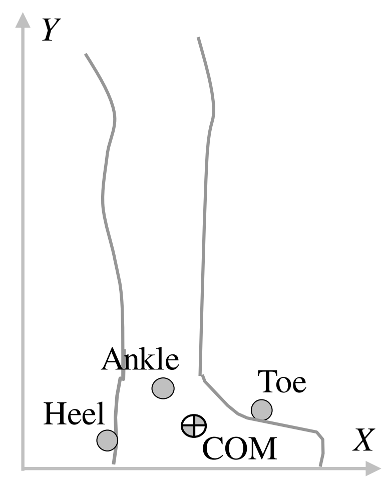
KIN 523 Winter 18
Homework Set 7 – Kinetics Pavol

Due: 3/09/18

Answer the following problems.
You must show all work, including formulas used, numbers substituted, and answers with units.
If Excel was used, e-mail a copy of the workbook to me.
If a computer program was used, either attach or e-mail me a copy of the program.
Your written portion must indicate what equations were used in Excel and/or your program.

We used the motion capture system and force plates in the Biomechanics Lab to record an individual's movements and the associated ground reaction forces as she performed a countermovement jump.



- We calibrated the motion capture system such that the global *X* axis was horizontal and the *Y* axis pointed vertically upward. $Y = 0$ corresponds to the top surface of the force plate.
- The participant faced in the +*X* direction during the jump (see figure).
- The kinematic data were recorded at 120 frames/s.

So far, we've performed the following analyses:

- Based on a residual analysis, the marker positions for the jumping trial were low-pass filtered with a 4th-order, Butterworth, no-lag filter with a cut-off frequency of 18 Hz. (Homework Set 3)
- From the heel and toe marker positions, we computed the segment angle of the left foot as a function of time during the jumping trial using the following convention:
  - The orientation of the foot during quiet standing was defined to be a foot angle of 0°
  - The foot angle > 0 for clockwise rotation of the foot from 0° (see figure).

  This is actually the angle of the shoe, but we've chosen to neglect the difference between them. (Homework Set 4)
- We computed the angular acceleration of the foot as a function of time, using the same sign convention as above. (Homework Set 4)
- Using the segment mass data of Vaughn et al. (1992) and the radius of gyration data of Zatsiorsky et al. (1990), we find:
  - Foot mass = 0.76 kg
  - Foot moment of inertia = 4390 kg mm^2 = 0.00439 kg m^2

  (Homework Set 5)
- We found the global *X* and *Y* positions of the foot center of mass (COM) by assuming that it was located at a fixed distance along the line from the heel to the toe marker. (Homework Set 5)

- You found the Y acceleration of the foot COM as a function of time, and I used a similar method to find the X acceleration of the foot COM as a function of time. (Homework Set 5)
- We found the ground reaction forces acting on the left foot in the global X and Y directions as a function of time. Positive X and Y forces were defined to act on the foot in the $+X$ and $+Y$ directions, respectively. For this homework, I low-pass filtered the ground reaction forces with a 4th-order, Butterworth, no-lag filter with a cut-off frequency of 18 Hz filter to match the kinematic data. I then set the reaction forces equal to 0 whenever the magnitude of the vertical component of force was less than 20 N. (Homework Set 6)
- We found the global X position of the center of pressure of the ground reaction force under the left foot as a function of time. This position was set to $X = 0.3$ m whenever the magnitude of the vertical component of the ground reaction force was less than 20 N. (Homework Set 6)

You are now going to find the resultant joint force (RJF) and resultant joint moment (RJM) acting at the left ankle during the countermovement jump. You must use the following conventions:

- A positive X component of the RJF represents an anteriorly-directed force on the foot;
- A positive Y component of the RJF represents a downward-directed force on the foot;
- A positive RJM corresponds to an ankle plantarflexion moment.

1. *(1.1 pts)* Draw a free body diagram that will allow you to compute the RJF and RJM acting at the ankle. In your diagram, each kinematic and kinetic variable described in the preceding text must be shown in the direction defined to be positive for that variable.

2. *(2 pts)* Based on your diagram in Problem 1, derive and report the three equations of motion for the linear and angular motion of the foot in the sagittal plane.

The comma-delimited file "JumpData.csv", posted on Canvas, contains the kinematic and kinetic data associated with the countermovement jump. The format of the file is as follows:

Column	Label	Description	Units
1	Frame	Frame number	
2	ANKX	Ankle X position	m
3	ANKY	Ankle Y position	m
4	FTANG	Foot angle	radians
5	FTACC	Foot angular acceleration	rad/s^2
6	COMX	Foot COM X position	m
7	COMY	Foot COM Y position	m
8	ACCX	Foot COM X acceleration	m/s^2
9	ACCY	Foot COM Y acceleration	m/s^2
10	FX	X component of GRF on the foot	N
11	FY	Y component of GRF on the foot	N
12	COPX	Center of pressure X position	m

In all cases, X and Y correspond to the global coordinate system. The data for FX, FY, and COPX have been down-sampled from 1080 Hz to 120 Hz by selecting every 9th value.

3. *(2 pts)* From the results of Problem 2, use the data in "JumpData.csv" to compute the RJF and RJM acting on the foot at the ankle, as a function of time. Express the RJF in the global (X, Y) coordinate system, using the convention described earlier.

4. *(0.8 pts)* Assume that the anatomical axes of the foot are defined such that, when the foot angle is 0°, the anteroposterior axis of the foot is aligned with the global X axis and the inferosuperior axis is aligned with the global Y axis. Using the foot angle data in "JumpData.csv" and the results of Problem 3, find the compressive and shear components of the RJF relative to the anatomical axes of the foot. Use the following convention:
   - A positive shear component represents an anteriorly-directed shear force on the foot;
   - A positive compressive component represents an inferiorly-directed force on the foot.

   Note: Given how the forces were defined, when the foot angle is 0°, the shear (RJF_S) and X components of the RJF (RJF_X) both point anteriorly, whereas the compressive (RJF_C) and Y components (RJF_Y) both point downward. As such, one of the following two conversion equations will apply:

   $$\begin{bmatrix} RJF_S \\ RJF_C \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} RJF_X \\ RJF_Y \end{bmatrix} \qquad \begin{bmatrix} RJF_S \\ RJF_C \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} RJF_X \\ RJF_Y \end{bmatrix}$$

   You need to determine which is correct. *Hint:* Draw what happens when the foot angle is 90°.

You do not need to turn in any tables or graphs for Problems 3 and 4.

5. *(1 pt)* Assume that our participant had a body mass of 56.3 kg and a body height of 157.7 cm. Normalize the RJM from Problem 3 to a proportion of (body weight x body height) and normalize the forces from Problem 4 to a proportion of body weight.

   From the results, turn in the following graphs:

   a. A graph of the normalized RJM vs. time for the entire duration of the trial;
   b. A graph of the normalized shear and compressive components of the RJF vs. time for the entire duration of the trial.

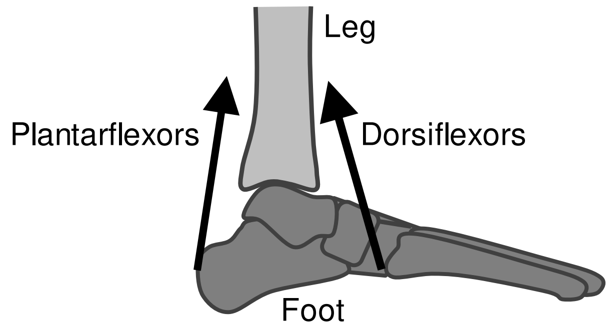
6. *(0.9 pts)* Based on your results, identify which muscle group(s) at the ankle are predominantly being used:

   a. Before take-off from the ground;
   b. While in the air;
   c. After touchdown

   If the predominant muscle group changes during a period, indicate during which portions of that period each muscle group is being used. Explain how you arrived at your answers.

7. *(1 pt)* The Achilles tendon connects the main plantarflexor muscles to the foot. When during the countermovement jump and landing was our participant at greatest risk of rupturing her Achilles tendon? Provide a specific time or range of times and justify your answer in terms of the present results and the properties and function of tendon.

8. *(1.2 pts)* The figure to the right shows a simplified version of the anatomy of the ankle joint. Based on the figure and the RJF and RJM you computed, what can you conclude about the joint contact force that acted on the talus (foot) from the tibia (leg) during the 200 ms after ground contact during landing? Answer in terms of the:



   a. Direction of the inferosuperior component…
   b. Magnitude of the inferosuperior component…
   c. Direction of the anteroposterior component…
   d. Magnitude of the anteroposterior component…

   …of the joint contact force relative to the anatomical axes of the foot. Justify each answer.