A New Image Segmentation Approach Based on the Louvain Algorithm

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***Abstract*—This paper presents an image segmentation strategy using an idea coming from the social networks analysis domain. This strategy relies on the use of community detection algorithms in order to cluster pixels that belong to the same group of information. The main issue with this approach is that community detection based image segmentation often leads to over-segmented results. In order to address this problem, we propose an algorithm that agglomerates homogeneous regions using their color properties. Our algorithm is tested on the publicly available Berkeley Segmentation Dataset and experimental results show that the proposed algorithm produces sizable segmentation and achieves object-level segmentation to some extent.**

I. Introduction

Image segmentation plays a major role in image processing applications [1]–[3]. The goal of image segmentation is not only to distinguish the interesting objects from the background, but also to identify them in an image. Particularly, image segmentation could benefit a variety of vision applications, for instance, object recognition, automatic driver assistance, and traffic control systems.

A variety of proposed algorithms has dealt with image segmentation in the literature. We can divide them into some main groups according to the underlying approaches, such as feature-based clustering, region-based segmentation methods, hybrid techniques and graph-based approaches.

In feature-based clustering approaches [4], color or texture can be considered as features. A specific distance metric system is defined to aggregate the feature samples into homogeneous regions without considering spatial informative features.

The region-based segmentation methods consider the similarity of pixels according to some homogeneity criteria. A pixel is assigned into an homogeneous region if it satisfies a given homogeneity criteria. Split-and-merge [5] and region-growing techniques [6] are examples of such methods.

The hybrid approaches combine region-based and feature-based techniques [7], [8]. These methods are widely applied in image segmentation strategies since they can rely on both local and global information.

Finally, graph-based approaches [9], [10] consider combinations of features and spatial information. In these approaches, an image is modeled as a graph in which vertices represent individual pixels and weighted edges describe the similarity of neighboring pixels.

A recent trend in image segmentation is based on community detection algorithms [1], [3], [4], [11]–[14]. A community is a group of nodes with dense internal connections and sparse connections with members of other communities. The general idea of those techniques is to highlight the similarity between the modularity criterion in network analysis and the image segmentation process. In fact, the larger the modularity of a network is, the more accurate the detected communities, *i.e.* the objects in the image, are [3], [4], [13], [15]. If the modeling of the image in a graph is well done then we can expect that a good partition in communities corresponds to a good segmentation of the image. The modularity of a partition is a scalar measures of the density of links inside communities as compared to links between communities, and its value fall into the interval [-1,1] [16].

Louvain method [16], a community detection algorithm, has received significant attention in the context of image segmentation [11], [12], [15]. However, it is still facing a problem of over-segmentation. In this paper, we propose a new image segmentation that incorporates Louvain method and an algorithm to agglomerate homogeneous regions into a segmentation processes in order to overcome the over-segmented problem. Each sub-segment obtained during the Louvain method phase represents a region. Then, the agglomeration of homogeneous regions algorithm operates by considering similarity distances of adjacent regions. The similarity distances are computed using the three color channels RGB individually.

The rest of this paper is organized as follows. In section II, we briefly review graph-based image segmentation methods. In section III, we introduce complex networks, the concept of community detection and Louvain algorithm to point out how community detection algorithms can be applied in image segmentation efficiently. Experiments on the publicly Berkeley Segmentation Data Set (BSDS500) are reported in section IV. Finally, our conclusions are presented in section V.

II. Related Work

In this section, we briefly review some well-known graph-based image segmentation methods.

Considering image segmentation problem from the perspective of graph partition has interested several researchers. In this approach, the image is regarded as an undirected weighted graph in which each pixel is represented as a node in the graph and edge weights measure the similarity distance between nodes. The graph is clustered by optimizing any adequate criteria, *i.e. minimum cut, normalized cut or related variants.*

The *Normalized Cut* [17] criterion provides a way of integrating global image information into the grouping process. For this work, given an affinity matrix W in which each entry represents the similarity of two pixels, *Normalized Cut* tries to solve the problem using an algorithm based on the generalized eigenvectors problem of the linear system from equation (1).

$$(D - W)z = \lambda Dz \tag{1}$$

D is a diagonal matrix with its diagonal entry $D_{ij} = \sum_j W_{ij}$. Then K-means clustering is applied to obtain a segmentation into regions. Cour, *et al.* approached Normalized Cuts with a variant namely Multiscale Normalized Cuts (NCuts). Sharon, *et al.* proposed an alternative to enhance the computational efficiency of Normalized Cuts. Beside, many image segmentation methods in this type were mentioned, for instance, Mumford and Shah proposed that the segmentation of an observed image u_0 is given by the minimization of functional:

$$F(u, C) = \int_{\Omega} (u - u_0)^2 dx + \mu \int_{\Omega \setminus C} |\bigtriangledown(u)|^2 dx + v|C| \tag{2}$$

where u is piecewise smooth in $\Omega \setminus C$ and μ, v are weighting parameters. In addition, several algorithms have been developed to minimize or simplify the energy by various approach strategies.

Felzenszwalb and Huttenlocher (Felz-Hutt) [9] attempt to partition image pixels into components. Constructing a graph in which pixels are nodes and edge weights measure dissimilarity between nodes (*e.g.* color differences), each node is initially placed in its own component. The internal difference of a component $Int(R)$ has been defined as the largest weight in the minimum spanning tree of R. Considering in non-decreasing order by weight of edges, each step of the algorithm merges components R_1 and R_2 connected by the current edge if the weight of the edge is less than:

$$min(Int(R_1) + \tau(R_1), Int(R_2) + \tau(R_2)) \tag{3}$$

where $\tau(R) = k/|R|$, k is a scale parameter that can be used to set a preference for component size.

Recently, complex networks analysis domain has been considered segmenting images and achieved outstanding results [1], [4], [12], [13]. The idea of community detection has been applied in image segmentation that offers a new perspective for researchers about image segmentation domain. Wenye Li [14], and Youssef, *et al.* [11] attempt to apply community detection problems in complex networks to solve image segmentation problems and investigate a new graph-based image segmentation as well as compare other methods. These works point out the potential perspective of community detection based image segmentation domain.

The image segmentation approaches of Ahmad Ali Abin *et al.* [4], and Oscar A. C. Linares *et al.* [1] are constructing weighted networks in which the small homogeneous regions *(super-pixels)* obtained by initial segmentation processes are nodes of graph and the computed similarity distance between are edge weights. One community detection method is applied to extract communities as segments.

Shijie Li, *et al.* [12], and Youssef Mourchid, *et al.* [13] propose using super-pixel and features to solve the over-segmentation problem. Both strategies initialize with an over-segmented image segmentation in which each subsegment represents a super-pixel. Then, they treat the over segmentation issue by different ways. Shijie Li, *et al.* solve it by reconstructing the neighborhood system for each region *(super-pixel)* and the histogram of states *(HoS)* texture feature. Then, they estimate the distribution of the color feature for each region. The similarity matrix W is computed and adaptively updated based on color feature and histogram of states *(HoS)* texture feature. Youssef Mourchid, *et al.* approaches the over-segmented problem in a quite similar way but they compute coefficients to adaptively update the similarity matrix W based on color feature and histogram of oriented gradients *(HOG)* texture feature.

III. Our Approach

We consider image from the perspective of a complex network and solve the image segmentation based on community detection. The Louvain algorithm has been applied for image segmentation widely [3], [11], [15]. However, The individual original Louvain method has not overcome the over-segmentation problem. Our algorithm is built on top of the Louvain method so as to avoid this drawback, and produce accurate results.

A. Complex Networks

A complex network is a graph (network) whose topological structure cannot be trivially described. It comprises properties that emerge as a consequence of global topological organization of the system. Complex network structures describe various systems of high technological and intellectual importance, such as the Internet, World Wide Web, financial, social, neural, and communication networks. One property that has attracted particular attention is the community structure of these networks.

The problem of community detection is usually defined as finding the best partition (or covering) of a network into communities of densely connected nodes, with the nodes belonging to different communities being only sparsely connected. Several algorithms have been proposed to find good partitions in term of a reasonably fast way. These algorithms can be divided into some main types such as, divisive algorithms that detect inter-community links and remove them from the network, agglomerative algorithms that merge similar or close nodes and more generally optimization methods are based on the maximization of an objective function [18]. The quality of partitions resulting from these methods is often measured by the modularity that has been introduced by Newman [19]. It is defined as follow:

$$Q = \sum_i (e_{ii} - a_i^2) \tag{4}$$

where e_{ii} denotes the fraction of edges in community i, and a_i if the fraction of ends of edges that belong to i. The value of modularity Q ranges in [-0.5,1] and higher values indicate stronger community structure of the network.

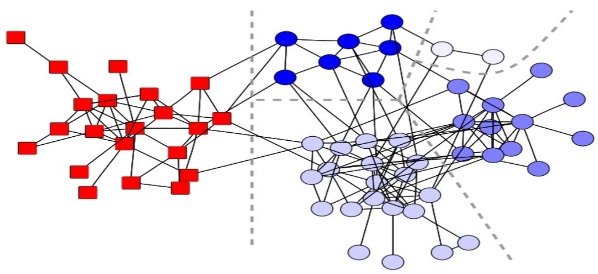

Figure 1. Community structure in the social network of bottle-nose dolphins population extracted using the algorithm of Girvan and Newman [20]. The squares and circles denote the primary split of the network into two groups and the circles are further subdivided into four smaller group as shown [21].

B. Generating a complex network from an image

Complex networks can be generated from images. Each image is represented as an undirected graph $G = (V, E)$, where V is a set of vertices ($V = \{v_1, v_2, ..., v_n\}$) and E is a set of edges $E = \{e_1, e_2, ..., e_k\}$. Each vertex $v_i \in V$ corresponds to an individual pixel and similarity/closeness of pixels are modeled as edges: an edge $e_{ij} \in E$ connects vertices v_i and v_j. A weight on each edge, w_{ij}, is a nonnegative value that measures the affinity between v_i and v_j. The higher affinity is the stronger relation between the pixels. In this paper, each node in the graph represents a pixel and edge weights are defined as:

$$w_{ij} = \begin{cases} 1 & if \quad d_{ij}^c \leq t \text{ for all color channels } c \\ nil & otherwise \end{cases} \tag{5}$$

where t is a threshold, d_{ij}^c is a measure of the similarity of pixels i and j intensity for color channel c (among R, G and B). It is defined by $d_{ij}^c = \left| I_i^c - I_j^c \right|$ where I_i^c and I_j^c represent the intensity of pixel i and j respectively for channel c.

Therefore, for a given pixel, links towards other pixels are created if and only other considered pixels are inside 20 neighboring pixels for rows and columns directions. Plus all distances d_{ij}^c of color channels must be lower than t for the edge to be considered. In this case, the weight is assigned $w_{ij} = 1$. Note that we could have put an edge and a weight that reflect the distance (both physical distance and color distance) in a more complex way but this is left for future investigations.

C. Louvain Algorithm

The Louvain method [16] is a hierarchical greedy algorithm that is designed to optimize the modularity on graphs or weighted graphs. Louvain algorithm consists of two phases that are repeated iteratively. Initially, every node is a singleton community. Next, during phase 1, all nodes are considered one by one. Each node is placed in its neighboring community, including its own one, that maximizes the static modularity gain. This process is repeated until no individual movement of node can improve the modularity. Therefore this first phase stops when the modularity reaches a local maximum. Then, phase 2 consists in building a new graph whose nodes are the communities found during the first phase. To build a new graph, links between nodes of the same community lead to self-loops while the weights of links between new nodes are computed by the sum of the weights of the links between nodes in the corresponding two communities.

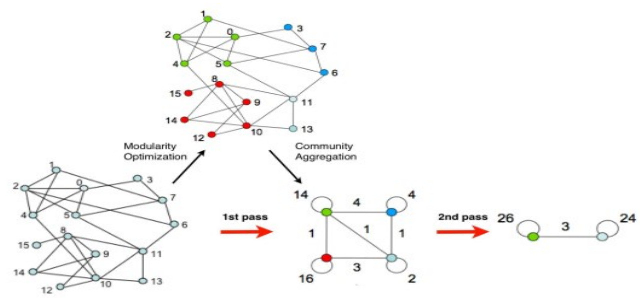

Figure 2. Process of community detection for Louvain method [16]

D. Aggregation of Regions Algorithm

Although the Louvain algorithm has been applied for image segmentation widely, it has not still overcome the over-segmentation problem. In general, modularity based image segmentation produces many homogeneous regions that could belong to one object in segmented object-level. However, community detection algorithms often deal with the time complexity so their characteristics are heuristics algorithms. Therefore, image segmentation based community detection often lead to over-segmentation. In order to solve this problem, homogeneous regions should be combined whenever its necessary.

Given an over-segmented image that consists of several homogeneous regions represented as a set of regions. The proposed algorithm can merge homogeneous regions in order to generate better segmented image results.

Algorithm ARA

Input: A set of regions $R = \{R_1, R_2, ..., R_n\}$

for each region $R_i \in R$ **do**
  for each region $R_j \in R$ **do**
    if R_i and R_j are adjacent regions **then**
      if $R_i < regthres$ *OR* $R_j < regthres$ **then**
        Merge region R_i and region R_j
      else
        Compute distance of similarity *d*(R_i,R_j) between region R_i and region R_j
        if *d*(R_i,R_j) $< threshold(t)$ **then**
          Merge region R_i and region R_j
        end if
      end if
    end if
  end for
end for

Output: The set of regions $R = \{R_1, R_2, ..., R_k\}$

In the algorithm ARA, the distance of similarity between region R_i and region R_j is computed as equation (8). Because the primary straightforward feature for image segmentation is color [12], [13]. Especially, it is more important when segmenting images using community detection because of its aggregation communities sharing color properties. In the proposed algorithm, the color images RGB are considered and focused on individual color channels. For each region, we construct a vector consisted of Mean and Standard deviation for every channel of colors as formulas (6), (7). Each region is represented by a 6 dimensional vector, *i.e.*, the region R_i and R_j the local color feature vector are $u_i, u_j \in R^6$.

$$Mean(R) = \frac{\sum_{i=1}^{n} C_i}{n} \tag{6}$$

$$SD(R) = \sqrt{\frac{\sum_{i=1}^{n} C_i^2}{n}} \tag{7}$$

where C_i is the color value channel of pixel i in image and n is the number of pixels in the set R.

The similarity of two adjacent regions R_i and R_j is computed by cosine similarities of a pair of vectors $u_i, u_j \in R^6$ that represented to two considering regions, as indicated in equation (8).

$$d(R_i, R_j) = Cosine(u_i, u_j) = \frac{u_i^{\intercal} u_j}{\|u_i\|.\|u_j\|} \tag{8}$$

E. Noise Removal

In our method, a primary technique that must be pointed out is noise removal process. As mentioned above, the results obtained from Louvain processes consist of over-segmented results, which decreases *Probabilistic Rand Index (PRI)* score when it will be evaluated. We proposed and evaluated some strategies to gain better results and obtain higher *PRI* scores. These strategies are divided into two phases of noise removal: preliminary removing noises and producing final segmentation results.

The preliminary removing noise process is a crucial part of our algorithm. It merges the small regions neglected when considering the similarity $d(R_i, R_j)$ between two regions. This merging process uses two adjacent regions and the number of pixels inside must be less than a threshold *regthres* in our algorithm. Empirically, we set the threshold for small regions $regthres$ = 200 pixels for testing and evaluating on the validation dataset. On a small test sample this value gives the best results but more general experimentations and an automatic selection of the threshold would be a improvement. More specifically, the modularity and therefore the Louvain method suffers from the resolution limit problem that tends to favor typical sizes for communities [22] and instability issues [23] that are both directly related to this problem. Furthermore, while the order in which the merging of small regions is performed can influence the result, we have not studied this question and in our method the order is purely random.

Although image noises have been reduced by the aggregation regions, the final segmentation images still contain noise that has to be removed to produce smooth images. A variety of noise reduction techniques have been introduced and are well-known in the literature. The primary techniques of image noise reductions are supported by Open Source Computer Vision Library such as: Averaging, Gaussian Blurring, Median Blurring and Bilateral Filtering. In our method, Median Blurring technique has been applied to smooth images and obtained efficient smoothing image results.

IV. Experimental Evaluation

This section provides experiments that were performed to assess our algorithm. To evaluate the proposed model, we used the *Berkeley Segmentation Data Set 500 (BSDS500)* [24] and evaluated it using the *Probabilistic Rand Index (PRI)* metric. The qualitative and quantitative evaluation are presented by some figures and comparative results table I.

A. Berkeley Segmentation Data Set 500

The Berkeley Segmentation Data Set has been built with the aim of providing an empirical basis for research on image segmentation and boundary detection. This dataset comprises 500 images, including 200 images for training, 200 images for testing and 100 images for validation. Each image has 481 x 321 pixels, which yields a graphs of 154401 vertices. BSDS500 also provides ground-truth segmentations that are manually generated by many human subjects. For every image, there are from 5 to 10 ground-truth segmentation maps. Supplying a benchmark for comparing different segmentation and boundary detection algorithms.

B. Probabilistic Rand Index

In general, evaluation segmentations metrics have been used to evaluate different image segmentation algorithms in the literature. Some common one include Variation of Information, Segmentation Covering and Probabilistic Rand Index. Among them, Probabilistic Rand Index is used for the evaluation of almost all algorithms on BSDS500 because of its advantages on multiple ground-truth.

The Probabilistic Rand Index [2], [25], [26] is a classical evaluation criteria for clusterings. PRI measures the probability that pair of pixels have consistent labels in the set of manual segmentations (ground-truth). Given a set of ground-truth segmentation $\{S_k\}$, the Probabilistic Rand Index is defined as:

$$PRI(S_{test}, \{S_k\}) = \frac{1}{T} \sum_{i<j} [c_{ij} p_{ij} + (1 - c_{ij})(1 - p_{ij})] \quad (9)$$

where c_{ij} is the event that the algorithm gave the same label to pixels i and j and p_{ij} corresponds to the probability of the pixels i and j having the same label, and is estimated by using sample mean of the corresponding Bernoulli distribution on the ground-truth dataset. T is the total number of pixel pairs. The PRI values range in [0,1] in which a larger value likely indicates a greater similarity between these segmentations.

C. Results

For qualitative evaluations, we present some images of the segmentation results in Figure 3. For these qualitative results, we can see that the proposed algorithm offers good results and produces sizable regions for all selected images. Figure 4 and figure 5 presents some segmentation results of our algorithm on some images. Our algorithm can aggregate homogeneous neighboring regions successfully even if pixels inside each region are dissimilar. However, we determine that the color feature hardly achieve instance image segmentation in case of the repetitive patterns of different colors in many homogeneous objects or the quite different color parts inside an entity.

From a quantitative point of view, we evaluated the segmentations results using the *Probabilistic Rand Index* (PRI) by comparing a test segmentation with multiple ground-truth images. We applied this measure on the validation set from the Berkeley segmentation dataset, and obtained the score of 0.819, while the ground-truth (segmentation made by human) got a score of 0.87 [12], [13], [27]. Detailed results are given in table I.

The evaluation results reflect the success of our agglomeration process for homogeneous regions. Our method exceeds all previous graph-based algorithms in term of PRI scores.

Methods	PRI
Human [27]	0.87
Our method	**0.819**
Youssef Mourchild's *(Fast multi-scalse (HOG))* [13]	0.811
gPb-owt-ucm [27]	0.81
Youssef Mourchild's *(Modularity optimization based on Danon (HOG))* [13]	0.803
Mean Shift [27]	0.78
Shijie Li's method *(L*a*b (HoS))* [12]	0.777
Felz-Hutt [27]	0.77
Canny-owt-ucm [27]	0.77
NCuts [27]	0.75
Shijie Li's method *(RGB (HoS))* [12]	0.749

Table I
COMPARATIVE RESULTS USING THE PRI INDEX ON THE BERKELEY SEGMENTATION DATASET [12], [13], [27]

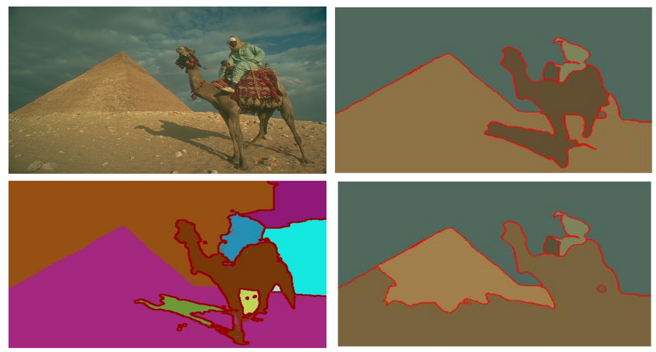

Figure 3. Visual Segmentation results of some methods: *Top left:* Origin image. *Top right:* Segmentation result of HoS method [12]. *Bottom left:* Our segmentation result *Bottom right:* Segmentation result of HOG method. [12]

V. CONCLUSION

This paper proposed a new segmentation approach based on community detection algorithms with the incorporation of an agglomeration of homogeneous regions. Our method is significantly accurate and produces efficient image segmentation results. The novelty in this paper is the consideration of color properties in order to build a 6-dimensional vector for each region and the proposal to apply cosine similarity distance for aggregation processes using only color properties. Hence, the time complexity has been reduced significantly compared to using 256 dimensional vector in some other techniques. Empirically, the threshold for agglomeration process is taking range from 0.990 to 0.999 because the regions that belong to one segment often have high color properties in common to each other. Extensive experiments have been performed, and the results show that the proposed algorithm can reliably segment the image and avoid over-segmentation in order to produce more accurate objects and enhance computing performance efficiently.

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Figure 4. Segmentation results of category Human: *Top:* Origin images. *Middle:* Segmentation results by applied Louvain method. *Bottom:* Segmentation result with the cooperative ARA algorithm

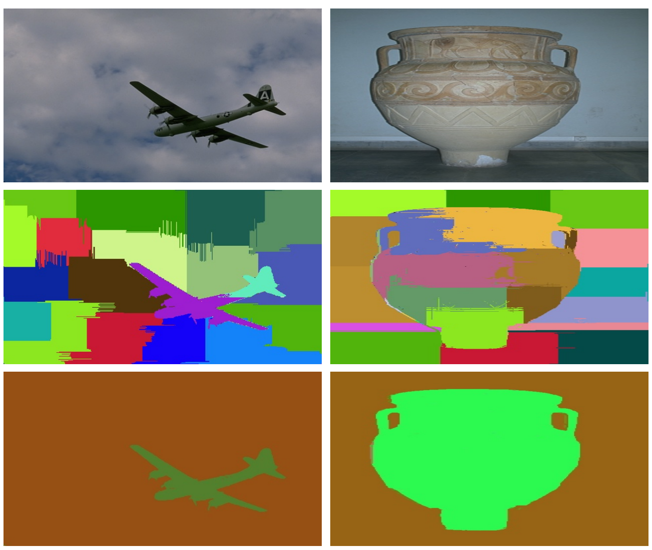

Figure 5. Segmentation results of category Objects: *Top:* Origin images. *Middle:* Segmentation results by applied Louvain method. *Bottom:* Segmentation result with the cooperative ARA algorithm

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