Homework #4 (HW4) — Assigned 3/15/22
BIOEN 5250/6250, Fall 2022 — Due online 3/29/22, 9 am MST

You must follow the homework guidelines on Canvas (Files/Homework/ BME_5250_Instructions_for_Preparing_Homework.pdf).
Please Submit completed homework on Gradescope before the beginning of class on the due date.

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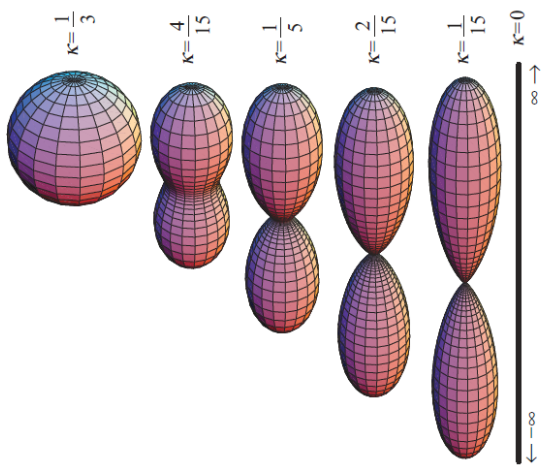
1. (Modeling fiber dispersion for transversely isotropic materials - 20 points) Soft tissues such as skin and arteries often exhibit distributions of fiber families. For instance, in the manuscript by Annaidh *et al.* 2012, the authors used a π-periodic von Mises distribution to describe the distribution of fiber density in skin as a function of angular orientation. The Holzapfel-Gasser-Ogden (HGO) hyperelastic constitutive model was developed specifically to model fiber dispersion (Gasser et al., J. Royal Soc Interface, 2005, see the pdf in Canvas). In this constitutive model, the standard structure tensor $\boldsymbol{a}^0 \otimes \boldsymbol{a}^0$, is replaced with an alternative tensor $\boldsymbol{H}$ that is a function of a fiber dispersion parameter κ:

$$\boldsymbol{H} = \kappa \boldsymbol{1} + \left(1 - 3\kappa\right) \boldsymbol{a}^0 \otimes \boldsymbol{a}^0 .$$

When $\kappa = 0, \boldsymbol{H} = \boldsymbol{a}^0 \otimes \boldsymbol{a}^0$, which is the standard form for a transversely isotropic material with one family of fibers aligned along a single direction. When $\kappa = 1/3, \boldsymbol{H} = \boldsymbol{1}$, which yields an initially isotropic distribution of fibers. If the distribution is described by the π-periodic von Mises distribution, the angular distribution density can be visualized as shown:

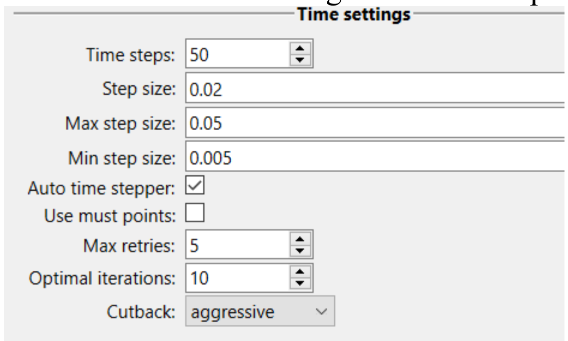


Construct a single-element finite element model with symmetry boundary conditions as you did in HW1, problem 5 to simulate the uniaxial elongation of skin along the *x*-direction to a stretch ratio of 1.5. Assign the HGO model to your element (Add Material → Uncoupled Elastic → Holzapfel-Gasser-Ogden). Use the following material coefficients and settings for your initial analysis, taken from Annaidh *et al.* 2012:

c	Shear modulus of ground matrix	0.2014 MPa
k_1	Fiber modulus	243.6 MPa
k_2	Fiber exponential coefficient	0.1327 (no units)
γ	Fiber mean orientation angle	0 degrees
κ	Fiber dispersion parameter	0 initially (no units)
k	Bulk modulus	100,000 MPa
Axes	Vector	
a	Primary fiber direction	1,0,0
d	Secondary fiber direction	0,1,0

Please see the FEBio manual page if you are interested in learning more about the HGO material:
https://help.febio.org/FebioUser/FEBio_um_3-4-4.1.2.6.html

Add an analysis “Step” for structural mechanics and change these timestep settings:

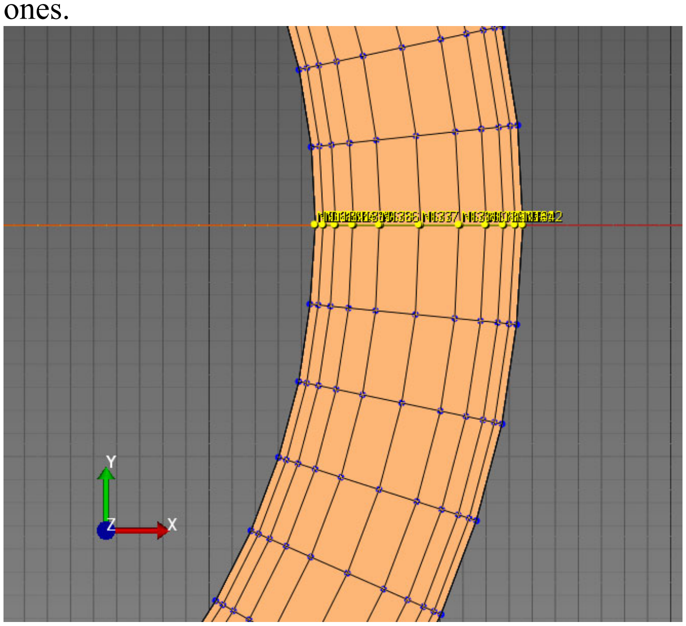


Make sure the “Cutback” setting is set to “aggressive”.

Run your finite element model for the 6 values of the dispersion parameter κ shown in the figure above and create an x-y graph of the uniaxial Cauchy stress versus stretch ratio (1.0 to 1.5) containing all 6 stress-stretch curves. Briefly interpret your findings (1-2 sentences). Include a pdf of one of your .feb files with your homework upload for this problem.

2. (Residual Stress - 30 points) The objective of this problem is to investigate the effect of pre-strain on circumferential stress in a finite element model of a carotid artery with ring opening angle of 90°. You will use two FEBio Studio models named “Artery.fsm” and “Artery_Residual_Stress.fsm” that are available on Canvas. The first model has 1 analysis step consisting of inflation via a luminal pressure, while the second model has 2 steps, the first pre-straining the ring to close, and the second applying the luminal pressure. Each analysis step has a time span of 1. The luminal pressure is 20.0 kPa (approximately 150 mmHg), and is considered high blood pressure. The constitutive model is a homogeneous isotropic Veronda-Westmann material that is nearly incompressible.

   a. Open the model Artery.fsm. Take some time to inspect the boundary conditions and applied loading in the model tree so you are familiar with the model setup. Then run the model on your local computer. Inspect the circumferential stresses through the wall thickness. Go to the unloaded configuration, Time = 1 (the final analysis step). Select the row of nodes along the positive x-axis using shift+click+drag. Make sure FEBio Studio is not also selecting the nodes behind the front ones.



Create a graph of the circumferential and radial stresses as a function of radial position ("Post → New Graph"). Use the graph *Type* "Time-Scatter". Select X-Position for the *X*-axis. Since the edges of the elements connected to the nodes you selected are tangent to the y coordinate direction, the circumferential stress is equivalent to the Y-Stress at those locations. So select Y-Stress for the y-axis.. Then hit *Tools*, and choose *Current time step*, so you only see data for the current state. Attach a screenshot of the graph, and describe the state of circumferential stress through the wall thickness. Do the same for the radial stress (X-Stress). Export the data for the circumferential stress as you will need it in the next part of the problem. Briefly describe the distributions of radial and circumferential stress, do they make sense to you based on the loading?

b. Now open the model "Artery_Residual_Stress.fsm". Again inspect the boundary conditions and applied loading in the model. Also inspect the two Steps in the "Step" section of the model tree. If you expand the items in each Step, you will see the different boundary conditions for closing the artery (time 0 to 1) versus inflating the artery (time 1 to 2). Then run the model on your local computer. Open the results plotfile and navigate to Attach a screenshot of the same circumferential stress graph at Time = 1, and describe the circumferential stress throughout the vascular wall after application of prestrain.

c. Now go to the loaded configuration, Time = 2 in the model "Artery_Residual_Stress.fsm", and plot the wall thickness position versus circumferential stress as you have done previously. Export the data for the circumferential stress. Using these data and the data from part a, create a plot containing the circumferential stress as a function of radial position through the arterial wall for both models in a graphing software of your choice. Label the graph, include a legend, and make sure to include units. Then, describe how pre-strain affected the distribution of circumferential stress through the thickness of the arterial wall in comparison to the model with no pre-strain.

3. (Skeletal Muscle, Hill Model, FEBio Studio – 30 points) In this problem we will use the Hill model and demonstrate its utility to represent the mechanical behavior of skeletal muscle. Follow the "Hill_Tutorial.pdf" file in Canvas to create a 1D Hill-type discrete element.

a. Run the tension-length model you created in the tutorial with 0%, 25%, 50%, 75%, and 100% activation levels. Provide a screen shot for each of the 5 cases showing graphs of X - displacement versus X- rigid force, as described in the tutorial.

Now we will replicate the quick-release experiments, where a muscle is tetanized and counteracts a set force (Use an activation of 100%). You will modify your model to add a force-velocity relationship to the Hill element, and then change the boundary conditions to drive the deformation by an applied force rather than a prescribed displacement. To measure the velocity, we will simply inspect the X-displacement at time = 1.

Force-Velocity

- Open the curve editor in your model, and expand *Discrete*, then *Muscle*, then *Fvl*, and left-click on "points." Add a point at (-1, 0), and then use the image named "force_velocity.png" to set the curve as you have done in the tutorial. Instead of driving the change in length by a prescribed displacement, we will use a prescribed force.

Boundary Conditions

- Delete the rigid constraint that prescribed the displacement of the right rigid body.
- Add a rigid constraint and name it "Force-Right." Apply it to the rigid material on the right, make sure it acts on the X-force and set the value to 50 N. In the curve editor, make sure this force is applied from time 0. The curve should be flat at a value of 1.

b. Now run the tension-velocity model with forces of: 50N, 40N, 30N, 20N, 10N. 5N, 2N. From each job, we are only concerned with measuring the velocity (or equivalently the displacement at time = 1 since the time interval is 1 second), that corresponds to the force we prescribed. Consider the shortening velocity as negative, and lengthening velocity as positive. Plot force versus velocity with the force on the Y-axis. (Hint: each job provides one point on this graph).

c. Describe the shape of the graph. Is it what you expected?

4. (Cardiac Mechanics - 20 points) The variation in fiber orientation through the myocardial wall is an important contributor to efficiency of the ventricular contraction. You will examine the effect of fiber orientation by running a model of the left ventricle. Connect to the Model Repository in FEBio Studio and navigate to "User Projects → Cardiac Mechanics Benchmark Problems. Download the model "Problem3VentricleContractionFineAL.feb" and open it in FEBio Studio. This model simulates the inflation pressure and contraction of the myocardium in the left ventricle during systole using a realistic variation of fiber orientation through the myocardial wall. The ventricular pressure and active contraction of the fiber material are ramped from time 0 to time 1 during the analysis. You can read about this benchmark problem here (Problem 3):

   https://febio.org/knowledgebase/case-studies/structural-mechanics/cardiac-mechanics-benchmark-problems/

   Investigate the material model used in the model tree. Note that it is a "solid mixture". Look up the two component material models in the solid mixture in the FEBio User's Manual so you understand how they work. Run this model without modification on your local computer. This should take about 5 minutes. Animate the model and note the axial twist and rise in the apex during systole resulting from the variation in fiber direction. Plot the Z - position of the apex on the epicardial surface (node 15851) as a function of time and include a screenshot here. What is the Z - position of the apex at time t = 1? How does your result compare to the result for FEBio in Figure 9 in the link above?