EXPERIMENTAL DESIGN EXAMPLES

Overview

In this set of notes, we will go over a few examples of designed experiments.

For each of these experiments, identify the design (e.g., randomized complete block design; nested design; design with nesting and crossing; split-plot design) and list the elements of experimental design.

You should do these as exercises and I provided brief solution keys in the second half of the notes.

Example 1

A fleet manager wishes to compare the wearability of 4 brands of tire: A, B, C, and D. Four cars are available for the experiment and 4 tires of each brand are available, for a total of 16 tires. The idea is to mount 4 tires on each of the 4 cars, ask the driver of each of the cars to drive his/her car for 20,000 miles and then to measure tread loss. We will measure tread loss in mils (.001 inches). We will designate the 4 cars as cars I, II, III, and IV.

Each car tests all four brands. Thus one tire from each brand is selected at random and randomly allocated to the 4 wheels of car I. Then one tire from each brand is selected and the four are randomly allocated to car II, and so forth. Here are the results of that design.

	Car I	Car II	Car III	Car IV
Brand assignment	**B**	**A**	**C**	**B**
	D	**D**	**B**	**A**
	C	**C**	**A**	**C**
	A	**B**	**D**	**D**

Example 2

An experiment was installed to compare the effects of 4 green manure treatments (barley, vetch, barley + vetch, and fallow) on the subsequent production of sugar beets at 2 levels of N (0, 120 lb/acre) with 3 replications. The N levels were assigned to the main plots as there was less interest in N effects than in the effects of the green manures and the interaction between N and the green manures.

Block I								Block II								Block III							
N 120				N 0				N 120				N 0				N 0				N 120			
BV	V	F	B	B	BV	F	V	F	BV	V	B	V	F	B	BV	F	BV	V	B	V	BV	B	F

Example 3

The surface finish of metal parts made on four machines is being studied. An experiment is conducted in which each machine is run by three different operators and two specimens from each operator are collected and tested. Because of the location of the machines, different operators are used on each machine, and the operators are chosen at random. The data are shown in the following table.

Oper-	Machine 1			Machine 2			Machine 3			Machine 4		
ator	1	2	3	1	2	3	1	2	3	1	2	3
	79	94	46	92	85	76	88	53	46	36	40	62
	62	74	57	99	79	68	75	56	57	53	56	47

Example 4

The self-inductance of coils with iron-oxide cores was measured under different temperature conditions of the measuring bridge. The coil temperature was held constant. Five coils were used for the experiment. The self-inductance of each coil was measured for each of four temperatures (22°, 23°, 24°, and 25°) for the measuring bridge. The temperatures were utilized in a random order for each coil. The data are percentage deviations from a standard.

	Coil				
Temperature	*1*	*2*	*3*	*4*	*5*
22	1.400	0.264	0.478	1.010	0.629
23	1.400	0.235	0.467	0.990	0.620
24	1.375	0.212	0.444	0.968	0.495
25	1.370	0.208	0.440	0.967	0.495

Source: H. Hamaker (1955), Experimental design in industry. *Biometrics* 11, 257–286.

Example 5

A research specialist for a large seafood company investigated bacterial growth on oysters and mussels subjected to three different storage temperatures. Nine cold storage units were available. Three storage units were randomly selected to be used for each of the storage temperatures. Oysters and mussels were stored for two weeks in each of the cold storage units. A bacterial count was made from a sample of oysters and mussels at the end of two weeks. The logarithm of bacterial count for each sample is shown in the table at the end of the exercise.

Storage Unit	*Temperature (°C)*	*Seafood**	*log (count)*
1	0	1	3.6882
1	0	2	0.3565
2	0	1	1.8275
2	0	2	1.7023
3	0	1	5.2327
3	0	2	4.5780
4	5	1	7.1950
4	5	2	5.0169
5	5	1	9.3224
5	5	2	7.9519
6	5	1	7.4195
6	5	2	6.3861
7	10	1	9.7842
7	10	2	10.1352
8	10	1	6.4703
8	10	2	5.0482
9	10	1	9.4442
9	10	2	11.0329

* 1 = oysters, 2 = mussels

Example 6

An experiment was conducted to compare the accuracy of two mass spectrometers in measuring the ratio of ^{14}N to ^{15}N. Two soil samples were taken from each of three plots of land treated with ^{15}N. Two subsamples of each sample were analyzed on each of the two machines.

The data are ratios of ^{14}N to ^{15}N (after multiplying by 1000).

Plot	*1*		*2*		*3*	
Sample	*1*	*2*	*3*	*4*	*5*	*6*
Machine A	3.833	3.819	3.756	3.882	3.720	3.729
	3.866	3.853	3.757	3.871	3.720	3.768
Machine B	3.932	3.884	3.832	3.917	3.776	3.833
	3.943	3.888	3.829	3.915	3.777	3.827

Source: D. Robinson (1987), Estimation and use of variance components. *The Statistician* 36, 3–14.

SOLUTION KEYS

Example 1

A fleet manager wishes to compare the wearability of 4 brands of tire: A, B, C, and D. Four cars are available for the experiment and 4 tires of each brand are available, for a total of 16 tires. The idea is to mount 4 tires on each of the 4 cars, ask the driver of each of the cars to drive his/her car for 20,000 miles and then to measure tread loss. We will measure tread loss in mils (.001 inches). We will designate the 4 cars as cars I, II, III, and IV.

Each car tests all four brands. Thus one tire from each brand is selected at random and randomly allocated to the 4 wheels of car I. Then one tire from each brand is selected and the four are randomly allocated to car II, and so forth. Here are the results of that design.

	Car I	Car II	Car III	Car IV
Brand assignment	**B**	**A**	**C**	**B**
	D	**D**	**B**	**A**
	C	**C**	**A**	**C**
	A	**B**	**D**	**D**

The key is to follow the randomization.

This is an RCBD design.

The treatment factor is the four brands of tire. We see that the four brands were randomized within each of the four cars—the cars serve as blocks.

Elements of experimental design:

The research question is to compare the wearability of 4 brands of tire.

The treatments are the four brands of tires (one factor, four levels).

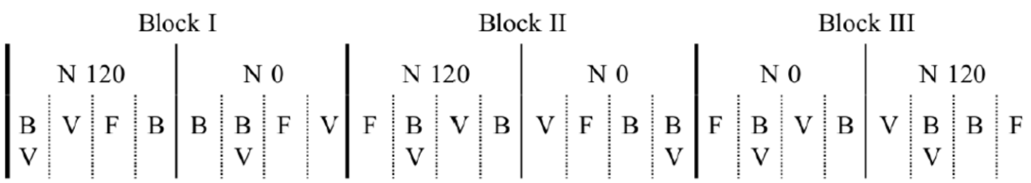
The four treatments are randomized to the four wheels of each car.

The experimental unit is thus one wheel of a car.

The measurement is the tread loss.

Example 2

An experiment was installed to compare the effects of 4 green manure treatments (barley, vetch, barley + vetch, and fallow) on the subsequent production of sugar beets at 2 levels of N (0, 120 lb/acre) with 3 replications. The N levels were assigned to the main plots as there was less interest in N effects than in the effects of the green manures and the interaction between N and the green manures.



This a split-plot design.

We have two levels of treatments, and correspondingly two levels randomization and two levels of experimental units.

At the whole-plot level, the two levels of N (I suppose it's Nitrogen?) were randomized to two main plots within each block in a RCBD design. The main plots are the experimental units for the treatment factor "N".

At the subplot level, the four manure treatments were randomized to the four subplots within each main plot. The subplots are the experimental units for the factor "manure".

Example 3

The surface finish of metal parts made on four machines is being studied. An experiment is conducted in which each machine is run by three different operators and two specimens from each operator are collected and tested. Because of the location of the machines, different operators are used on each machine, and the operators are chosen at random. The data are shown in the following table.

Operator	Machine 1			Machine 2			Machine 3			Machine 4		
	1	2	3	1	2	3	1	2	3	1	2	3
	79	94	46	92	85	76	88	53	46	36	40	62
	62	74	57	99	79	68	75	56	57	53	56	47

This is a nested design: the operators are nested under machine.

For this type of design, one common question of interest is to attribute the total variation in the data to different sources: i.e., figure out the variance components corresponding to random error, operator and machine.

In this problem, the fours machines can be viewed as a fixed effect factor if we are only interested in making inference about these four machines, not viewing them as a random sample from a larger pool of machines.

Example 4

The self-inductance of coils with iron-oxide cores was measured under different temperature conditions of the measuring bridge. The coil temperature was held constant. Five coils were used for the experiment. The self-inductance of each coil was measured for each of four temperatures (22°, 23°, 24°, and 25°) for the measuring bridge. The temperatures were utilized in a random order for each coil. The data are percentage deviations from a standard.

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Source: H. Hamaker (1955), Experimental design in industry. *Biometrics* 11, 257–286.

This is an RCBD design.

The research question seems to be one wants to study the effect of the temperature conditions of the measuring bridge on self-inductance measurements.

Since the temperatures were randomized for each coil. The coils serve as a blocking factor.

If this were a consulting project, I would ask the client to clarify the research question.

Example 5

A research specialist for a large seafood company investigated bacterial growth on oysters and mussels subjected to three different storage temperatures. Nine cold storage units were available. Three storage units were randomly selected to be used for each of the storage temperatures. Oysters and mussels were stored for two weeks in each of the cold storage units. A bacterial count was made from a sample of oysters and mussels at the end of two weeks. The logarithm of bacterial count for each sample is shown in the table at the end of the exercise.

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* 1 = oysters, 2 = mussels

This is (kind of) a split-plot design.

The three levels of temperatures were randomized to the 9 storage units.

Within each storage unit, we have two types of seafood. But the question did not say whether the two samples of seafood were randomly selected from larger populations, or whether there is some randomization when we put the two types of seafood into the storage units. Without randomization, the two food types could be potentially confounded with, e.g., the locations in the stroage unit.

If this were a real consulting project, I would ask the client to clarify on the sampling and randomization regarding the two types of seafood.

Example 6

An experiment was conducted to compare the accuracy of two mass spectrometers in measuring the ratio of ^{14}N to ^{15}N. Two soil samples were taken from each of three plots of land treated with ^{15}N. Two subsamples of each sample were analyzed on each of the two machines.

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This is a design with crossed and nested factors. Soil samples are nested under plots. The two machines are crossed with both sample and plots.

By sampling plots and soil samples, we want to see whether the mass spectrometers give consistent readings for a wide range of samples.

Summary

The key to correctly identify a design is to follow the randomization.

In experiments with multiple factors, the randomization might be different for different factors (such as in a split-plot design).