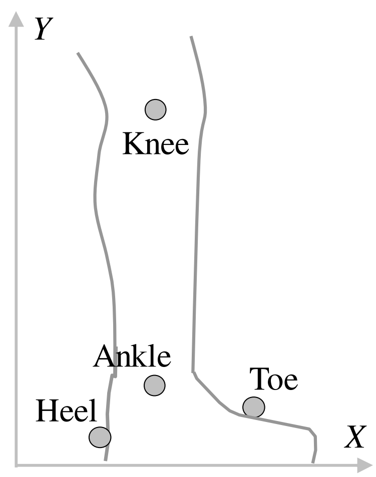
KIN 523 — Winter 18
Homework Set 8 – Work, Power, Energy, and Force Estimation — Pavol

Due: 3/16/18

Answer the following problems.
You must show all work, including formulas used, numbers substituted, and answers with units.
If Excel was used, e-mail a copy of the workbook to me.
If a computer program was used, either attach or e-mail me a copy of the program.
Your written portion must indicate what equations were used in Excel and/or your program.

We used the motion capture system and force plates in the Biomechanics Lab to record an individual's movements and the associated ground reaction forces as she performed a countermovement jump. In previous homework sets, you've performed the following analyses:



- You computed the segment angles of the foot and leg as a function of time. (Homework Set 4)
- You computed the ankle angle and ankle angular velocity as a function of time. The following convention was used:
    - o An ankle angle of 0° corresponded to the relative orientation of the foot and leg in the anatomical position;
    - o Positive values of the ankle angle and angular velocity corresponded to dorsiflexion.

  (Homework Set 4)
- It was determined that takeoff from the ground occurred at 1.059 s (frame 128) and touchdown occurred at 1.513 s (frame 182). (Homework Set 6)
- You computed the resultant joint moment at the ankle as a function of time. Ankle plantarflexion moments were defined to be positive. (Homework Set 7)
- You computed the resultant joint force at the ankle as a function of time, and converted this force into its compressive and shear components relative to the anatomical axes of the foot. The following convention was used:
    - o A positive shear component represented an anteriorly-directed shear force on the foot;
    - o A positive compressive component represented an inferiorly-directed force on the foot.

  (Homework Set 7)

I computed the angular velocities of the foot and leg segments from the corresponding segment angles, using the methods of Homework Set 4. I used the following convention:

- o For the foot, positive angular velocities correspond to clockwise rotation (see figure);
- o For the leg, positive angular velocities correspond to counterclockwise rotation (see figure).

Note: This convention for the leg is opposite to the one used in Homework Set 4.

The comma-delimited file "HW8_Data.csv", posted on Canvas, contains selected kinematic and kinetic data associated with the countermovement jump. The format of the file is as follows:

Column	Label	Description	Units
1	Frame	Frame number	
2	AANK	Ankle angle	deg
3	WANK	Ankle angular velocity	deg/s
4	RJM	Resultant ankle moment	Nm
5	RJFS	Shear component of resultant ankle force	N
6	RJFC	Compressive component of resultant ankle force	N
7	WFOOT	Foot angular velocity	deg/s
8	WLEG	Leg angular velocity	deg/s

These data were sampled at 120 Hz.

1. *(1.4 pts)* Using the data in "HW8_Data.csv", compute and graph the instantaneous power being produced at the ankle as a function of time during the trial. On the graph, mark and label the periods over which each of the following were occurring:

   a. Plantarflexors generating power

   b. Plantarflexors absorbing power

   c. Dorsiflexors generating power

   d. Dorsiflexors absorbing power

   Clearly indicate the start and end of each period. Some of the above may occur over multiple periods. If so, mark and label each of these periods. It is also possible that some of the above will not occur. You do not need to mark periods during which the power being generated or absorbed is close to zero ($|P| < 18$ W), although you may do so if you wish.

   Note #1: Be careful to use a consistent sign convention for all variables when calculating power.

   Note #2: To get the correct units for power (i.e. Watts), angular velocity must be in rad/s.

2. *(0.8 pts)* From the answer to Problem 1, compute the work done by the ankle muscles over the period from the start of the data (frame 0) until take-off (frame n). When performing the numerical integration, use the following approximation:

$$\int_{t0}^{tn} P(t)dt \approx \left[\frac{P(t_0)}{2} + \sum_{i=1}^{n-1} P(t_i) + \frac{P(t_n)}{2}\right]\Delta t$$

   where t_i and $P(t_i)$ are the time and power output corresponding to frame i, respectively, and Δt is the sampling period (i.e. time between consecutive frames).

   Did the ankle musculature act to increase or decrease the energy of the body over the period from the start of the trial until take-off? Explain how you arrived at this answer.

3. *(1.7 pts)* On the same graph, plot the angular velocities of the foot and leg segments as a function of time. For the periods of time *before takeoff* and *after landing*, mark and label on the graph the periods over which the resultant ankle moment was performing each of the following:

   a. Increasing the energy of the foot
   b. Decreasing the energy of the foot

   Mark below the curve

   c. Increasing the energy of the leg
   d. Decreasing the energy of the leg

   Mark above the curve

   e. Transferring energy from the foot to the leg
   f. Transferring energy from the leg to the foot

   As in Problem 1, clearly indicate the start and end of each period. Some of the above may occur over multiple periods, some may never occur, and more than one of the above may occur at the same time. You do not need to mark periods during which the angular velocity of the segment is close to zero ($|\omega| < 20$ deg/s) or during which the resultant ankle moment was close to zero ($|RJM| < 5$ Nm)

   Note: Remember Newton's 3rd Law when determining the direction in which the resultant ankle moment acts on each segment.

4. *(2.4 pts)* Based on the results of Problems 1-3, describe the function of the ankle plantarflexors over the course of a countermovement jump and landing in terms of:

   i) Are they acting to generate or absorb power at the ankle?
   ii) Are they acting to speed up or slow down the motion of the ankle?
   iii) What motion (i.e. plantarflexion, dorsiflexion) are they acting to speed up or slow down?
   iv) Are they increasing or decreasing the energy of the foot?
   v) Are they increasing or decreasing the energy of the leg?
   vi) Are they transferring energy from one segment to the other and, if so, in which direction is the energy being transferred?

   These six factors should be discussed for each of the following phases of the jump:

   a. The initial downward countermovement (frames 0-103);
   b. The upward motion leading to takeoff (frames 103-128);
   c. The downward motion following touchdown (frames 182-220);
   d. The return to standing (frames 220-288).

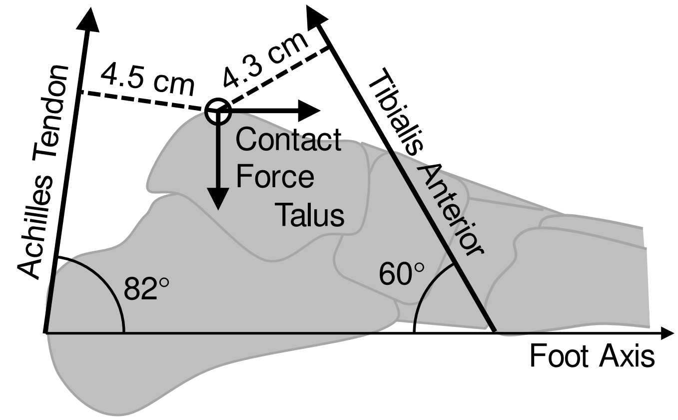
   Note: This problem is requesting 24 specific pieces of information, and no justification needs to be provided for your answers. It is suggested that you answer this problem as an ordered list.

Assume that the anatomy of the ankle joint is modeled as shown in the figure below.

There are two muscle groups:

- Triceps surae (medial gastrocnemius, lateral gastrocnemius, and soleus, all acting through the Achilles tendon)
- Tibialis anterior

For simplicity, assume that the moment arms and the angles of insertion of the Achilles tendon and tibialis anterior relative to the foot remain constant, as shown in the figure.



5. *(0.9 pts)* Assume that only one muscle group was active at a time. Based on the data in "HW8_Data.csv", compute and graph (on separate graphs) the forces generated by the triceps surae and tibialis anterior muscle groups versus time during the trial.

6. *(1.4 pts)* Assume that muscle produces a maximum stress of 30 N/cm^2 when fully activated under isometric conditions and that the physiological cross-sectional areas of our individual's triceps surae muscles are as follows:
   - Soleus: 69 cm^2
   - Medial Gastrocnemius: 26 cm^2
   - Lateral Gastrocnemius: 10 cm^2

   Using these data and the results of Problem 5, compute and report the following for the frame in which the triceps surae muscles generate the most force:
   a. The muscle force as a percentage of the maximum isometric strength of the triceps surae;
   b. The force being produced by each of the three triceps surae muscles, based on the "equal stress" assumption (i.e. at any instant, the stresses in all synergist muscles are equal).

   Note: Stress = Force / Area

7. *(1.4 pts)* Using the data in "HW8_Data.csv" and the forces computed in Problem 5, compute the compressive and shear components of the joint contact force acting on the talus at the ankle during the trial. Use the following convention:
   - o A positive shear component represents an anteriorly-directed force on the talus;
   - o A positive compressive component represents an inferiorly-directed force on the talus.

   Turn in two graphs:
   a. The compressive components of the resultant joint force and joint contact force vs. time;
   b. The shear components of the resultant joint force and joint contact force vs. time.

Note #1: The approach we used in Problems 5-7 represents a vast simplification relative to the state of the art, as described in class, for estimating muscle and joint contact forces.

Note #2: Notice that the resultant joint moment allowed for reasonable inference as to what muscles were being used and to what extent, without need for mathematical modeling. However, the resultant joint force was a poor estimate of the joint contact force. To obtain reasonable estimates of skeletal loading requires mathematical modeling or direct, invasive measurement.