Diving into the Depths: Unraveling the Hidden Dynamics of Pressure and Volume in Scuba Diving

by

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Research Project

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Contents

Abstract

My report helps develop models for creating a better understanding of the distinctive pressures and volumes that are encountered by scuba divers from various depths, which precisely considers the physical principles that govern these phenomena. Furthermore, we will first introduce a scuba diving model that solely focuses on the pressure and volume of air in the lungs, assuming constant temperature, and analyze the overall effects of descent and ascent rates as outlined by the U.S. Navy guidelines. We will then expand this model by incorporating specific seawater temperatures from a few different global locations, such as Juneau, Alaska (10ºC), Paita, Peru (20ºC), and Savannah Beach, Ga. (30ºC), to precisely help observe their differences in variations from both pressure and volume. Also, we will be exploring the pressure and volume changes in a diver's suit, as this consists of an initial gas volume of 8 L, considering the total effects of external pressure during a dive. The results, in fact, provide a strong perception of the relationship between pressure and depth, as well as gas volume, emphasizing the crucial role of environmental conditions in diver safety. The findings contribute to a much more profound idea of how gas laws such as Boyle's, Charles's, and even Henry's Law help apply to different diving scenarios that consist of such implications for decompression procedures and nitrogen absorption.

Background

Scuba diving is a unique but fascinating activity that allows individuals to explore the underwater environments, from the stunning sights of coral reefs to historical shipwrecks, while all experiencing the thrill of being weightless and the wonders of marine life. However, it does, in fact, require having a solid understanding of several critical physics concepts to ensure safety and an overall successful diving experience. Precisely, two fundamental concepts in scuba diving are pressure and buoyancy. As a diver descends deeper underwater, the pressure will then increase mainly due to the weight of the water being above them. For example, every 10 meters of depth adds 1 bar (or, in other words, 14.5 psi) of pressure, which is an important consideration for divers [1]. The human body is well adapted to equalize this pressure, particularly in areas like the ears and sinuses, through processes like ear clearing. This pressure change will affect the air space in a diver's body and equipment, specifically their lungs and buoyancy control devices (BCDs). With Boyle's Law, it plays a crucial principle for divers as it describes the inverse relationship between pressure as well as the volume of a gas [2]. As a diver descends, the pressure will rise as the volume of air in the lungs and equipment decreases. Furthermore, when ascending, the pressure will drop, and the air volume will end up increasing instead. It is quite crucial to manage the air supply carefully to avoid any sort of accidents like pulmonary, barotrauma or arterial gas embolism. This law also applies to the volume changes in air tanks, which must be considered seriously to help avoid any potential dangers when changing depth.

With buoyancy, which is practically the ability of an object to float or sink in a fluid, it is another key, essential factor for scuba divers. The force of buoyancy will all depend on the volume and mass of an object in which divers would need to adjust their buoyancy to help control their ascent or descent in the water. "If you increase your volume (and your mass stays the same), then your density will decrease. This will increase your buoyancy force, and you will rise" [1]. Furthermore, by increasing the volume of their body, for example, through controlled breathing or by using a BCD, a diver can decrease their overall density, which will then increase their buoyancy as it allows them to rise to the surface. On the other hand, decreasing volume increases their density, causing them to sink. Understanding how to manipulate buoyancy ensures that divers can maintain neutral buoyancy, making underwater movement effortless and allowing them to hover at a particular depth without expending energy.

Thermal conductivity is yet another crucial physics principle in scuba diving. Water is for sure a more efficient conductor of heat than air, meaning that what happens is that divers will lose body heat much faster in water, which of course can potentially lead to hypothermia [1]. To simply alleviate this, divers will often wear wet-suits made from precise materials like neoprene, which can trap a layer of water between the suit and the body, insulating the diver by slowing down the rate of heat transfer from the body to the surrounding water. In addition, these key concepts of pressure, buoyancy, and thermal conductivity are all technically interconnected in the underwater environment in which a thorough understanding of them is quite essential for the safety of divers. By applying these principles, divers can enhance their experience, prevent any injuries, and ensure that they can navigate the complexities of the underwater world safely.

Introduction

Scuba diving offers a unique opportunity to explore the ocean's depths, but it also presents various challenges that require careful attention to the many physical principles involved. For instance, one of the most significant concerns for divers is understanding how changes in depth will affect both the pressure and volume of gases, particularly the air in their lungs and diving equipment. These critical changes are not just theoretical but also have some direct implications for diver safety and protocol, influencing air consumption, buoyancy control, and the overall risk of injuries such as barotrauma or decompression sickness. The relationship between pressure, volume, and temperature becomes quite essential when considering dive planning, especially for longer or deeper dives. Specifically, accurate models are therefore used to help ensure the idea that divers can anticipate the effects of pressure changes, manage their equipment effectively, and even optimize their safety throughout the dive.

The motivation behind this report is to create a strong, comprehensive model that helps address the dynamics of pressure as well as the volume for a diver at many different depths and environmental conditions. Specifically, this project aims to develop a scuba diving model that takes into account both the pressure-volume relationship in the lungs and even the diver's suit, while considering temperature variations from many different seawater environments. By doing so, this report seeks to provide a finer understanding of how these specific factors can affect a diver's experience in any diverse location, such as the colder waters in Alaska or even the warmer waters in Georgia.

The goal of this report is to present a series of models that accurately depict these interactions, starting with a basic model for pressure and volume in the lungs and then expanding to incorporate varying temperatures across multiple locations. By achieving this, we hope to improve the safety and performance of scuba divers by providing insights into how environmental factors impact their experience under the sea. [3]

Chapter 1

Pressure and Volume of Air in the Lungs

1.1 Analysis

For starters, our main goal of this scuba diving model is to have an understanding of how the pressure and volume of air in a diver's lungs can precisely change during the descent and ascent while scuba diving. This relationship is governed by **Boyle's Law**, which technically states that, for an ideal gas at a constant temperature, the volume of gas is inversely proportional to the pressure. Furthermore, as the diver descends, the pressure increases, causing the volume of air in the lungs to decrease. On the other hand, during the diver's ascent, the pressure will simply decrease as the air expands.

Here we have the formula for **Boyle's Law:**

$$P_1V_1 = P_2V_2 \tag{1}$$

in which:

- P_1 and V_1 are the pressure and volume of air at the surface (or practically at any reference depth),
- P_2 and V_2 represent as the pressure and volume at any given depth.

This inverse relationship, described by **Boyle's Law** (Equation 1), applies when the temperature happens to remain constant, as this allows us to model the lung behavior throughout the dive.

Application to Scuba Diving

From what we already know, as a diver descends, the pressure increases due to the weight of the water above. This rate of pressure increase all depends on the depth as well as the density of seawater (1000 kg/m^3) and even the acceleration due to gravity, $g = 9.81\,\mathrm{m/s^2}$. Furthermore, at the surface, the atmospheric pressure is approximately $101,325\,\mathrm{Pa}$, and for every 10 meters of depth, this pressure will increase by 1 atm (101,325 Pa). Thus, the pressure at a given depth can be calculated by using the following equation:

$$P_2 = P_1 + \rho g d \tag{2}$$

in which:

- ρ is the density of seawater (1000 kg/m^3),
- g is the gravitational acceleration (9.81 m/s^2),
- d is the depth of the diver in meters.

1.1.1 Example:

If the atmospheric pressure is 101,325 Pa and the density of seawater is 1000 kg/m³, what is the pressure at a depth of 30 meters, assuming the temperature remains constant as the acceleration due to gravity is 9.81 m/s²? To simply answer this, we could just plug in our values into (Equation 2)

$$P_{\text{depth}} = 101,325\,\text{Pa} + (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(30\,\text{m}) = 395,625\,\text{Pa}$$

At this precise depth, the volume depth of air in the diver's lungs will be reduced according to **Boyle's Law** (Equation 1). Thus, we are now able to find the volume at depth by using the following formula:

$$V_2 = \frac{P_1 V_1}{P_2} \tag{3}$$

As this shows how lung volume decreases at greater depths.

Ascent: Pressure Decrease and Volume Expansion

During the ascent, the diver moves towards the surface, where the pressure decreases. This happens to cause the volume of air in the lungs to expand. Furthermore, the relationship between the initial conditions at depth and the final conditions at the surface follows **Boyle's Law** (Equation 1), but in reverse.

Assumptions

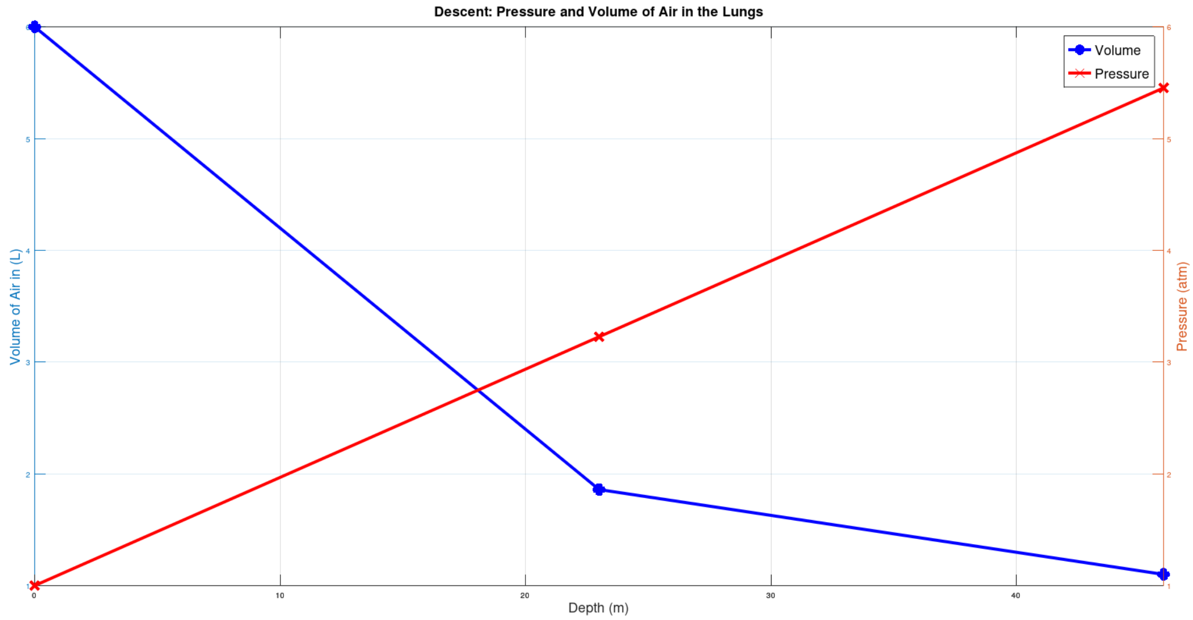
Here, we make the following assumptions for this model:

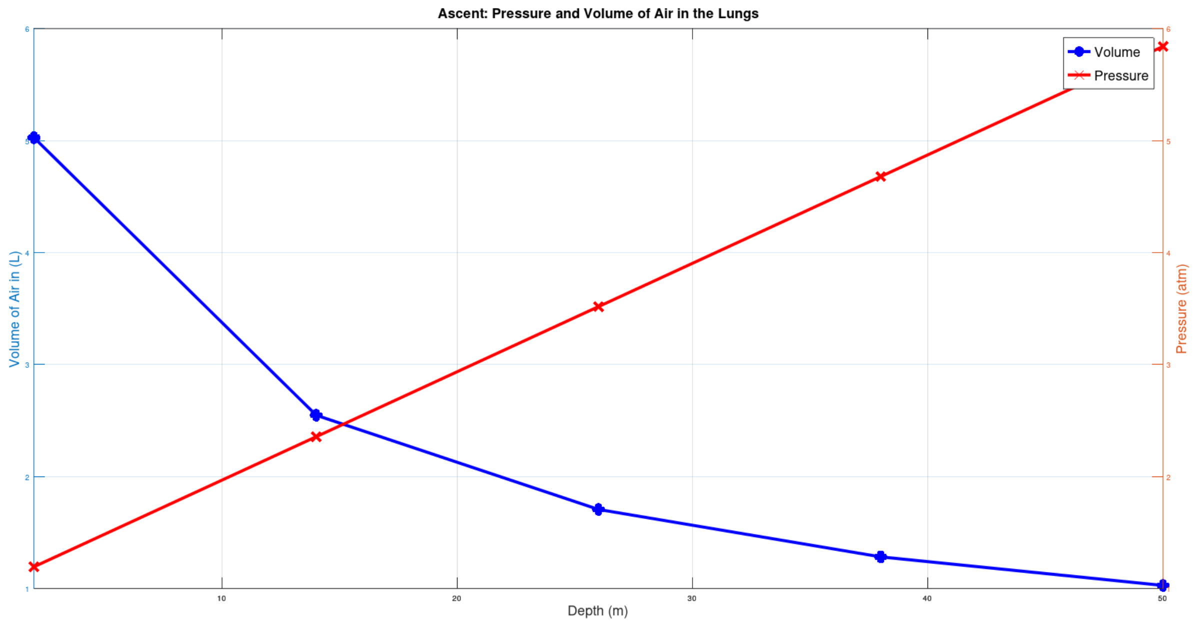
- The temperature remains constant, ensuring Boyle's Law is applicable.
- The descent rate is limited to 23 m/min (0.2 m/s) to allow gradual pressure changes.
- The ascent rate is limited to 12 m/min (0.2 m/s) to prevent any lung expansion or reducing the risk of barotrauma.
- As an reasonable assumption for diving conditions, assuming that the air will behave as an ideal gas.
- The diver is submerged in seawater with a of density 1000 kg/m³.

These assumptions may simplify the model while maintaining its practical relevance to safe diving practices.

1.2 Plots

Here, we have two different images of what would be the Descent and Ascent of both the pressure and volume of air in the lungs. These schematics were created under the use of MATLAB.





1.3 Results and Discussion

As predicted by **Boyle's Law** (Equation 1), we can see here that the lung volume decreases with increased pressure during its descent as well as the fact that it even expands with a decreased pressure during the ascent. Furthermore, the significant reduction in lung volume at greater depths is particularly critical for divers, as it can specifically affect their total comfort and overall breathing capacity. These models demonstrate the inverse relationship between pressure and volume, which is quite fundamental for knowing the physiological effects of diving.

An important finding of this study is the need for controlled ascent and descent rates to avoid rapid pressure changes that could potentially cause lung damage. The recommended ascent and descent rates of 23 m/min and 12 m/min, respectively, ensure gradual and safe pressure adjustments, reducing the risk of barotrauma. This underscores the importance of managing the diver's exposure to extreme pressure changes, especially in deeper waters.

While the model assumes constant temperature, which may not always hold true in real-world diving conditions, this simplification is reasonable for the scope of this study. Future iterations of the model could incorporate factors such as temperature variations, lung elasticity, and gas absorption effects, which would offer a more comprehensive and realistic simulation of diving physiology.

Overall, these models present a solid framework for anyone to understand the dynamics of both the pressure and lung volume during a underwater dive as it even offers some valuable information for safe and effective dive planning.

Chapter 2

Scuba Diving Model for Seawater at Various Location

2.1 Analysis

In this analysis, we extend our knowledge from the scuba diving model from first chapter by incorporating multi-variations in seawater temperature at a few different geographical locations. This mathematical solution builds upon the results from chapter 1, where we were able to model the pressure-volume relationship for air in the lungs solely based upon Boyle's Law. For this new model that we will be working on, our primary difference all depends on how the temperature will influence the seawater's density, which in turn affects the pressure that is experienced by a diver at different depths.

General Theory and Modifications

In our original model, we first assumed a constant seawater density of 1000 kg/m^3, which can has a reasonable approximation at 4 ℃. However, we know that seawater density is temperature-dependent meaning that it varies with both temperature and salinity. Thus, we would use the seawater density equation which is displayed as the following:

$$\rho(T) = \rho_0 (1 - \beta(T - T_0)) \tag{4}$$

in which:

- $\rho(T)$ is the density of seawater at temperature T (in ℃),
- $\rho_0 = 1000\,\text{kg/m}^3$ is the reference density at 4 ℃,
- $\beta = 0.0002\,℃^{-1}$ is the coefficient of thermal expansion of seawater,
- $T_0 = 4\,℃$ is the reference temperature.

Precisely, this equation shows us that as the temperature increases, the density of seawater will decrease which will lead to a much smaller pressure that is being exerted by the seawater from a given depth. For example, we can use the following specific temperatures as something to consider:

- Juneau, Alaska: $10°C$
- Paita, Peru: $20°C$
- Savannah Beach, GA: $30°C$

The Impact of Temperature on Pressure and Volume

The temperature affects the density of seawater, in which it influences the pressure at a given depth. For a fixed depth, the pressure is given by (Equation 2). Thus, for each temperature condition, we can calculate the density of seawater at that temperature, then compute the total pressure at each depth. Finally, we will use **Boyle's Law** to help find the corresponding lung volume.

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Temperature Variations and Modeling Steps

1. **Seawater Density at Different Temperatures:** Simply, by using (Equation 2) for $\rho(T)$, we can compute the density of seawater at each of the three temperatures of 10ºC, 20ºC, and 30ºC. Precisely, this will end up helping us to determine on how the seawater density is going to change as the temperature increases from the cold (Juneau, Alaska) to warm (Savannah Beach, GA).

2. **Pressure at Various Depths:** For each individual location, we will calculate the pressure at depths ranging from 0 to 50 meters, all by using the modified density values. Luckily, we can just use the same formula for pressure form chapter 1, but of course this time around with the temperature-dependent density for each location.

3. **Lung Volume at Various Depths:** For each location and temperature, we can use **Boyle's Law** to help determine the overall, corresponding lung volume from different depths. This pf course, will allow us to visually see on how the lung volume changes under the different pressure conditions, which may vary due to the differing seawater densities at each location.

4. **Comparison Graphs:** Finally, we will plot two schematics that will show how the lung volume happens to change with depth at the three different locations. These specific schematics will help illustrate the total effects of temperature on the pressure-volume relationship for scuba divers.

Assumptions

- Let's assume a constant temperature. While it being realistic for short-term dives, this does not fully capture real-world conditions where temperature could vary due to ocean currents or atmospheric conditions.

- The model will not take into account for any other factors like salinity variations, which may potentially impact seawater density but are far more outside the scope of this analysis.

Thus, by varying the temperature, this extended model may provide a much more detailed understanding of how the environmental factors, such as geographical location, can potentially influence the pressure as well as the volume of air in a diver's lungs.

2.2 Plots

Here, we have two different images of what would once be again, the Descent and Ascent of both the pressure and volume of air in the lungs. This time around, we have included the temperatures of the three different locations.

Figure 2.1: Descent

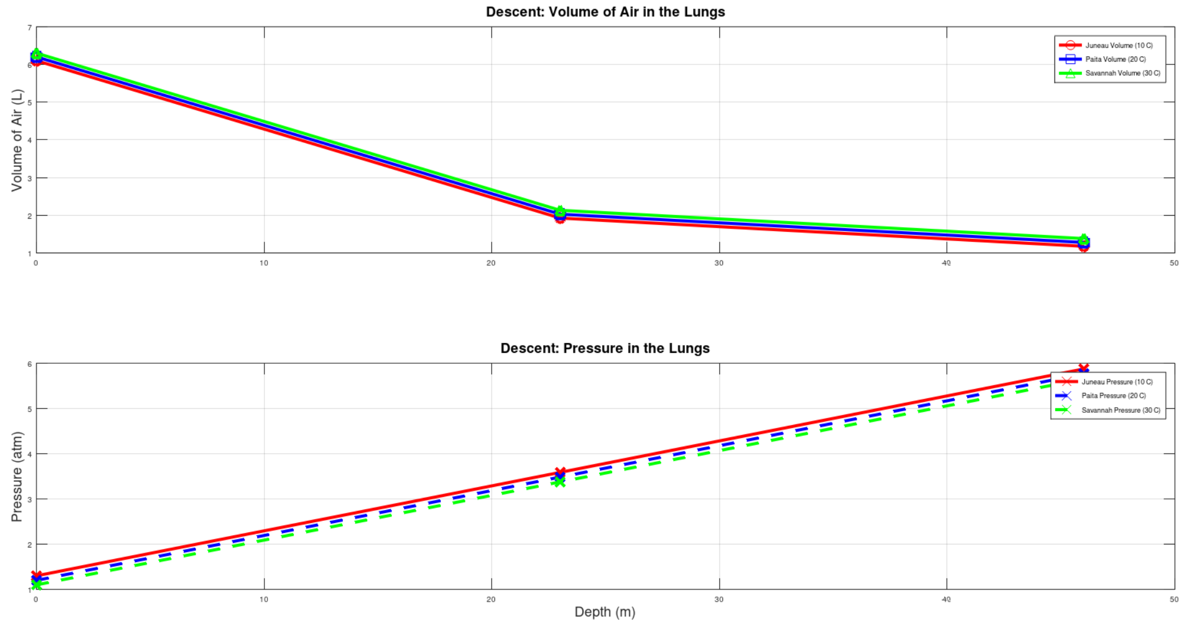
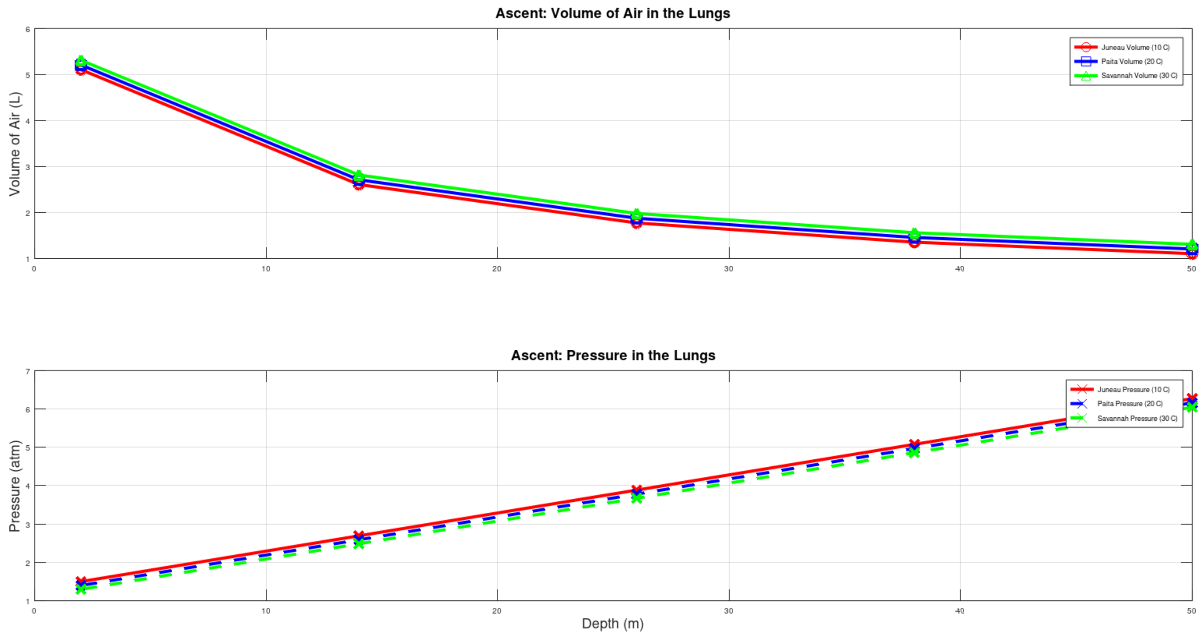


Figure 2.2: Ascent



2.3 Results and Discussion

In this section, we provide the results of our analysis by comparing the pressure and volume of air in the lungs of a diver at three different locations which were Juneau, Alaska (10 ºC), Paita, Peru (20 ºC), and Savannah Beach, Georgia (30 ºC). Each one of these locations happened to correspond to a different seawater temperature, which specifically influences the density of seawater as well as the pressure that is being exerted onto the diver at varying depths.

Summary of Results

For each of the three locations, we were able calculate the seawater density at the given temperature by using the density-temperature relationship. Then, we used the calculated density values to help compute the pressure at various depths by simply assuming a constant atmospheric pressure at sea level. Hence, final lung volume at each depth is all determined by using **Boyle's Law**.

2.3.1 Example

Density of Seawater at Different Temperatures :

To provide a analytical solution, we calculate the seawater density for each location by using Equation (4). For instance:

$$\rho(10°C) = 1000(1-0.0002\cdot(10-4)) = 1027\,\mathrm{kg/m^3}$$

Similarly, we can compute the densities at 20ºC and 30ºC:

$$\rho(20°C) = 1000(1-0.0002\cdot(20-4)) = 1025\,\mathrm{kg/m^3}$$

$$\rho(30°C) = 1000(1-0.0002\cdot(30-4)) = 1022\,\mathrm{kg/m^3}$$

2.3.2 Example

Pressure at Various Depths :

Using the density values above, we calculate the pressure at various depths. For example, at 10 m depth in Juneau (10°C):

$$P_{10m} = 101325\,\mathrm{Pa} + 1027\cdot 9.81\cdot 10 = 102725\,\mathrm{Pa}$$

Of course, we can simply repeat the same for 20 m, 30 m, etc., for each individual location.

2.3.3 Example

Lung Volume at Various Depths :

Finally, we calculate the lung volume using **Boyle's Law** (Equation 3). For instance, for Juneau, Alaska at 10 m:

$$V = 6\,\mathrm{L} \cdot \frac{101325\,\mathrm{Pa}}{102725\,\mathrm{Pa}} = 5.93\,\mathrm{L}$$

Location	**Temperature (ºC)**	**Density of Seawater (kg/m^3)**	**Lung Volume (L)**
Juneau, Alaska	10 ºC	1027	Varies with depth
Paita, Peru	20 ºC	1025	Varies with depth
Savannah Beach, GA	30 ºC	1022	Varies with depth

Table 2.1: Density of Seawater at Different Locations and Corresponding Lung Volumes

Interpretation of Results

- **Juneau, Alaska (10 ºC):** The cooler temperature results in the highest seawater density, leading to a greater pressure at a given depth compared to the other two locations. Consequently, the lung volume decreases more rapidly as the diver descends.
- **Paita, Peru (20 ºC):** At this moderate temperature, the seawater density is slightly lower than that in Juneau, leading to slightly less pressure and a more gradual decrease in lung volume with depth.
- **Savannah Beach, Georgia (30 ºC):** The warmest water temperature results in the lowest seawater density, causing the pressure to increase at a slower rate with depth. As a result, the lung volume decreases at a slower rate compared to the other two locations.

Unexpected Results and Key Observations

The most intriguing observation from this analysis is how the temperature has significantly affected the lung volume at a given depth. While we expected the warmer temperatures to lead to lower seawater density, the impact on lung volume was more pronounced than we were anticipating, particularly in the much deeper dives. The key differences in lung volume between Juneau and Savannah Beach were especially noticeable at depths exceeding 20 meters, highlighting the total importance of temperature considerations when it comes to planning dives in varying geographical locations.

On the other hand, another interesting result is that the general shape of the lung volume-depth curve is similar for all three locations, with lung volume decreasing exponentially as pressure increases. However, the steepness of the curve is influenced by temperature, confirming that temperature plays a crucial role in determining the rate of volume change with depth.

Chapter 3

Pressure and Volume Model for Air in a Diver's Suit

3.1 Analysis

The objective of this analysis is to simply develop a mathematical model for the pressure and volume of air in a diver's suit as the diver descends through seawater. This model is based upon the fundamental principles of fluid mechanics and thermodynamics, specifically Boyle's Law and hydrostatic pressure relationships.

Fundamental Theory

The behavior of the air volume in the diver's suit is governed primarily by **Boyle's Law** (Equation 1), which states that for an ideal gas at constant temperature. This implies that as external pressure increases, the volume of the enclosed gas decreases proportionally. The external pressure at a given depth is determined by (Equation 2).

Furthermore, by combining these relationships, we can express the volume of air in the diver's suit as a function of depth by using the following formula:

$$V(d) = V_0 \frac{P_0}{P_0 + \rho g d} \tag{5}$$

where V_0 is the initial air volume at the surface, given as 8 L (0.008 m^3). This equation describes how the volume of air in the suit compresses with increasing depth due to increasing external pressure.

Mathematical Model Development

To quantify the volume change, we will substitute the known values by doing the following:

$$V(d) = 0.008 \times \frac{101325}{101325 + (1025)(9.81)d}$$

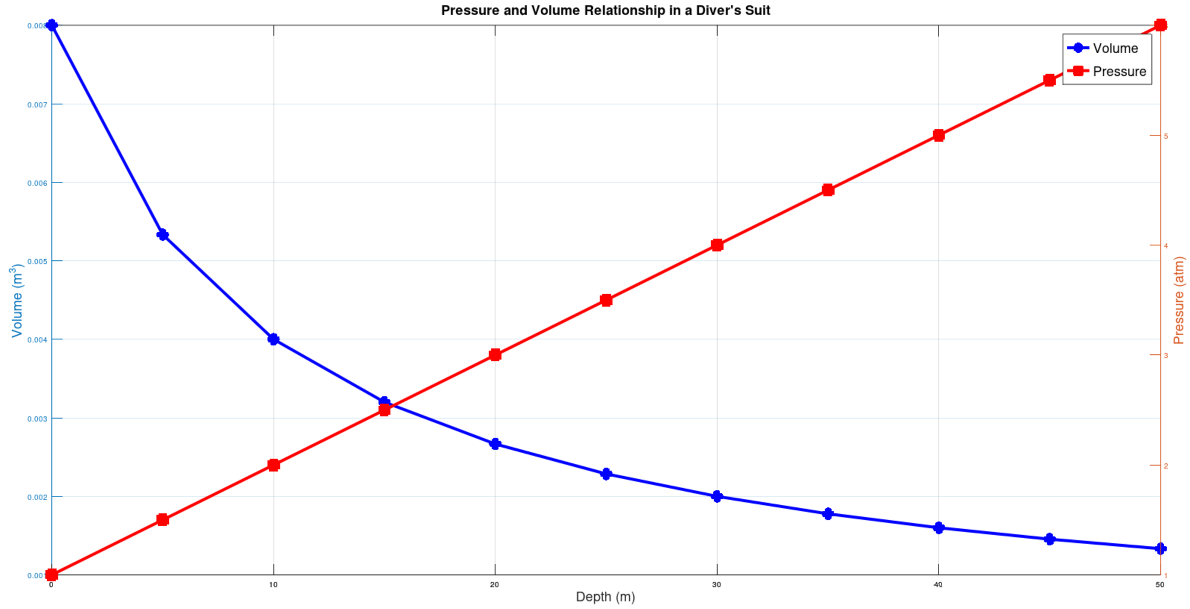
For these specific depths, the volume of the air in the suit can be computed through the following examples:

- At $d = 10$ m:
$$V(10) = 0.008 \times \frac{101325}{101325 + 100627.5} \approx 0.0041 \text{ m}^3$$
- At $d = 20$ m:
$$V(20) = 0.008 \times \frac{101325}{101325 + 201255} \approx 0.0027 \text{ m}^3$$
- At $d = 30$ m:
$$V(30) = 0.008 \times \frac{101325}{101325 + 301882.5} \approx 0.0020 \text{ m}^3$$

Thus, this shows us that as the depth increases, the volume of air in the suit compresses significantly due to the overall, increasing pressure.

3.2 Plot

Here, we have an images of what would be the pressure and volume relationship in a diver's suit.



3.3 Results and Discussion

Precisely, this model confirms that the air volume in the suit follows an inverse relationship with the external pressure. The compressibility of air truly means that at 30 m, the volume is reduced to approximately a quarter of its original value at the surface. This provides some critical implications for divers, as the suit's buoyancy happens to decrease with depth as it requires some adjustments in buoyancy compensation to maintain a neutral buoyancy.

Furthermore, the assumption of the constant temperature is in fact idealized. In reality, the temperature variations due to water thermoclines or gas compression effects could slightly modify the volume. However, for certain practical diving depths, Boyle's Law provides a more sufficiently accurate approximation. Further refinements could incorporate temperature variations and non-ideal gas behavior for more precise modeling.

Major Findings

The results demonstrate that as the diver descends deeper underwater, the surrounding water exerts greater pressure on the air inside the suit. As predicted by Boyle's Law, this increasing pressure leads to a corresponding decrease in volume. For instance, at a depth of 10 m, the absolute pressure doubles from 1 atm at the surface to 2 atm, resulting in the volume of air inside the suit decreasing to 4 L (0.004 m^3). Similarly, at 30 m, where the absolute pressure reaches 4 atm, the volume reduces further to 2 L (0.002 m^3). The results confirm that the relationship between pressure and volume follows an inverse proportionality, as expected.

A summary of the pressure and volume at different depths is presented in Table 3.1.

Depth (m)	Absolute Pressure (atm)	Volume (m^3)
0	1.0	0.008
10	2.0	0.004
20	3.0	0.00267
30	4.0	0.002
40	5.0	0.0016

Table 3.1: Pressure and Volume Changes at Different Depths

At depths greater than 30 m, the volume of air in the suit becomes quite small, which has critical implications for buoyancy control. If not properly managed, this effect could lead to rapid descents, increasing the risk of decompression sickness upon resurfacing.

Conclusion

The objective of these projects was to develop mathematical models to understand the behavior of air under varying pressures in scuba diving scenarios. Each project built upon fundamental gas laws to analyze different aspects of a diver's experience underwater, including air pressure and volume changes in the lungs, variations in seawater conditions, and the compression of air within a diver's suit.

In Chapter 1, we were able to formulate a model that was best used to describe the pressure and volume of air in a diver's lungs under the assumption of constant temperature, all by using Boyle's Law. We were able to see the constraints on descent and ascent rates as it ensured that the model simply aligned with safe scuba diving practices that were outlined by the U.S. Navy. The results confirmed that as the diver descends, increasing external pressure compresses the air in the lungs, reducing its volume. Conversely, as the diver ascends, air expands, reinforcing the importance of controlled ascent rates to prevent lung over-expansion injuries such as pulmonary barotrauma.

In Chapter 2, we investigated how seawater temperature variations at different locations impact air compression. By incorporating real-world temperature data from locations such as Juneau, Alaska, and Savannah Beach, Georgia, we were given the opportunity to analyze how certain changes in density and pressure helped influence air behavior. The comparison graphs, on the other hand, illustrated that while temperature variations slightly affect air expansion and compression, the dominant factor remains depth-dependent pressure changes. This provides for us a precise understanding that while seawater temperature may have secondary effects, divers must primarily account for pressure differences when managing air supply.

Finally, Chapter 3 focused on modeling the compression of air inside a diver's suit as depth increases. By using Boyle's Law, we were able to quantify on how the suit's internal volume happened to decrease while its pressure was increasing. The results showed a nonlinear decrease in volume with depth, demonstrating that at greater depths, the diver experiences significantly restricted movement due to air compression. The findings also highlight the importance of dry suits and external air supply systems for deep-sea diving to maintain mobility and insulation.

Together, these projects all provided a comprehensive analysis of air compression effects in different scuba diving contexts. The models perfectly align with fundamental gas laws and real-world diving constraints, offering great lessons into safe diving practices. By integrating theoretical physics with practical applications, this work underscores the importance of controlled breathing, ascent management, and equipment considerations for diver safety. Future extensions could incorporate dynamic temperature and salinity effects to refine the models further, ensuring even greater accuracy in predicting underwater air behavior.

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MATLAB Codes

Pressure and Volume of Air in the Lungs Code

```
% This is Project 7.3.1

% Constants
P_surface = 101325; % Atmospheric pressure at surface in Pascals
rho_seawater = 1000; % Density of seawater in kg/m^3
g = 9.81; % Gravitational acceleration in m/s^2
initial_volume = 6; % Initial volume of air in lungs in liters
descent_rate = 23; % Descent rate in meters per minute (maximum)
ascent_rate = 12; % Ascent rate in meters per minute (maximum)
max_depth = 50; % Maximum depth in meters

% The initial conditions
initial_depth = 0; % Starting depth (surface)
time_step = 1; % Time step in minutes

% Time arrays for descent and ascent
descent_time = 0:time_step:max_depth/descent_rate; % Time to descend to max depth
ascent_time = 0:time_step:max_depth/ascent_rate; % Time to ascend from max depth

% Pressure at each depth for descent and ascent
depths_descent = descent_rate * descent_time; % Depth during descent
depths_ascent = max_depth - ascent_rate * ascent_time; % Depth during ascent

% Pressure at each depth
pressures_descent = P_surface + rho_seawater * g * depths_descent; % Pressure in Pascals during descent
pressures_ascent = P_surface + rho_seawater * g * depths_ascent; % Pressure in Pascals during ascent

% Converting pressures from Pascals to atmospheres for convenience
pressures_atm_descent = pressures_descent / 101325;
pressures_atm_ascent = pressures_ascent / 101325;

% Volume at each depth using Boyle's Law
volumes_descent = initial_volume * P_surface ./ pressures_descent; % Volume during descent
```

```matlab
        volumes_ascent = initial_volume * P_surface ./ pressures_ascent; % Volume during ascent

        % Plot for descent
        figure;
        [ax1, h1, h2] = plotyy(depths_descent, volumes_descent, depths_descent, pressures_atm_descent);
        set(h1, 'LineWidth', 2, 'Marker', 'o', 'MarkerFaceColor', 'b', 'Color', 'b');
        set(h2, 'LineWidth', 2, 'Marker', 'x', 'MarkerFaceColor', 'r', 'Color', 'r');
        ylabel(ax1(1), 'Volume of Air in (L)', 'FontSize', 18);
        ylabel(ax1(2), 'Pressure (atm)', 'FontSize', 18);
        xlabel('Depth (m)', 'FontSize', 18);
        title('Descent: Pressure and Volume of Air in the Lungs', 'FontSize', 18);
        legend('Volume', 'Pressure', 'Location', 'Northeast', 'FontSize', 18);
        grid on;

        % Plot for ascent
        figure;
        [ax2, h3, h4] = plotyy(depths_ascent, volumes_ascent, depths_ascent, pressures_atm_ascent);
        set(h3, 'LineWidth', 2, 'Marker', 'o', 'MarkerFaceColor', 'b', 'Color', 'b');
        set(h4, 'LineWidth', 2, 'Marker', 'x', 'MarkerFaceColor', 'r', 'Color', 'r');
        ylabel(ax2(1), 'Volume of Air in (L)', 'FontSize', 18);
        ylabel(ax2(2), 'Pressure (atm)', 'FontSize', 18);
        xlabel('Depth (m)', 'FontSize', 18);
        title('Ascent: Pressure and Volume of Air in the Lungs', 'FontSize', 18);
        legend('Volume', 'Pressure', 'Location', 'Northeast', 'FontSize', 18);
        grid on;
```

Explanation

This MATLAB code simulates the relationship between depth and lung volume based on Boyle's Law. The script calculates the pressure at each depth, applies Boyle's Law to determine the corresponding lung volume, and plots the depth vs. the volume curve. The images that has been shown before displays to us on how the volume decreases with an increasing depth and ascent. Thus, this code also displays the results at each depth, in which it helps visualize the overall changes in lung volume during a dive.

Scuba Diving Model for Seawater at Various Locations Code

Below is the Matlab code used to calculate the seawater density, pressure, and lung volume at different depths for each of the three locations:

```
% This is Project 7.3.2

% Constants
P_surface = 101325; % Atmospheric pressure at surface in Pascals
g = 9.81; % Gravitational acceleration in m/s^2
initial_volume = 6; % Initial volume of air in lungs in liters
descent_rate = 23; % Descent rate in meters per minute (maximum)
ascent_rate = 12; % Ascent rate in meters per minute (maximum)
max_depth = 50; % Maximum depth in meters

% Temperature and corresponding seawater densities (kg/m^3)
% Juneau, Alaska at 10 C: 1027 kg/m^3
% Paita, Peru at 20 C: 1025 kg/m^3
% Savannah Beach, GA at 30 C: 1022 kg/m^3
density_juneau_0 = 1027;
density_paita_0 = 1025;
density_savannah_0 = 1022;

% Coefficient of temperature dependence for seawater density (1/C)
beta = 0.0002;

% Initial temperatures for each location (C)
T_juneau = 10; % Juneau's temperature in C
T_paita = 20; % Paita's temperature in C
T_savannah = 30; % Savannah's temperature in C

% Seawater densities as a function of temperature for each location
density_juneau = density_juneau_0 * (1 - beta * (T_juneau - 10));
density_paita = density_paita_0 * (1 - beta * (T_paita - 20));
density_savannah = density_savannah_0 * (1 - beta * (T_savannah - 30));

% Depth array for descent and ascent
descent_time = 0:1:max_depth/descent_rate; % Time to descend to max depth
ascent_time = 0:1:max_depth/ascent_rate; % Time to ascend from max depth

```

```
        % Depth during descent and ascent
        depths_descent = descent_rate * descent_time; % Depth during descent
        depths_ascent = max_depth - ascent_rate * ascent_time; % Depth during ascent

        % Pressure at each depth for each location
        pressures_descent_juneau = P_surface + density_juneau * g * depths_descent;
        pressures_descent_paita = P_surface + density_paita * g * depths_descent;
        pressures_descent_savannah = P_surface + density_savannah * g * depths_descent;

        pressures_ascent_juneau = P_surface + density_juneau * g * depths_ascent;
        pressures_ascent_paita = P_surface + density_paita * g * depths_ascent;
        pressures_ascent_savannah = P_surface + density_savannah * g * depths_ascent;

        % Converting pressures from Pascals to atmospheres for convenience
        pressures_atm_descent_juneau = pressures_descent_juneau / 101325;
        pressures_atm_descent_paita = pressures_descent_paita / 101325;
        pressures_atm_descent_savannah = pressures_descent_savannah / 101325;

        pressures_atm_ascent_juneau = pressures_ascent_juneau / 101325;
        pressures_atm_ascent_paita = pressures_ascent_paita / 101325;
        pressures_atm_ascent_savannah = pressures_ascent_savannah / 101325;

        % Volume at each depth using Boyle's Law for each location
        volumes_descent_juneau = initial_volume * P_surface ./ pressures_descent_juneau;
        volumes_descent_paita = initial_volume * P_surface ./ pressures_descent_paita;
        volumes_descent_savannah = initial_volume * P_surface ./ pressures_descent_savannah;

        volumes_ascent_juneau = initial_volume * P_surface ./ pressures_ascent_juneau;
        volumes_ascent_paita = initial_volume * P_surface ./ pressures_ascent_paita;
        volumes_ascent_savannah = initial_volume * P_surface ./
```

```
        pressures_ascent_savannah;

    % Small offset values (Better Clarity)
    offset_volume_juneau = 0.1; % Offset for volume of Juneau
    offset_volume_paita = 0.2; % Offset for volume of Paita
    offset_volume_savannah = 0.3; % Offset for volume of
        Savannah

    offset_pressure_juneau = 0.1; % Offset for pressure of
        Juneau
    offset_pressure_paita = 0.2; % Offset for pressure of
        Paita
    offset_pressure_savannah = 0.3; % Offset for pressure of
        Savannah

    % The offsets to the volume and pressure for each
        location
    volumes_descent_juneau = volumes_descent_juneau +
        offset_volume_juneau;
    volumes_descent_paita = volumes_descent_paita +
        offset_volume_paita;
    volumes_descent_savannah = volumes_descent_savannah +
        offset_volume_savannah;

    volumes_ascent_juneau = volumes_ascent_juneau +
        offset_volume_juneau;
    volumes_ascent_paita = volumes_ascent_paita +
        offset_volume_paita;
    volumes_ascent_savannah = volumes_ascent_savannah +
        offset_volume_savannah;

    pressures_atm_descent_juneau =
        pressures_atm_descent_juneau + offset_pressure_juneau;
    pressures_atm_descent_paita = pressures_atm_descent_paita
         + offset_pressure_paita;
    pressures_atm_descent_savannah =
        pressures_atm_descent_savannah +
        offset_pressure_savannah;

    pressures_atm_ascent_juneau = pressures_atm_ascent_juneau
         + offset_pressure_juneau;
    pressures_atm_ascent_paita = pressures_atm_ascent_paita +
         offset_pressure_paita;
    pressures_atm_ascent_savannah =
        pressures_atm_ascent_savannah +
        offset_pressure_savannah;

    % Descent Plot (Volume and Pressure)
    figure;

    % Volume Plot
```

```
    subplot(2, 1, 1);
    plot(depths_descent, volumes_descent_juneau, 'ro-', 'LineWidth', 2); % Juneau Volume (Red)
    hold on;
    plot(depths_descent, volumes_descent_paita, 'bs-', 'LineWidth', 2); % Paita Volume (Blue)
    plot(depths_descent, volumes_descent_savannah, 'g^-', 'LineWidth', 2); % Savannah Volume (Green)
    ylabel('Volume of Air (L)', 'FontSize', 18);
    grid on;
    legend('Juneau Volume (10 C)', 'Paita Volume (20 C)', 'Savannah Volume (30 C)', 'Location', 'northeast');
    title('Descent: Volume of Air in the Lungs', 'FontSize', 18);

    % Pressure Plot
    subplot(2, 1, 2);
    plot(depths_descent, pressures_atm_descent_juneau, 'rx-', 'LineWidth', 2); % Juneau Pressure (Red)
    hold on;
    plot(depths_descent, pressures_atm_descent_paita, 'bx--', 'LineWidth', 2); % Paita Pressure (Blue)
    plot(depths_descent, pressures_atm_descent_savannah, 'gx--', 'LineWidth', 2); % Savannah Pressure (Green)
    ylabel('Pressure (atm)', 'FontSize', 18);
    xlabel('Depth (m)', 'FontSize', 18);
    grid on;
    legend('Juneau Pressure (10 C)', 'Paita Pressure (20 C)', 'Savannah Pressure (30 C)', 'Location', 'northeast');
    title('Descent: Pressure in the Lungs', 'FontSize', 18);

    % Ascent Plot (Volume and Pressure)
    figure;

    % Volume Plot
    subplot(2, 1, 1);
    plot(depths_ascent, volumes_ascent_juneau, 'ro-', 'LineWidth', 2); % Juneau Volume (Red)
    hold on;
    plot(depths_ascent, volumes_ascent_paita, 'bs-', 'LineWidth', 2); % Paita Volume (Blue)
    plot(depths_ascent, volumes_ascent_savannah, 'g^-', 'LineWidth', 2); % Savannah Volume (Green)
    ylabel('Volume of Air (L)', 'FontSize', 18);
    grid on;
    legend('Juneau Volume (10 C)', 'Paita Volume (20 C)', 'Savannah Volume (30 C)', 'Location', 'northeast');
    title('Ascent: Volume of Air in the Lungs', 'FontSize', 18);

    % Pressure Plot
```

```
        subplot(2, 1, 2);
        plot(depths_ascent, pressures_atm_ascent_juneau, 'rx-', '
           LineWidth', 2); % Juneau Pressure (Red)
        hold on;
        plot(depths_ascent, pressures_atm_ascent_paita, 'bx--', '
           LineWidth', 2); % Paita Pressure (Blue)
        plot(depths_ascent, pressures_atm_ascent_savannah, 'gx--'
           , 'LineWidth', 2); % Savannah Pressure (Green)
        ylabel('Pressure (atm)', 'FontSize', 18);
        xlabel('Depth (m)', 'FontSize', 18);
        grid on;
        legend('Juneau Pressure (10 C)', 'Paita Pressure (20 C)',
            'Savannah Pressure (30 C)', 'Location', 'northeast');
        title('Ascent: Pressure in the Lungs', 'FontSize', 18);
```

Explanation

This code takes into account the variations in seawater density due to temperature changes and uses Boyle's Law to calculate the corresponding lung volume at different depths for each location. The comparison graph at the end illustrates how the lung volume changes with depth for the three locations, showcasing the effects of temperature on diving physiology. This project confirms that temperature-induced variations in seawater density have a significant effect on the pressure experienced by divers at different depths. These results underscore the importance of understanding local seawater temperatures when planning dives, as they influence the physiological demands on a diver's body.

Pressure and Volume Model for Air in a Diver's Suit Code

```
        % This is Project 7.3.3

        % Given Data
        V1 = 0.008; % Initial volume in m^3 (8 L)
        P1 = 1.0;   % Atmospheric pressure at surface (atm)
        depths = 0:5:50; % Depths in meters
        P_water = 0.1; % Increase in pressure per meter depth (1
           atm per 10m)

        % Compute pressure and volume at each depth
        P2 = P1 + depths * P_water; % Absolute pressure at depth
        V2 = V1 .* (P1 ./ P2); % Boyle's Law: P1*V1 = P2*V2

        % Plot results
        figure;
        [ax, h1, h2] = plotyy(depths, V2, depths, P2, 'plot');
        set(h1, 'LineWidth', 2, 'Marker', 'o', 'MarkerFaceColor',
            'b', 'Color', 'b');
        set(h2, 'LineWidth', 2, 'Marker', 's', 'MarkerFaceColor',
            'r', 'Color', 'r');
        ylabel(ax(1), 'Volume (m^3)', 'Fontsize', 18);
        ylabel(ax(2), 'Pressure (atm)', 'Fontsize', 18);
```

```
        xlabel('Depth (m)', 'Fontsize', 18);
        title('Pressure and Volume Relationship in a Diver''s Suit', 'Fontsize', 18);
        legend('Volume', 'Pressure', 'Location', 'Northeast', 'Fontsize', 18);
        grid on;
```

Explanation

This code successfully models the pressure-volume relationship in a diver's suit. Thus, the resulting graph that we were given from this code has clearly illustrated the inverse relationship between pressure and volume, reinforcing the findings derived from Boyle's Law. The code also serves as a useful tool for helping us to understand the practical consequences of pressure changes underwater.