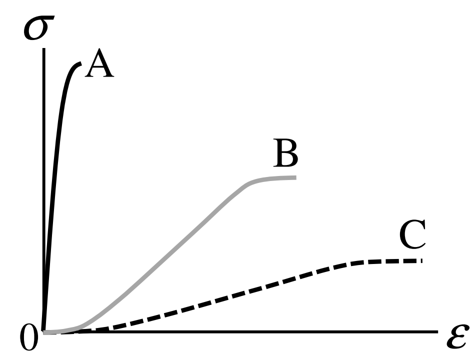
BIOE 599 – Physiology for Engineers
Homework Set 2

Due: 3/02/18 at 7 pm to Mike Pavol

Answer the following questions.

- You must show your work.
- If a computer program was used, include a listing of the program with your homework submission.
- If Excel was used, email a copy of the workbook to the instructor.

1. A tendon, a ligament, and a sample of cortical bone were loaded in tension to the point of failure, and the resulting stress-strain behaviors are shown in the figure to the right. Based on this figure, which tissue (i.e. A, B, or C)…
    a. Is the tendon?
    b. Is the cortical bone?
    c. Is the strongest?
    d. Is the stiffest in its linear elastic region?
    e. Is the toughest?
    f. Is the most ductile?



2. The diaphysis of a 75 kg person's femur is 26 cm long, has an outer diameter of 2.5 cm, and an inner diameter of 1.5 cm. Assume the diaphysis can be modeled as a linear elastic, hollow tube with material properties as given in the lecture notes.
    a. What is the largest bending moment that can be applied to the femoral diaphysis without causing a fracture?
    b. What is the largest torque that can be applied about the longitudinal axis of the femoral diaphysis without causing a fracture?
    c. Due to exercise-related remodeling, the outer diameter of the diaphysis increases by 4% to 2.6 cm, while the inner diameter remains 1.5 cm. By what percentages do the answers to parts (a) and (b) change?
    d. Due to inactivity-related remodeling, the inner diameter of the diaphysis increases by 4% to 1.56 cm, while the outer diameter remains 2.5 cm. By what percentages do the answers to parts (a) and (b) change?
    e. In terms of factors that influence bone remodeling in response to mechanical loading, why do bone formation and resorption tend to occur to greater extent at the periosteal and endosteal surfaces, respectively?
    f. Based on the answers to parts (c) and (d), what are the mechanical advantages of the preferential remodeling described in part (e)?

3. The person from problem 2 receives an artificial hip. The femoral component of the implant fills the medullary cavity of the proximal portion of the diaphysis. The implant is made of a titanium alloy with an elastic modulus of 100 GPa.
   a. Before the person received the implant, what average stress did the bone of the proximal diaphysis experience when a longitudinal compressive force of 1 times body weight was applied to the femur?
   b. If the same longitudinal compressive force of 1 times body weight is applied to the femur after the person receives the implant, what average stress does the bone experience within the portion of the diaphysis that contains the implant?
   c. Based on the answers to parts (a) and (b), why might loosening of the implant occur over time?

4. Osteoporosis is a disease in which a loss of bone increases the risk of fracture, with fractures of the spinal vertebrae being the most common. Because bone adapts to mechanical loading, exercise has been proposed as an intervention against osteoporosis. Answer the following questions related to this topic.
   a. The L4 vertebral body of a typical young adult has a bone mineral density (BMD) of 1.1 g/cm^2 and an ultimate stress of 6.7 MPa under compression. The L4 vertebral body of a 70 kg osteoporotic woman has a BMD of 0.6 g/cm^2. Estimate the ultimate stress of this woman's L4 vertebral body under compression.
   b. Rowing is an exercise associated with high BMD of the lumbar vertebrae, and it is estimated to apply peak compressive forces of 4.6 times body weight to the lumbar spine. The body of the L4 vertebra of a 75 kg young adult is 3 cm in height, has a cross-sectional area of 10 cm^2, and an elastic modulus of 300 MPa in the longitudinal direction. Estimate the peak longitudinal strain this vertebral body will undergo during rowing.
   c. The table below lists the loading of the L4 vertebral body for seven exercise protocols in which a person is performing an upright row against an applied resistance. In protocol 7, the person performs the exercise while standing on a vibrating platform during the first session.

	Loading				
Protocol	Conditions per Day	# of Cycles	# of Freq.	Frequency (Hz)	Peak-to-Peak Strain
1	1	12	1	0.5	.005
2	1	24	1	0.5	.005
3	2	12	1	0.5	.005
		12	1	0.5	.005
4	1	12	1	1	.005
5	1	12	1	0.5	.01
6	1	6	1	1	.01
7	2	6	2	0.5	.003
				20	.00009
		12	1	0.5	.005

Use the Turner (1998) theoretical model of mechanical adaptation to compute the daily loading stimulus (S) associated with each exercise protocol, assuming a proportionality constant of 1 in the equation for S.

d. For each of the following, choose the more effective strategy for increasing the daily loading stimulus (S) on the L4 vertebral body, other factors being equal. In each case, state which results from part (c) support your argument.
   i. A) double the number of repetitions performed, or B) double the peak-to-peak strain of the vertebra during the repetitions (e.g. by increasing the applied resistance)
   ii. A) double the number of repetitions performed, or B) double the frequency at which the vertebra is loaded (e.g. by performing repetitions more quickly)
   iii. A) double the peak-to-peak strain of the vertebra during the repetitions, or B) double the frequency at which the vertebra is loaded
   iv. A) double the number of repetitions performed within an exercise session, or B) add a second, identical exercise session later in the day.

5. A person exhibits elevated levels of circulating parathyroid hormone (PTH) in the blood. Which of the following is most likely the plasma calcium concentration of this individual, and how do you know this?
   A) 8 mg/dL
   B) 10 mg/dL
   C) 12 mg/dL

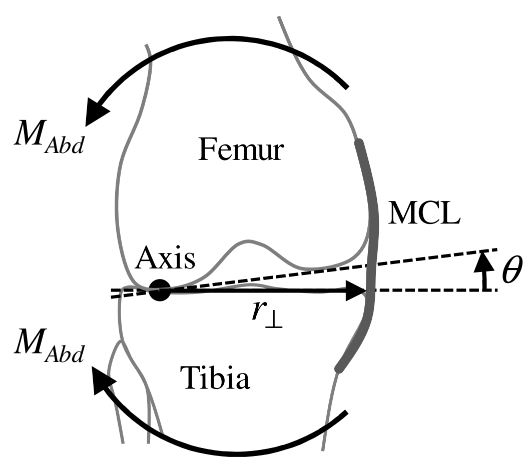
6. The medial collateral ligament (MCL) connects the femur and tibia at the medial aspect of the knee. A frontal view of the relevant anatomy in the case of an externally applied knee abduction moment (M_{Abd}) is shown in the figure below. Assume the MCL has a moment arm ($r_\perp$) of 5.5 cm about the instantaneous axis of rotation, and the length (l) of the MCL is given by:

$$l = 5\text{ cm} + (5.5\text{ cm/rad})\theta \qquad \text{for } \theta \geq 0$$

where θ is the knee abduction angle (in rad).

Assume the MCL has the following material properties:

- Slack length (l_0) = 5.1 cm
- Cross-sectional area (A) = 3 mm^2
- ε_{toe} = 0.03
- Tangent modulus = 270 MPa
- σ_{lin} = 36 MPa
- ε_{lin} (= ε at σ_{lin}) = 0.15



Answer parts (a)-(e) on the next page based on these data.

a. We will model the MCL using Model 3 from the slide on "Models of Tendon & Ligament" in the "Tendon & Ligament" lecture notes. By enforcing the constraint that both the stress-strain relationship and its slope must be continuous at ε_{toe}, one can determine that k_2 = 64.01 for the MCL. Solve for the values of k_1, k_3, and k_4.

b. Using the results of part (a), compute and graph the moment (in Nm) produced by the MCL about the instantaneous axis of rotation as a function of the knee abduction angle (θ) over the range of 0-10°.

c. Does the moment in part (b) act in the knee abduction or the knee adduction direction? Briefly explain the reasoning behind this answer.

d. What basic function does the moment from part (b) serve?

e. Determine the knee abduction angle (θ) at which the strain in the MCL will reach ε_{lin}. What will happen to the MCL if θ exceeds this angle?

For comparison, the peak vertical force acting on the foot from the ground during landing from a vertical jump is approximately 2 times body weight. If a 70 kg person lands such that this force is directed 0.5 cm lateral to the instantaneous axis of rotation in the figure, it will produce an external knee abduction moment (M_{Abd}) of 6.9 Nm.

7. A simple model of viscoelasticity was presented in the lecture notes on "Mechanics of Biological Materials". We will use this model to model the behavior of the Achilles tendon. For this tendon, the model has the following properties:

- l_0 = 220 mm
- k_1 = 200 N/mm
- k_2 = 467 N/mm
- b = 15000 N·s/mm

Simulate and graph the following behaviors, and answer the associated questions.

a. The tendon force versus length in response to the following length changes:

i. $l = 225 - 5\cos(2\pi \cdot 0.05t)$ mm for $0 \le t \le 20$ s

ii. $l = 225 - 5\cos(2\pi \cdot 0.5t)$ mm for $0 \le t \le 2$ s

iii. $l = 225 - 5\cos(2\pi \cdot 5t)$ mm for $0 \le t \le 0.2$ s

In all three cases, set $F = 0$ at $t = 0$. Graph all three traces on the same figure. What two properties of a viscoelastic tissue are illustrated in the figure? What should the figure look like for a viscoelastic tissue?

b. The tendon force versus length in response to the following length change:

$$l = 225 - 5\cos(2\pi \cdot 0.05t) \text{ for } 0 \le t \le 60 \text{ s}$$

where $F = 0$ at $t = 0$. What changes do you observe between the first and third loading cycle?

c. The tendon force versus time in response to the following length change:

$$l = \begin{cases} 225 - 5\cos(2\pi \cdot 0.5t) \text{ mm} & 0 \le t \le 1 \text{ s} \\ 230 \text{ mm} & 1 \text{ s} \le t \le 200 \text{ s} \end{cases}$$

where $F = 0$ at $t = 0$. What property of a viscoelastic tissue is illustrated by this simulation? What should the figure look like for a viscoelastic tissue?

d. The tendon length versus time in response to the following change in force:

$$F = \begin{cases} 1000 - 1000\cos(2\pi \cdot 0.5t)\ \text{N} & 0 \le t \le 1\ \text{s} \\ 2000\ \text{N} & 1\ \text{s} \le t \le 200\ \text{s} \end{cases}$$

where $l = 220$ mm at $t = 0$. What property of a viscoelastic tissue is illustrated by this simulation? What should the figure look like for a viscoelastic tissue?

Notes:

- In this problem, the argument of the cosine function is in radians.
- The ode45 function in MATLAB is recommended for performing the simulations.
- Because you've been given an expression for the input as a function of time, you can find an expression for the time-derivative of the input as a function of time. With this, it is straightforward to find an expression for the time-derivative of the output as a function of time.

8. The external and internal ion concentrations for Sodium, Potassium, and Chloride for the Aplysia giant nerve cell are as follows:

	Concentration	
Ion	External	Internal
Potassium	10 mM	280 mM
Sodium	485 mM	61 mM
Chloride	485 mM	51 mM

a. Determine the Nernst potential for each ionic species above.

b. Is there any (one) membrane potential that could exist for which all three ions would be at equilibrium? Why or why not?

c. If the resting potential of the cell is -49 mV, which ions are at equilibrium and which are not? For ions that are not at equilibrium, in what direction will each ionic species move? Why?

d. What must be true for the cell at rest for the net ionic movement through the cell membrane? Why?

9. This problem deals with a comparison of the total charge that must move across a cell membrane during an action potential, as compared to the total charge that resides within the cell. Consider a cylindrical muscle fiber of 10 microns diameter.

a. If we recall that Q=CV where Q is charge, C can be the membrane capacitance, and V the membrane voltage change (excursion) during an action potential (e..g. from -80 mV up to a peak of 20 mV, how much charge must move across the cell membrane for a unit length of the cell (1 cm). Assume that the membrane capacitance is 1 micorFarad per cm.

b. Now please calculate the total charge within a unit length of the same cell just by Potassium (a major ionic constituent in the cell). Assume that intracellular Potassium concentration is 124 mM.

c. What is the ratio of the charge calculated above for (a) versus (b)? What is the implication in terms of the cell's function in terms of creating action potentials?