KIN 523 Winter 18
Homework Set 5 – 3D Kinematics & Anthropometry Pavol

Due: 2/22/18

Answer the following problems.
You must show all work, including formulas used, numbers substituted, and answers with units.
If Excel was used, e-mail a copy of the workbook to me.
If a computer program was used, either attach or e-mail me a copy of the program.
Your written portion must indicate what equations were used in Excel and/or your program.

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1. We used a motion capture system to record a classmate's movements. The motion capture system was calibrated using a right-handed *XYZ* coordinate system such that the global *Y* axis pointed vertically upward. Our participant performed a static trial of quiet standing, followed by several countermovement jumps. The individual faced in the +*X* direction during all trials. All trials were recorded at 120 frames/s.

   We attached 3 reflective markers to the left foot, at the heel (HEE), 2nd metatarsal (TOE), and 5th metatarsal head (MT5). The global *XYZ* positions of these markers during the static trial were, on average:

$$\vec{r}_{HEE} = \begin{bmatrix} 0.155 \\ 0.035 \\ -0.132 \end{bmatrix} m \qquad \vec{r}_{TOE} = \begin{bmatrix} 0.303 \\ 0.063 \\ -0.124 \end{bmatrix} m \qquad \vec{r}_{MT5} = \begin{bmatrix} 0.309 \\ 0.033 \\ -0.191 \end{bmatrix} m$$

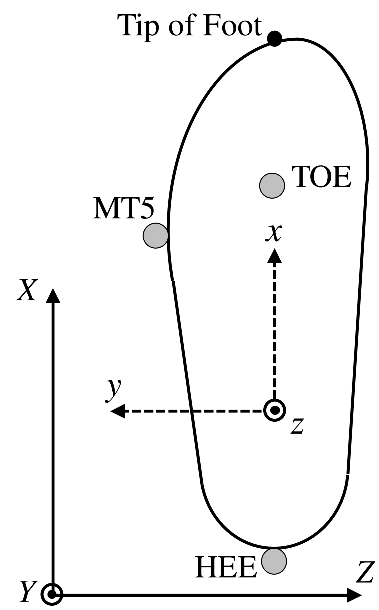
   From these markers, I defined a marker-based $x'y'z'$ coordinate system for the foot, in which:
   - The origin is located at the HEE marker;
   - The x' axis runs from the HEE to the TOE marker;
   - The z' axis is perpendicular to the plane of the HEE, TOE, and MT5 markers and points from plantar to dorsal;
   - The $x'y'z'$ axes form a right-handed coordinate system.

   Based on this convention, I used the methods from Homework Set 1 to find the orientations of the unit vectors of the marker-based $x'y'z'$ coordinate system of the foot during the static trial. The orientations of these unit vectors in the global *XYZ* coordinate system were:

$$\hat{\imath}' = \begin{bmatrix} 0.981 \\ 0.186 \\ 0.053 \end{bmatrix} \qquad \hat{\jmath}' = \begin{bmatrix} 0.125 \\ -0.400 \\ -0.908 \end{bmatrix} \qquad \hat{k}' = \begin{bmatrix} -0.147 \\ 0.898 \\ -0.415 \end{bmatrix}$$

   I have also defined an anatomical *xyz* coordinate system of the foot such that:
   - The *x* axis ($\hat{\imath}$ unit vector) points anteriorly;
   - The *y* axis ($\hat{\jmath}$ unit vector) points left;
   - The *z* axis ($\hat{k}$ unit vector) points from plantar to dorsal (distal to proximal).

During the static trial, we were careful to position our participant so her foot was aligned with the global *XYZ* axes (see figure). We will also assume that the insole of shoe angled downward at 5° from heel to toe. The orientations, in the global *XYZ* coordinate system, of the unit vectors of the anatomical *xyz* coordinate system of the foot during the static trial were thus:



$$\hat{\imath} = \begin{bmatrix} 0.996 \\ -0.087 \\ 0 \end{bmatrix} \qquad \hat{\jmath} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \qquad \hat{k} = \begin{bmatrix} 0.087 \\ 0.996 \\ 0 \end{bmatrix}$$

We also know that, during the static trial, the tip of the participant's foot was located anterior to the HEE by a distance equal to her foot length plus ½ the marker diameter, and was at floor height. The participant's foot length was 25.4 cm, the marker diameter was 9.5 mm, and floor height corresponded to $Y = 0$ in the global coordinate system.

a. *(0.5 pts)* Find the global *XYZ* position of the tip of the participant's foot during the static trial.

b. *(0.7 pts)* Using the answer to part (a), find the position of the tip of the foot with respect to the TOE marker within the marker-based $x'y'z'$ coordinate system of the foot.

c. *(0.9 pts)* Find the rotational transformation matrix (RTM) that will transform the unit vectors of the anatomical *xyz* coordinate system of the foot to the marker-based $x'y'z'$ coordinate system of the foot.

We now want to analyze one of the participant's jumping trials. At the peak height of her jump during this trial, the global *XYZ* positions of the markers attached to the foot were:

$$\vec{r}_{HEE} = \begin{bmatrix} 0.277 \\ 0.417 \\ -0.134 \end{bmatrix} m \qquad \vec{r}_{TOE} = \begin{bmatrix} 0.408 \\ 0.348 \\ -0.164 \end{bmatrix} m \qquad \vec{r}_{MT5} = \begin{bmatrix} 0.373 \\ 0.308 \\ -0.214 \end{bmatrix} m$$

Again using the methods from Homework Set 1, I found the corresponding orientations of the unit vectors of the marker-based $x'y'z'$ coordinate system of the foot at the peak height of the jump. The orientations of these vectors in the global *XYZ* coordinate system were:

$$\hat{\imath}' = \begin{bmatrix} 0.870 \\ -0.453 \\ -0.196 \end{bmatrix} \qquad \hat{\jmath}' = \begin{bmatrix} -0.451 \\ -0.567 \\ -0.689 \end{bmatrix} \qquad \hat{k}' = \begin{bmatrix} 0.201 \\ 0.688 \\ -0.697 \end{bmatrix}$$

d. *(0.7 pts)* Using the answer to part (b), find the global *XYZ* position of the tip of the foot when the participant was at the peak height of her jump.

Note: This will give us an estimate of how high the tip of her foot was from the ground at the peak height of her jump, even though we did not put a marker at the tip of the foot.

e. *(0.9 pts)* Using the answer to part (c), find the orientations, in the global *XYZ* coordinate system, of the unit vectors of the anatomical *xyz* coordinate system of the foot when the participant was at the peak height of her jump.

   Note: This information can later be used to compute the ankle angle.

2. From the recorded marker positions, the methods presented in class were also used to compute the orientations of the anatomical *xyz* coordinate systems of the left thigh and leg segments during the jumping trial from Problem 1. The unit vectors of these coordinate systems were defined using the same convention as in Problem 1. Assume that, at the knee, the leg moves relative to the thigh by a Cardan rotation sequence of *y*, *x*, *z*. For the left knee, positive rotations will correspond to flexion, abduction, and external rotation of the leg with respect to the thigh, respectively.

   At the instant of ground contact, the unit vectors of the thigh (*T*) and leg (*L*) relative to the global coordinate system were as follows:

$$\hat{\imath}_T = \begin{bmatrix} 0.962 \\ 0.110 \\ -0.251 \end{bmatrix} \qquad \hat{\jmath}_T = \begin{bmatrix} -0.258 \\ 0.053 \\ -0.965 \end{bmatrix} \qquad \hat{k}_T = \begin{bmatrix} -0.093 \\ 0.992 \\ 0.080 \end{bmatrix}$$

$$\hat{\imath}_L = \begin{bmatrix} 0.991 \\ -0.103 \\ 0.085 \end{bmatrix} \qquad \hat{\jmath}_L = \begin{bmatrix} 0.091 \\ 0.050 \\ -0.995 \end{bmatrix} \qquad \hat{k}_L = \begin{bmatrix} 0.098 \\ 0.993 \\ 0.059 \end{bmatrix}$$

   a. *(0.8 pts)* From the unit vectors, find the rotation matrix to convert a point from the thigh coordinate system to the leg coordinate system at the instant of ground contact. Show the resulting 3 x 3 matrix of numbers.

   b. *(1 pt)* Using the results of part (a) and the corresponding rotation matrix expressed in terms of the joint angles (given in your notes), compute and report the values of the knee flexion, abduction, and external rotation angles at the instant of ground contact.

   c. *(0.5 pts)* Based on the assumed *y*, *x*, *z* rotation sequence, determine and report the global *XYZ* components of the three unit vectors that correspond to the knee flexion axis, the knee abduction axis (*i.e.* line of nodes), and the knee external rotation axis at the instant of ground contact.

   d. *(0.3 pts)* For the left knee, positive rotations about the *x*, *y*, and *z* axes correspond to abduction, flexion, and external rotation, respectively. To what anatomical joint movements do positive rotations about the *x*, *y*, and *z* axes correspond for the right knee?

   e. *(0.5 pts)* Injuries to the anterior cruciate ligament (ACL) of the knee are of major concern to athletes. Non-contact injuries to the ACL typically occur shortly after ground contact during tasks involving large accelerations (e.g. cutting, landing). The ACL acts as a restraint to anterior translation of the tibia and knee hyperextension. It also acts as a secondary restraint to knee internal and external rotation, abduction, and adduction. *(continued on next page)*

Cadaver studies suggest that some of the greatest forces on the ACL occur when the knee is loaded when it is near full extension (<20° flexion) and in internal rotation and/or abduction (Markolf et al. 1995). In a prospective study, Hewett et al. (2005) found that females who went on to injure their ACL had greater knee abduction angles at ground contact (5° vs. -3° in uninjured) during a drop-jump task. Given this information and the results of part (b), assess the extent to which the individual recorded does or does not exhibit risk factors for ACL injury during landing.

3. To determine the forces and moments acting at the individual's ankle during her countermovement jump, we will need to know the mass of her foot and the foot's moment of inertia about its center of mass in the sagittal plane.

   Three sets of sources give the following anthropometric data for the foot:

   1) Dempster & Gaughran (1967); from 7 older male cadavers
      - Mass: $0.0145\ m_{BODY}$
      - Radius of Gyration: $0.475\ l_{ANK\text{-}MTH2}$

   2) Zatsiorsky et al. (1990); from 15 young adult women
      - Mass: $0.0129\ m_{BODY}$
      - Radius of Gyration: $0.299\ l_{FOOT}$

   3) Vaughn et al.(1992), Hinrichs (1985); from 12 older male cadavers
      - Mass: $0.0083\ m_{BODY} + (2.545 \times 10^{-7}\ \text{kg/mm}^3)\ l_{FOOT}\ h_{ANKLE}\ w_{ANKLE} - 0.065\ \text{kg}$
      - Moment of inertia: $(67.508\ \text{kg mm})\ l_{FOOT} - (42.725\ \text{kg mm})\ h_{ANKLE} - 10542\ \text{kg mm}^2$

   For our individual, who was a young adult woman:

   m_{BODY} = body mass = 56.3 kg
   l_{FOOT} = foot length = 254 mm
   $l_{ANK\text{-}MTH2}$ = length from ankle to 2nd metatarsal head = 142 mm
   h_{ANKLE} = ankle height = 91 mm
   w_{ANKLE} = ankle width = 61 mm

   All body measurements were made with shoes on.

   a. *(1 pt)* For each of the three sets of sources, determine and report: i) the mass of the foot and ii) the foot's moment of inertia about its center of mass.

   b. *(0.5 pts)* Which values for foot mass and moment of inertia would you use in your analysis, and why? Give the reasons that the values you chose were the most appropriate, and the reasons that each set of values that you did not choose were less appropriate.

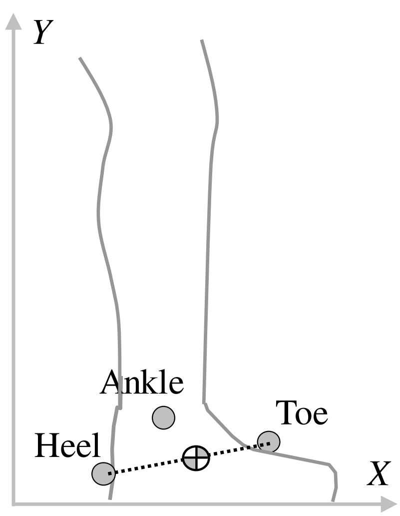
4. To determine the forces and moments acting at the individual's ankle during the countermovement jump, we will also need to know the position and acceleration of the foot center of mass.

   The comma-delimited file "Jump_HW5.csv", posted on Canvas, contains the positions of selected markers during the jumping trial from Problems 1 and 2. The format of the file is as follows:

Column	Label	Description	Units
1	Frame	Frame number	frames
2	ANKX	Ankle marker X position	m
3	ANKY	Ankle marker Y position	m
4	HEEX	Heel marker X position	m
5	HEEY	Heel marker Y position	m
6	TOEX	Toe marker X position	m
7	TOEY	Toe marker Y position	m

   The marker positions were low-pass filtered with a 4th-order, Butterworth, no-lag filter with a cut-off frequency of 18 Hz.

   a. *(1 pt)* For simplicity, assume that the foot center of mass is located along the line from the heel marker to the toe marker (see figure), at a distance of 40.1% of foot length from the heel marker. The length of our participant's foot was 25.4 cm. Determine the global (*X*, *Y*) position of the foot center of mass as a function of time during the jumping trial.

      Create a graph in which you show the global (*X*, *Y*) trajectories of 4 points over the course of the jumping trial: (i) the heel marker, (ii) the toe marker, (iii) the ankle marker, and (iv) the foot center of mass. All four trajectories should be plotted on the same graph (and labeled), with the *X* location plotted on the horizontal axis and the corresponding *Y* location plotted on the vertical axis.

   b. *(0.7 pts)* Using the results of part (a), compute and then graph the acceleration of the foot center of mass in the global *Y* direction as a function of time during the jumping trial. (*Hint: See the "Planar Kinematics" notes*)