Brain potentials to mathematical syntax problems

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Abstract

Anterior negativities obtained during sentence processing have never been unambiguously reported in the mathematical domain. The reason for this might be that the tasks explored in the mathematical domain have been far from resembling those typically yielding language-related anterior negativities. To test this hypothesis, we explored three mathematical aspects: Order-relevant information, a parenthesis indicating the onset of an embedded calculation, and violations of the type of symbol displayed. Results yielded parieto-occipital instead of frontal negativities. Late posterior positivities were also found, largely comparable to linguistic P600 in topography, but dissociable in functional terms. Our data suggest that language-related anterior negativities may indeed reflect language-specific resources of the human brain and support recent claims that language and mathematical domains are more independent than previously thought.

Descriptors: LAN, P600, Syntax, Arithmetic, Language

One of the major current challenges for cognitive neurosciences is to understand the functional neurophysiology of human language. Within this issue, a primary goal is to discern to what extent human language is a specific cognitive ability within our species' brain or whether, in contrast, it is sharing resources with other cognitive functions. A good approach to explore human language is the use of event-related brain potentials (ERPs) elicited during sentence comprehension.

As a matter of fact, different ERP components have been shown to substantiate at least two main distinctive features of the human language: syntactic and semantic processes (Hauser, Chomsky, & Fitch, 2002; Schoenemann, 1999). When the type of manipulated information is semantic, the so-called N400 effect is the main finding (Kutas & Hillyard, 1980), a negative-going component between roughly 250 and 550 ms and usually larger over central and posterior electrode sites, slightly right-sided (Kutas & Besson, 1999). Typically, this component increases in amplitude with the difficulty of semantically integrating a word into its context (Chwilla, Brown, & Hagoort, 1995).

In the syntactic domain, the main findings are anterior negativities and posterior positivities. Anterior negativities, typically labeled as LAN (left anterior negativity), resemble the N400 in latency, though negativities appearing as early as between 100 and 200 ms, named ELAN (early LAN), have also been reported. Word category violations are the anomalies most frequently associated with ELAN (e.g., Friederici & Mecklinger, 1996; Friederici, Pfeifer, & Hahne, 1993), whereas other grammatical anomalies, including morphosyntactic violations (e.g., Coulson, King, & Kutas, 1998), usually evoke a LAN. Anterior negativities may reflect highly automatic first-pass parsing processes, the detection of a morphosyntactic mismatch and/or, the inability to assign the incoming word to the current phrase structure (Friederici, 2002). An alternative interpretation is the reflection of working memory load necessary during these situations (King & Kutas, 1995; Weckerly & Kutas, 1999). However, it has been recently demonstrated that anterior negativities to grammatical violations and those related to linguistic working memory are dissociable (Martín-Loeches, Muñoz, Casado, Melcón, & Fernández-Frías, 2005), the former being usually more left-lateralized and conspicuous in time, the latter being more bilateral or around the midline and of longer duration.

A second syntax-related component, labeled P600, is a late (around 600 ms) positivity that has been found for syntactic violations (e.g., Osterhout & Holcomb, 1992) and structurally ambiguous or garden path sentences (Frisch, Schlesewsky, Saddy, & Alpermann, 2002). It has been suggested that the P600 is an indicator of greater syntactic processing costs due to a necessary revision and reanalysis of a structural mismatch (Münte, Heinze, Matzke, Wieringa, & Johannes, 1998). Recent reviews on ERP components related to semantic and syntactic processes are provided by Friederici (2004) and Kutas, Van Petten, and Kluender (in press). To the extent that these language-related ERP components can also be obtained for other cognitive domains, the question of the degree of specificity of the processes involved in language processing might be solved.

For the time being, however, it appears relatively unambiguous that N400 and P600 would not reflect language-specific

processes. The N400 has been obtained to incorrect results of mathematical operations (Jost, Henninghausen, & Rösler, 2004; Niedeggen & Rösler, 1999), being equated to linguistic N400 in terms of topography, latency, and functional response (Niedeggen, Rösler, & Jost, 1999). Further evidence against the language specificity of the N400 comes from studies using pictures (McPherson & Holcomb, 1999) or odor–picture incongruence (Castle, Van Toller, & Milligan, 2000).

Similarly, a late component unambiguously equated to the linguistic P600 both in topography and latency has been reported for several types of harmonic or diatonic violations during the processing of musical information (e.g., Patel, 2003; Patel, Gibson, Ratner, Besson, & Holcomb, 1998). A late posterior positivity has also been reported to violations of mathematical sequencing and to operations with an incorrect ending, this positivity being identified with the P600 by several authors (Núñez-Peña & Honrubia-Serrano, 2004; Núñez-Peña, Honrubia-Serrano, & Escera, 2005), although in this case a direct contrast with linguistic P600 is still missing. Anomalies belonging to other nonlinguistic domains, such as geometric patterns (Besson & Macar, 1987), or abstract visual structures (Lelekov-Boissard & Dominey, 2002), also seem to yield a P600.

However, anterior negativities observed in nonlinguistic domains and identifiable as those obtained in the linguistic domain have been not only rare, but also problematic, in our view, to unequivocally equate with LAN, ELAN, or negativities related to language working memory. In this regard, what has been found along the latency range of language-related anterior negativities during the harmonic or diatonic musical violations that yield a P600 is a frontal negativity with a right-sided distribution (Gunter, Schmidt, & Besson, 2003; Koelsch, Gunter, Wittfoth, & Sammler, 2005; Loui, Grent-'t-Jong, Torpeay, & Woldorff, 2005; Patel et al., 1998). The anterior negativities associated with language, in contrast, have never displayed such a lateralization (for reviews, see, e.g., Friederici, 2004; Hagoort, Brown, & Osterhout, 1999).

Núñez-Peña and Honrubia-Serrano (2004) reported an anterior negativity to violations or number sequencing, but its identification to linguistic LAN remained open due to its unclear distribution (only a midline frontal—Fz—electrode covered the frontal portions of the scalp). Jost, Beinhoff, Henninghausen, and Rösler (2004) have recently reported a left anterior negativity to certain mathematical problems—zero multiplication—presumably involving a rule-based processing step. However, its latency was in the order of 1100 to 1900 ms, that is, very late when compared to linguistic LAN, and it was preceded by a long-lasting positivity, the pattern being rather the opposite for linguistic LAN (e.g., Friederici, 2002; Friederici et al., 1993). A narrowly similar result was obtained by Hoen and Dominey (2000) in the frame of a letter-sequencing task. It must also be raised that, contrasting with the case of the N400 or the P600, anterior negativities obtained in nonlinguistic domains have never been directly contrasted with their linguistic counterparts.

It appears, therefore, that further research is needed to establish whether the anterior negativities obtained for syntactic manipulations can indeed be found in nonlinguistic domains. The answer to this question is of the highest interest to determine the existence of brain and cognitive systems specific for human language, provided that they exist.

In our view, mathematical cognition is an excellent frame in search of nonlinguistic anterior negativities unambiguously equivalent to those obtained in the language domain. Dehaene and Cohen (1995) have proposed a model of mathematical processing implying a large overlap with language functions, this model being supported in its most part by a number of subsequent studies (e.g., Houdé & Tzourio-Mazoyer, 2003; Kadosh et al., 2005; Kong et al., 2005; Van Harskamp & Cipolotti, 2001). Overall, these lines of evidence support the idea that mathematical reasoning is making use of language-related brain areas in its most part, comprising Broca's area as well as the supramarginal and angular giri, even if additional brain regions, usually in the right hemisphere, are also recruited during these operations.

Even so, as has been probed above, ERP studies in the mathematical domain have not yielded anterior negativities unambiguously identifiable as those obtained for language. Conceivably, this state of affairs might be due to the fact that the tasks yielding nonlinguistic negativities have been far from resembling those typically yielding language-related anterior negativities, that is, violations of recursiveness and structure dependency or situations in which working memory is greatly demanded for these processes. Instead, the tasks used have involved either the break of a linear sequence (e.g., Núñez-Peña & Honrubia-Serrano, 2004) or the application of an unusual arithmetic rule (e.g., Jost, Beinhoff, et al., 2004).

Interestingly, a recent work by Varley, Klessinger, Romanowski, and Siegal (2005) has shown that subjects with brain lesions over their left-hemisphere perisylvian regions clearly impaired in grammatical language processing demonstrated proficiency in computationally equivalent mathematical problems, operations requiring recursiveness and structure dependency ("mathematical syntax" problems, in the words of Brannon's, 2005, comment on this study). Varley et al.'s (2005) work provides us, on the one hand, with previously researched mathematical operations that could be consensually considered as comparable to linguistic grammatical operations. On the other, this work also permits us to anticipate that anterior negativities obtained in the frame of linguistic syntax may not be obtained in the frame of mathematical syntax. The independence of both domains was the main finding of Varley et al. (2005), in consonance with very recent proposals (Gelman & Butterworth, 2005) that are starting to criticize Dehaene and Cohen's (1995) view of a large overlap between language and mathematical reasoning. This appears, therefore, to be a currently controversial issue.

In Varley et al. (2005), the patients were unable to differentiate between the statements "Mary hit John" and "John hit Mary," but successfully solved mathematical operations structurally dependent in this general way, as the difference between "90–60" and "60–90," or between "90/30" and "30/90." Accordingly, some of our trials depended entirely on the specific order of the operands. It has been shown that variables disambiguating word order or marking a change in word order relative to one assumed or most expected can elicit anterior negativities, particularly at the point at which a cue reveals this order. These cues can be a function word or an affix (e.g., Rösler, Pechmann, Streb, Röder, & Hennighausen, 1998; Matzke, Mai, Nager, Rüsseler, & Münte, 2002) or the conceptual features of a word (e.g., an inanimate noun appearing at the initial position is less plausibly the subject of that sentence; Casado, Martín-Loeches, Muñoz, & Fernández-Frías, 2005). Consequently, we expect functionally equivalent anterior negativities to mathematical problems in which small numbers precede larger ones in the frame of tasks encompassing subtractions or divisions.

Similarly, in Varley et al.'s (2005) study the subjects were unable to comprehend sentences with embedded clauses, such as

"The reporter that attacked the senator admitted the error," but were proficient in computing expressions with similar embedding, such as "36/(3 × 2)." Accordingly, some of our trials included embedded calculations. These embedded calculations implied that the number appearing at first position had to be maintained into working memory without completing a calculation, which cannot proceed until the embedded calculation is concluded. In our view, this resembles previous examples from linguistic literature in which an element in a sentence containing an embedded relative clause must be kept active in working memory until it is connected to the one to which it is related, the storage of incomplete head dependencies in phrase structure involving greater storage costs (Gibson, 1998). The word marking the onset of a relative clause (like "that" in the previous example) has been seen to imply greater reading difficulties (King & Just, 1991; Wanner & Maratsos, 1978) and to yield anterior negativities presumably related to linguistic working memory (e.g., King & Kutas, 1995; Martín-Loeches et al., 2005). The parenthesis marking the appearance of an embedded calculation in the mathematical tasks appears to us to highly resemble these situations. Thereafter, an anterior negativity is expected at this point.

Finally, although in Varley et al. (2005) this was not contemplated, it appears of the highest interest to explore the effects on ERPs of a mathematical violation conceivably equivalent to a word category grammatical violation. In mathematical tasks, arithmetic symbols are treated as "function" words that determine the relationships between the "content" elements whose semantic meaning is supported elsewhere (Dehaene & Cohen, 1995; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). In our view, presenting an operating symbol where a number should appear is to some extent equivalent to displaying a function or open-class word where a close-class word should appear and, thus, conceivably resembles a word category violation. We expect anterior negativities to these violations. Furthermore, it has also been demonstrated that in the linguistic domain, working memory is overtly more taxed as the distance between an element already kept in working memory and the new appearing material increases (Ford, 1983; King & Just, 1991), which has been seen to affect anterior negativities to grammatical violations (Vos, Gunter, Kolk, & Mulder, 2001). Consequently, we expected that if arithmetic violations resembling word category violations yielded anterior negativities, those occurring at late positions within longer problems would be affected in comparison to those occurring at earlier positions.

It is the main aim of the present study to explore the possibility that several situations in the mathematical domain may yield ERP modulations, particularly anterior negativities, analogous to those obtained in the linguistic domain. To achieve this goal, we presented our participants with mathematical problems containing situations presumably akin to those yielding anterior negativities in the linguistic domain, namely, (a) order-relevant information determining the type of final result, (b) the presence of a parenthesis indicating the onset of an embedded calculation, and (c) violations of the type of element appearing within a problem. If one or several of these manipulations yielded anterior negativities, they could be directly contrasted in terms of latency and topography with anterior negativities obtained in the frame of linguistic studies closely similar in structure and design performed by the present group of authors (Casado et al., 2005; Martín-Loeches et al., 2005). The same procedure will also be applied to expected late posterior positivities, this being a secondary aim of the present study.

Methods

Participants

Thirty native Spanish people (28 women, mean age 21.2 years, range 18–39) participated in the experiment. All of them were right-handed, with average handedness scores of +82, ranging from +50 to +100, according to the Edinburgh Handedness Inventory (Oldfield, 1971). All participants had normal or corrected-to-normal vision, and had no history of reading difficulties or neurological or psychiatric disorders. Participants were paid with course credit for participating in the experiment.

Materials

Order-relevant condition. Two different conditions were used in the present study. In one (order-relevant condition), we attempted to explore the effects on ERP of information determining a final negative or fractional result, this fact directly depending on the order of the operands. In this condition, the experimental material consisted of 96 mathematical problems with two operands. All of the operands included in the problems were between 1 and 10. Half (48) of the problems were subtractions and the other half were divisions. Within either group of subtractions or divisions, in half of the problems the first number was ≥ 5 and the second number was ≤ 4, so that the correct result resulted in either a positive value in subtractions or a non-fractionary number in divisions. In the other half of the problems the first number was ≤ 4 and the second number was ≥ 5, so that the correct result was negative in subtractions and a fractionary number in divisions. Every problem was followed by a result that could be correct (half of them) or incorrect. The incorrect displayed results could belong to one of two types: (a) It would have been a correct result in case of inverting the order of presentation of the operands (half of the incorrect results) or (b) other but the correct result in either order, the number between 1 and 10. Examples are given below:

Correct operations:

- $9 - 1 = 8$ (subtraction); $6/2 = 3$ (division) [first number ≥ 5, second number ≤ 4]
- $2 - 8 = -6$ (subtraction); $3/6 = 0.5$ (division) [first number ≤ 4, second number ≥ 5]

Incorrect operations:

- $5 - 1 = -4$ (subtraction); $5/3 = 0.6$ (division) [first number ≥ 5, second number ≤ 4; correct if inverting the order of the operands]
- $6 - 4 = 5$ (subtraction); $8/4 = 3$ (division) [first number ≥ 5, second number ≤ 4; unrelated incorrect result]
- $3 - 7 = 4$ (subtraction); $9/3 = 0.33$ (division) [first number ≤ 4, second number ≥ 5; correct if inverting the order of the operands]
- $1 - 8 = -2$ (subtraction); $4/6 = 0.5$ (division) [first number ≤ 4, second number ≥ 5; unrelated incorrect result].

Resembling the fillers used in the frame of language studies, we also included 48 nonexperimental mathematical problems with the aim of avoiding subjects' anticipation of the structure of the experimental operations. These problems were longer (three operands) and included sums and multiplications in addition to subtractions and divisions, these operations appearing randomly within each problem. Half of the results presented after each of

these nonexperimental problems were incorrect. No constraint was introduced here regarding the operands appearing within a problem and their order, with the exception that they had to be between 1 and 10.

All the problems were presented element (either number or symbol) by element on the center of a computer screen. Every element appearing before the equal sign (=) had a duration of 300 ms and an interstimulus blank interval of 200 ms. The equal sign also lasted 300 ms, followed by a blank period of 700 ms, after which the result (either correct or incorrect) was presented for 2 s. After the presentation of the result, a white square appeared for 1 s, followed by the first operand of the following problem (Figure 1A).

Embedded calculation and violation condition. In the other condition (embedded calculation and violation condition) we attempted to explore the effects of the presence of a parenthesis indicating an embedding and of mathematical violations that could be equivalent to word category violations. For this condition, 128 experimental mathematical problems were used, pertaining to one of either two types: (a) short problems: 64 problems with two operands, using the four basic mathematical operations (addition, subtraction, multiplication, and division), and (b) long problems: 64 operations with three operands, also using the four basic mathematical operations but now combined with parentheses comprising the last two operands of the problem. In either type of problem, the operands were always between 1 and 10. Half of the problems within each of the two types included a violation, consisting of presenting a mathematical operation symbol where a number should appear. The violation's position was always that corresponding to the last number of the problem. Every solvable problem (i.e., without violation) was followed by the displaying of a result that could be correct

(50%) or incorrect. The incorrect result could be any number between 1 and 100, as was the case for the result appearing after a problem with a violation. An example of each type of problem is given below:

Short problems:

Without violation:

Correct displayed result: $4+8 = 12$
Incorrect displayed result: $5 \times 6 = 25$

With violation:

$5 \div \; - \; = 9$

Long problems:

Without violation:

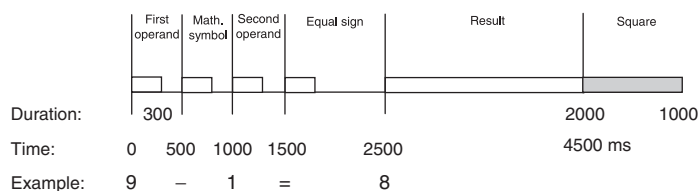Correct displayed result: $8 - (5 \times 1) = 3$
Incorrect displayed result: $6+(3 \times 5) = 15$

With violation:

$4 \div (10 - \times) = 3$

As in the order-relevant condition, nonexperimental (filler) problems were also included. There were 128 and they belonged to one of either two groups: (a) Half of them never included parentheses, but comprised either three or four (this balanced) operands and (b) the other half had always four operands and the last three numbers were embedded between parentheses. Again, the operands were always between 1 and 10. Within either group of nonexperimental operations, half of them included a violation by substituting the last operand by an operation symbol. Again,
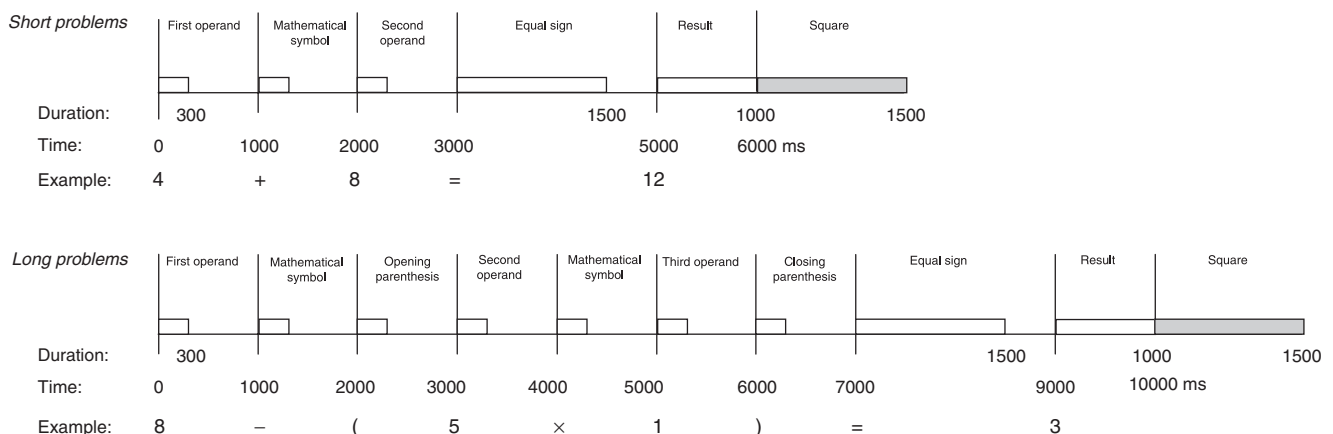

Figure 1. Examples of the problems presented to subjects, together with a schematic representation of the stimulation procedures. Two conditions were used: order-relevant condition (A) and embedded calculation and violation condition (B).

every solvable problem was followed by the displaying of a result that could be correct (50%) or incorrect. The incorrect result could be any number between 1 and 100, as was the case for the result appearing after a violation.

All the operations were presented element by element on the center of a computer screen. Every element appearing before the equal sign (=) had a duration of 300 ms and an interstimulus blank interval of 700 ms. The equal sign lasted 1500 ms, followed by a blank period of 500 ms, after which the result (either correct or incorrect) was displayed for 1 s. After the presentation of the result, a white square appeared for 1500 ms, and followed by the first operand of the next problem (Figure 1B).

In both the order-relevant and the embedded calculation and violation conditions, all the stimuli were matched in visual aspects. They were presented white on black and controlled by SuperLab software. All the stimuli were between 0.7° and 1.3° high, and between 1.1° and 6° wide. Finally, the same number could never appear in two successive problems.

Procedure

Each experimental session lasted for about 3 h, and subjects were given short breaks during the recordings. Participants were told that they would be presented with a series of mathematical problems and that they had to perform a judgment about the correctness of the result displayed at the end of every problem, and that when an operation was unsolvable (trials with violations), the result was considered incorrect. They were instructed to press one of two buttons when the displayed result was correct and the other button when the result was incorrect or followed an unsolvable operation, and to give their responses just at the moment at which the result appeared in the computer screen. The session started with a short practice block that did not include any of the experimental problems. For each subject and condition, the mathematical problems were extracted and ordered randomly from the corresponding pools of experimental and nonexperimental operations. Order of presentation of the two conditions was balanced across subjects.

Electrophysiological Recordings

The electroencephalogram (EEG) was recorded from 27 tin electrodes embedded in an electrode cap (ElectroCap International). All electrodes were referenced online to the right mastoid, and re-referenced off-line to the average of the left and right mastoids. Bipolar horizontal and vertical electrooculograms (EOG) were recorded for artifact monitoring. Electrode impedances were always kept under 3 kΩ. The signals were recorded continuously with a bandpass from 0.01 to 40 Hz and a sampling rate of 250 Hz.

Data Analysis

First, the continuous EEG was divided into epochs as described below (they depended on the condition analyzed), and all the epochs belonging to trials with subjects' errors were eliminated. Artifacts were automatically rejected by eliminating those epochs that exceeded ± 100 μV. Additionally, a visual inspection was performed to eliminate epochs with too many blinks, excessive muscle activity, or other artifacts. Off-line correction of smaller eye movement artifacts was also made, using the method described by Gratton, Coles, and Donchin (1983).

In the order-relevant condition, epochs from − 100 to 800 ms relative to stimulus onset were made after collapsing subtractions and divisions and were constructed for each of the elements

(either operands or mathematical signs) included in the experimental problems, separately. Each problem comprised five elements, including the equal sign and the displayed result. Correct and incorrect displayed results were collapsed, as the main interest was on ERP modulations as a function of the size of the first operand relative to the second one. Epochs were made for problems with a first operand ≤ 4 (then, second operand ≥ 5) and problems with a first operand ≥ 5 (second operand ≤ 4) separately. Repeated-measures analyses of variance (ANOVAs) were performed to analyze ERP amplitudes using two factors: Value of the first operand (two levels: ≤ 4 or ≥ 5) and 27 Electrodes. Analyses of amplitudes and, therefore, ANOVAs were performed on consecutive 50-ms-wide windows.

In the embedded calculation and violation condition, epochs from − 200 to 1100 ms relative to stimulus onset were constructed only for the elements of main interest, separately. These were (a) the second number of short problems without violation, (b) the violating symbol substituting that number in short problems with violation, (c) the third number in long problems without violation, (d) the violating symbol substituting that number in long problems with violation, and (e) the opening parenthesis in the long problems without violations (those in the problems with violations were considered theoretically and functionally equivalent and not included in separate analysis after a visual confirmation of identical results). Epochs in the problems without violation (a and c) were made disregarding whether the result displayed at the end of the problem was correct or not, as the variables of interest appeared before and did not depend on this factor. Overall repeated-measures ANOVAs were first performed to analyze ERP amplitudes using three factors: Problem Length (long, short), Violation (absent, present), and 27 Electrodes. Separate ANOVAs were also performed comparing the opening parenthesis in long problems without violation with the second operand in short problems without violation, as well as with the violation in short problems. Analyses of amplitudes and, consequently, ANOVAs were performed on consecutive 100-ms-wide windows, starting at 100 ms after stimulus onset and ending until the end of the epochs. The Greenhouse–Geisser correction was always applied.

Finally, profile analyses (McCarthy & Wood, 1985) were performed to assess differences in scalp topographies independent of overall ERP amplitude (Rugg & Coles, 1995). Topography comparisons were made between components that could be interpreted to be the same, both within this study, within and between conditions, as well as between the data obtained here and others from previous language studies performed by our group.

Results

Behavioral Data

In the order-relevant condition, correct and incorrect displayed problems' results were collapsed. Thereafter, the percentage of errors was 20.2% for the problems with a first operand ≤ 4 (second operand ≥ 5) and 12.6% for the problems with a first operand ≥ 5 (second operand ≤ 4), this difference being significant, $t(1,29) = 4.8$, $p < .0001$. Reaction times measured from the onset of the result were 1081.19 and 951.38 ms, respectively, again differing significantly, $t(1,29) = 70.8$, $p < .001$. Clearly, problems starting with small numbers were more difficult to the subjects.

In the embedded calculation and violation condition, correct and incorrect displayed results were collapsed for problems without violation. Thereafter, the percentage of errors was 3.8% for the short problems without violation, 1.4% for the short problems with a violation, 11.4% for the long problems without violation, and 1.3% of the long problems with a violation. An ANOVA revealed strong main effects of Problem Length, Violation, and their interaction, $F(1,29) = 23.48$, 25.12, and 21.31, respectively, p alwaysc $<.0001$. Reaction times where computed from the onset of the result, yielding 887 ms for the short problems without violation, 900 ms for the short problems with a violation, 973 ms for the long problems without violation, and 664 ms for the long problems with a violation. An ANOVA indicated again main effects of Problem Length, Violation, and their interaction, $F(1,29) = 100.5$, 13.35, and 36.13, respectively, p between $<.001$ and $<.0001$. Apparently, the easiest problems for the subjects were the long ones with a violation.

Electrophysiology

Order-relevant condition. When comparing a small (≤ 4) first operand with a large one (≥ 5), a parieto-occipital negativity for small operands was observed starting at about 200 ms and resolving about 250 ms later (Figure 2). A central or centro-parietal positivity, slightly left-lateralized, followed, starting at about 400 ms and lasting until the end of the epoch. These effects were small, however, and only the time windows between 200 and 500 ms yielded significant effects of value of first operand by electrode interaction, $F(26,754)$ between 2.5 and 6.4, ε between .147 and .177, p between $<.05$ and $<.001$. Accordingly, only the initial parieto-central negativity appears statistically supported.

When the second element of each problem, that is, the subtraction or division signs, was compared according to the value of the first operand of the problem, the result was a long-lasting central or centro-parietal negativity, slightly left-lateralized, for symbols preceded by small numbers. This started very early and lasted for about 600 ms, then starting a parietal positivity by the end of the epoch. Significant effects for the value of first operand and for its interaction with electrode were found in the 50–100-ms window, $F(1,29) = 4.8$, $p < .05$, and $F(26,754) = 3.1$, $\varepsilon = .168$, $p < .05$, respectively. After that, only the 200–250 ms window, those between 300 and 500 ms, and 650–700 ms window yielded significant effects of value of First Operand × Electrode interactions, $F(26,754)$ between 2.5 and 7.7, ε between .158 and .192, p between $<.05$ and $<.001$. The very early onset of the negative fluctuation, as well as its discontinuous statistical effects and its distribution, suggest that this might be the result of baselining at this element the nonsignificant centro-parietal positivity appearing for the preceding element. The very late positivity appears to be a reflection of processes linked to the following stimulus.

In the comparison of the second number in each problem, we observed a long-lasting positivity for second (and large) numbers
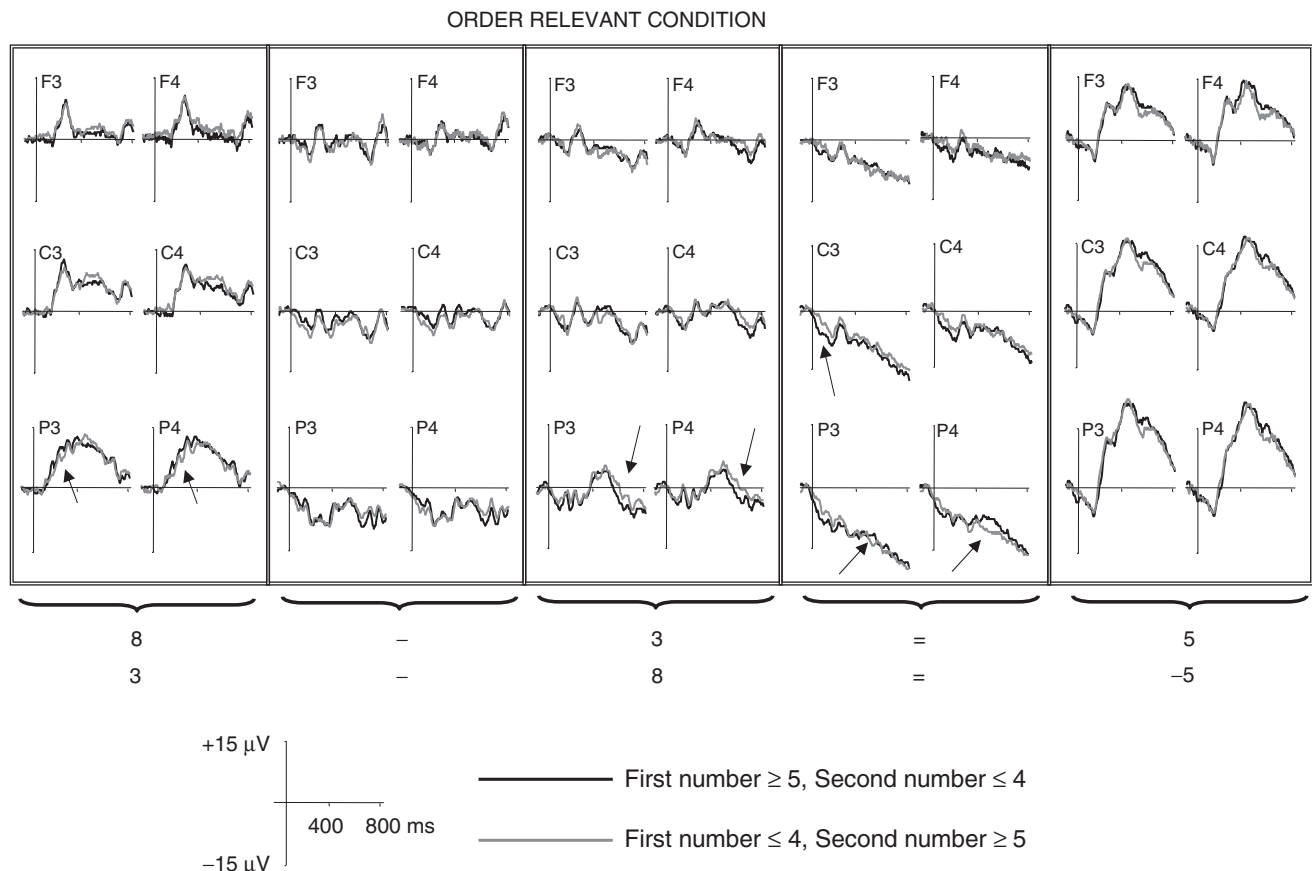
ORDER RELEVANT CONDITION

Figure 2. Order-relevant condition. Grand average ERP ($n = 30$) time-locked to the onset of each element in the problem, comparing problems starting with a small (≤ 4) first operand (then, second operand ≥ 5) with problems starting with a large one (≥ 5, second operand ≤ 4). For this and subsequent figures only a selection of electrodes is displayed, and arrows are depicted pointing to relevant results supported statistically.

preceded by small (first) operands, which started at about 100 ms and lasted until the end of the epoch. First, this positivity appeared more parietal, thereafter being rather parieto-occipital, the voltage slightly increasing progressively. With the exception of the 200–250 ms window, all the windows from 150 ms to the end of the epoch yielded significant effects of the interaction between Value of first operand and Electrode, $F(26,754)$ between 2.7 and 7.1, ε between .118 and .184, p between <.05 and <.001). There was also a significant main effect of value of first operand in the 50–100-ms and 150–200-ms windows, as well as in those between 550 ms and the end of the epoch, $F(1,29)$ between 3.8 and 8.4, p between <.05 and <.01.

When the equal sign was compared according to the value of the first operand of the problem, a mainly central positivity, starting with the onset of the epoch and lasting about 250 ms, was found for signs in operations starting with small numbers. This was followed by a parieto-occipital negativity covering from 250 ms to the end of the epoch. In parallel, statistical analyses revealed two groups of results. On the one hand, significant main effects of Value of first operand were found only in the windows from 50 to 200 ms, $F(1,29)$ between 3.0 and 9.1, p between <.05 and <.01. On the other hand, significant effects of the interaction between Value of first operand and Electrode were found in the windows from 250 ms to the end of the epoch, $F(26,754)$ between 2.2 and 4.0, ε between .127 and .209, p between <.05 and <.01.

The comparison of the problems' results as a function of the value of first operand revealed identical results for both types of operations along the first 400 ms, following thereafter a negativity in problems starting with small numbers. However, the absence of any significant result in the corresponding ANOVAs fairly supports no clear differences at this point as a function of the value of first operand.

The main results for this condition could be summarized as follows. The initial presence of a small number, making more probable that the second number will be a larger one, initiates a process reflected in a parieto-occipital negativity from 200 to about 450 ms. Later, a large parieto-occipital positivity is the more remarkable result, appearing when it is unambiguous that a second number is larger than the first one in the frame of a subtraction or a division problem. Finally, the equal sign indicating the end of a problem with first operand ≤ 4 evoked a central positivity and a parieto-occipital long-lasting negativity.

Embedded calculation and violation condition. The effects of length and violations will be analyzed first. By looking at Figure 3, it can be observed that violations, regardless of the length of the problem, displayed an initial negativity, roughly from 200 to 450 ms and with parieto-occipital distribution, followed by a long-lasting but conspicuous parietal positivity from about 500 to about 1300 ms. At variance with the preceding negativity, which was similar regardless of problem length, this parietal positivity appeared noticeably larger in amplitude for longer problems, although the peak latency was the same (about 650 ms) in both lengths. Length alone effects appear to be reflected in a small long-lasting positivity for longer problems, starting at about 400 ms and covering the remaining period of the epoch. This positivity displayed an overall distribution, although it tended to be rather fronto-central in a late period.

Overall ANOVA results are summarized in Table 1. These confirm the existence of Violation or Violation × Electrode effects from 100 to 1100 ms independent of length effects; Length

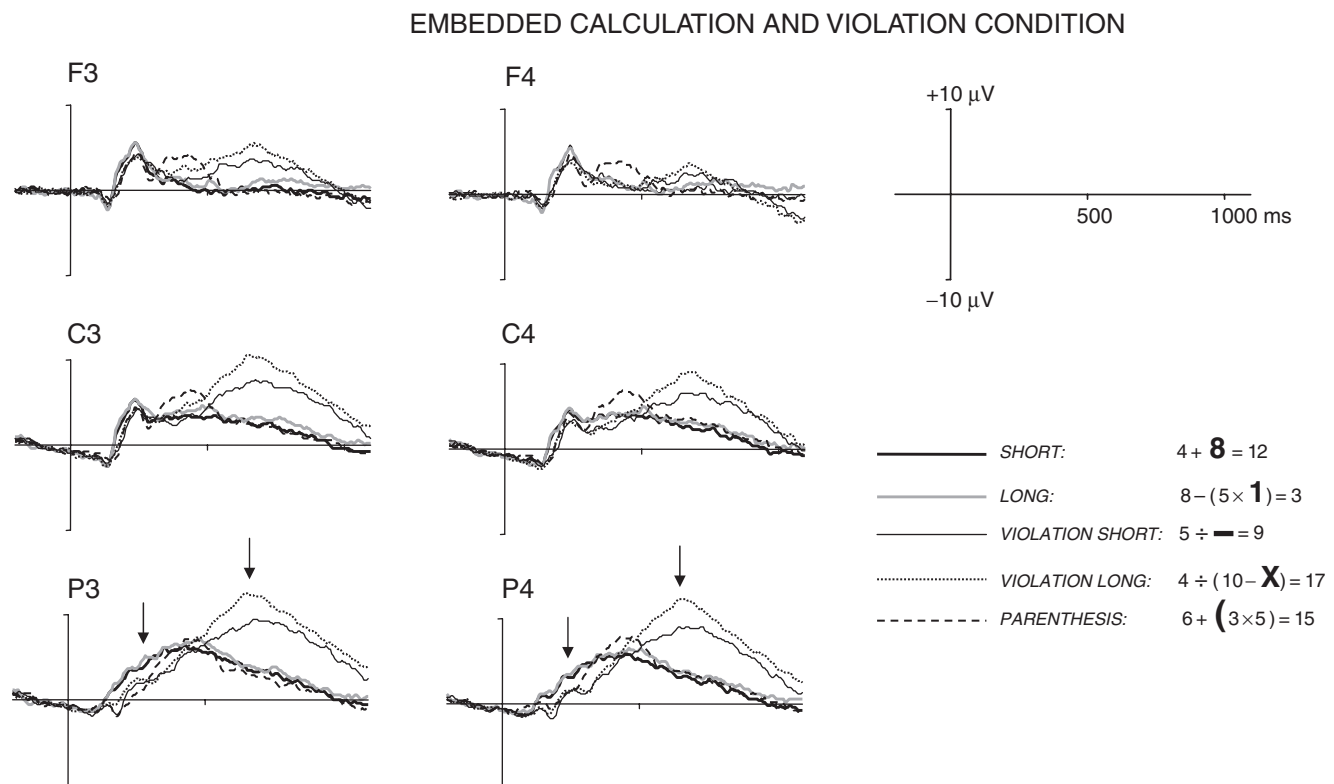## EMBEDDED CALCULATION AND VIOLATION CONDITION

Figure 3. Embedded calculation and violation condition. Grand average ERP time-locked to the onset of each relevant element for the effects of problem length, violation, and an opening parenthesis indicating the onset of an embedded calculation.

Table 1. *Overall ANOVA Results in the Embedded Calculation and Violation Condition (F Values)*

	d.f.	100–200	200–300	300–400	400–500	500–600	600–700	700–800	800–900	900–1000	1000–1100
Violation	1,29	28.1****	62.7****	27.7****		23.1****	75.7****	97.2****	77.6****	49.0****	8.4**
Violation × Electrode	26,754	5.28***	8.2***	25.2****	5.5***		31.6****	39.8****	33.3****	33.5****	32.7****
Length	1,29					6.2*	9.3*				
Length × Electrode	26,754				9.7****	8.6****	9.8****	9.8****	4.7**	4.8**	4.4**
Violation × Length	1,29						11.0**				
Violation × Length × Electrode	26,754						6.3****	3.2*	3.2*	4.6*	2.8*

*p<.05; **p<.01; ***p<.001; ****p<.0001. Only significant results are displayed.

or Length × Electrode effects independent of violation effects from 500 to 1100 ms, and interactions between both effects, mainly in interaction with Electrode, starting at 600 ms and ending at 1100 ms. Further step-down analyses follow. When comparing violations (operation symbols substituting an expected operand) and correctly appearing operands in short problems, there were both significant main effects of Violation and of the Violation × Electrode interaction across all the analyzed windows, with the exception of the 400–500-ms window for violation, $F(1,29)$ between 14.2 and 62.5, p between <.01 and <.0001 for violation alone effects; $F(26,754)$ between 3.2 and 23.1, ε between .080 and .130, p between <.05 and <.0001 for the interaction. The same comparison in longer problems revealed identical pattern, $F(26,754)$ between 11.6 and 69.8, p between <.01 and <.0001 for violation alone effects; $F(26,754)$ between 3.1 and 37.3, ε between .105 and .147, p between <.05 and <.0001 for the interaction. When comparing violations as a function of problem length, significant differences started in the 400–500-ms window and covered the whole set of subsequent windows for the length by electrode interaction, $F(26,754)$ between 3.2 and 12.8, ε between .115 and .147, p between <.05 and <.0001, length main effects appearing only from 500 to 800 ms, $F(1,29)$ between 6.0 and 15.7, p between <.01 and <.0001. Finally, when comparing correctly appearing operands in short and

long problems, only the windows 400–500 and 500–600 ms yielded significant results, these corresponding to Length × Electrode interactions, $F(26,754) = 4.4$ and 3.5, ε = .134 and .146, respectively, <.05 in both cases.

On the whole, it seems that mathematical violations like those used here evoke an initial posterior negativity, independent of the length of the problem, followed by a posterior positivity that interacts with problem's length in the sense of increasing its amplitude when the problem is larger. Length alone effects, on the other hand, seem to manifest in a widely distributed small positivity between 400 and 600 ms.

The effects of the appearance of an opening parenthesis indicating an embedded problem are analyzed at this time (Figure 4). When this parenthesis was compared to numbers in the same problem's positions (i.e., second operand in short problems), we could observe, first, a negativity from 200 to 400 ms with parieto-occipital distribution, strikingly very similar to that found for violations. Thereafter, a fronto-central positivity started at about 350 ms and seemed to resolve by 600 ms. Results of the ANOVA confirmed these differences. We found significant main effects of the Type of element (parenthesis, number) in the 100–500-ms segments, $F(1, 29)$ between 5.5 and 16.8, p between <.05 and <.0001, and in interaction with Electrode from 200 to 600 ms, and from 900 to 1100 ms, $F(26,754)$ between 3.1 and 5.6,

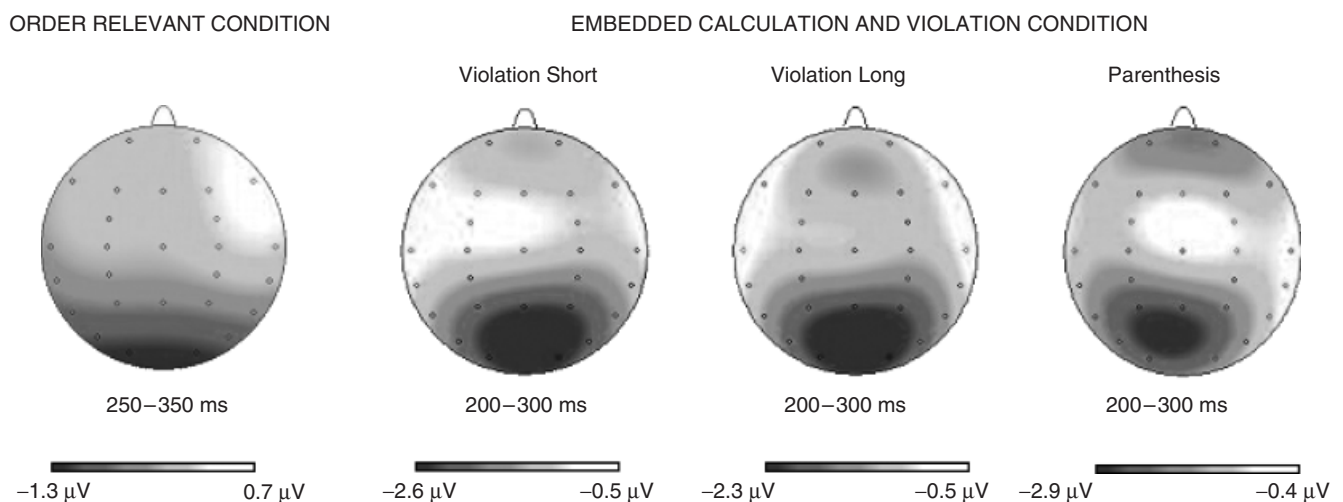ORDER RELEVANT CONDITION EMBEDDED CALCULATION AND VIOLATION CONDITION

Figure 4. Early posterior negativities found in the order-relevant condition for small numbers (≤ 4) starting a problem (left) and in the embedded calculation and violation condition for violations in short problems, violations in long problems, and opening parentheses indicating the onset of embedded calculations. For this and subsequent figures, the maps are computed from the mean amplitude in the corresponding time windows and interpolated with spherical splines, using the algorithm described in Perrin, Bertrand, and Echallier (1989). Also for this and subsequent figures, individual scales based on the particular maximum and minimum values are used in the maps.

ε between .110 and .138, *p* between <.05 and <.001. When the opening parenthesis was compared with the violation in short problems, remarkable differences started to manifest at about 300 ms. These consisted, first, in the already described fronto-central positivity for the parenthesis, followed by the large positivity in the violation. Even so, small amplitude differences also appeared for the early parieto-occipital negativity common to both elements. This is supported by significant effects of the interaction between Type of element (parenthesis, violation) across all the analyzed windows, from 100 to 1100 ms, $F(26,754)$ between 2.8 and 19.3, ε between .029 and .135, *p* between <.05 and <.0001. Significant main effects of Type of element were found to start at 300 ms and cover all the remaining subsequent windows, $F(1,29)$ between 6.4 and 56.7, *p* between <.05 and <.0001.

Summarizing, the appearance of a parenthesis indicating the existence of an embedded calculation evokes an initial posterior negativity similar to that evoked by violations although of slightly larger amplitude. Then, a fronto-central positivity, not present for violations, follows.

Within-study and between-studies comparisons of posterior negativities and posterior positivities. Importantly, and for the purposes of the present study, we were not able to find any frontal negativity despite the high resemblance of certain situations here explored with conditions that reliably yield anterior negativities in the frame of linguistic studies. What we have observed instead in the two conditions are posterior negativities with a similar latency to that reported for linguistic anterior ones, as well as later posterior positivities seemingly resembling in latency and topography the language-related P600.

An early parieto-occipital negativity was found in the order relevant-condition for small first numbers, as well as for violations regardless of the length of the problem, and for the appearance of an opening parenthesis. All these negativities were compared by means of profile analyses. For these comparisons, similar 100-ms-wide windows corresponding to the segment of highest amplitude value of each negativity were analyzed to per-

form amplitude measurements, which were always the result of the subtraction between the trials yielding the negativity and the corresponding control trials (Figure 4). Overall repeated-measures ANOVA results with unscaled data yielded significant effects of Type of negativity (4 levels: one negativity from the order-relevant condition, three from the embedded calculation and violation conditionviolation in short problems, violation in long problems, and parenthesis) × Electrode interaction, $F(78,2262) = 3.7$, ε = .094, *p*<.05. When the data were scaled, this interaction continued to be significant, $F(78,2262) = 3.4$, ε = .101, *p*<.05. Subsequent pairwise profile analyses revealed that whereas all the early posterior negativities in the embedded calculation and violation condition (evoked by both violations regardless of length and by parenthesis) did not differ significantly between them, the negativity in the order-relevant condition was significantly different in topography whenever compared to those in the other condition, $F(26,754)$ between 2.5 and 2.6, ε between .129 and .143, *p* always <.05. Accordingly, the early negativity in the order-relevant condition appears to involve different neural and, consequently, cognitive processes to those involved in the embedded calculation and violation condition. In the latter, in contrast, the same early processes would be involved both when a violation occurs regardless of problem's length as well as when an opening parenthesis appears.

In the embedded calculation and violation condition, the two violations yielded a late posterior positivity whereas the parenthesis yielded a fronto-central positivity. Although the similarity in topography between the positivities for the violations as well as their disparity with the one for the parenthesis were evident to the eye (Figure 5), profile analyses were performed to support this depiction. Again, similar 100-ms-wide windows centered in the peak of each corresponding positivity were analyzed to perform amplitude measurements, being always the result of the subtraction between the stimulus yielding the positivity and the corresponding control stimulus. Overall repeated-measures ANOVA results with scaled data yielded significant effects of Type of positivity (3 levels: violation in short problems, violation in

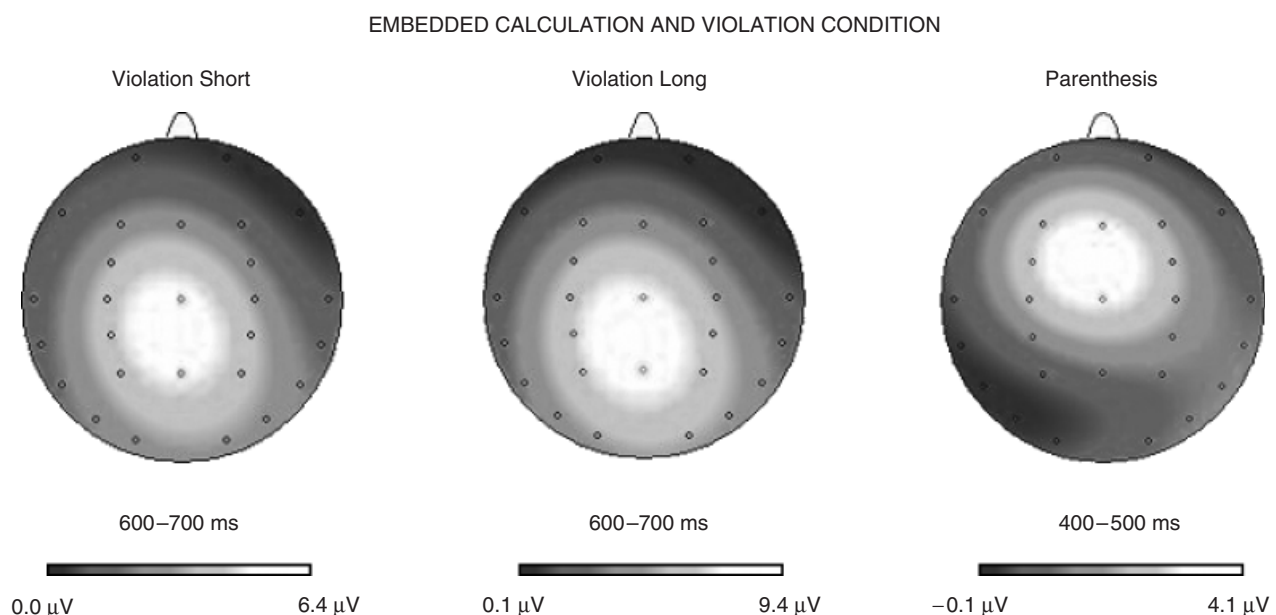## EMBEDDED CALCULATION AND VIOLATION CONDITION

Figure 5. Late positivities in the embedded calculation and violation condition for violations in short problems, violations in long problems, and opening parentheses indicating the onset of embedded calculations.

Table 2. *Comparisons between Late Posterior Positivities in Present (Mathematic) and Previous (Linguistic) Studies by Means of Profile Analyses (F Values)*

	Posterior positivities		Comparisons			
	Condition yielding the positivity	Type of process	2	3	4	5
1	Semantic info. reverses sentence order (Casado et al., 2005)	Linguistic	—	—	—	3.2*
2	Syntactic info. reverses sentence order (Casado et al., 2005)	Linguistic		—	5.1**	—
3	Word category violation (Martín-Loeches et al., 2005)	Linguistic			—	3.6**
4	Order-relevant condition	Mathematical				3.3*
5	Embedded calculation and violation condition (violation)	Mathematical				

$F(26,1560)$ in the comparisons 1 vs. 3, 2 vs. 3, 3 vs. 4, and 3 vs. 5; $F(26,1508)$ in the remaining comparisons.
*$p < .05$; **$p < .01$. Only significant results are displayed.

long problems, and parenthesis) × Electrode interaction, $F(52,1508) = 4.0$, $\varepsilon = .081$, $p < .05$. Subsequent pairwise profile analyses revealed that whereas the late posterior positivities evoked by both violations did not differ significantly, the positivity evoked by the parenthesis was significantly different in topography when compared to those in either violation, $F(26,754) = 4.7$ and 2.9; $\varepsilon = .147$ and 0.129, $p < .001$ and $.05$ for the violations in short and long problems, respectively.

As mentioned in the introduction, we also wanted to directly compare the late posterior positivities here obtained for mathematical operations with P600 obtained in the frame of linguistic tasks. We have obtained these components in previous linguistic studies that were, to a large extent, similar in structure and design to those used here for mathematical problems. In Casado et al. (2005) we obtained P600 in conditions in which the order of the constituents of a sentence reversed according to either semantic or syntactic information of the words. Although slightly dissimilar, the two P600 resulted topographically the same in statistical terms regardless of the type of information determining word order. The two P600 of that study will be used here. In Martín-Loeches et al. (2005) we obtained a P600 to word-category violations regardless of sentence length. The three P600 s to word-category violations obtained in that study were identical in both amplitude and topography, so only the one for short sentences will be used here for simplicity reasons. Accordingly, these

three unambiguously linguistic P600 s will be compared with the posterior positivities here obtained in the order-relevant and the embedded calculation and violation conditions (only for short problems in the latter, because short and long problems did not differ in topography) by means of profile analyses.

For these comparisons, similar 100-ms-wide windows centered in the peak of each corresponding posterior positivity from each study were analyzed to perform amplitude measurements, which were always the result of the subtraction between the condition yielding the positivity and the corresponding control condition. All the studies have used a linked mastoids reference and, because Casado et al. (2005) and Martín-Loeches et al. (2005) used more than the same 27 electrodes used here, the additional electrodes were skipped for the present analyses. Overall between-subjects ANOVA results with unscaled data yielded significant effects of Type of positivity (5 levels: two from the Casado et al., 2005, study and one from the Martín-Loeches et al., 2005, study, one from the order-relevant condition, and one from the embedded calculation and violation condition) × Electrode interaction, $F(104,3822) = 3.6$, $\varepsilon = .124$, $p < .0001$. When the data were scaled, this interaction continued to be significant, $F(104,3822) = 2.8$, $\varepsilon = .129$, $p < .0001$. Subsequent pairwise profile analyses are displayed in Table 2. The topographies compared are shown in Figure 6. Interestingly, the picture is somewhat complex. On the one hand, results manifest that the
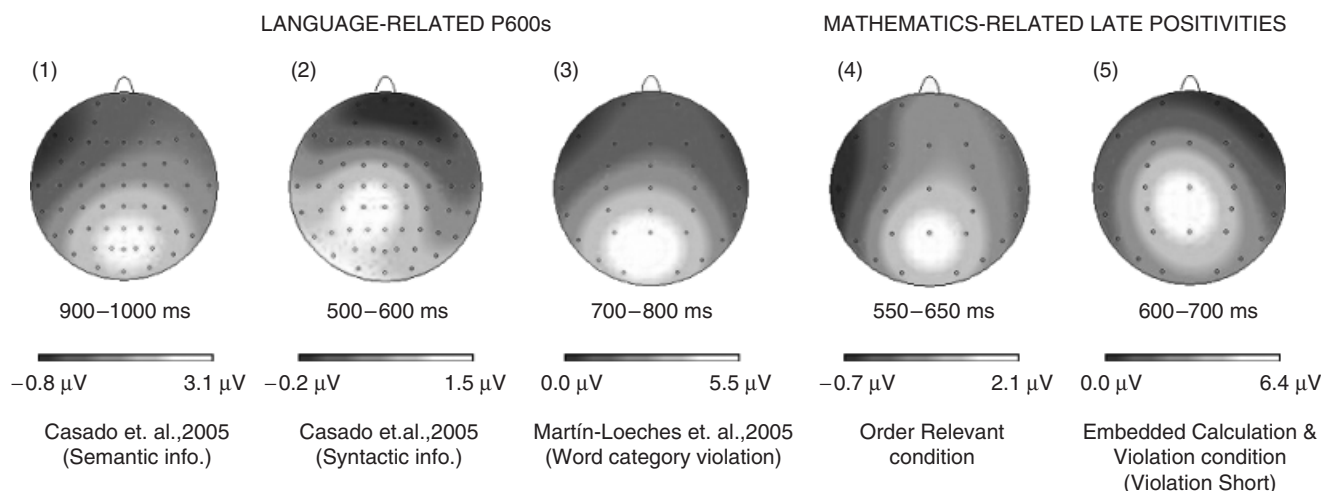
LANGUAGE-RELATED P600s   MATHEMATICS-RELATED LATE POSITIVITIES

(1)	(2)	(3)	(4)	(5)
900–1000 ms	500–600 ms	700–800 ms	550–650 ms	600–700 ms
−0.8 µV 3.1 µV	−0.2 µV 1.5 µV	0.0 µV 5.5 µV	−0.7 µV 2.1 µV	0.0 µV 6.4 µV
Casado et. al.,2005 (Semantic info.)	Casado et.al.,2005 (Syntactic info.)	Martín-Loeches et. al.,2005 (Word category violation)	Order Relevant condition	Embedded Calculation & Violation condition (Violation Short)

Figure 6. Language-related late posterior positivities (P600) found in previous studies for word-order relevant information (maps 1 and 2) and a word category violation (map 3), compared with the late posterior positivities found in the present study for the order-relevant condition (map 4) and the violation in short problems of the embedded calculation and violation condition (map 5). Map numbers match those in Table 2.

late positivities obtained in language tasks (positivities 1 to 3 in Table 2 and Figure 6) did not differ significantly in topography between them. On the other hand, some of these linguistic positivities (1 and 3) did not differ significantly when compared to the positivity in the order-relevant condition (positivity 4), but did so when compared to the positivity evoked by a mathematical violation (positivity 5), the latter being nonetheless comparable in topography to the P600 in Casado et al. (2005) evoked by syntactic information reversing sentence order (positivity 2). Accordingly, a large degree of overlap between the generators involved in linguistic P600 and those yielding the late positivities in mathematical problems could be assumed, even if subtle but significant differences suggest that the generators are not exactly the same. In turn, the positivities caused by mathematical manipulations differed significantly in topography between them, which indicates that the processes involved at this stage in both mathematical situations appeared to some extent dissociable.

Discussion

The main objective of the present study was to obtain, in the frame of mathematical problems, anterior negativities that could be unambiguously identifiable as those obtained in the language domain. For these purposes, the mathematical tasks explored here incorporated several features apparently equivalent to those that have yielded anterior negativities in the language domain. The most important result in this regard has been that we were unable to find anterior negativities.

First, some of our trials depended importantly on the specific order of the operands. In the language domain, variables disambiguating word order or marking a change in word order relative to one assumed or most expected can elicit anterior negativities (e.g., Casado et al., 2005; Matzke et al., 2002). Consequently, we expected functionally equivalent anterior negativities to mathematical problems in which numbers between 1 and 4 were presented at the first position, as they would be most probably followed by a larger second number. A parieto-occipital negativity was found instead.

Second, some of our trials included embedded calculations. The parenthesis marking the appearance of an embedded calculation in the mathematical tasks here used appeared to us to highly resemble situations in the language domain in which a word marking the onset of a relative clause has been seen to yield anterior negativities presumably related to linguistic working memory (e.g., King & Kutas, 1995; Martín-Loeches et al., 2005). We expected, therefore, functionally equivalent anterior negativities to parentheses marking the onset of an embedded calculation. However, a posterior negativity followed by a fronto-central positivity was obtained instead.

Third, some trials included what could be considered as a parallel to word category violations by presenting an operating symbol where a number should appear. We expected anterior negativities to these violations. We also expected that those negativities occurring at late positions within long problems would be affected in comparison to those occurring at earlier positions, as is the case in the language domain (Vos et al., 2001). However, not only we did not find anterior negativities but the posterior negativities we did find for mathematical violations were the same regardless of problem length. Furthermore, in Martín-Loeches et al. (2005) it was shown that language-related anterior negativities to grammatical viola-

tions and to the onset of embedded clauses significantly differ in topography, whereas the early negativities here found for mathematical violations and those for the parentheses displayed the same topography.

Thus, considering present and previous results in which anterior negativities to nonlinguistic manipulations have not been unambiguously identifiable as those obtained for language, it is our conclusion that anterior negativities obtained in the linguistic domain appear language specific, and that no firm evidence against this assertion could be alleged for the time being.

Overall, what we have found instead of anterior negativities has been posterior negativities. The early posterior (parieto-occipital) negativities common to violations and to an opening parenthesis could reflect, in our view, some type of working memory processes. These situations, in parallel with analog situations in the linguistic domain, would noticeably tap on working memory to the extent that they involve particular difficulties within a mathematical problem. These working memory processes appear nevertheless specific for these situations, different from those possibly reflected by the posterior negativity in the order-relevant condition for small numbers starting a problem. In the later situation, working memory processes can also be adduced, because a small initial number notably increases the difficulty of the problem, and the parieto-occipital distribution of the early negativity suggests some overlap with the regions and processes involved during violations and opening parentheses. But the nonidentical distribution, as confirmed statistically, is suggestive of some subprocesses specifically involved by the order-relevant condition. Even so, the posterior distribution of all these negativities appears compatible with Dehaene and Cohen's (1995; see also Dehaene, 2005) proposal that cortical areas grossly localized to the vicinity of the parieto-occipito-temporal junction of both hemispheres constitute the main relevant regions in mathematical reasoning, these regions being involved in the representation of magnitudes, essential to afford comparisons.

Mathematical problems also yielded late posterior positivities. Overall, the degree of similarity between these positivities and P600s obtained in the linguistic domain appears noteworthy. Several of the comparisons between domains failed to differ significantly in topography. However, whereas the language-related P600s compared here were not significantly different in topography between them, this was not the case for the mathematics-related late positivities. Further, the functional response of the late positivity to mathematical violations diverges from that of the linguistic domain in the sense that whereas the latter is not affected by sentence length (Kaan, 2002; Phillips, Kazanina, & Abada, 2004), our mathematical late posterior positivity to violations was significantly affected by problem length, increasing its amplitude in longer problems. A review of the literature reveals that the P600 is a controversial component. Some authors (e.g., Patel, 2003), suggest that this component may reflect nonlinguistic processes of knowledge-based structural integration. Others have claimed that this could be a member of the P300-family (e.g., Osterhout, McKinnon, Bersick, & Corey), although several studies (e.g., Frisch, Kotz, von Cramon, & Friederici, 2003) support that at least partially different neural structures generate both P300 and the P600. In this line, a suggested possibility is that the P600 reflects the composite activity of multiple independent generators, each of which is responsible for a separate subprocess (e.g., Münte et al., 1998). This later interpretation may fit well with our results.

In our view, if the late posterior positivities do, in fact, reflect the activity of multiple independent generators, it appears plausible that most of these generators are common to linguistic tasks whereas mathematical reasoning would recruit a large amount of these generators even if differentially as a function of the condition performed. Whatever the case, the interpretation of late posterior positivities as reflecting some type of overall structural integration costs could still hold, in our opinion. In this regard, some type of structural problems may plausibly occur in the operations starting with small numbers of the order-relevant condition and in the violations of the embedded calculation and violation condition. This would not be the case, however, for the appearance of a parenthesis, and indeed this did not yield a late posterior positivity but, rather, a central positivity earlier in time.

Other results here obtained will only tentatively be discussed. Two of these findings were a central positivity and a long-lasting parieto-occipital negativity elicited by the equal sign in the order-relevant condition for problems starting with small numbers. They might reflect the additional operations needed to solve these difficult problems as soon as all the essential information is at hand. In this regard, the very early onset of the central positivity may indicate that these operations actually started before the equal sign appeared, because the main information was already available. Thereafter, the subsequent long-lasting parieto-occipital negativity could plausibly reflect operations occurring within the parieto-occipito-temporal areas that, as already mentioned, have been suggested as major regions essential to afford comparisons (Dehaene & Cohen, 1995), where a final result could then unfold and be accomplished.

Finally, another observed result was a small long-lasting positivity as pure length effect in the embedded calculation and violation condition. Considering both its low magnitude and its undefined distribution, further research appears necessary to establish the reliability and meaning of this result.

Conclusion

Obviously, language and mathematical reasoning cannot totally overlap; otherwise they would be one and the same cognitive ability. But whereas most of previous data are compatible with mathematical reasoning using language areas in addition to other nonlinguistic brain regions, the absence of unequivocal anterior negativities related to syntax or language working memory during mathematical reasoning implies, in our view, that at least certain brain areas/functions are employed by the former but not by the latter. This would be, on the one hand, a step toward the identification of language-specific resources of the human brain. For the time being, these resources would be reflected importantly by anterior negativities of the ERPs obtained during sentence comprehension. On the other hand, our data provide further evidence supporting recent claims that language and mathematical domains may indeed be more remarkably dissimilar than previously thought.

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