

21.13

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Additional Problem

For the following function $f(x)$,

```
f=@(xx) sin(50.*xx).*xx.^2
```

Develop an “m” to compute its derivative on the interval x in $(0,1)$. Assign the interval, the spacing h , and the function values using a command like:

```
x=linspace(0,1,n);
```

```
h = 1/n;
```

```
y = f(x);
```

Then, assign a “new” variable, xx , by

```
xx=x(2:n-1);
```

Assess the numerical derivatives only at the points “ xx ”; the advantage of using the interval defined by “ xx ” is that the necessary values of the function f will always be defined, regardless of which kind of derivative you need to compute. In other words, there is always an $f(1)$ and $f(n)$ defined because of the way that “ xx ” is defined.

Using this as a starting point, please do the following.

- Compute the derivative of the function f for the points “ xx ” using $n = 100$. Compute the lowest-order forward, backward, and centered difference approximations to the function, and plot them on a single plot.
- For the three kinds of derivatives computed in (a), compute the true percent relative error, and plot it for each value of “ xx ”. Note that the derivative of $f(x)$ is:

```
g=@(xx) 50*cos(50.*xx).*xx.^2 + 2*sin(50.*xx).*xx
```
- Compute the derivative of the function f using the forward difference scheme only, but with $n=10, 20, 40$, and 100 . Plot these using **points**, along with the exact derivative given by “ g ” above (as a **line**).
- Compute the true percent relative error for each value of “ xx ” associated with these values of n , and plot all four cases on one plot (plot as points).
- Using the centered difference, develop a code to compute the $O(h^4)$ accurate derivative. Feel free to restrict the domain like you did for “ xx ” to make this computation more tractable. Plot the results for the two derivatives on a single plot using $n = 50$. Compute the true relative percent error for the $O(h^2)$ and $O(h^4)$ centered difference schemes, and plot them.