KIN 523: Homework 1

Due on Thursday, January 18, 2018

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Problem 1

a. $\hat{\imath} = \begin{bmatrix} 0.9866 \\ 0.0996 \\ -0.1292 \end{bmatrix}$, $\hat{\jmath} = \begin{bmatrix} -0.1578 \\ 0.3825 \\ -0.9104 \end{bmatrix}$, $\hat{k} = \begin{bmatrix} -0.0413 \\ 0.9186 \\ 0.3931 \end{bmatrix}$. Refer to lines 5 through 39 in the code below.

b. $WJC = \begin{bmatrix} 281.7 \\ 3.5 \\ 1128.1 \end{bmatrix}$ in global XYZ coordinate system. Refer to lines 12 and 18 in the code below.

c. $WJC = \begin{bmatrix} -8.1391 \\ 20.8544 \\ -7.8006 \end{bmatrix}$ in local hand xyz coordinate system. Refer to line 51 in the code below.

d. $\vec{x} = \begin{bmatrix} 0.1292 \\ 0.9104 \\ -0.3931 \end{bmatrix}$ in the $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ directions, respectively.

$$\vec{e} = e_x\hat{\imath} + e_y\hat{\jmath} + e_z\hat{k} \rightarrow A\vec{x} = \vec{e}, \text{ where } A = \begin{bmatrix}\hat{\imath}, \hat{\jmath}, \hat{k}\end{bmatrix} \text{ from Problem 1a, } \vec{x} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}, \text{ and } \vec{e} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}.$$

To find the components of $\vec{e}$ in the $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ directions, I solved $\vec{x} = A^{-1}\vec{e}$. Refer to lines 54 through 58 in the code below.

```
1  %% Problem 1
2  close all;
3  clear;
4  
5  % r_MH3: 3rd metacarpal head (knuckle of the middle finger)
6  % r_MH5: 5th metacarpal head (knuckle of the pinky)
7  % r_WRI: lunate bone at the wrist
8  
9  % global XYZ positions of the three markers, in mm
10 r_MH3 = [366.0; 10.9; 1139.5];
11 r_MH5 = [350.4; -29.1; 1125.4];
12 r_WRI = [292.7; 3.5; 1149.1];
13 
14 % wrist joint center offset from WRI marker
15 offset = [-11; 0; -21];
16 
17 % actual wrist joint center position in global XYZ
18 WJC = r_WRI + offset;
19 
20 % u vector from WRI to MH3 markers
21 u = r_MH3 - r_WRI; % x-axis
22 norm_u = norm(u, 2); % magnitude of u
23 
24 % v vector from WRI to MH5 markers
25 v = r_MH5 - r_WRI; % vector b/w WRI and MH5
26 norm_v = norm(v, 2); % magnitude of v
27 
28 % w vector perpendicular to the v and u vectors
29 w = cross(u, v, 1); % y-axis
```

```
norm_w = norm(w, 2); % magnitude of w

% unit vectors of hand xyz coordinate system
i = u / norm_u; % unit vector in x
j = w / norm_w; % unit vector in y
k = cross(i, j, 1); % unit vector in z

% cat vectors for data processing ahead
hand_marks = [r_MH3, r_MH5, r_WRI];
unit_vecs = [i, j, k];

% transpose of unit vector matrix
tp_unit_vecs = transpose(unit_vecs);

% global XYZ origin
XYZ = [0; 0; 0];
X = [1; 0; 0];
Y = [0; 1; 0];
Z = [0; 0; 1];

% conversion of wrist joint center position in local hand xyz
WJC_xyz = tp_unit_vecs * (WJC - r_WRI);

% unit vector for hand's anterior axis in global XYZ
e_XYZ = [0; 0; -1];

% values of e_x, e_y, e_z corresponding to the components of e_XYZ in the
% i, j, k directions
e_ijk = unit_vecs \ e_XYZ;
```

Problem 2

a. Table of the global XYZ components of each unit vector, $\hat{i}$, $\hat{j}$, and $\hat{k}$, at each sample time step during the movement trial. Refer to lines 5 through 39 in the code below.

Time	$\hat{i}_X$	$\hat{i}_Y$	$\hat{i}_Z$	$\hat{j}_X$	$\hat{j}_Y$	$\hat{j}_Z$	$\hat{k}_X$	$\hat{k}_Y$	$\hat{k}_Z$
0.0000	0.0780	-0.9862	0.1463	0.2602	-0.1215	-0.9579	0.9624	0.1128	0.2471
0.0333	0.0769	-0.9861	0.1475	0.2568	-0.1233	-0.9586	0.9634	0.1115	0.2437
0.0667	0.0736	-0.9865	0.1464	0.2443	-0.1245	-0.9617	0.9669	0.1065	0.2319
0.1000	0.0735	-0.9874	0.1398	0.2473	-0.1178	-0.9618	0.9662	0.1052	0.2355
0.1333	0.0627	-0.9880	0.1412	0.2314	-0.1233	-0.9650	0.9708	0.0932	0.2209
0.1667	0.0567	-0.9895	0.1327	0.1961	-0.1193	-0.9733	0.9789	0.0812	0.1873
0.2000	0.0592	-0.9920	0.1113	0.1611	-0.1005	-0.9818	0.9852	0.0761	0.1539
0.2333	0.0514	-0.9953	0.0823	0.1603	-0.0731	-0.9844	0.9857	0.0638	0.1558
0.2667	0.0432	-0.9983	0.0402	0.1509	-0.0333	-0.9880	0.9876	0.0488	0.1492
0.3000	0.0437	-0.9989	-0.0143	0.1379	0.0202	-0.9902	0.9895	0.0413	0.1386
0.3333	0.0399	-0.9965	-0.0728	0.0990	0.0765	-0.9921	0.9943	0.0323	0.1017
0.3667	0.0301	-0.9880	-0.1512	0.0808	0.1532	-0.9849	0.9963	0.0175	0.0844
0.4000	0.0260	-0.9690	-0.2458	0.0758	0.2471	-0.9660	0.9968	0.0065	0.0799

0.4333	0.0342	-0.9403	-0.3387	0.0381	0.3399	-0.9397	0.9987	0.0192	0.0475
0.4667	0.0441	-0.8934	-0.4471	-0.0106	0.4471	-0.8944	0.9990	0.0442	0.0102
0.5000	0.0371	-0.8520	-0.5223	-0.0432	0.5208	-0.8526	0.9984	0.0542	-0.0175
0.5333	0.0287	-0.7820	-0.6226	-0.1423	0.6134	-0.7769	0.9894	0.1108	-0.0937
0.5667	0.0216	-0.7011	-0.7127	-0.1870	0.6974	-0.6918	0.9821	0.1482	-0.1161
0.6000	-0.0020	-0.6128	-0.7903	-0.2153	0.7720	-0.5981	0.9766	0.1689	-0.1335
0.6333	-0.0204	-0.5109	-0.8594	-0.2372	0.8375	-0.4923	0.9713	0.1938	-0.1383
0.6667	-0.0336	-0.4113	-0.9109	-0.2241	0.8913	-0.3942	0.9740	0.1909	-0.1221
0.7000	-0.0470	-0.2906	-0.9557	-0.1606	0.9465	-0.2799	0.9859	0.1404	-0.0912
0.7333	-0.0921	-0.1575	-0.9832	-0.0221	0.9875	-0.1561	0.9955	0.0073	-0.0944
0.7667	-0.0075	-0.0524	-0.9986	0.2651	0.9628	-0.0525	0.9642	-0.2651	0.0067
0.8000	0.0524	0.0391	-0.9979	0.5416	0.8384	0.0613	0.8390	-0.5436	0.0228
0.8333	0.1216	0.1124	-0.9862	0.7505	0.6398	0.1654	0.6496	-0.7603	-0.0065
0.8667	0.1746	0.1488	-0.9733	0.8437	0.4870	0.2258	0.5076	-0.8606	-0.0405
0.9000	0.2192	0.1477	-0.9644	0.8797	0.3976	0.2609	0.4220	-0.9056	-0.0428
0.9333	0.2549	0.1159	-0.9600	0.9025	0.3279	0.2792	0.3471	-0.9376	-0.0211
0.9667	0.3027	0.1075	-0.9470	0.9306	0.1809	0.3181	0.2056	-0.9776	-0.0453
1.0000	0.3354	0.0810	-0.9386	0.9406	0.0266	0.3384	0.0524	-0.9964	-0.0672
1.0333	0.3367	0.0183	-0.9414	0.9391	-0.0792	0.3343	-0.0684	-0.9967	-0.0438
1.0667	0.3437	-0.0333	-0.9385	0.9194	-0.1918	0.3435	-0.1914	-0.9809	-0.0353
1.1000	0.3610	-0.0789	-0.9292	0.8917	-0.2627	0.3687	-0.2732	-0.9616	-0.0244
1.1333	0.3953	-0.1190	-0.9108	0.8552	-0.3141	0.4122	-0.3352	-0.9419	-0.0224
1.1667	0.4796	-0.1194	-0.8693	0.8203	-0.2907	0.4925	-0.3115	-0.9493	-0.0415
1.2000	0.5246	-0.1620	-0.8358	0.7569	-0.3606	0.5450	-0.3896	-0.9186	-0.0666
1.2333	0.6618	-0.1566	-0.7331	0.6974	-0.2300	0.6787	-0.2749	-0.9605	-0.0430
1.2667	0.7282	-0.1818	-0.6608	0.5831	-0.3422	0.7368	-0.3601	-0.9219	-0.1431
1.3000	0.7943	-0.1881	-0.5777	0.5129	-0.3021	0.8035	-0.3257	-0.9345	-0.1435
1.3333	0.8503	-0.1962	-0.4883	0.4236	-0.2956	0.8563	-0.3124	-0.9349	-0.1683
1.3667	0.9033	-0.1866	-0.3862	0.3265	-0.2849	0.9012	-0.2782	-0.9402	-0.1965
1.4000	0.9458	-0.1629	-0.2811	0.2525	-0.1758	0.9515	-0.2044	-0.9708	-0.1252
1.4333	0.9679	-0.1530	-0.1996	0.1619	-0.2281	0.9601	-0.1924	-0.9615	-0.1960
1.4667	0.9840	-0.1491	-0.0979	0.0627	-0.2244	0.9725	-0.1669	-0.9630	-0.2114
1.5000	0.9885	-0.1511	0.0071	-0.0347	-0.1810	0.9829	-0.1473	-0.9718	-0.1842
1.5333	0.9868	-0.1523	0.0555	-0.0789	-0.1523	0.9852	-0.1416	-0.9765	-0.1623
1.5667	0.9838	-0.1461	0.1039	-0.1227	-0.1264	0.9844	-0.1307	-0.9812	-0.1423
1.6000	0.9790	-0.1476	0.1407	-0.1619	-0.1428	0.9764	-0.1240	-0.9787	-0.1637
1.6333	0.9756	-0.1437	0.1661	-0.1896	-0.1701	0.9670	-0.1107	-0.9749	-0.1932
1.6667	0.9724	-0.1476	0.1808	-0.2084	-0.1999	0.9574	-0.1052	-0.9686	-0.2252
1.7000	0.9688	-0.1408	0.2042	-0.2313	-0.2154	0.9487	-0.0896	-0.9663	-0.2413
1.7333	0.9607	-0.1225	0.2493	-0.2692	-0.1900	0.9441	-0.0683	-0.9741	-0.2155
1.7667	0.9583	-0.1237	0.2577	-0.2788	-0.2054	0.9381	-0.0631	-0.9708	-0.2313
1.8000	0.9565	-0.1203	0.2658	-0.2851	-0.1920	0.9391	-0.0620	-0.9740	-0.2180
1.8333	0.9545	-0.1080	0.2780	-0.2941	-0.1873	0.9372	-0.0491	-0.9763	-0.2105
1.8667	0.9551	-0.1048	0.2770	-0.2923	-0.1832	0.9386	-0.0476	-0.9775	-0.2056
1.9000	0.9567	-0.0978	0.2740	-0.2875	-0.1743	0.9418	-0.0444	-0.9798	-0.1949
1.9333	0.9553	-0.0958	0.2797	-0.2919	-0.1552	0.9438	-0.0470	-0.9832	-0.1762
1.9667	0.9564	-0.0963	0.2756	-0.2870	-0.1370	0.9481	-0.0536	-0.9859	-0.1587
2.0000	0.9566	-0.0975	0.2746	-0.2859	-0.1321	0.9491	-0.0563	-0.9864	-0.1543

Problem 2 continued on next page...

b. Table of the global XYZ components of the unit vector, $\vec{e}$, at each sample time during the movement trial. Refer to lines 64 through 79 in the code below.

Time	$\vec{e}_X$	$\vec{e}_Y$	$\vec{e}_Z$
0	-0.1313	-0.2823	-0.9503
0.0333	-0.135	-0.2835	-0.9494
0.0667	-0.1481	-0.2827	-0.9477
0.1	-0.1452	-0.2762	-0.9501
0.1333	-0.1628	-0.2765	-0.9471
0.1667	-0.1989	-0.2684	-0.9425
0.2	-0.2329	-0.2496	-0.9399
0.2333	-0.2349	-0.2203	-0.9467
0.2667	-0.2452	-0.1785	-0.9529
0.3	-0.2578	-0.127	-0.9578
0.3333	-0.2956	-0.0718	-0.9526
0.3667	-0.3142	0.0049	-0.9494
0.4	-0.3194	0.0972	-0.9426
0.4333	-0.3534	0.1804	-0.9179
0.4667	-0.3966	0.2743	-0.8761
0.5	-0.427	0.3427	-0.8368
0.5333	-0.5147	0.4138	-0.7509
0.5667	-0.5535	0.4861	-0.6763
0.6	-0.5801	0.5572	-0.5941
0.6333	-0.6003	0.6203	-0.5049
0.6667	-0.5912	0.6832	-0.4285
0.7	-0.5398	0.769	-0.3424
0.7333	-0.4233	0.8758	-0.2321
0.7667	-0.1386	0.974	-0.1794
0.8	0.17	0.982	-0.0821
0.8333	0.4436	0.8958	0.0257
0.8667	0.5912	0.8009	0.0958
0.9	0.6633	0.737	0.1297
0.9333	0.7181	0.682	0.1384
0.9667	0.8056	0.5629	0.185
1	0.8791	0.4263	0.2132
1.0333	0.9254	0.322	0.1999
1.0667	0.9566	0.2067	0.2053
1.1	0.9658	0.1286	0.2252
1.1333	0.9614	0.0689	0.2664
1.1667	0.9312	0.0931	0.3523
1.2	0.91	0.0119	0.4143
1.2333	0.8285	0.1479	0.5401
1.2667	0.7665	0.0274	0.6417
1.3	0.6976	0.068	0.7133
1.3333	0.6182	0.073	0.7826
1.3667	0.5233	0.0861	0.8478
1.4	0.4324	0.2005	0.8791
1.4333	0.3481	0.1505	0.9253
1.4667	0.2499	0.155	0.9558

1.5	0.154	0.1977	0.9681
1.5333	0.1113	0.2255	0.9679
1.5667	0.0667	0.2517	0.9655
1.6	0.0278	0.2356	0.9715
1.6333	-0.0031	0.2098	0.9777
1.6667	-0.0227	0.1796	0.9835
1.7	-0.0501	0.1655	0.9849
1.7333	-0.0942	0.1941	0.9765
1.7667	-0.1052	0.1786	0.9783
1.8	-0.1116	0.1925	0.9749
1.8333	-0.1251	0.1993	0.9719
1.8667	-0.124	0.2039	0.9711
1.9	-0.1207	0.2138	0.9694
1.9333	-0.1239	0.2328	0.9646
1.9667	-0.1166	0.2504	0.9611
2	-0.1145	0.2548	0.9602

To find the XYZ components of $\vec{e}$ in the global XYZ coordinate system, I solved $A\vec{x} = \vec{e}$ at each sample time, where $A = \left[\hat{i}, \hat{j}, \hat{k}\right]$ at each sample time from Problem 2a, and $\vec{x} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$ from Problem 1d.

c. Table of the global XYZ positions of the wrist joint center (WJC) at the start of the movement trial (t=0). Refer to lines 57 through 62 in the code below.

Time	WJC_X	WJC_Y	WJC_Z
0	131.157	-654.5617	1641.2146

d. i. At the start of the trial, the right shoulder is in horizontal shoulder abduction in the transverse plane and in shoulder abduction in the frontal plane, where the hand is roughly at shoulder height (or the arm is parallel to the floor).

ii. From the start orientation, the right shoulder adducts in the frontal plane until the hand reaches the side of the body (medial direction), then the shoulder flexes in the sagittal plane until the hand is roughly at shoulder height again.

iii. The right hand/wrist starts at shoulder height with the palmar side facing downwards towards the floor, and then moves medially to the side of the body in the frontal plane while keeping the palmar side of the hand/wrist facing medially (palm facing body). At the side of the body, the hand/wrist (radioulnar) supinates (external rotation) and the palm faces anteriorly (palm faces forward). Sequentially, the wrist moves away from the body as the shoulder flexes in the sagittal plane with the palmar side of the hand/wrist facing forward/up (palm faces away from body) until the hand/wrist is roughly at shoulder height again and the dorsum side of the hand/wrist is facing downwards towards the floor (palmer side facing upwards).

```
%% Problem 2
close all;
clear;
```

```matlab
% load parameters
load('hand.mat');

% global XYZ positions of the three markers, in mm
MH3 = [MH3X, MH3Y, MH3Z];
MH5 = [MH5X, MH5Y, MH5Z];
WRI = [WRIX, WRIY, WRIZ];

% wrist joint center position in local hand xyz
% from problem 1c
WJC_xyz = [-8.13912848115047; 20.8543822348700; -7.80059800074728];

% u vector from WRI to MH3 markers
U = MH3 - WRI; % x-axis
norm_U = vecnorm(U, 2, 2); % magnitude of u

% v vector from WRI to MH5 markers
V = MH5 - WRI; % vector b/w WRI and MH5
norm_V = vecnorm(V, 2, 2); % magnitude of v

% w vector perpendicular to the v and u vectors
W = cross(U, V, 2); % y-axis
norm_W = vecnorm(W, 2, 2); % magnitude of w

% unit vectors of hand xyz coordinate system
I = U ./ norm_U; % unit vector in x
J = W ./ norm_W; % unit vector in y
K = cross(I, J, 2); % unit vector in z

% cat vectors for data processing ahead
HAND_MARKS = [MH3, MH5, WRI];
UNIT_VECS = [I, J, K];

% table for problem 2a
table_2a = [Time, UNIT_VECS];

% transpose of unit vectors
tp_I = transpose(I);
tp_J = transpose(J);
tp_K = transpose(K);

% reshape matrix of unit vectors
UNIT_VECS_stacked = cat(3, I, J, K);
UNIT_VECS_rs = permute(UNIT_VECS_stacked, [2, 3, 1]);
UNIT_VECS_rs2 = permute(UNIT_VECS_stacked, [3, 2, 1]);

% global XYZ origin
XYZ = [0, 0, 0];
X = [1, 0, 0];
Y = [0, 1, 0];
Z = [0, 0, 1];

% conversion of wrist joint center position in global XYZ
```

```
WJC_XYZ = (transpose([I(1, :); J(1, :); K(1, :)]) * WJC_xyz) + transpose(WRI(1, :));
WJC_XYZ = transpose(WJC_XYZ);

% table for problem 2c
table_2c = [Time(1,:) , WJC_XYZ];

% components of e_XYZ in the i, j, k directions; from problem 1d
e_ijk = [0.129213880302701; 0.910385524181768; -0.393067386713025];

% create empty array
E = [];

% global XYZ components of e in the global XYZ coordinate system at each
% sample time
for rows = 1:size(UNIT_VECS_rs, 3)
    e_IJK = UNIT_VECS_rs(:, :, rows) * e_ijk;
    E = cat(2, E, e_IJK);
end
E = transpose(E);

% table for problem 2b
table_2b = [Time, E];
```

Problem 3

a. Sagittal-plane analysis: Degree of knee flexion during the squat phase of a backflip.

b. Frontal-plant analysis: Foot pronation in footstrike during walking/running, including subtalar eversion, ankle dorsiflexion, and forefoot abduction.

c. Transverse-place analysis: Hand and forearm movement with a computer mouse on a flat surface (e.g. office desk).

d. 3D (multi-planar) analysis: Energy flow and upper limb joint kinetics/kinematics (combinations of elbow and shoulder flexion/extension and rotations) during the serve of tennis players, including joint forces and torques of the racket arm.