Homework 1 – Vector Math and Describing Human Motion

Due: 1/18/18

Answer the following problems.

You must show all work, including formulas used, numbers substituted, and answers with units. If Excel was used, e-mail a copy of the workbook to me.

If a computer program was used, either attach or e-mail me a copy of the program.

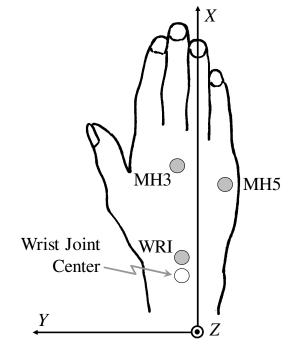
Your written portion must indicate what equations were used in Excel and/or your program.

You are performing a 3D analysis of hand motion during a task. You have placed three markers on the dorsal surface of the right hand (see figure):

- MH3: 3rd Metacarpal Head (knuckle of the middle finger)
- MH5: 5th Metacarpal Head (knuckle of the pinky)
- WRI: Lunate Bone at the wrist

The global coordinate system has been defined such that X points forward, Y points left, and Z points up.

1. You begin by filming a static trial in which the hand is held stationary with the palm facing straight down. In this position, the global XYZ positions of the three markers, in mm. are:



$$\vec{r}_{MH3} = \begin{bmatrix} 366.0 \\ 10.9 \\ 1139.5 \end{bmatrix} \text{ mm}$$
 $\vec{r}_{MH5} = \begin{bmatrix} 350.4 \\ -29.1 \\ 1125.4 \end{bmatrix} \text{ mm}$ $\vec{r}_{WRI} = \begin{bmatrix} 292.7 \\ 3.5 \\ 1149.1 \end{bmatrix} \text{ mm}$

$$\vec{r}_{MH5} = \begin{vmatrix} 350.4 \\ -29.1 \\ 1125.4 \end{vmatrix}$$
mn

$$\vec{r}_{WRI} = \begin{bmatrix} 292.7 \\ 3.5 \\ 1149.1 \end{bmatrix}$$
mm

- a. (2 pts) Use the global XYZ positions of the markers during the static trial to construct a local hand xyz coordinate system.
 - The origin of the hand coordinate system should be at the wrist marker;
 - The hand x axis should point from the wrist to the 3rd metacarpal head marker;
 - The hand y axis should be perpendicular to the plane of the markers and should point from dorsum to palm (i.e. generally downward);
 - The hand xyz axes should form a right-handed coordinate system.

Based on this convention, find the orientation of the three unit vectors of the hand xyz coordinate system (i.e. $\hat{i}, \hat{j}, \hat{k}$) in terms of the [X,Y,Z] components of each unit vector in the global XYZ coordinate system during the static trial.

- b. (0.5 pts) During the static trial, the actual wrist joint center was -11 mm from the wrist marker in the global X direction and -21 mm (= 1/2 of the wrist thickness + 1/2 of the marker diameter) from the wrist marker in the global Z direction. What was the [X,Y,Z] position of the wrist joint center in the global XYZ coordinate system?
- c. (0.7 pts) What was the [x,y,z] position of the wrist joint center in the local hand xyz coordinate system during the static trial? *Hint: This involves converting the position from the global XYZ coordinate system to the local hand xyz coordinate system.*
- d. $(0.9 \, pts)$ Assume that the true anterior axis of the hand pointed in the global -Z direction during the static trial. The unit vector for the hand's anterior axis, \vec{e} , is thus [0,0,-1] in the global XYZ coordinate system during the trial. This unit vector can also be expressed as a linear combination of the unit vectors \hat{i} , \hat{j} , \hat{k} of the local hand xyz coordinate system:

$$\vec{e} = e_x \hat{i} + e_y \hat{j} + e_z \hat{k} \tag{1}$$

where e_x , e_y , and e_z correspond to the components of \vec{e} in the \hat{i} , \hat{j} , and \hat{k} directions, respectively. Using the results of problem 1(a), find the values of e_x , e_y , and e_z . Hint: There is a vector operation that allows you to determine the component of a vector in the direction of a unit vector; use it.

This basic procedure is used in biomechanics research to calibrate a kinematic model of a participant. It establishes relationships between marker placements and both the positions of anatomical landmarks and the orientations of body segment axes.

2. You now perform a dynamic trial in which you film the hand at 30 frames/s as it is moved through space. During the movement, the person stood facing in the global +*X* direction and moved the hand through motion at the right shoulder only, keeping the elbow fully extended and the radioulnar and wrist joints in their neutral (i.e. anatomical reference) orientations.

A comma-delimited text file "Hand.csv" has been posted to Canvas with the resulting marker position data in the global *XYZ* coordinate system. The first row of the file contains header information. Each subsequent row comprises a time-sample of data:

Column	Label	Contents	Units
1	Time	Time	sec
2	MH3X	Global <i>X</i> position of MH3	mm
3	MH3Y	Global <i>Y</i> position of MH3	mm
4	MH3Z	Global Z position of MH3	mm
5	MH5X	Global <i>X</i> position of MH5	mm
6	MH5Y	Global <i>Y</i> position of MH5	mm
7	MH5Z	Global Z position of MH5	mm
8	WRIX	Global <i>X</i> position of WRI	mm
9	WRIY	Global <i>Y</i> position of WRI	mm
10	WRIZ	Global Z position of WRI	mm

- a. (1.5 pts) Using the global [X,Y,Z] positions of the markers in "Hand.csv", reconstruct the three unit vectors of the local hand xyz coordinate system at each sample time. Use the same axis definitions as in problem 1(a). Turn in a table that shows the global [X,Y,Z] components of each unit vector (i.e. \hat{i},\hat{j},\hat{k}) at each sample time during the trial.
 - Note: If you use Excel, enter the equation for the first row of data using the cell name (e.g. B2) in place of the data value, and then copy and paste the equation to the remaining rows. Excel will automatically change the row number in the equation.
- b. $(0.9 \, pts)$ Using the results of problems 1(d) and 2(a), along with equation (1), determine the orientation of the unit vector for the hand's anterior axis, \vec{e} , in the global XYZ coordinate system at each sample time during the trial. Turn in a table that shows the global [X,Y,Z] components of the unit vector \vec{e} at each sample time.
 - *Note: You may include the answers to problems 2(a) and 2(b) in the same table.*
- c. (0.7 pts) Assume that the [x,y,z] position of the wrist joint center in the local hand xyz coordinate system is always the same as was computed in problem 1(c). Use that result and the results of problem 2(a) to compute the global [X,Y,Z] position of the wrist joint center at the start of the movement (i.e. at time = 0). Hint: This involves converting the position from the local hand xyz coordinate system to the global XYZ coordinate system.
- d. (1.8 pts) Based on the results of problems 2(a) and 2(b), qualitatively describe:
 - i) The manner in which the right shoulder was rotated, with respect to the anatomical position, at the start of the trial;
 - ii) The sequence of rotations that occurred at the right shoulder over the course of the trial;
 - iii) The translation (i.e. linear motion) of the right wrist with respect to the body over the course of the trial

In each of (i) - (iii), you must use the correct anatomical terms.

Note: The components from problems 2(a) and 2(b) will tell you how the unit vectors of the hand are oriented relative to the global XYZ coordinate system. For example, if the \hat{i} unit vector of the hand, expressed in the global XYZ coordinate system, is:

$$\hat{i} = \begin{bmatrix} 0\\0.52\\-0.85 \end{bmatrix}$$

then the fingers are pointing mainly down (-Z) but also a little to the left (+Y). Another of the unit vectors will tell you which way the palm is facing.

These basic procedures are used in biomechanics research to derive the locations of anatomical landmarks and the orientations of body segment axes from the marker data collected during movement.

- 3. (1 pt) When performing a kinematic analysis, it is critical to select an appropriate approach. For each of the following types of kinematic analysis, provide an example of an athletic skill, an occupational task, an activity of daily living, or an aspect/component of one of these (e.g. ankle eversion during stance phase in running) for which the analysis would be appropriate. You may not use any of the examples presented in class.
 - a. Sagittal-plane analysis
 - b. Frontal-plane analysis
 - c. Transverse-plane analysis
 - d. Three-dimensional (multiplanar) analysis