Test Problems

Problem 1: 1D Head on Collision with Stationary Target

A cart with a mass of 0.6 kg moving on a frictionless 1D track to the right at an initial speed of 3.14 ms^{-1} undergoes an elastic collision with an initially stationary cart of mass 1 kg. What is the final velocities, in unit vector notation, for both carts?

Results:

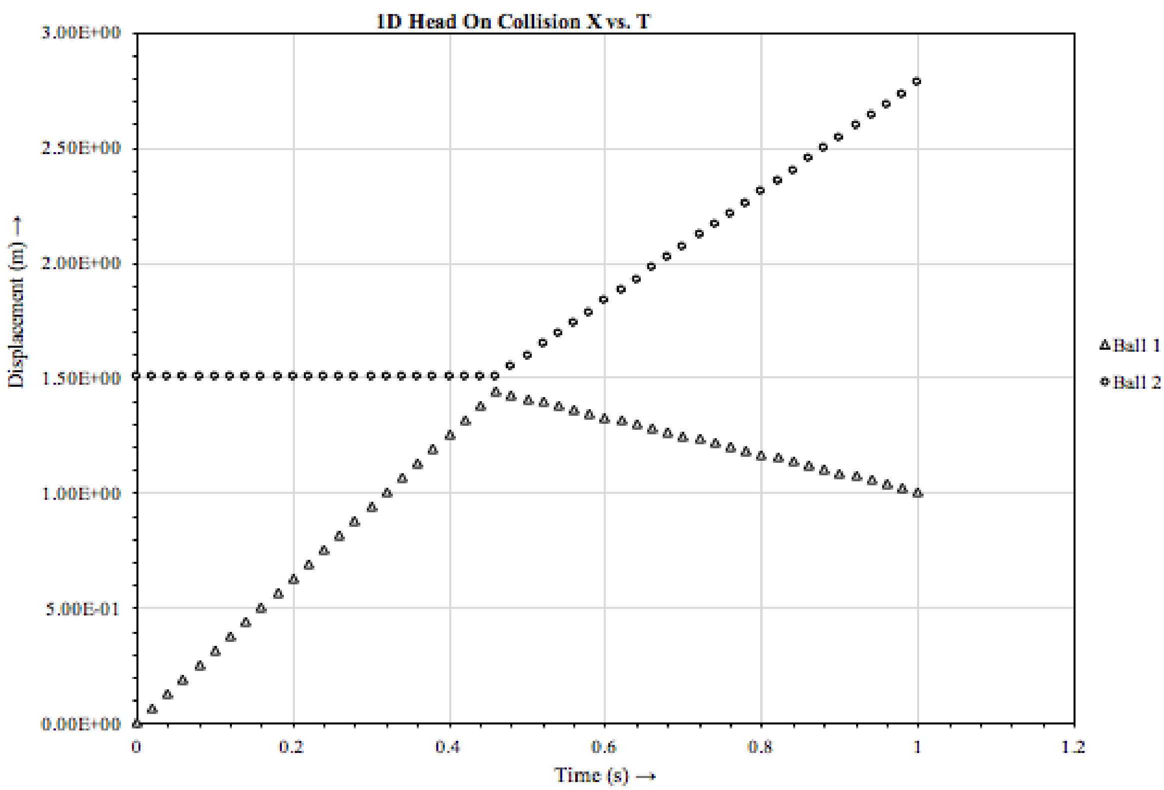

Fig 2. 1D head-on collision of two stress balls, x vs. t. The computer simulation modelled the collision between an object at rest, ball 2, and another object, ball 1, in which they collide head-on. The position (m) of each ball was graphed against time (s). No error is provided as the graph as values are calculated through simulation based on given parameters.

	Mass 1 Final v_x (ms^{-1})	**Mass 1 Final v_y (ms^{-1})**	**Mass 2 Final v_x (ms^{-1})**	**Mass 2 Final v_y (ms^{-1})**
Worked Answer	-0.78500	0.000	2.35500	0.000
Simulation	-0.80500	0.00052	2.36725	-0.00046
Residual	-0.02000	0.00052	0.01225	-0.00046

Fig 3. Initial and Final Velocities for Head-on Collision with Stationary Target. The above table depicts the worked solutions for Problem 1, the computed solutions from the simulation, as well as the residual value below it.

As can be seen above, the velocity of both objects is only changed by the collision, as would be expected to be observed in a situation without friction. Because the collision occurs in a comparatively small timeframe, it is difficult to observe the intermediate velocities over this interval. However, we can see how the velocities have been changed by the impact and can observe how the impulse has changed the velocities after the collision. We see similar results between the worked answer and the simulated output, attributable to the error of assuming a constant instead of changing velocity over a small interval.

Problem 2: 2D Glancing Collision with Stationary Target

Two 0.1 kg bodies, *A* and *B*, collide. *B* is stationary before the collision. The velocity for *A* is v_A= (4i + 2.2 j) ms^{-1}. Ball A has a final velocity of (1.25i -1j) ms^{-1}. What is the final velocity for *B*?

Results:

	Mass 1 Final v_x (ms^{-1})	Mass 1 Final v_y (ms^{-1})	Mass 2 Final v_x (ms^{-1})	Mass 2 Final v_y (ms^{-1})
Worked Answer	1.250	-1.000	2.734	3.261
Simulation	1.207	-1.043	2.792	3.242
Residual	0.043	0.043	-0.058	0.019

Fig 4. Initial and Final Velocities for Glancing Collision with Stationary Target. The above table depicts the worked solutions for Problem 2, the computed solutions from the simulation, as well as the residual value below it for both of the objects.

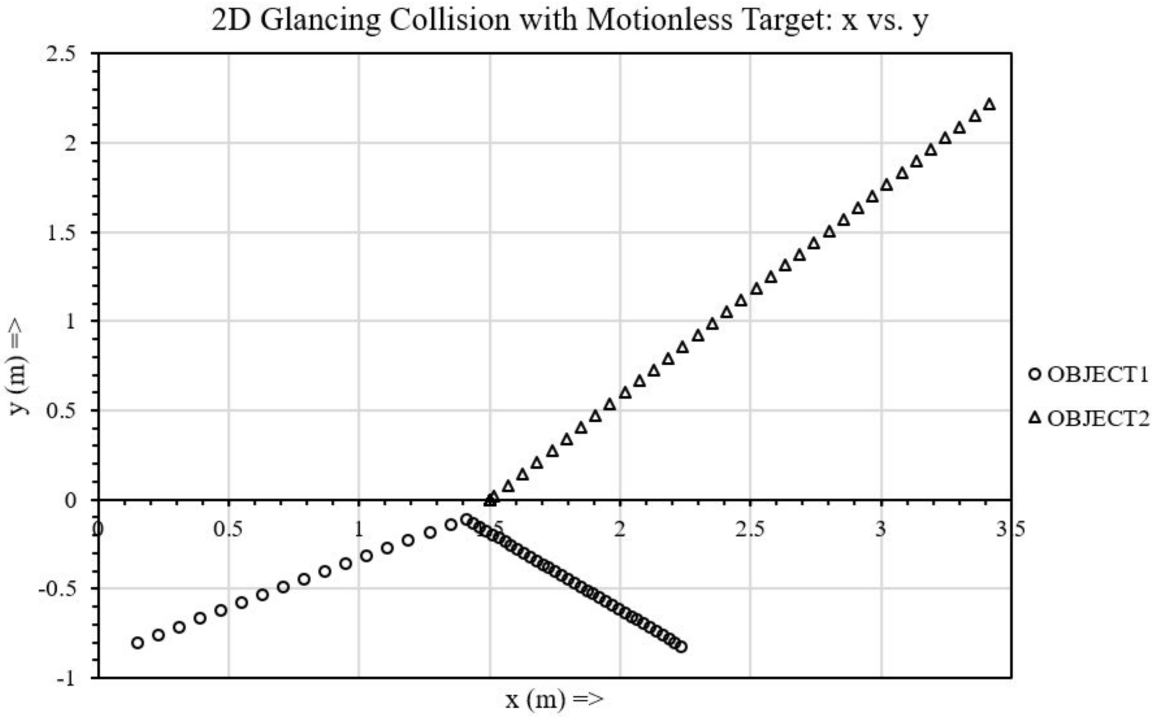

Fig 5. 2D glancing collision of two stress balls, x vs. y. The computer simulation modelled the collision between a ball at rest, object 2, and another ball, object 1, in which they collide at an angle. The vertical position (m) of the balls are graphed against the respective horizontal positions. No error is provided as the graph as values are calculated through simulation based on given parameters.

By inspection, it can be seen that the velocity of both objects is constant when not in the collision, without external forces or acceleration considered. It is again not possible to observe intermediate velocities during the collision as the time interval over which the collision occurs is comparatively small. However, it is possible to observe the trends of the balls' behaviour before and after the collision. It is observable that the position of object 2 does not change until the collision, indicating it is at rest, while object 1 is approaching object 2. After the collision, object 2 begins to move, and object 1 continues to move with a change in direction of velocity. As with the first problem, the simulation produces reasonably accurate values — accurate within a decimal point.

Problem 3: 2D Glancing Collision with Moving Target

Ball *B*, moving in the direction at a speed of v_B = (1i + 0.3j) ms^{-1} collides with a moving ball A which is moving at a speed of v_A= (-0.5i - 0.5j) ms^{-1}. Both A and B have a mass of 1 kg. After the collision, ball B has a velocity of (1i + 0.15j) ms^{-1}. Find the velocity of ball A.

	Mass 1 Final v_x (ms^{-1})	Mass 1 Final v_y (ms^{-1})	Mass 2 Final v_x (ms^{-1})	Mass 2 Final v_y (ms^{-1})
Worked Answer	-0.564	-0.345	1.000	0.150
Simulation	-0.571	-0.348	1.071	0.148
Residual	0.007	0.003	-0.071	0.002

Fig 6. Initial and Final Velocities for Glancing Collision with Moving Target. The above table depicts the worked solutions for Problem 3, the computed solutions from the simulation, as well as the residual value below it for both of the objects.

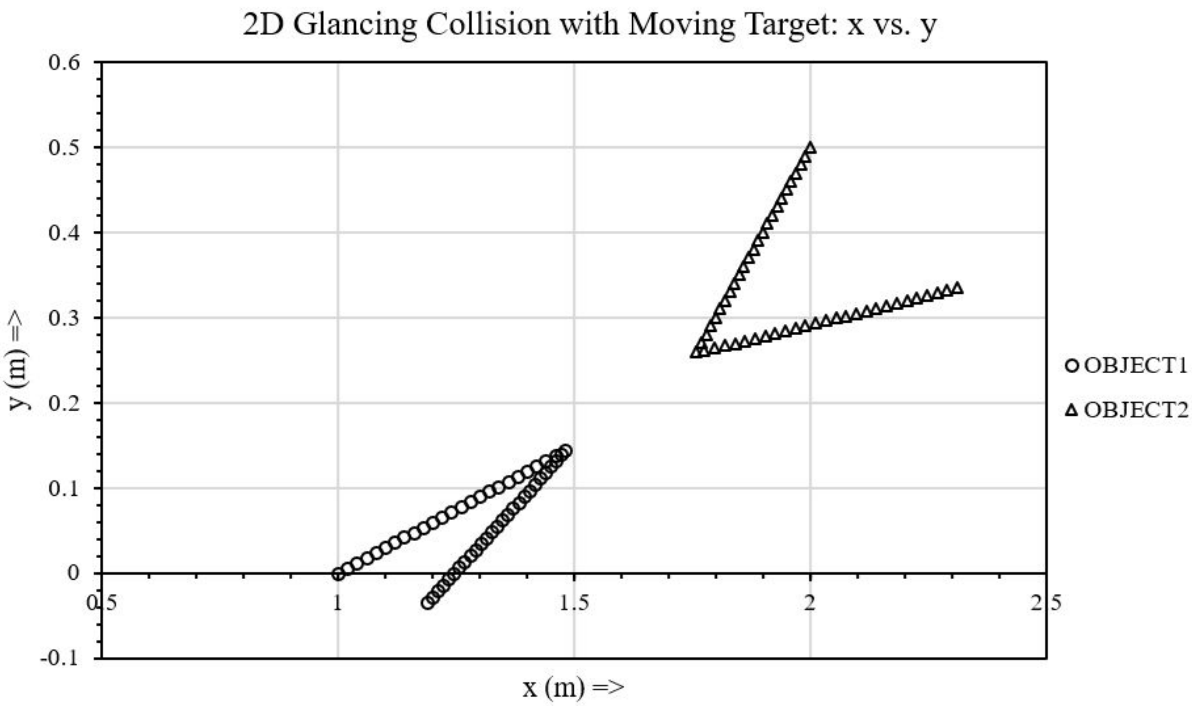

Fig 7. 2D glancing collision of two stress balls, x vs. y. The computer simulation modelled the collision between two balls, object 2, and object 1, in which they collide at an angle. The vertical position (m) of the balls are graphed against the respective horizontal positions. No error is provided as the graph as values are calculated through simulation based on given parameters.

For this graph again, the passage of time can be inferred by observing the path of the two balls. It is observed that the paths do not appear to meet due to the paths plotting the centre of the balls (the edges would contact in the middle of the closest points). Again, the path of each ball follows the same direction before and after the collision, which is the same as the direction of the velocity of each. The actual collision still takes place over a short amount of time as witnessed before which corresponds with the observation of witnessing two colliding stress balls. Again, the simulation produces fairly accurate results, and the error is as would be expected based on the use of successive approximation.

Two-Dimensional Glancing Collision with Rotation

According to the original simulation, the rotational aspect of the experimentation was not included. However, to give a realistic model of the situation in which two stress balls collide, the rotation that occurs upon impact must be considered. Yet, it's a highly complex situation to analyze due to the different factors that affect the collision and the resulting rotations. The following is an attempt at analyzing the collision's rotational aspect.

The first attempt at modelling the rotational aspect examined Conservation of Energy, Conservation of Linear Momentum, and Conservation of Angular Momentum. This gave three different equations where the left hand side was before impact, consisting of purely translational motion, and the right hand side contained rotational elements alongside the translational elements. Each equation is then separated into its x and y components. From this, it can concluded that the purely translational kinetic energy was split into rotational and translational kinetic energy after impact. This would indicate that although the path in which the balls would travel when spinning is the exact same direction as the original simulation, it would hypothetically be at a lower velocity. This left four different unknowns that had to be computed: the final angular velocity of ball 1 and ball 2, as well as the final translational velocity of ball 1 and ball 2. That left three equations with four unknowns — thus, unsolvable. There was an attempt made to redefine the angular velocity of one of the balls in terms of the other variables but the system remained unsolvable. Additionally, another problem occurred in the algebra. While manipulating the equations to cancel out the Force variable for torque, the change in time, *dt,* also got cancelled out which cannot be correct. That would mean the calculations are not time-dependent and that simply does not make sense.

Hence, a different approach had to be taken. In a Cartesian reference frame, when examining the angle at which the two stress balls collide with each other, the problem can be broken down into further components. The point at which the object collides, *dA*, can be analogous to that of two parallel plates which are in contact. There are two components of the force applied in contact — tangentially and the radially with respect to the two objects. Previously, in the method described in the original attempt, the radial force was all that was accounted for, however, it is the tangential force that causes rotation. This tangential force, which is solely caused by friction, imparts a torque. From $\tau = r \times F$, where F is the force of friction, and since friction is always perpendicular to r, $\tau = rF$. From this, we can rewrite τ and find the angular acceleration.

$$\alpha_1 = \frac{\mu_k a_2}{r_1}$$

$$\alpha_2 = \frac{\mu_k a_1}{r_2}$$

Where:

α_1, α_2 = the angular acceleration of mass 1 and 2 respectively (rad s^{-2})
r_1, r_2 = the radii of mass 1 and 2 respectively
μ_k = the coefficient of kinetic friction

Further, the angular velocity can be found over the temporal resolution of the simulation.

For the linear acceleration of the mass, we must consider the net force. The net force on one of the masses is equal to the sum of the spring force from the other ball (the radial force), and the tangential force. Breaking the equation into the components, and solving for the acceleration, we get

$$a_1 = \frac{-k(r_1 + r_2 - d)(m_2 + \mu_k m_1)}{2m_1 m_2(1 - \mu_k^2)}$$

$$a_2 = \frac{-k(r_1 + r_2 - d)(m_1 + \mu_k m_2)}{2m_1 m_2(1 - \mu_k^2)}$$

Where:

a_1, a_2 = the acceleration of mass 1 and 2 respectively (ms^{-2})
m_1, m_2 = the masses of mass 1 and 2 respectively (kg)
d = the distance between the centre of masses of the 2 objects
k = the spring constant, 1302 (Nm)

Finally, by solving for the x and y components of a_1 and a_2, the respective velocities and position can be computed.

Rotation Scenario Tested:

2D Glancing Collision with Moving Targets

Ball *B*, moving in the direction at a speed of v_B = (1i + 0.3j) m/s collides with a moving ball A which is moving at a speed of v_A= (-0.5i - 0.5j) m/s. Ball *A* has a mass of 1 kg, and ball B has a mass of 1.5 kg. After the collision, ball B has a velocity of (0.75i + 0.025j) m/s. Find the velocity of ball A.

In the simulation, it is assumed that the two balls are not spinning initially, and that both have a radius of 0.15m.

	Mass 1 Final v_x (ms^{-1})	**Mass 1 Final v_y (ms^{-1})**	**Mass 1 Final ω (rad s^{-1})**	**Mass 2 Final v_x (ms^{-1})**	**Mass 2 Final v_y (ms^{-1})**	**Mass 2 Final ω (rad s^{-1})**
Worked Answer (without rotation)	-0.872	-0.488	N/A	0.75	0.025	N/A
Simulation (with rotation)	-0.701	-0.4014	4.373	0.944	0.095	5.151

Fig 8. Initial and Final Velocities for Glancing Collision with Moving Target. The above table depicts the worked solutions for Problem 3, the computed solutions from the simulation, as well as the residual value below it for both of the objects. It includes the angular components of the velocity as found through the computer simulation.

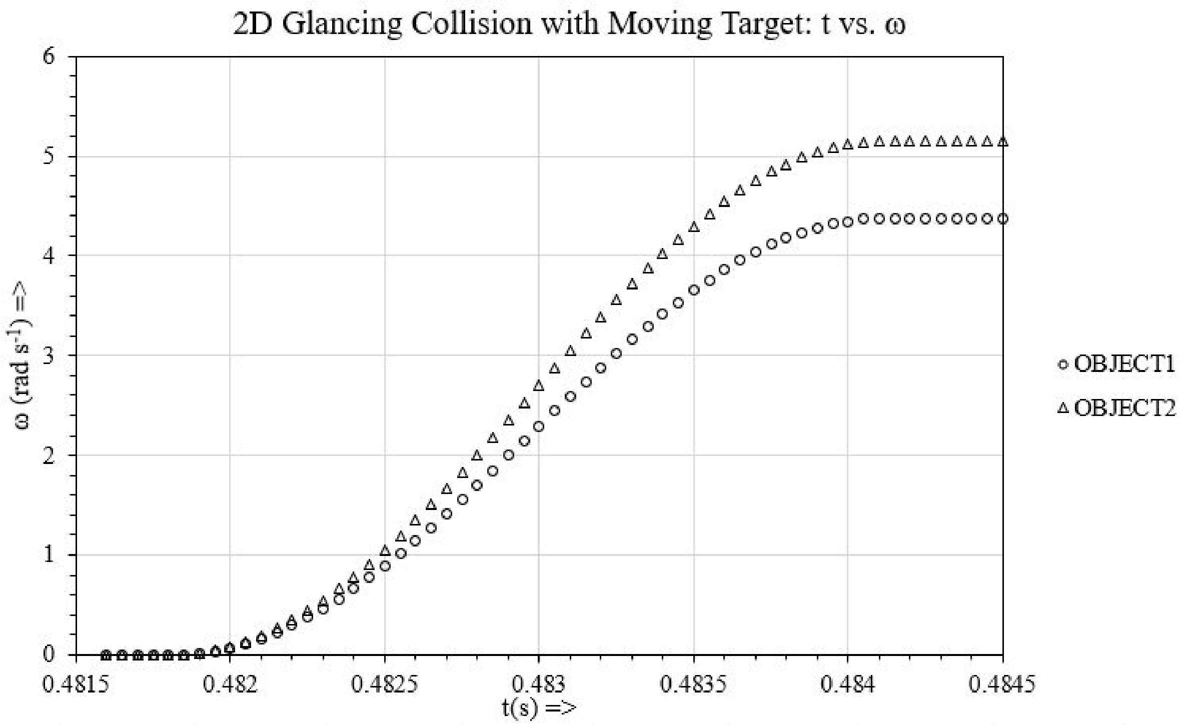

Fig 9. 2D glancing collision of two stress balls, time vs. angular velocity. The computer simulation modelled the collision between a ball at rest, object 2, and another ball, object 1, in which they collide at an angle.The angular velocity of the two stress balls upon collision is graphed against time.

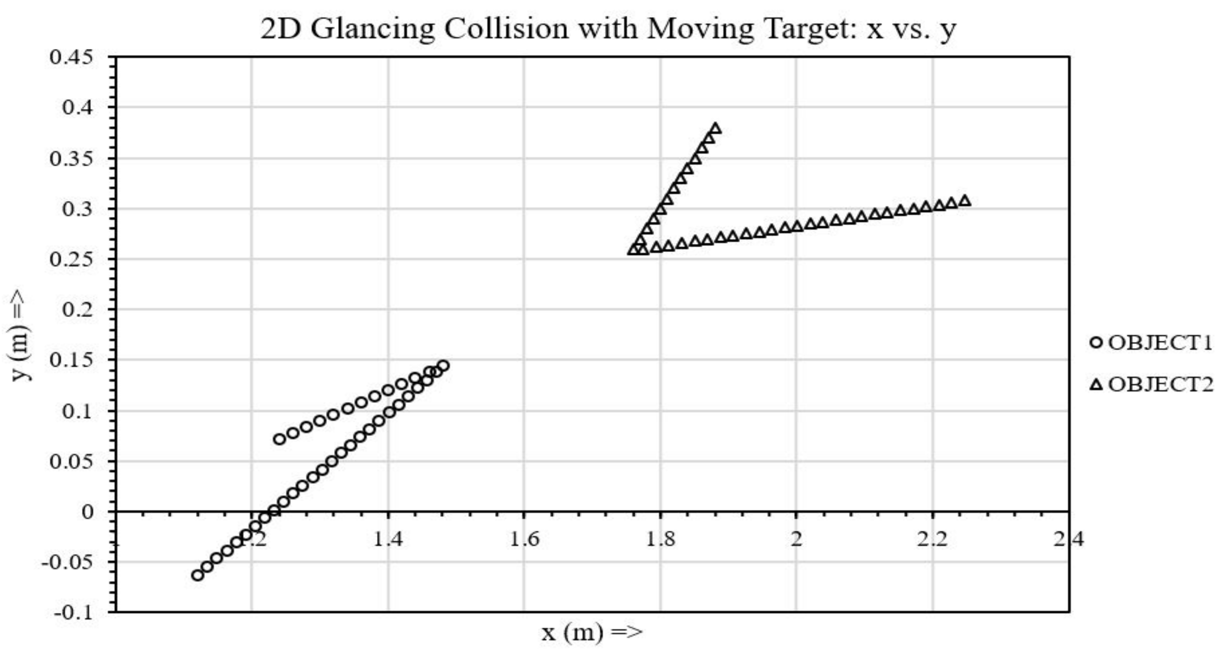

Fig 10. 2D glancing collision of two stress balls, x vs. y. The computer simulation modelled the collision between a ball at rest, object 2, and another ball, object 1, in which they collide at an angle. The vertical position (m) of the balls are graphed against the respective horizontal positions. No error is provided as the graph as values are calculated through simulation based on given parameters.