1. **Multiples of 3 and 5**

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

1. **Even Fibonacci numbers**

Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

1. **Largest prime factor**

The prime factors of 13195 are 5, 7, 13 and 29.

What is the largest prime factor of the number 600851475143 ?

1. **Largest palindrome product**

A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is 9009 = 91 × 99.

Find the largest palindrome made from the product of two 3-digit numbers.

1. **Smallest multiple**

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

1. **Sum square difference**

The sum of the squares of the first ten natural numbers is,

12 + 22 + ... + 102 = 385

The square of the sum of the first ten natural numbers is,

(1 + 2 + ... + 10)2 = 552 = 3025

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is 3025 − 385 = 2640.

Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

1. **10001st prime**

By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13.

What is the 10 001st prime number?

1. **Largest product in a series**

The four adjacent digits in the 1000-digit number that have the greatest product are 9 × 9 × 8 × 9 = 5832.

73167176531330624919225119674426574742355349194934  
96983520312774506326239578318016984801869478851843  
85861560789112949495459501737958331952853208805511  
12540698747158523863050715693290963295227443043557  
66896648950445244523161731856403098711121722383113  
62229893423380308135336276614282806444486645238749  
30358907296290491560440772390713810515859307960866  
70172427121883998797908792274921901699720888093776  
65727333001053367881220235421809751254540594752243  
52584907711670556013604839586446706324415722155397  
53697817977846174064955149290862569321978468622482  
83972241375657056057490261407972968652414535100474  
82166370484403199890008895243450658541227588666881  
16427171479924442928230863465674813919123162824586  
17866458359124566529476545682848912883142607690042  
24219022671055626321111109370544217506941658960408  
07198403850962455444362981230987879927244284909188  
84580156166097919133875499200524063689912560717606  
05886116467109405077541002256983155200055935729725  
71636269561882670428252483600823257530420752963450

Find the thirteen adjacent digits in the 1000-digit number that have the greatest product. What is the value of this product?

## Special Pythagorean triplet

A Pythagorean triplet is a set of three natural numbers, a < b < c, for which,

a2 + b2 = c2

For example, 32 + 42 = 9 + 16 = 25 = 52.

There exists exactly one Pythagorean triplet for which a + b + c = 1000.  
Find the product abc.

## Summation of primes

The sum of the primes below 10 is 2 + 3 + 5 + 7 = 17.

Find the sum of all the primes below two million.

## Largest product in a grid

In the 20×20 grid below, four numbers along a diagonal line have been marked in red.

08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08  
49 49 99 40 17 81 18 57 60 87 17 40 98 43 69 48 04 56 62 00  
81 49 31 73 55 79 14 29 93 71 40 67 53 88 30 03 49 13 36 65  
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91  
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80  
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50  
32 98 81 28 64 23 67 10 **26** 38 40 67 59 54 70 66 18 38 64 70  
67 26 20 68 02 62 12 20 95 **63** 94 39 63 08 40 91 66 49 94 21  
24 55 58 05 66 73 99 26 97 17 **78** 78 96 83 14 88 34 89 63 72  
21 36 23 09 75 00 76 44 20 45 35 **14** 00 61 33 97 34 31 33 95  
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92  
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57  
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58  
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40  
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66  
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69  
04 42 16 73 38 25 39 11 24 94 72 18 08 46 29 32 40 62 76 36  
20 69 36 41 72 30 23 88 34 62 99 69 82 67 59 85 74 04 36 16  
20 73 35 29 78 31 90 01 74 31 49 71 48 86 81 16 23 57 05 54  
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 19 67 48

The product of these numbers is 26 × 63 × 78 × 14 = 1788696.

What is the greatest product of four adjacent numbers in the same direction (up, down, left, right, or diagonally) in the 20×20 grid?

1. **Highly divisible triangular number**

The sequence of triangle numbers is generated by adding the natural numbers. So the 7th triangle number would be 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28. The first ten terms would be:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

Let us list the factors of the first seven triangle numbers:

**1**: 1  
**3**: 1,3  
**6**: 1,2,3,6  
**10**: 1,2,5,10  
**15**: 1,3,5,15  
**21**: 1,3,7,21  
**28**: 1,2,4,7,14,28

We can see that 28 is the first triangle number to have over five divisors.

What is the value of the first triangle number to have over five hundred divisors?

1. **Large sum**

Work out the first ten digits of the sum of the following one-hundred 50-digit numbers.

37107287533902102798797998220837590246510135740250  
46376937677490009712648124896970078050417018260538  
74324986199524741059474233309513058123726617309629  
91942213363574161572522430563301811072406154908250  
23067588207539346171171980310421047513778063246676  
89261670696623633820136378418383684178734361726757  
28112879812849979408065481931592621691275889832738  
44274228917432520321923589422876796487670272189318  
47451445736001306439091167216856844588711603153276  
70386486105843025439939619828917593665686757934951  
…

1. **Longest Collatz sequence**

The following iterative sequence is defined for the set of positive integers:

*n* → *n*/2 (*n* is even)  
*n* → 3*n* + 1 (*n* is odd)

Using the rule above and starting with 13, we generate the following sequence:

13 → 40 → 20 → 10 → 5 → 16 → 8 → 4 → 2 → 1

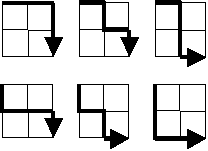
It can be seen that this sequence (starting at 13 and finishing at 1) contains 10 terms. Although it has not been proved yet (Collatz Problem), it is thought that all starting numbers finish at 1.

Which starting number, under one million, produces the longest chain?

**NOTE:** Once the chain starts the terms are allowed to go above one million.

1. **Lattice paths**

Starting in the top left corner of a 2×2 grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner.



How many such routes are there through a 20×20 grid?

1. **Power digit sum**

215 = 32768 and the sum of its digits is 3 + 2 + 7 + 6 + 8 = 26.

What is the sum of the digits of the number 21000?

1. **Number letter counts**

If the numbers 1 to 5 are written out in words: one, two, three, four, five, then there are 3 + 3 + 5 + 4 + 4 = 19 letters used in total.

If all the numbers from 1 to 1000 (one thousand) inclusive were written out in words, how many letters would be used?

**NOTE:** Do not count spaces or hyphens. For example, 342 (three hundred and forty-two) contains 23 letters and 115 (one hundred and fifteen) contains 20 letters. The use of "and" when writing out numbers is in compliance with British usage.

1. **Maximum path sum I**

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

**3**  
**7** 4  
2 **4** 6  
8 5 **9** 3

That is, 3 + 7 + 4 + 9 = 23.

Find the maximum total from top to bottom of the triangle below:

75  
95 64  
17 47 82  
18 35 87 10  
20 04 82 47 65  
19 01 23 75 03 34  
88 02 77 73 07 63 67  
99 65 04 28 06 16 70 92  
41 41 26 56 83 40 80 70 33  
41 48 72 33 47 32 37 16 94 29  
53 71 44 65 25 43 91 52 97 51 14  
70 11 33 28 77 73 17 78 39 68 17 57  
91 71 52 38 17 14 91 43 58 50 27 29 48  
63 66 04 68 89 53 67 30 73 16 69 87 40 31  
04 62 98 27 23 09 70 98 73 93 38 53 60 04 23

**NOTE:** As there are only 16384 routes, it is possible to solve this problem by trying every route. However, [Problem 67](https://projecteuler.net/problem=67), is the same challenge with a triangle containing one-hundred rows; it cannot be solved by brute force, and requires a clever method! ;o)

## Counting Sundays

You are given the following information, but you may prefer to do some research for yourself.

* 1 Jan 1900 was a Monday.
* Thirty days has September,  
  April, June and November.  
  All the rest have thirty-one,  
  Saving February alone,  
  Which has twenty-eight, rain or shine.  
  And on leap years, twenty-nine.
* A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.

How many Sundays fell on the first of the month during the twentieth century (1 Jan 1901 to 31 Dec 2000)?

## Factorial digit sum

*n*! means *n* × (*n* − 1) × ... × 3 × 2 × 1

For example, 10! = 10 × 9 × ... × 3 × 2 × 1 = 3628800,  
and the sum of the digits in the number 10! is 3 + 6 + 2 + 8 + 8 + 0 + 0 = 27.

Find the sum of the digits in the number 100!