Python Phd Course Report

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The Lorenz system is a set of differential equations proposed by Edward Lorenz in 1963. When the equations are graphed, they produce a 3D plot of a strange attractor that forms the shape of a butterfly. In this project, we will use a basic numerical differentiator to solve the Lorenz system of equations and plot the results, as a set of 2D and 3D plots. In addition, we will color the plots using a colormap, save the produced plots and results in PNG and CSV files, and create tests to verify the functionality of the project.

The differential equations that describe the Lorenz system are:

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

$$\frac{dz}{dt} = xy - \beta z$$

where x, y, and z are the state variables, and σ, ρ, and β are the system parameters.

To solve the differential equations, we will use the Euler method, which is a simple numerical method to solve ordinary differential equations. Essentially, we will use a small time step on a discrete time space, and on each time step, we will update the state variables by adding the product of the time step and their derivatives.

$$x_{n+1} = x_n + \frac{dx}{dt} t_\delta$$

$$y_{n+1} = y_n + \frac{dy}{dt} t_\delta$$

$$z_{n+1} = z_n + \frac{dz}{dt} t_\delta$$

To color the plots, we will calculate the magnitude of the velocity vector of the differential equations and use it on a scatter plot with a colormap.

$$v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

One of the problems of the Lorenz system is that it is not stable in regards to the initial conditions. Since no initial conditions were provided as part of the project, we will use random ones, generated between 0 and 1. To ensure reproducibility, we pin Numpy's random seed to 128 and use the `numpy.random.rand` function to generate the initial conditions.

Project Structure

Structure

The project will be implemented in Python, using the NumPy and Matplotlib libraries. The project will be structured in the following way:

- `lorenz.py`: The main module that will contain the implementation of the Lorenz system.
- `diff.py`: The module that will contain the implementation of the Euler method.
- `tests.py`: The test module that will contain the tests for the two functions.

The tests can be ran using:

```
pytest tests.py
```

Standard NumPy arrays will be used to handle the history of the state variables, and the results will be saved in CSV files. Given their trivial size (5000), there is no need to use more complex data structures.

Installation

To ensure the reproducibility of the project, we will use a virtual environment combined with the `pip-tools` tool `pip-compile`. We begin by placing the dependencies in the `requirements.in` file:

```
pandas
numpy
matplotlib
jupyterlab
pytest
```

Then, we compile the dependencies to the `requirements.txt` file, where they are frozen to the exact version:

```
# Only run to update dependencies
# Skip for just running the code
pip install pip-tools
pip-compile
```

Finally, we install the dependencies in a virtual environment:

```
python -m venv venv
# substitute with venv\Scripts\activate.bat on Windows
source venv/bin/activate
pip install -r requirements.txt
```

This ensures that years down the line, the project can be easily reproduced even if the dependencies have had breaking changes.

A gitignore file is included to ignore the virtual environment. However, it does not ignore the output plots, as they are used in this document in markdown form and are needed for this readme to function.

Data Analysis

To execute the differentiation and plotting of the Lorenz system, we will use a Jupyter notebook, called `lorenz.ipynb`, which will contain the examples and visualizations of the Lorenz system.

The notebook was created in Visual Studio Code. However, barring that, it can be run in Jupyter Lab:

```
venv/bin/jupyter lab
```

The project saves the plots and results in the `./res` folder. Plots are saved as PNG files in `./res/fig`, and results are saved as CSV files in `./res/fig`. The file `./res/csv/hyper.csv` contains the hyperparameters that were used in each system.

For both this readme and the notebook, I will export them as PDFs and include them in the project folder. Therefore, no files need to be run to see the results of the project.

```
pandoc lorenz.ipynb -o lorenz.pdf
pandoc README.md -o report.pdf
```

Experimental Setup

As per the project description, we will use the following hyperparameters to generate a set of Lorenz systems. For all runs, we will use a time delta of 0.01 and 5000 points, for a total time of 50s.

Index	σ	β	ρ
1	10	8/3	6
2	10	8/3	16
3	10	8/3	28
4	14	8/3	28
5	14	13/3	28

For the plots themselves, we will use a subplot of 1x3 for the 2D plots (XY, XZ, YZ), and a square plot for the 3D plot. For both plots, we will use scatter plots with a colormap, where the color is determined by the magnitude of the velocity vector.

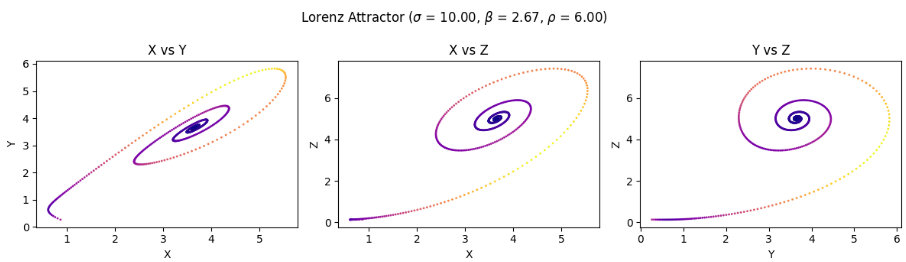
Performance

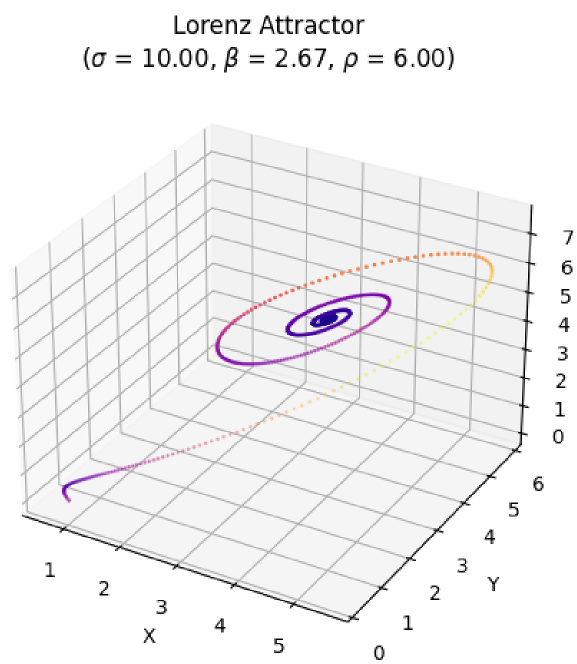
After evaluating the performance of the system, it was found that just 4% of the time was spent on the Euler method (38ms), while the rest of the time was spent on plotting (920ms), with plotting taking 71% of the time (882ms) and saving taking 25% of the time (234ms) respecively. This is due to using a scatter plot instead of a line plot, which was required to add the colormap. As such, optimizing the differentiation method would not provide a meaningful improvement in performance.

5000 points were used, as that resulted in the most legible and appealing plots.

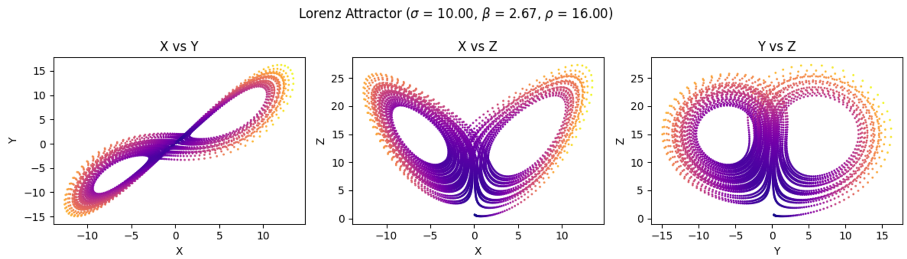
Results

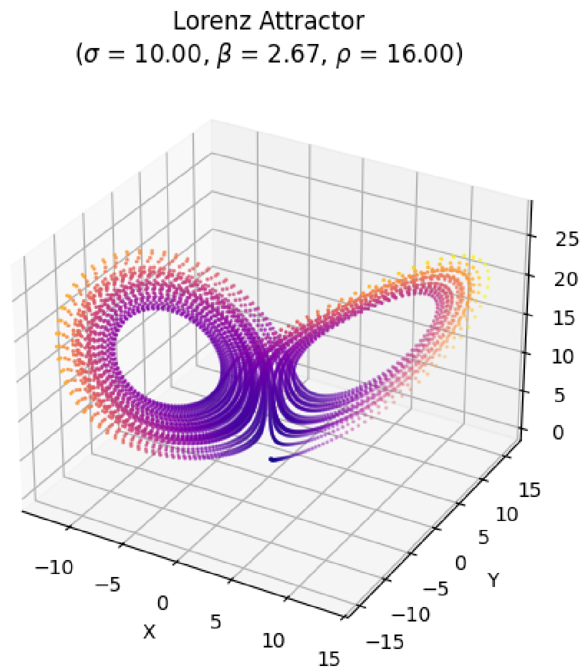
Run 1



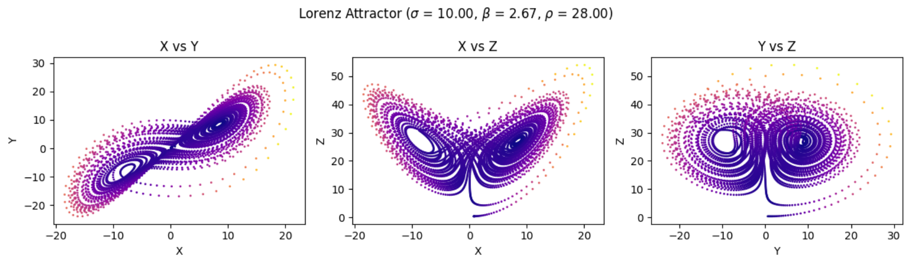


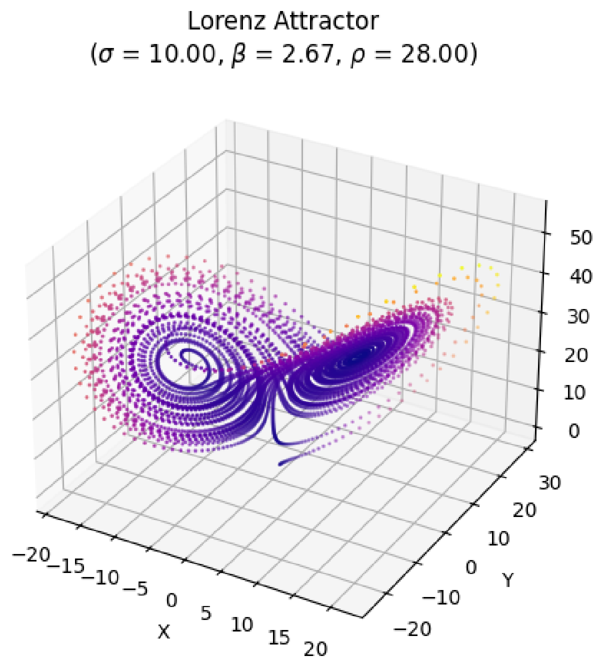
Run 2


Lorenz Attractor (σ = 10.00, β = 2.67, ρ = 16.00)
X vs Y
X vs Z
Y vs Z
X
Y
Z

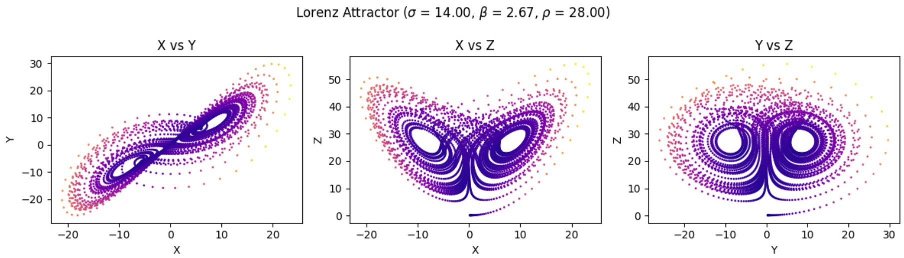

Lorenz Attractor
(σ = 10.00, β = 2.67, ρ = 16.00)
X
Y

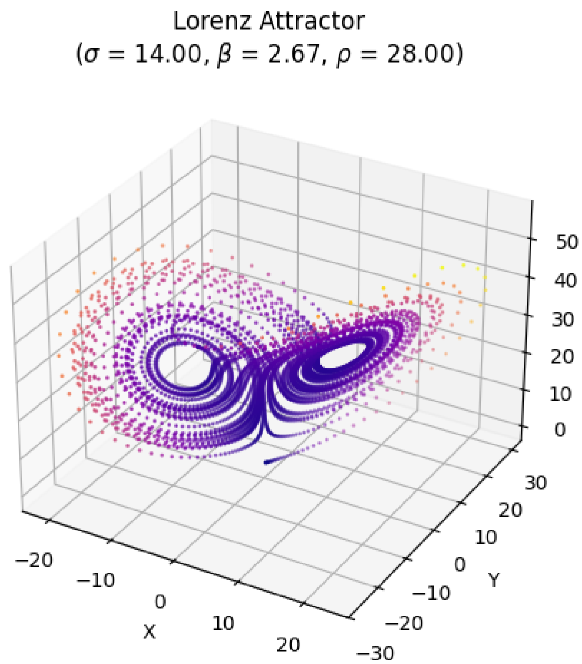
Run 3


Lorenz Attractor (σ = 10.00, β = 2.67, ρ = 28.00)
X vs Y
X vs Z
Y vs Z

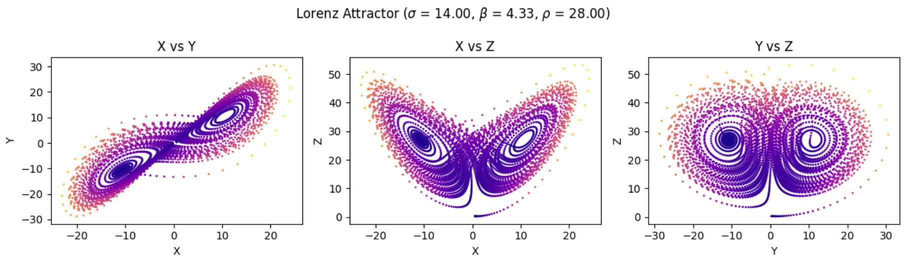

Lorenz Attractor
(σ = 10.00, β = 2.67, ρ = 28.00)

Run 4

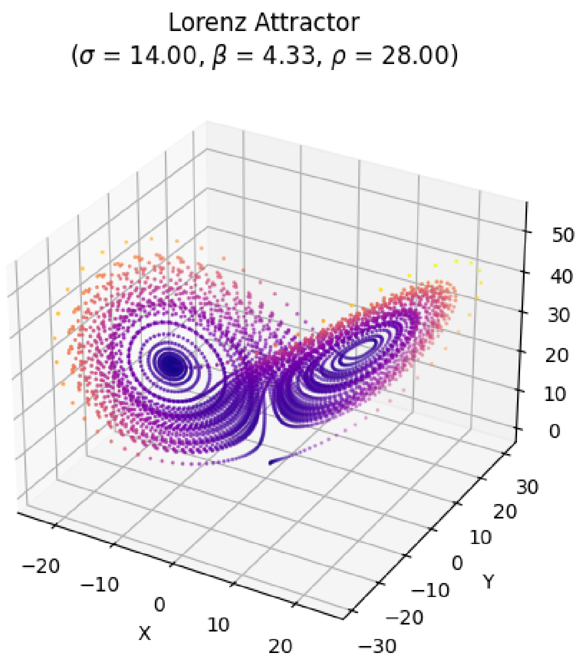

Lorenz Attractor (σ = 14.00, β = 2.67, ρ = 28.00)
X vs Y
X vs Z
Y vs Z
X
Y
Z


Lorenz Attractor
(σ = 14.00, β = 2.67, ρ = 28.00)
X
Y

Run 5



Lorenz Attractor (σ = 14.00, β = 4.33, ρ = 28.00)
X vs Y
X vs Z
Y vs Z
X
Y
Z



Lorenz Attractor
(σ = 14.00, β = 4.33, ρ = 28.00)
X
Y