calculate that $\hat{\zeta}_{1,j} \le \upsilon_{1,j} = (5/3)$ for $j = 1, 2, 3$.

For the second step, the design parameters are selected as $c_2 = 5$, $k_2 = 5$, $\rho_2 = 5 > |z_2(0)| = 1.10$, $\eta_2 = 2$, and $\Gamma_2 = \mathrm{diag}(3, \ldots, 3)$, with $l_1 = 3$. The initial values of the adaptive weight estimators are set as $\hat{\zeta}_2(0) = [\hat{\zeta}_{2,1}(0), \ldots, \hat{\zeta}_{2,l_2}(0)]^T$, where $\hat{\zeta}_{2,j}(0) = 0$ for $j = 1, 2, \ldots, l_2$, $l_2 = 27$. Accordingly, the approximation set for the second step is determined as $\Omega_{\mathrm{NN}_2} = [-\rho_1, \rho_1] \times [-\rho_2, \rho_2] \times [-\upsilon_{1,1}, \upsilon_{1,1}] \times [-\upsilon_{1,2}, \upsilon_{1,2}] \times [-\upsilon_{1,3}, \upsilon_{1,3}] = [-5, 5] \times [-5, 5] \times [-(5/3), (5/3)] \times [-(5/3), (5/3)] \times [-(5/3), (5/3)]$. The RBF vector centers δ_2, are uniformly distributed over

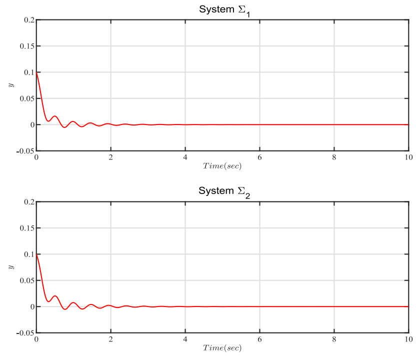

Fig. 3. Temporal evolution of the output y.

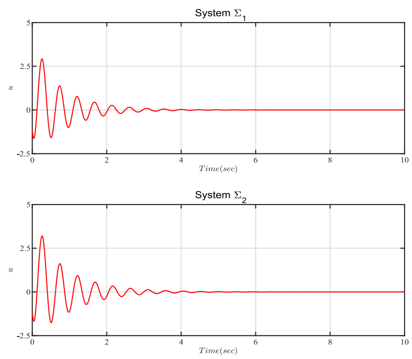

Fig. 4. Temporal evolution of the control input u.

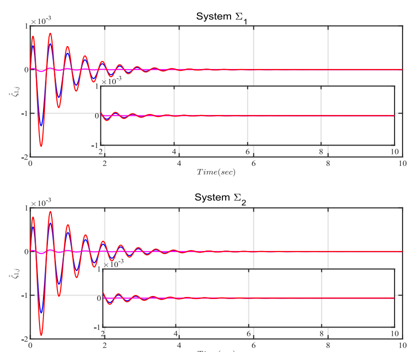

Fig. 5. Temporal evolution of the NN weight estimators $\hat{\zeta}_{1,j}$, $j = 1, 2, 3$.

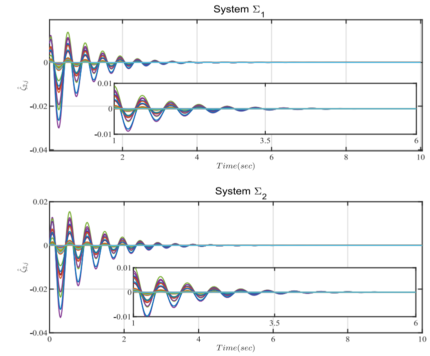

Fig. 6. Temporal evolution of the NN weight estimators $\hat{\zeta}_{2,j}$, $j = 1, \ldots, 27$.

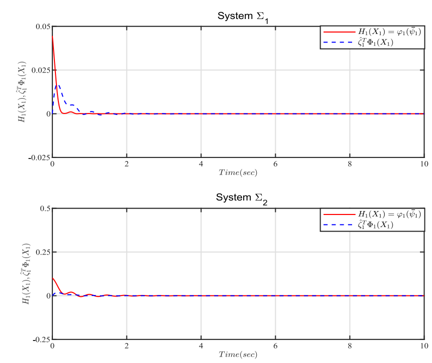

Fig. 7. Temporal evolution of the system unknown nonlinearities $H_1(X_1)$ and its approximation $\hat{\zeta}_1^T \Phi_1(X_1)$.

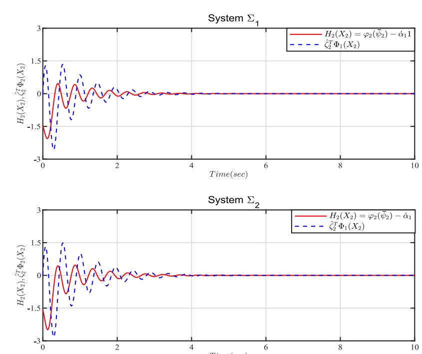

Fig. 8. Temporal evolution of the system unknown nonlinearities $H_2(X_2)$ and its approximation $\hat{\zeta}_2^T \Phi_2(X_2)$.

Ω_{NN_2}, with the Gaussian function width selected as $\sigma_2 = 2$. In this approximation set Ω_{NN_2}, the unknown nonlinearity $H_2(X_2) = \varphi_2(\bar{\psi}_2) - \dot{\alpha}_1$ is approximated by the second RBF NN.

The simulation results are provided in Figs. 3–8. In Fig. 3, the output y is shown to converge to a neighborhood around zero. Fig. 4 presents the control input u. The adaptive NN weight estimators $\hat{\zeta}_{1,j}$ for $j = 1, 2, 3$ and $\hat{\zeta}_{2,j}$ for $j = 1, \ldots, 27$ are shown in Figs. 5 and 6, respectively. Figs. 7 and 8 illustrate the approximation of the unknown system nonlinearities using the RBF NN. As observed in Figs. 7 and 8, the learned functions $\hat{\zeta}_1^T \Phi_1(X_1)$ and $\hat{\zeta}_2^T \Phi_2(X_2)$ accurately approximate the system's unknown nonlinearities, $H_1(X_1) = \varphi_1(\bar{\psi}_1)$ and $H_2(X_2) = \varphi_2(\bar{\psi}_2) - \dot{\alpha}_1$, respectively. The simulation results indicate the following.

1) The NN approximation sets $\Omega_{\text{NN}_i} = [-\rho_1, \rho_1] \times \cdots \times [-\rho_i, \rho_i] \times [-\upsilon_{1,1}, \upsilon_{1,1}] \times \cdots \times [-\upsilon_{1,l_1}, \upsilon_{1,l_1}] \times [-\upsilon_{i-1,1}, \upsilon_{i-1,1}] \times \cdots \times [-\upsilon_{i-1,l_{i-1}}, \upsilon_{i-1,l_{i-1}}]$ can be determined in advance.
2) The output y converges to a neighborhood of zero.
3) All signals in the closed-loop system remain uniformly bounded.

To further demonstrate the advantages of the proposed scheme in terms of approximation accuracy, a comparative simulation was conducted. Three controllers were applied to the system (57): 1) the proposed BF-based adaptive NN control (BF-ANNC) scheme from this article; 2) the BLF-based adaptive NN control (BLF-ANNC) scheme from [28]; and 3) the CANNC approach from [14]. The simulation results in Figs. 9 and 10 demonstrate that the proposed BF-ANNC method more accurately approximates the unknown system nonlinearities and achieves faster convergence of

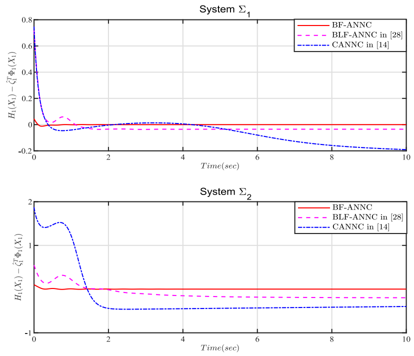

Fig. 9. Temporal evolution of the approximation error $H_1(X_1) - \hat{\zeta}_1^T \Phi_1(X_1)$ of the system nonlinearities under the proposed BF-ANNC method, the BLF-ANNC method in [28], and the CANNC method in [14].

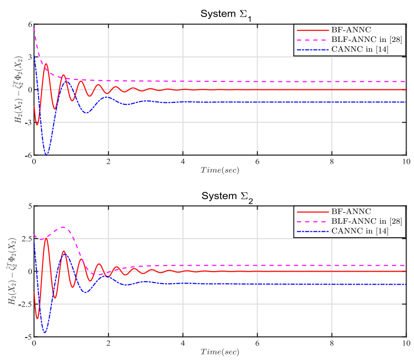

Fig. 10. Temporal evolution of the approximation error $H_2(X_2) - \hat{\zeta}_2^T \Phi_2(X_2)$ of the system nonlinearities under the proposed BF-ANNC method, the BLF-ANNC method in [28], and the CANNC method in [14].

the approximation errors compared to the BLF-ANNC and CANNC methods.

Example 2: To demonstrate the practical application of the proposed adaptive NN control scheme, we consider an inverted pendulum system, whose dynamics are described as [39]

$$\begin{aligned} \dot{\psi}_1 &= \psi_2 \\ \dot{\psi}_2 &= \mathcal{F} + \mathcal{G}u \\ y &= \psi_1 \end{aligned} \tag{59}$$

where $\mathcal{F} = [g\sin(\psi_1) - [\mathcal{M}_p \mathcal{L} \psi_2^2 \cos(\psi_1)\sin(\psi_1)/\mathcal{M}_c + \mathcal{M}_p]/ \quad \mathcal{L}([4/3] - (\mathcal{M}_p \cos^2(\psi_1)/\mathcal{M}_c + \mathcal{M}_p))]$, $\mathcal{G} = [(\cos(\psi_1)/ \quad \mathcal{M}_c + \mathcal{M}_p)/\mathcal{L}((4/3) - [\mathcal{M}_p \cos^2(\psi_1)/ \mathcal{M}_c + \mathcal{M}_p])]$. ψ_1 represents the angle of the velocity of the pendulum, ψ_2 denotes its angular velocity. $\mathcal{M}_c$ is the mass of a cart, $g = 9.8[m/s^2]$ represents the gravitational acceleration, $\mathcal{L}$ is the half length of a pole, and $\mathcal{M}_p$ is the mass of the a pole. u represents the demanded force.

The simulation is conducted with system parameters set to $\mathcal{M}_c = 1[kg]$, $\mathcal{M}_p = 0.1[kg]$ and $\mathcal{L} = 0.5[m]$. The initial conditions of the system, as described in (59), are $[\psi_1(0), \psi_2(0)]^T = [0.1, 0]^T$. To demonstrate the universal applicability and effectiveness of the proposed BF-ANNC scheme, the controller from Example 1 is directly applied to the system (57) without modifying the design parameters, i.e., $c_1 = c_2 = 5$, $k_1 = k_2 = 5$, $\rho_1 = \rho_2 = 5$, $\Gamma_1 = \Gamma_2 = \text{diag}(3, \ldots, 3)$, $\eta_1 = 3$ and $\eta_2 = 2$. The simulation results

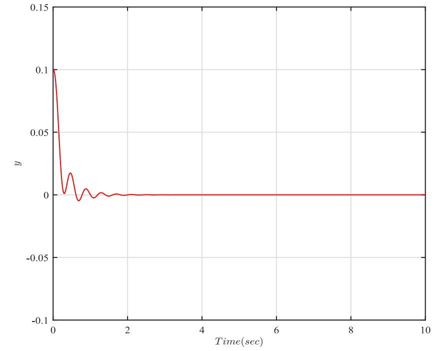

Fig. 11. Temporal evolution of the output y.

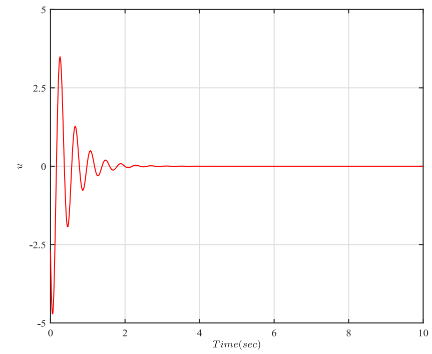

Fig. 12. Temporal evolution of the control input u.

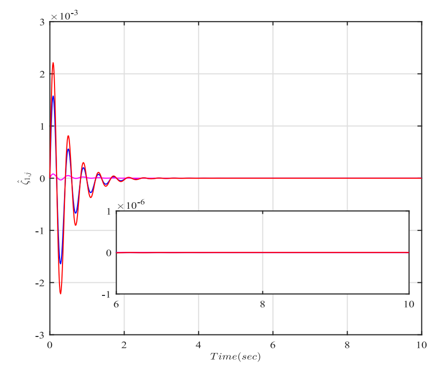

Fig. 13. Temporal evolution of the NN weight estimators $\hat{\zeta}_{1,j}$, $j = 1, 2, 3$.

are presented in Figs. 11–15. Specifically, Fig. 11 illustrates the output response y, and Fig. 12 shows the corresponding control input u. Fig. 13 depicts the response of the adaptive NN weight estimators $\hat{\zeta}_{1,j}$, $j = 1, 2, 3$. Fig. 14 presents the response of the adaptive NN weight estimators $\hat{\zeta}_{2,j}$, $j = 1, \ldots, 27$. Finally, the unknown nonlinearities and the learned functions are shown in Fig. 15.

VI. Conclusion

This article addresses the problem of predetermining the NN approximation set in the adaptive control of strict feedback uncertain systems. By integrating signal substitution techniques, BF, and backstepping, we proposed an innovative constructive scheme that enables the NN approximation set to be determined in advance at each step of the recursive design process. Simulation results demonstrated the effectiveness and practicality of the developed adaptive NN control approach.

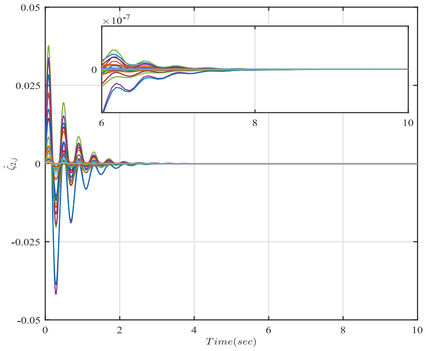

Fig. 14. Temporal evolution of the NN weight estimators $\hat{\zeta}_{2,j}, j = 1, \ldots, 27$.

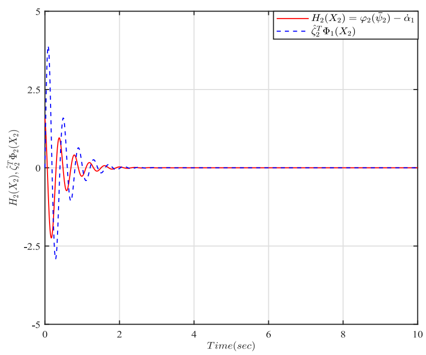

Fig. 15. Temporal evolution of the system unknown nonlinearities $H_2(X_2)$ and its approximation $\hat{\zeta}_2^T \Phi_2(X_2)$.

Several important issues remain open for further exploration. From an implementation and computational perspective, the proposed controller in (7)–(12) requires repeated differentiation of virtual control variables, leading to a significant increase in complexity as the system order grows. To address this "explosion of complexity," one potential solution is the application of dynamic surface control techniques, which will be investigated in future research. Another promising direction is the extension of the proposed control algorithm to more general nonlinear systems, such as pure-feedback or high-order nonlinear systems. Practical applications of this approach could include systems like magnetic levitation ball systems or DC-DC buck converters [40].

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