* Show all of your work for full credit and **BOX YOUR ANSWERS**. Write symbolic forms of all equations used in your solution.
* When asked to use MATLAB for simulation, submit the commented and concise code used to create the plots and simulation. If you don't include this, you will not receive full credit.
* You must program and write up your own assignment, but you can and are encouraged to discuss the approaches to the problems with your classmates. If you discuss the problems with your classmates, you must put their names on the top of your assignment.

Problem 0: Optional

For those of you not familiar with MATLAB, see the CMU MATLAB page:
http://www-2.cs.cmu.edu/afs/cs/local/matlab/common/www/index.html
You should be able to:

• Enter matrices
• Simple matrix operations
• Create simple M-files
• Create simple plots
• Load and save MATLAB variables to MAT-files and ASCII files.
• You should also become comfortable using toolbox functions (e.g. if you are told to use a function (say, lsim), you should be able to look up what it does using "help" and "type" and understand how to use it).

Problem 1

In this problem, you will investigate a model to account for a muscle's dynamics. The model is shown in Figure 1. The elastic elements are linear springs with force (here, we use tension, T) proportional to change of length, T_k = k(x – x_rest). The viscous element is a linear damper with tension proportional to rate of change of length, T_b = b x_dot . The "contractile component" generates a tension that depends only on time, T_o(t) .

(a) Develop a differential equation that relates the measured tension T(t) to the "contractile component" force T_o (t) and the total length X_total (t) .

(b) Assuming that the muscle operates under isometric conditions (no muscle length change since rest), how does the differential equation in (a) change?

(c) Assuming that the muscle operates under isometric conditions as in (b) and the input T_o (t) is a step input with a magnitude of 1 N, solve for this isometric muscle's step response.

(d) Assuming that the muscle operates under isometric conditions again, solve for a muscle "twitch response". That is, simulate the time-course of the measured force, T(t), in response to a pulse To (t) (with a magnitude of 1 N) that lasts 5 milliseconds. You can use the piece-wise technique, shifting property of Laplace transforms, or approximation of pulse response.

(e) Finally, simulate this twitch response using MATLAB. Scale the amplitude of T_o(t) so that the peak of the twitch is approximately 0.1 N. Choose values of b and k so that the time course of T(t) resembles that of the pulse shown in Kandel et al. Figure 34-4A (you need to simulate only one of the pulses and the plot can be an approximate of the figure); What are the values of b and k that allows similar response to Figure 34-4A? Plot the measured force vs. time.

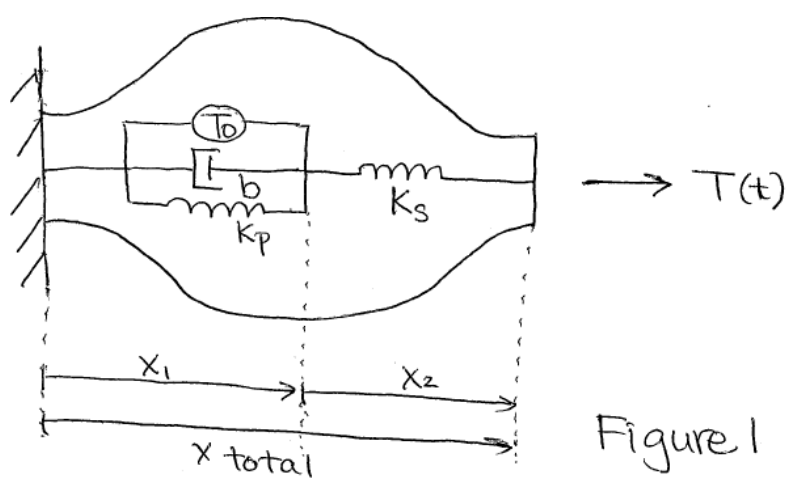

Figure 1

Problem 2

In this problem, you are to investigate another model to account for the dynamics of muscle. The second model is shown in Figure 2. It's similar to the fIrst model but a linear "fIlter" representing the excitation-contraction dynamics has been added between the muscle \alpha motomeuron input, \alpha , and the "contractile component" tension T_o(t) .

(a) Develop a differential equation that relates the "contractile component" force T_o (t) to the muscle a - motomeuron input, \alpha assuming T_o (0) = O.

(b) Again assuming the muscle operates under isometric conditions, use MATLAB to simulate a muscle "twitch response". That is, simulate the time-course of the measured tension, T(t), in response to a pulse of \alpha that lasts 5 milliseconds. Choose values of b and k so that the time-constant of the mechanical system is identical to the time constant of the linear fIlter, and choose the time-constant such that the time-course of T(t) resembles that of the pulses shown in Kandel et aI. Figure 34-4A. Scale the amplitude of \alpha so that the peak of the twitch is approximately 0.1 N. Plot the measured force vs. time. Report on the values of b, k and the time-constant.

(c) Using the same b, k, and the time-constant you found in (b), simulate the response of this muscle model to a train of pulses at 10Hz, 20Hz, and 80Hz. Plot the measured force vs. time. (hint: You can use lsim in MATLAB)

(d) Comment briefly on the merits and drawbacks of the inodel in this problem compared to the model in Problem #1.

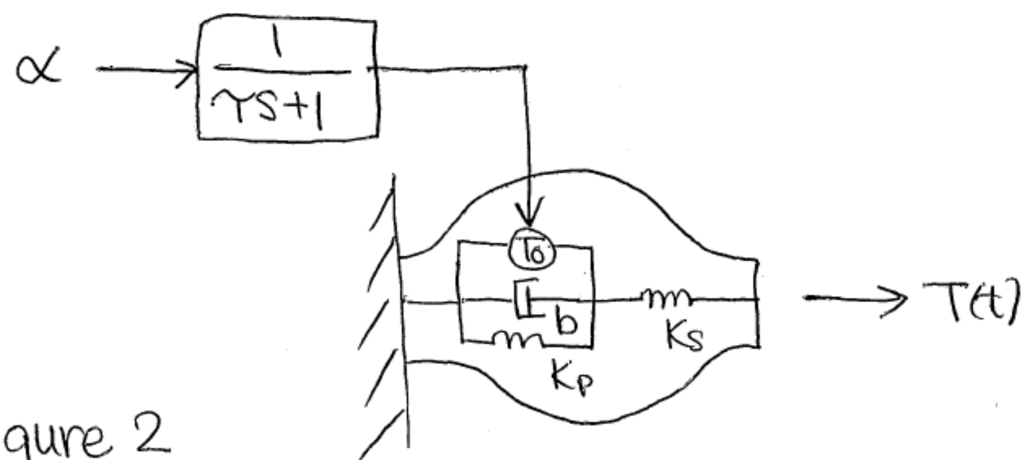

Figure 2

Problem 3

Using the muscle response you found in Problem #2 (c) for a train of pulses at 80 Hz, show that you can add or subtract pulses to change the amplitude of the tension T(t).

(a) Add one pulse at approximately 200 milliseconds into the response and plot the response in MATLAB.

(b) Subtract one pulse at approximately 200 milliseconds into the response and plot the response in MATLAB.

(c) Add one extra pulse between the first and second pulses at the beginning of the response and plot the response in MATLAB.

(d) Comment qualitatively what happens to the amplitude of the tension T(t) for (a), (b), and (c).

Problem 4

In this problem, you are to explore the steady-state relation between force, length, and activation of muscle. To keep things simple, assume that muscle tension, T, is the sum of two components, Ta + Tp , as follows.

The active component, T_a , due to the sliding filaments is a parabolic function of length which reaches a maximum value, Tmax at length l_0 and drops to zero at lengths 0.5 l_0 and 1.5 l_0 (this is a crude model of the static behavior due to the sliding filaments.). This active tension curve is also proportional to the neural activation, \alpha, which is a dimensionless parameter (0<= \alpha <= 1).

The passive component, T_p, due to the connective tissues in the muscle is an exponential function of length that is zero for lengths less than l_0 and rises exponentially to a value equal to the maximum active tension at length 1.5 l_0. That is, $\mathrm{T_p} = \frac{\mathrm{T_{max}}}{\exp(\mathrm{k_s})-1}\left(\exp\left(\mathrm{k_s}\frac{\Delta \mathrm{x_p}}{\Delta \mathrm{x_{pmax}}}\right) - 1\right)$ for l >= l_0 and zero otherwise. Here $\Delta\mathrm{x_{pmax}} = 0.5\,\mathrm{l\_0}, \mathrm{k_s} = 3, \Delta\mathrm{x_p} = \mathrm{l} - \mathrm{l_0}$.

(a) Using MATLAB, plot the force vs.length curve for this muscle model for the length range 0.5 l_0 <= l <= 1.5 l_0 and for activation values of *\alpha* = 0, 0.5, and 1.

(b) Suppose that in an experiment this muscle is used to support a weight against gravity. If the muscle is maximally activated, for what range of lengths can a weight be supported in stable, static equilibrium? (the stability of static equilibrium can be established by examining the local slope of the force-displacement relation $\frac{\delta F}{\delta l}$.

(c) Repeat (b) for a sub-maximal muscle activation corresponding to *a =0.5.*

(d) Explain the difference between (b) and (c).

(e) Qualitatively explain whether the unstable condition is realistic.