CHE 581: Assignment 5

Due on Friday, March 1, 2019

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Contents

Problem 22.1

a Analytically

$\frac{dy}{dt} = yt^2 - 1.1y$

$\frac{dy}{dt} = y(t^2 - 1.1)$

$\frac{dy}{y} = (t^2 - 1.1)dt$

$\int \frac{dy}{y} = \int (t^2 - 1.1)dt$

$ln(y) = (\frac{t^3}{3} - 1.1t)$

$e^{ln(y)} = e^{(\frac{t^3}{3} - 1.1t)}$

$y = e^{(\frac{t^3}{3} - 1.1t)}$

b Euler's Method with $h = 0.5$, 0.25

$f(t, y) = \frac{dy}{dt}$

For $h = 0.5$:

$y(0) = 1$

$y(0.5) = y(0) + f(0, 1)(0.5)$
$y(0.5) = e^{\frac{(0)^3}{3} - 1.1(0)} + ((1)(0)^2 - 1.1(1))(0.5)$
$y(0.5) = 0.4500$

$y(1) = y(0.5) + f(0.5, y(0.5))(0.5)$
$y(1) = 0.2587$

For $h = 0.25$:

$y(0) = 1$

$y(0.25) = y(0) + f(0, 1)(0.25)$
$y(0.25) = 0.7250$

$y(0.5) = y(0.25) + f(0.25, y(0.25))(0.25)$
$y(0.5) = 0.5370$

Problem 22.1 continued on next page...

$y(0.75) = y(0.5) + f(0.5, y(0.5))(0.25)$
$y(0.75) = 0.4229$

$y(1) = y(0.75) + f(0.75, y(0.75))(0.25)$
$y(1) = 0.3660$

c Midpoint Method with $h = 0.5$

$y(0) = 1$

$y(0.25) = y(0) + f(0, 1)(\frac{0.5}{2})$
$y(0.25) = -0.7522$

$y(0.5) = y(0) + f(0.25, y(0.25))(0.5)$
$y(0.5) = 0.6239$

$y(0.75) = y(0.5) + f(0.5, y(0.5))(\frac{0.5}{2})$
$y(0.75) = -0.2641$

$y(1) = y(0.5) + f(0.75, y(0.75))(0.5)$
$y(1) = 0.4919$

d 4th-Order RK Method with $h = 0.5$

$y(0) = 1$

$k_{1,1} = f(0, 1)$
$k_{1,1} = -1.100$
$y_{mid1,1} = y(0) + k_{1,1}(\frac{0.5}{2})$
$y_{mid1,1} = 0.7250$
$k_{2,1} = f(0.25, y_{mid1,1})$
$k_{2,1} = -0.7522$
$y_{mid2,1} = y(0) + k_{2,1}(\frac{0.5}{2})$
$y_{mid2,1} = 0.8120$
$k_{3,1} = f(0.25, y_{mid2,1})$
$k_{3,1} = -0.8424$
$y(0.5) = y(0) + k_{3,1}(0.5)$
$y(0.5) = 0.5788$
$k_{4,1} = f(0.5, y(0.5))$
$k_{4,1} = -0.4920$
$\phi_1 = \frac{1}{6}(k_{1,1} + 2k_{2,1} + 2k_{3,1} + k_{4,1})$
$\phi_1 = -0.7969$

$y(0.5) = y(0) + \phi_1(0.5)$
$y(0.5) = 0.6016$

$k_{1,2} = f(0.5, 0.6016)$
$k_{1,2} = -0.5113$
$y_{mid1,2} = y(0.5) + k_{1,2}(\frac{0.5}{2})$
$y_{mid1,2} = 0.4737$
$k_{2,2} = f(0.75, y_{mid1,2})$
$k_{2,2} = -0.2546$
$y_{mid2,2} = y(0.5) + k_{2,2}(\frac{0.5}{2})$
$y_{mid2,2} = 0.5379$
$k_{3,2} = f(0.75, y_{mid2,2})$
$k_{3,2} = -0.2891$
$y(1) = y(0.5) + k_{3,2}(0.5)$
$y(1) = 0.4570$
$k_{4,2} = f(1, y(1))$
$k_{4,2} = -0.0457$
$\phi_2 = \frac{1}{6}(k_{1,2} + 2k_{2,2} + 2k_{3,2} + k_{4,2})$
$\phi_2 = -0.2741$

$y(1) = y(0.5) + \phi_2(0.5)$
$y(1) = 0.4645$