KIN 523 Winter 18
Homework Set 3 – Signal Processing Pavol

Due: 2/06/18

Answer the following problems.
You must show all work, including formulas used, numbers substituted, and answers with units.
If Excel was used, e-mail a copy of the workbook to me.
If a computer program was used, either attach or e-mail me a copy of the program.
Your written portion must indicate what equations were used in Excel and/or your program.

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1. You have recorded a person's hip height at 60 frames/s during a sit-to-stand task. You now wish to low-pass filter the data using a 4th-order, no-lag, Butterworth filter with a cut-off frequency of 5 Hz. Assume that the data consist of a time-series $X(kT)$, where k is the sample number and T is the sample period.

   a. *(0.8 pts)* What is the equation for filtering the data in the forward direction? You must show the calculations used to find the values of the coefficients in the filter equation (i.e. using the "*butter*" function in MATLAB to find the coefficients is *not* acceptable).

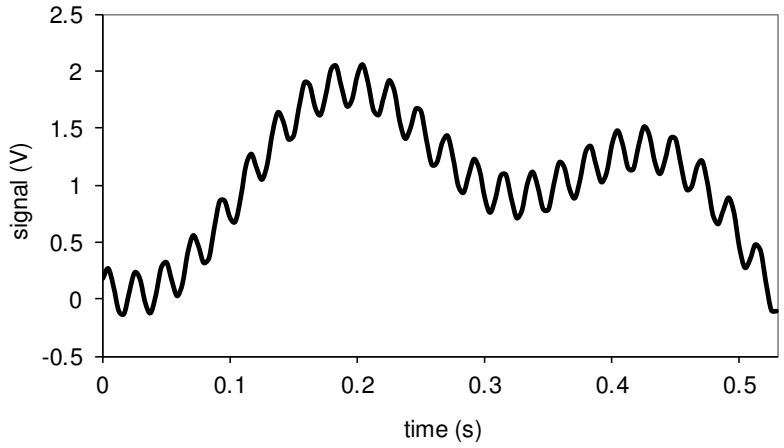
   b. *(0.5 pts)* The table below shows the hip height data, $X(kT)$, over a subset of samples k. The values $Y_1(kT)$ correspond to the output of the filter in part (a) while filtering $X(kT)$ in the forward direction. The values $Y_2(kT)$, correspond to the filtering of $Y_1(kT)$ in the backward direction using the same filter. Fill in the two missing values of the table.

k	$X(kT)$ (mm)	$Y_1(kT)$ (mm)	$Y_2(kT)$ (mm)
24	559.65	538.17	559.56
25	569.50	547.19	573.45
26	589.97	?	?
27	610.15	571.20	608.56
28	626.80	587.31	629.36

2. *(0.8 pts)* You are analyzing the mechanics of the knee as an individual lands on one foot following a jump. You collected motion capture data at 60 Hz and ground reaction force data at 1080 Hz. In one trial, the peak vertical ground reaction force occurred at a time of 0.7417 s. The corresponding knee flexion angles computed from the motion capture data are shown in the table to the right. Use linear interpolation to determine the knee flexion angle at the time of peak vertical ground reaction force. *Be sure to show the formula you used.*

Time (s)	Knee Angle (deg)
0.7000	30.2
0.7167	31.3
0.7333	33.9
0.7500	37.8
0.7667	43.2

3. You have performed a pilot data collection to determine what sample rate to use in your study. You collect a trial of data at 240 Hz and get the signal shown below:



Assume that you perform a fast Fourier transform of the signal over a period of 0.533 s (=128 sample points). The frequency amplitude spectrum you would obtain is contained in the comma-delimited file "FFT.csv" posted on Canvas, where the first column contains the frequency in Hz, and the second column contains the amplitude in volts. The first row of the file contains header information.

a. *(1.2 pts)* In your study, you plan to low-pass filter the collected data with a cut-off frequency of 6 Hz. Based on this and the frequency amplitude spectrum:

   i) What is the minimum sample rate to avoid aliasing?

   ii) What is the recommended sample rate to capture the shape of the filtered data?

   iii) What sample rate would you use?

   Justify your answers.

b. *(0.8 pts)* Based on the frequency amplitude spectrum, what is the power of the signal:

   i) at 0 Hz?

   ii) at 3.75 Hz?

4. We need to low-pass filter the data that we recorded during the countermovement vertical jumps that were performed in the lab. The comma-delimited file "FilterData.csv" posted on Canvas contains the *Y* position (height, in meters) vs. time of the markers at the ankle (LANK) and second metatarsal (LTOE) of the left foot during one of the trials. These data were collected at 120 frames/s. Time (in seconds) is in the first column, LANK is in the second column, and LTOE is in the third column. The first row of the file contains header information.

a. *(1.3 pts)* If the LTOE data are low-pass filtered using a 4th-order, no-lag Butterworth filter with a cut-off frequency of 10 Hz, what is the value of the residual between the filtered data and the original, raw data?

   Note: In determining the filter coefficients, compare the value of Ωc to that from problem 1; it might save you some time.

b. *(1 pt)* Using the same basic process of part (a), the residuals were determined for LANK for cut-off frequencies ranging from 1 to 35 Hz. The comma-delimited file "Residuals.csv" posted on Canvas contains these residual values for LANK as a function of cut-off frequency. Based on the residual analysis procedure described in class, what is an appropriate filter cut-off frequency for LANK? *Show the graph you used and how you determined the cut-off frequency from the graph. Give your answer to the nearest multiple of 1 Hz.*

c. *(1.5 pts)* Low-pass filter the LANK data using a 4th-order, no-lag, Butterworth filter with the cut-off frequency determined in part (b). Turn in one graph in which both the raw and the filtered LANK data are plotted vs. time over the entire trial, a second graph in which these data are plotted only over the period from 0.3 to 0.8 s, and a third graph in which the data are plotted over the period from 1.5 to 1.75 s. The latter two graphs must be scaled so differences between the raw and filtered data are clearly visible.

d. *(0.5 pts)* The period up to 0.86 s corresponds to the initial downward countermovement, during which the left heel remained on the floor until 0.76 s. Initial contact of the left foot with the ground on landing occurred at 1.52 s. From observing the participant's movements, it is clear that she landed on her toes. It is also clear that either just before or just after the participant's heels contacted the ground, she bounced back up onto her toes slightly and then slowly lowered her heels back to the ground, coming to rest. Based on this information and the results of part (c), is the cut-off frequency you found in part (b) appropriate? Why or why not?

Note: You are allowed to use the filtering functions in MATLAB for parts (a) and (c).

5. Cubic splines are often used to fill gaps in motion capture data.

   a. *(0.8 pts)* What are two primary reasons that cubic splines are preferred over linear interpolation when filling in gaps in motion capture data?

   b. *(0.8 pts)* List two general circumstances under which it would be inappropriate to use a cubic spline to fill a gap in motion capture data, and explain why it would be inappropriate to do so under these circumstances.