Homework #5 (HW5) Assigned 3/30/22
BIOEN 5250/6250, Spring 2022 Due online 4/12/22, 9 am MST

You must follow the homework guidelines on Canvas (Files/Homework/5250_instructions_for_preparing_homework.pdf). Submit completed homework on Canvas before the beginning of class on the due date.

1. (Biphasic Theory – 40 points) Consider a biphasic mixture made up of intrinsically incompressible fluid and solid constituents. Assume that each phase is present everywhere in the mixture, so that at a point in the mixture, the local volume dV and mass dm can be written:

$$dV = dV^s + dV^w \text{ and } dm = dm^s + dm^w$$

where superscripts s and w denote solid and fluid, V and m are the total tissue volume and mass, V^s and m^s are the true volume and mass of the solid, and V^w and m^w are the true volume and mass of the fluid.

   a. Provide expressions for the true densities of the solid fluid and apparent densities of the solid fluid , in terms of dV, dV^s, dV^w, dm, dm^s, and dm^w.

   b. Given our incompressibility assumptions, which of the quantities in part (a) may change with deformation of the mixture?

   c. Using the definitions given in the lecture notes for volume fractions of the solid ϕ^s and fluid ϕ^w, and the general expression for conservation of mass:

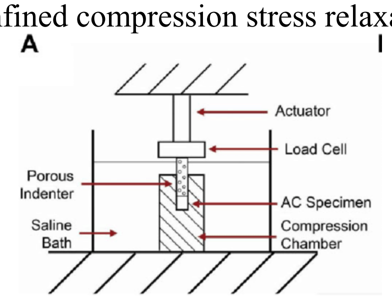
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

   derive the continuity equation for the biphasic system in terms of the solid and fluid volume fractions:

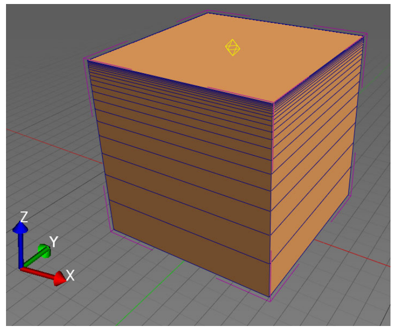
$$\nabla \cdot (\phi^s \boldsymbol{v}^s + \phi^w \boldsymbol{v}^w) = 0$$

   Make sure to show all of your work.

2. (Biphasic Theory, inspecting the effect of aggregate modulus and permeability – 60 points) In this problem, you will use FEBio to simulate a confined compression stress relaxation test of a biphasic material:



Download the file Biphasic_Confined_Compression.fsm from Canvas. This finite element model is a cube with side length of 1 mm, that is compressed from the top surface. A 20% compression is applied on the top surface via a rigid nodeset. The boundary conditions are used to simulate uniaxial confined compression.



The bottom surface is fixed in the Z direction, and all nodes in the model are fixed in the X and Y directions. Additionally, we need boundary conditions for the fluid phase. By default, FEBio assumes fluid cannot exit the material unless we set a fluid boundary condition. Here, we assume the fluid can only be exuded through the top surface. This is the boundary condition labeled "FixedFluidPressure1" in the model tree.

Since this test results in nonzero fluid velocity and solid displacement only along one coordinate axis, this is a one-dimensional problem. We can simulate this test using a stack of hexahedral elements that are constrained so that the nodes can only displace along one axis.

The cartilage is represented as a biphasic material with a Holmes-Mow hyperelastic constitutive model for the solid phase (*Holmes-Mow*) and a constant permeability constitutive model (*perm-const-iso*):

https://help.febio.org/FebioUser/FEBio_um_3-4-4.1.3.9.html

https://help.febio.org/FebioUser/FEBio_um_3-4-4.8.2.1.html

The biphasic material is initially setup with a solid volume fraction of 0.2, a Young's modulus of 2 MPa, and permeability of 0.0003 $\frac{mm^4}{N*s}$.

The displacement is ramped up from 0.0 to 0.2 mm (20% compression) over 200 seconds and then held constant out to 1,2000 seconds to observe stress relaxation.

a. Run the model without changing any parameters. You can ignore the warning, "Material "Indenter" is not assigned to any parts". Plot the effective fluid pressure for nodes 1, 4 and 21 (bottom, middle and top) as a function of time. Describe your results and interpret the observed variations in effective fluid pressure with position and time.

b. Generate a similar plot for nodes 1, 4 and 21 for the z-component of the infinitesimal strain tensor. Describe your results and interpret the observed variations in strain with position and time. Consider your results in light of the applied effective z-strain of 20%.

c. Now you will examine the effect of permeability on the effective fluid pressure and z component of the stress. Run the simulation two more times with the *permeability* property of the material set to 3e-3 and 3e-2. Generate Z-stress vs time and effective fluid pressure vs time data of the bottom element (element 1) for all 3 simulations (3 simulations, 2 graphs, 3 lines each) using an external graphing package. Make sure to include a legend and units. Comment on the effect of permeability on Z-stress and fluid pressure.