STELLAR MODEL SIMULATION

A Sun-like star was modelled in Matlab to find the temperature as a function of the distance from the centre (r). Variable pressure and mass was computed by solving the Hydrostatic Equilibrium Equation and the Continuity Equation. For simplicity, the star's density was varied linearly as a linear stellar model, as opposed to a polytrope. Hence, the equation for linear density is:

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \qquad [1]$$

Where:

$\rho(r)$ = density (kgm^{-3})
ρ_0 = density at the centre of the Sun (150 000 kgm^{-3})
r = distance from centre of star (m)
R = radius of the Sun (638 000 000 m)

From the Continuity Equation,

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r) \qquad [2]$$

Where:

$m(r)$ = mass (kg)

Substituting [1] into [2] and integrating, since we know that *m(0) = 0*, the mass as a function of *r* can be solved.

$$m(r) = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) \qquad [3]$$

For pressure, from the Hydrostatic Equilibrium Equation,

$$\frac{dP(r)}{dr} = \frac{-Gm(r)\rho(r)}{r^2} \qquad [4]$$

Where:

$P(r)$ = pressure ($kgm^{-1}s^{-2}$)

Substituting [1] and [3] into the Hydrostatic Equilibrium Equation, we can solve for *P(r)*.

$$\mathrm{P(r)} = -4\pi \mathrm{G}\rho_0^2 \int \left(\frac{\mathrm{r}}{3} - \frac{\mathrm{r}^2}{4\mathrm{R}}\right)\left(1 - \frac{\mathrm{r}}{\mathrm{R}}\right) \mathrm{dr} \qquad [5]$$

Since we have *P(R)=0*, then we can find pressure as a function of r.

$$\mathrm{P(r)} = -4\pi \mathrm{G}\rho_0^2 \left(\frac{\mathrm{r}^4}{16\mathrm{R}^2} - \frac{7\mathrm{r}^3}{36\mathrm{R}} + \frac{\mathrm{r}^2}{6}\right) + \frac{5}{36}\pi \mathrm{G}\rho_0^2 \mathrm{R}^2 \qquad [6]$$

The $\frac{5}{36}\pi G\rho_0^2 R^2$ portion of [6] is from the constant of integration solved with the initial condition *P(R)=0*.

Furthermore, if we assume that the star largely follows the Ideal Gas Law,

$$P(r) = \frac{k}{\mu m_H}\rho(r)T(r)$$

Where: [7]

$T(r)$ = temperature (K)
μ = mean molecular weight (0.61 u)
m_H = mass of a Hydrogen atom (1.67E-27 kg)
k = Stefan-Boltzmann constant (1.38E-23 JK^{-1})

Then, solving for *T(r)*, the temperature can be written as a function *r*.

$$T(r) = \frac{\mu m_H P(r)}{k\rho(r)}$$

[8]

$\rho(r)$ and $P(r)$ are from [1] and [6] respectively.

In Matlab, [1], [3], and [6] were written as function of *r*, and substituted to solve in [8]. Graphing [8] in Matlab, the following curve is obtained.

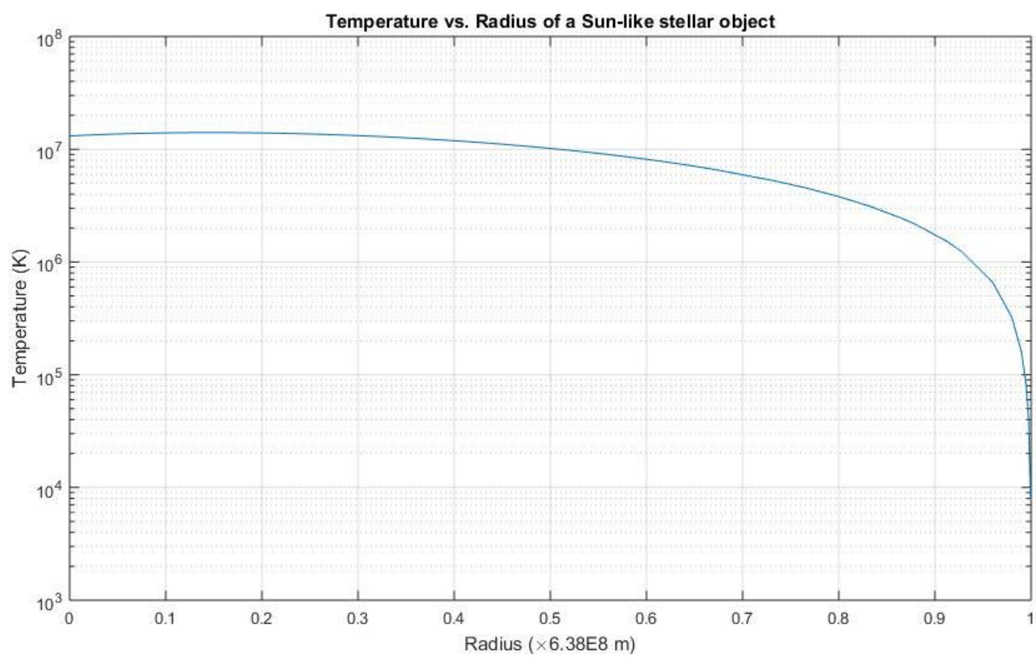

Figure 1. Temperature vs. Radius of a Sun-like stellar object. The temperature as computed by [8] in Matlab is graphed. A coefficient of 0.1 was multiplied to reach the initial conditions of the Sun at core and surface temperatures. The model assumes a linear density and the Ideal Gas law, and varies as a function of pressure and mass from the Hydrostatic Equilibrium Equation and the Continuity Equation respectively.

Figure 1 appears to switch energy transport modes somewhere around 0.75R, when the graph begins to slope downwards more steeply. However, if we were to actually graph $\frac{dT_R}{dr}$ (radiative export) and $\frac{dT_C}{dr}$ (connective export) in Matlab, we find that convective heat transfer is preferred starting from about 0.05R, which is largely unrealistic. Thus, utilizing the Ideal Gas Law clearly does not account for the two types of energy transports. Regardless, the curve in Figure 1 can be interpreted as switching from radiative heat export to convective heat export at around 0.75R. This is within reason of the 0.7R switch observed in the Sun.

The curve in Figure 1 is comparable in shape to the Dalsgaard Model 1 as calculated by NASA.

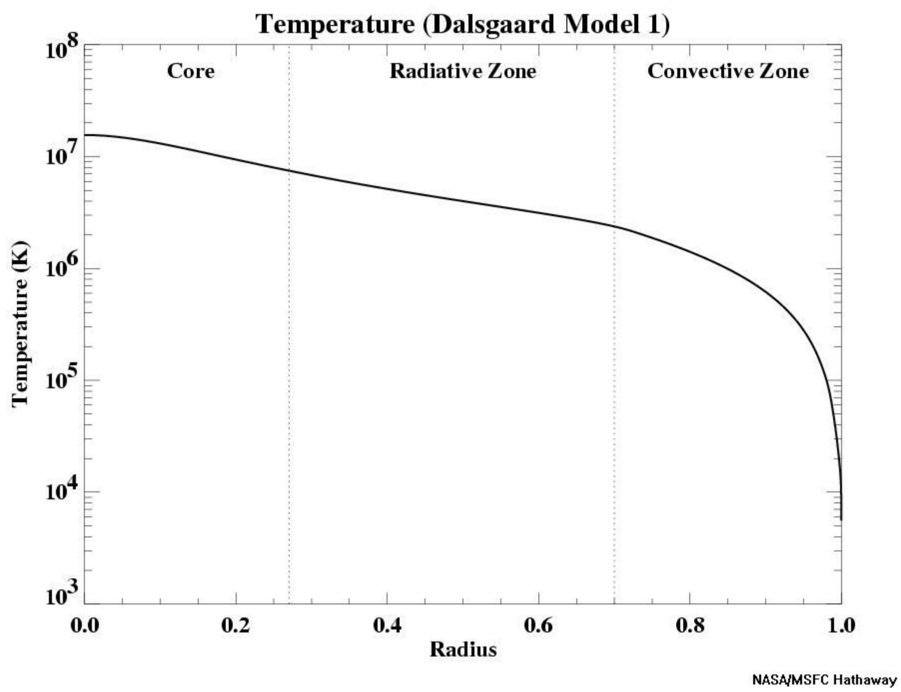

Figure 2. Temperature vs. Radius of the Sun using the Dalsgaard Model 1. The temperatures of the Sun as calculated from NASA Marshall Space Flight Sun is graphed against the radius of the Sun.

There are several reasons for the discrepancy between our values in Figure 1 as compared to Figure 2.

Firstly, a linear density was used in this simulation for simplicity. Although this is fairly accurate from 0R to roughly 0.9R, the density drops rapidly afterwards. This would, in effect, make the temperature drop less sudden near the surface of the star as observed in Figure 1. Hence, a linear density model would be unrealistic for a real star.

Further, there is likely some floating-point error within the Matlab simulation due to both the extremely small and large numbers and constants. For instance, the pressure function based on [6] in Matlab returns P(R) = 160 when it should mathematically be 0.

Lastly, Figure 2 optimizes between radiative export (T_R) and convective export (T_C). For a star, the preferred method of heat transfer is always the lesser of $\frac{dT_R}{dr}$ and $\frac{dT_C}{dr}$. This can be observed in Figure 2, where from 0R to 0.7R, the preferred method of heat transfer is radiative export. From 0.7R to the surface, the graph dips more steeply where convective export takes over. However, in our calculations in Figure 1, utilizing the Ideal Gas law did not account for these two modes of heat export. Hence, curve obtained was continuous rather than contiguous.