

Lunar Impact Ejecta Benchmark and Models

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1 Executive Summary

2 Characteristics of the 17 March 2013 Event

2.1 LRO Observations: Robinson et al. 2015

Here we summarize the findings from [Robinson et al. \[2015\]](#). Their assessment used the impact-scaling model of [Holsapple \[1993\]](#) to constrain the impact event parameters, with a fixed rim-to-rim crater diameter.

2.1.1 Crater Morphology

The March 2013 crater has a rim-to-rim diameter of $D_{rim} = 18.8^{+1.1}_{-1.2}$ m with a depth of approximately 2-3 m. The transient crater diameter was estimated to be $D = 14$ m, see Figure 1 for an illustration.



Figure 1: Example of a simple excavation crater, <http://keith.aa.washington.edu/craterdata/scaling/index.htm>.

2.1.2 Impact Event Parameters

Impactor densities were chosen to be either cometary 1 g/cm^3 , chondritic 3.4 g/cm^3 , or iron-nickel 6 g/cm^3 with plausible impact velocities of 5-60 km/s. The diameter of the impactor was between 0.28 and 1.10 m with a mass between 33 and 702 kg. For this given range of impact velocities and densities, the kinetic energy of the impactor was between $6.4 \times 10^9 \text{ J}$ and $6.0 \times 10^{10} \text{ J}$.

2.2 MEO Observations: Moser et al. 2014, ACM

Here we summarize the findings from [Moser et al. \[2014\]](#). They used both Gault's crater scaling law [[Gault, 1974](#)] and Holsapple's online calculator [[Holsapple, 1993](#)] in their analysis.

2.2.1 Correlation with Meteor Activity

[Moser et al. \[2014\]](#) found correlation with the Virginid meteor shower complex (EVI/NVI) with an observed meteor shower on 17 March 2013 and the lunar impact that was seen by NASA MSFC. According to the [Meteor Data Center](#), the eta Virginids, EVI, have a speed of 26.6 km/s to 34.2 km/s and the Northern March Virginids, NVI, have a speed of 23.0 km/s. However, given the cluster of five fireballs that were seen on 17 March 2013, the speed was $v_g = 25.6 \pm 0.8$ km/s and matched closer to the EVI orbital elements. The Tisserand number was 3.1 ± 0.2 , which is right on the line of being either cometary (< 3) or asteroidal (> 3). It was found that the lunar impact angle from the horizontal was $\theta_h = 56^\circ$ for a Virginid meteor.

2.2.2 Impact Event Parameters

Assuming a regolith density of $\rho_t = 1500$ kg/m³, a porosity of 40%, and cohesion strength of 0.1 Mpa, the impactor densities ranged from 1000 kg/m³ to 3300 kg/m³, the mass was between 11 kg and 66 kg, and the kinetic energy ranged from 3.6×10^9 J to 2.2×10^{10} J. The duration of the impact was estimated to be 1.03 s with a nominal diameter of 22 ± 3 cm, assuming $\rho_p = 3000$ kg/m³.

2.2.3 Luminous Efficiency

From [Moser et al. \[2014\]](#), if we assume the impactor was associated with the Virginids, the luminous efficiency η_λ is in the range $7.5^{+4.5}_{-2.5} \times 10^{-4} < \eta_\lambda < 1.5^{+0.8}_{-0.5} \times 10^{-3}$, depending on the regolith density, $1500 \text{ kg/m}^3 < \rho_t < 2100 \text{ kg/m}^3$, and impactor density, $1000 \text{ kg/m}^3 < \rho_p < 3300 \text{ kg/m}^3$. This range of luminous efficiencies is consistent with [Bouley et al. \[2012\]](#) ($\eta_\lambda = 5 \times 10^{-4}$) and [Moser et al. \[2011\]](#) ($\eta_\lambda = 1.3 \times 10^{-3}$).

3 Lunar Regolith Properties

4 Spheral Benchmark of 17 March 2013 Event

4.1 Getting Started and Running on Computer Clusters

4.1.1 Installing on MEO Cluster

```
1 References :
2 If gcc is too old, follow this if you have CentOS:
3 https://ahelpme.com/linux/centos7/how-to-install-new-gcc-and-
   development-tools-under-centos-7/
4
5
6 To install Spheral and test installation :
7
8 cd
9 mkdir Spheral
```

```
10 mkdir github-Spheral
11 cd github-Spheral
12 git clone https://github.com/jmikeowen/spheral
13 cd spheral/src/
14 scl enable devtoolset-7 bash
15 ./boot
16 mkdir BUILD
17 cd BUILD
18 ../configure --prefix=/home/adestefa/Spheral --with-opt=3 --
    with-compilers=gnu --with-dbc=none
19 make -j 20
20 cd /home/adestefa/github-Spheral/spheral/tests
21 /home/adestefa/Spheral/bin/ats -n 20 -e /home/adestefa/Spheral
    /bin/python integration.ats
22
23
24
25 To install Visit:
26
27 cd
28 mkdir download-visit
29 cd download-visit/
30 wget http://portal.nersc.gov/project/visit/releases/2.13.3/
    visit2_13_3.linux-x86_64-rhel7.tar.gz
31 wget http://portal.nersc.gov/project/visit/releases/2.13.3/
    visit-install2_13_3
32 chmod 755 visit-install2_13_3
33 ./visit-install2_13_3 2.13.3 linux-x86_64-rhel7 /home/adestefa
    /visit-2.13.3
34 cd
35 echo "export PATH=$PATH:/home/adestefa/visit-2.13.3/bin" >> .
    bashrc
```

4.1.2 Running on MEO Cluster

4.1.3 Running on LLNL Cluster

The connection and login details described below is assumed to be on a Windows machine.

1. Connect to VPN using Cisco AnyConnect Secure Mobility Client
 - VPN name: vpn.llnl.gov
 - Group name: llnl.vpnc
2. Login with OUN (destefano2) and CZ PIN + token # (RSA SecurID)
3. SSH into LLNL server head node using Putty

- Host name: rzgw.llnl.gov
 - Option: Connection → SSH → X11 → Enable X11 forwarding¹
 - Option: X display location → localhost:0
 - Login: LC username (destefan)
 - Password: RZ PIN + token # (RZ)
4. SSH into RZ specific server
 - ssh -XY destefan@rztopaz
 - Password: RZ PIN + token # (RZ)
 5. Check current jobs for a user
 - squeue -u <username>
 - Add - -start to see begin ETA
 6. Load Spheral module
 - ml Spheral/exp
 7. Debug Spheral Python script
 - srun -n <# of CPUs, max 36 per node> -p pdebug python -i <python_script.py>
 - Add any variable definitions after the python_script.py, such as - -impactAngle=56, - -rImpactor=25.2, etc. See the halfSphere_amdvX.py file for more variable options.
 8. Grab compute nodes for continuous debugging
 - mxterm <# of nodes, max 8> <# of CPUs, =36×# of nodes, max 288> <time, max 60 (minutes)> -q pdebug
 - After nodes have been grabbed, do steps 6 and 7 as before
 9. Submit batch script
 - msub <script.msub>
 10. Check drive space quota
 - quota -v

¹Need to download either VcXsrv (works better), or Xming. In VcXsrv, which will be called VLaunch on the desktop, need to deselect **Native opengl** on the last tab.

4.2 Porosity ϕ_0 Study

4.3 Max Strength Y_m Study

5 Spheral Based Lunar Ejecta Modeling

6 MEM3 Based Lunar Ejecta Modeling

The Meteoroid Engineering Model (MEM) describes the sporadic meteoroid complex, or the background meteoroid environment, and does not include meteor showers. The impactor masses range from 1 μg to 10 g. Larger impactor masses must be dealt with differently [e.g., see [Neukum et al., 2001](#); [Brown et al., 2002](#)], *which we omit here for now*². We use output from MEM in order to estimate the number of particles per area per year greater than a certain mass (or the particle flux mass spectrum) due to secondary ejecta from meteoroid impacts on the Moon. The risk due to impacts on the Moon is driven by secondary ejecta and not the primary meteoroid flux.

We begin by first describing the algorithm at which we plan to use to compute the particle flux mass spectrum at a given point on the Moon. In essence, the algorithm is based on the reverse Monte-Carlo idea. We then go into detail about how to compute each step of the algorithm, either borrowing from the literature or making our own derivations.

6.1 Algorithm

1. For a given location on the Moon, compute the particle flux mass spectrum
2. For each source location
 - Defines the distance D from the source required to compute the ejecta velocity $v = v(D, \gamma)$.
3. For each ejecta angle γ
 - Completely defines the ejecta velocity $v = v(D, \gamma)$
4. For each meteoroid impact angle α (from MEM output)
 - At the moment, we will sum over the azimuthal angle and assume isotropic azimuthal secondary ejecta (this is not the case for impact angles less than 30° from the horizon).
5. For each meteoroid impact speed U
6. For each impactor density δ
 - We only need to compute this once for each target material, and can factor out as a constant. The density is given as an output in MEM.

²At some point we will need to account for impactor masses larger than 10 g in order to understand the tail of the distribution in terms of the risk analysis.

7. For each impactor mass m_p
 - We can integrate this out. At first glance, we will get a list of hypergeometric functions, but we only need to evaluate these once and factor out as a constant. The mass distribution is given in MEM Eq. (2.1).
8. For each ejecta particle size m_e
 - We can integrate the particle size distribution, but we will need to keep track of each mass size m_e .

6.2 Regolith Size Distribution

For relatively small impact sizes (craters $< 30 - 50$ m), we can generally assume the secondary ejecta follows that of the original regolith. The cumulative distribution function (CDF) of the particle sizes can be fit to many observations, as shown in Figure 2.

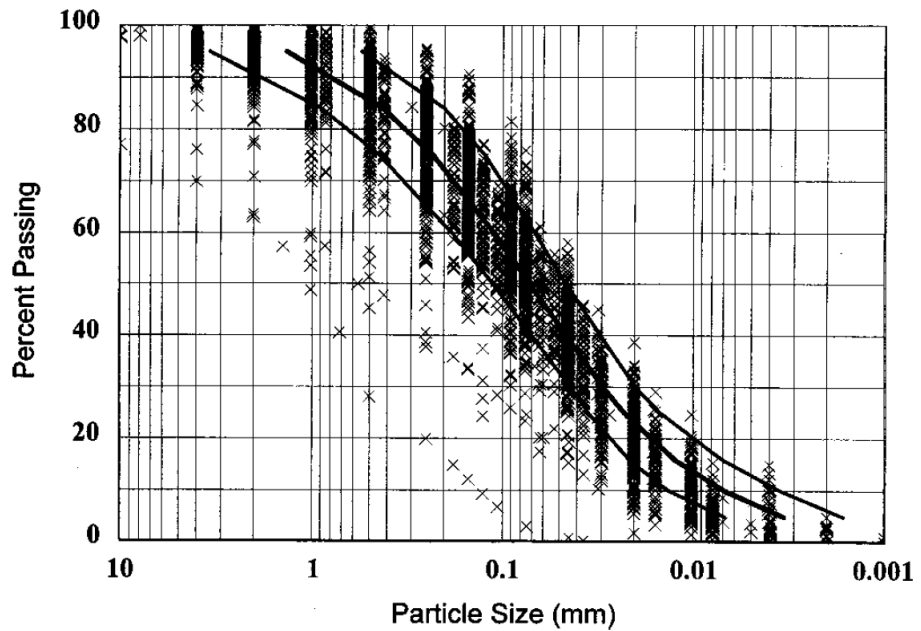


Figure 2: Geotechnical particle size distribution: middle curve showing the average distribution; left-hand and right-hand curves showing ± 1 standard deviation [Carrier III, 2003].

In order to parametrize the CDF from Figure 2, we make a fit to the model equation

$$C_{moon} = 1 - \exp\left(\frac{-1}{ax^b + cx^d}\right), \quad (1)$$

which is an exponential distribution with two scales defined by a and c , with x in units of mm. In SciDAVis, we make the fit with the x-axis on a logarithmic scale to give equal weight to both small and large scaled particles. The results for the curve fit are shown in Figure 3. We found that a simple exponential distribution with a single scale was insufficient, hence the reason we opted for a two-scaled exponential distribution.

```
[12/30/2019 5:40:09 PM      Plot: "Graph2"]
Non-linear fit of dataset: Table2_2, using function: 100*(1-exp(-1/(a*(10^x)^b+c*(10^x)^d)))
Y standard errors: Unknown
Nelder-Mead Simplex algorithm with tolerance = 0.0001
From x = -2.482108151 to x = 0.134259956
a = 0.0548044684398436 +/- 0.00569348851446585
b = -1.01472478485333 +/- 0.0196858460040614
c = 0.337499285612252 +/- 0.0049924392579481
d = -0.251808155531636 +/- 0.0188844274561765

-----
Chi^2 = 8.25978651917647
R^2 = 0.999849854428033
-----
Iterations = 89
Status = success
-----
```

Figure 3: Non-linear fit of Figure 2 (the average distribution) with Eq. 1 in SciDAVis, giving the constants for a , b , c , and d .

To compute the probability distribution function (PDF), we can simply take the derivative of the CDF with respect to x , which results in the following equation:

$$P_{moon} = -A \frac{abx^{b-1} + cdxd^{d-1}}{(ax^b + cx^d)^2} \exp\left(\frac{-1}{ax^b + cx^d}\right), \quad (2)$$

where A is the normalization constant. In theory, this should be equal to 1, but since we are not taking our particle size from 0 to infinity, we need to compute the value of A . If we assume the particle size can range from 0.001 mm to 10 mm, then $A = 1.02218$.

Since our goal is to compute the particle flux mass spectrum, we need to *count* the number of particles from the PDF that make up the mass of M from Eq. 6. We first need to know the total volume per complete PDF, which is given by

$$V_{P_{moon}}(x_{min}) = \frac{4}{3}\pi \int_{x_{min}}^{x_{max}} dx x^3 P_{moon}(x), \quad (3)$$

where $x_{min} = 0.001$ mm and $x_{max} = 10$ mm. An exact analytic solution of Eq. 3 is quite difficult, but we notice that the integrand $x^3 P_{moon}(x)$ very closely resembles a linear equation within the desired range, see Figure 4.

Therefore, we make the approximation

$$x^3 P_{moon}(x) \sim p_0 x + p_1, \quad (4)$$

where $p_0 = 1/11.48$ and $p_1 = 1/232.6$. The total volume of particles with mass greater

Figure 4: Linear fit to the integrand $x^3 P_{moon}(x)$.

than $m_0 = \frac{4}{3}\pi\rho x_0^3$ is given by

$$V_{P_{moon}}(x_0) = \frac{4}{3}\pi \left[\frac{1}{22.96} (x_{max}^2 - x_0^2) - \frac{1}{232.6} (x_{max} - x_0) \right], \quad (5)$$

where $V_{P_{moon}}(x_{min}) = 18.06377 \text{ mm}^3$. Given the density of the regolith ρ , we can then compute the total mass of the particles from the PDF.

6.3 Ejected Mass from an Impactor

From [Housen and Holsapple \[2011\]](#), we can compute the mass ejected faster than v in terms of impactor properties, given by

$$M(v; \rho; m, \delta, U, \alpha) = C_4 m \left[\frac{v}{U\Theta(\alpha)} \left(\frac{\rho}{\delta} \right)^{\frac{3\nu-1}{3\mu}} \right]^{-3\mu}, \quad (6)$$

where

- v : secondary ejecta speed,
- ρ : target density,
- m : projectile mass,
- δ : projectile density,
- U : projectile speed,
- α : projectile impact angle (from horizon),

and

$$C_4 = \frac{3k}{4\pi} C_1^{3\mu}, \quad (7)$$

where the constants k , C_1 , ν , and μ depend on the specific material properties, see Table 3 of [Housen and Holsapple \[2011\]](#). The impact angle modification equation $\Theta(\alpha)$ can be chosen to be

$$\Theta(\alpha) = \begin{cases} 1 \\ \sin \alpha \\ \sin(\sqrt{\alpha_0^2 + \alpha^2}), \alpha_0 \sim 5^\circ - 15^\circ. \end{cases} \quad (8)$$

6.4 Ejecta Mass Distribution Function

The mass ejected from the crater, $M(> v)$, from Eq. (6), is the total mass ejected from an impact at velocities greater than v . However, we would like to know how this ejecta is distributed in speed and solid angle so we can map the ejecta to a particular surface location on the Moon. We can then set Eq. (6) in terms of the integral over the distribution functions of speed and solid angle as

$$M(> v) = \int_v^\infty \int_0^{2\pi} \int_0^{\pi/2} \sin \alpha d\alpha d\beta dv' F(\alpha) G(\beta) H(v'), \quad (9)$$

where α is the zenith angle, β is the azimuth angle, and v is the ejecta speed. Just to note, at the secondary impact location, the zenith angle will be the same as the ejected zenith angle. However, the azimuth angle (or bearing) will be modified due to travel across the spherical surface, see Section 6.9.

The speed distribution $H(v')$ is defined as

$$\int_v^\infty dv' H(v') = v^{-3\mu}, \quad (10)$$

which is the speed dependent term of Eq. (6). We can then solve the speed distribution explicitly as

$$H(v) = 3\mu v^{-(3\mu+1)}. \quad (11)$$

For the angular dependent terms $F(\alpha)$ and $G(\beta)$, they technically should depend on speed as well as particle size [e.g., [Rival and Mandeville, 1999](#)]. For simplicity, we will assume the angular distribution is independent of speed and particle size but will be dependent on impact zenith and azimuth angle with respect to the bearing.

Adopting the zenith and azimuth distributions from [Rival and Mandeville \[1999\]](#), we have the following equations: The zenith distribution is given by

$$F(\alpha) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{(\alpha - \alpha_{max})^2}{2\sigma^2} \right], \quad (12)$$

where α_{max} is defined as

$$\alpha_{max} = \begin{cases} \frac{\alpha_{max60} - \alpha_{max0}}{\pi/3} \alpha_i + \alpha_{max0} & \text{for } \alpha_i \leq \pi/3 = 60^\circ \\ \alpha_{max60} & \text{for } \alpha_i > \pi/3 = 60^\circ \end{cases}, \quad (13)$$

for α_i the impact zenith angle, and [see [Miller, 2017](#)]

$$\alpha_{max0} = \frac{\pi}{6} = 30^\circ, \quad (14)$$

$$\alpha_{max60} = \frac{4\pi}{9} = 80^\circ, \quad (15)$$

$$\sigma = \frac{\pi}{60} = 3^\circ, \quad (16)$$

where the peak ejecta angle is shifted from 30° of zenith for a normal impact to 80° of zenith for oblique impacts ($> 60^\circ$).

The azimuth distribution is given by [[Rival and Mandeville, 1999](#)]

$$G(\beta) = \begin{cases} \frac{1}{2\pi} \left[\frac{3\alpha_i}{2\pi - 3\alpha_i} \cos(\beta - \beta_i) + 1 \right] & \text{for } \alpha_i \leq \pi/3 = 60^\circ \\ \frac{1}{\sigma' \sqrt{2\pi}} \exp \left[-\frac{(\beta - \beta_i)^2}{2\sigma'^2} \right] & \text{for } \alpha_i > \pi/3 = 60^\circ \end{cases}, \quad (17)$$

where

$$\sigma' = \frac{\pi}{36} = 5^\circ, \quad (18)$$

for β_i the impact azimuth angle $+\pi$.

6.4.1 Zenith Distribution Function

To complete the integral over the zenith distribution, namely

$$\int_{\alpha_0(v)}^{\alpha_1(v)} d\alpha \sin \alpha F(\alpha), \quad (19)$$

we need to choose a distribution function for $F(\alpha)$ to allow for analytic solutions. We will therefore look for an alternate distribution from Eq. (12), given by the form

$$F(\alpha) = (1 - \cos \alpha)^{1/a} \cos^a \alpha, \quad (20)$$

where the exponent a can be defined in terms of the peak angle α_{max} as

$$a = \frac{\cos \alpha_{max}}{1 - \cos \alpha_{max}}, \quad (21)$$

such that $F'(\alpha_{max}) = 0$ and $F''(\alpha_{max}) < 0$. We then can do a variable substitution (chosen so the domain goes from $\alpha \in [0, \pi/2]$ to $x \in [0, 1]$)

$$1 - x = \cos \alpha, \quad (22)$$

$$dx = \sin \alpha d\alpha, \quad (23)$$

so that Eq. (19) becomes

$$\int_{x_0(v)}^{x_1(v)} dx x^{1/a} (1 - x)^a, \quad (24)$$

where the equations $x_0(v)$ and $x_1(v)$ are a linear function of v and an implicit function of the distances D_0 and D_1 , respectively. The integral in Eq. (24) is the incomplete beta function

$$\int_{x_0(v)}^{x_1(v)} dx x^{1/a} (1-x)^a = \beta(x_1(v); 1/a+1, a+1) - \beta(x_0(v); 1/a+1, a+1). \quad (25)$$

For small differences in D_0 and D_1 , we can roughly assume small differences in $x_0(v)$ and $x_1(v)$ so that we can write Eq. (25) in terms of a derivative, where we evaluate the derivative at the midpoint

$$\begin{aligned} & \beta(x_1(v); 1/a+1, a+1) - \beta(x_0(v); 1/a+1, a+1) \\ &= \frac{\beta(x_0(v) + \Delta x; 1/a+1, a+1) - \beta(x_0(v); 1/a+1, a+1)}{\Delta x} \Delta x \\ &\approx \Delta x \frac{d}{dx} \beta(x_0(v); 1/a+1, a+1) \Big|_{x_0(v) \rightarrow x_0(v) + \Delta x/2} \\ &= \Delta x [1 - x_0(v)]^a x_0^{1/a}(v) \Big|_{x_0(v) \rightarrow x_0(v) + \Delta x/2} \\ &= \Delta x(v) \left[1 - \frac{x_0(v) + x_1(v)}{2} \right]^a \left[\frac{x_0(v) + x_1(v)}{2} \right]^{1/a}, \end{aligned} \quad (26)$$

where $\Delta x(v) = x_1(v) - x_0(v)$, and (note, the v 's are normalized by v_{esc} , emitted for clarity)

$$x_0(v) = m_0 v + b_0, \quad (27)$$

$$x_1(v) = m_1 v + b_1, \quad (28)$$

$$\Delta x(v) = (m_1 - m_0)v + b_1 - b_0. \quad (29)$$

The coefficients m_0, m_1 and b_0, b_1 are implicit functions of the distances D_0, D_1 . For the j -th distance D_j and the i -th speed v_i , the m and b coefficients can be written as

$$m_{j,i}^{\pm} = \frac{v_{i+1} - v_i}{x_{j,i+1}^{\pm} - x_{j,i}^{\pm}}, \quad (30)$$

$$b_{j,i}^{\pm} = v_i - m_{j,i}^{\pm} \cdot x_{j,i}^{\pm}, \quad (31)$$

where

$$x_{j,i}^{\pm} = 1 - \cos \alpha_{j,i}^{\pm}, \quad (32)$$

$$\cos \alpha_{j,i}^{\pm} = \frac{\cot \alpha_{j,i}^{\pm}}{\sqrt{1 + \cot^2 \alpha_{j,i}^{\pm}}}, \quad (33)$$

for (using Eq. (56))

$$\cot \alpha_{j,i}^{\pm} = v_i^2 \cot \left(\frac{D_j}{2r_m} \right) \pm \sqrt{v_i^4 \cot^2 \left(\frac{D_j}{2r_m} \right) + 2v_i^2 - 1}. \quad (34)$$

There are three regions of the zenith angle-space, and hence the x -space, where we have:

Region I: For all valid distances D_j and D_{j+1} , use $m_{j,i}^+$, $b_{j,i}^+$, $m_{j+1,i}^+$ and $b_{j+1,i}^+$

Region II: For $D_j < \pi r_m$ and $D_{j+1} \geq \pi r_m$, use $m_{j,i}^-$, $b_{j,i}^-$, $m_{j+1,i}^+$ and $b_{j+1,i}^+$

Region III: For $D_j < \pi r_m$ and $D_{j+1} < \pi r_m$, use $m_{j,i}^-$, $b_{j,i}^-$, $m_{j+1,i}^-$ and $b_{j+1,i}^-$

6.5 Meteoroid Projectile Mass Distribution

From the MEM3 User Guide, we get the $g(m)$ flux of meteoroids larger than a limiting mass m , originally from [Grün et al. \[1985\]](#). The Grün interplanetary flux equation is given by

$$g(m) = (c_4 m^{\gamma_4} + c_5)^{\gamma_5} + c_6 (m + c_7 m^{\gamma_6} + c_8 m^{\gamma_7})^{\gamma_8} + c_9 (m + c_{10} m^{\gamma_9})^{\gamma_{10}}, \quad (35)$$

where the constants are $c_4 = 2.2 \times 10^3$, $c_5 = 15$, $c_6 = 1.3 \times 10^{-9}$, $c_7 = 10^{11}$, $c_8 = 10^{27}$, $c_9 = 1.3 \times 10^{-16}$, $c_{10} = 10^6$; and the exponents are $\gamma_4 = 0.306$, $\gamma_5 = -4.38$, $\gamma_6 = 2$, $\gamma_7 = 4$, $\gamma_8 = -0.36$, and $\gamma_{10} = -0.85$. Eq. 35 is applied to MEM's mass range and is shown in Figure 5.

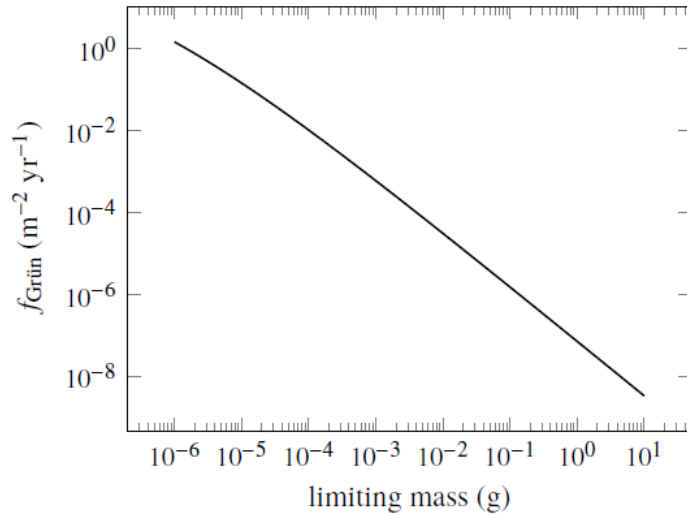


Figure 5: The Grün interplanetary meteoroid flux as a function of limiting particle mass [[Moorhead et al., 2019](#), Figure 1].

The mass flux $dg(m)/dm$ and Eq. 6 should be integrated over the mass range $m_{min} = 10^{-6}$ g to $m_{max} = 10^1$ g in order to account for all impactor mass sizes, which we call G_m given as

$$G_m = \int_{m_{min}}^{m_{max}} dm \frac{dg(m)}{dm} m. \quad (36)$$

The mass flux $dg(m)/dm$ can be fit to a double power law

$$\frac{dg(x)}{dx} = \frac{1}{ax^b + cx^d}, \quad (37)$$

where the fit parameters are shown in Figure 6, using a log-log scale to capture the small and large masses correctly.

```
[1/2/2020 4:13:33 PM      Plot: "Graph4"]
Non-linear fit of dataset: Table2_2, using function: log10(1/(a*(10^x)^b+c*(10^x)^d))
Y standard errors: Unknown
Nelder-Mead Simplex algorithm with tolerance = 0.0001
From x = -5.924382925 to x = 0.935617075
a = 321,865,117,982,837 +/- 0
b = 2.32111297269978 +/- 0.0005958076665748
c = 1,406,901,447,961.14 +/- 0
d = 1.81651108891357 +/- 0.00063978048941211

Chi^2 = 0.00183119809759917
R^2 = 0.999998212686885

Iterations = 95
Status = success
```

Figure 6: Non-linear fit of Figure 5 with Eq. 37 in SciDAVis, giving the constants for a , b , c , and d .

To integrate Eq. (36), we use the following solutions

$$\int dx \frac{x}{ax^b + cx^d} = -\frac{1}{a(b-2)x^{b-2}} {}_2F_1 \left[1, \frac{b-2}{b-d}; \frac{b-2}{b-d} + 1; -\frac{c}{a}x^{d-b} \right], \quad (38)$$

$$= -\frac{1}{c(d-2)x^{d-2}} {}_2F_1 \left[1, \frac{d-2}{d-b}; \frac{d-2}{d-b} + 1; -\frac{a}{c}x^{b-d} \right], \quad (39)$$

where Eq. 38 is more appropriate for small x if $d-b > 0$ and Eq. 39 is more appropriate for large x if $d-b > 0$. If the sign of $d-b$ is flipped, then the small and large scale equations are swapped.

6.6 Meteoroid Projectile Density Distribution

The meteoroid density has two components, a low and a high density contribution, as shown in Figure 7. To take into account this particular distribution in computing the particle flux mass spectrum, we should integrate Figure 7 against Eq. 6. Since the meteoroid density components can be written in terms of log-normal distributions

$$F_\delta(x) = \frac{A}{\sigma\sqrt{2\pi}x} \exp \left[-\frac{(\ln x - \mu_\delta)^2}{2\sigma^2} \right], \quad (40)$$

the integration entails computing the moments of a log-normal distribution. The α -moment is given by

$$F_{\delta}^{\alpha}(A, \mu_{\delta}, \sigma) = A \exp\left(\alpha\mu_{\delta} + \frac{1}{2}\alpha^2\sigma^2\right). \quad (41)$$

Inserting these results into Eq. 6, the functional form of the projectile density contribution can be written as

$$F_{\delta} = F_{\delta}^{3\nu-1}(A_{low}, \mu_{low}, \sigma_{low}) + F_{\delta}^{3\nu-1}(A_{high}, \mu_{high}, \sigma_{high}), \quad (42)$$

where the fit parameters for the low and high density components are shown in Figures 8 and 9. Since the meteoroid density is given in units of *fraction per 50 kg m⁻³*, we need to divide the A constants by 50 in order to give correct units.

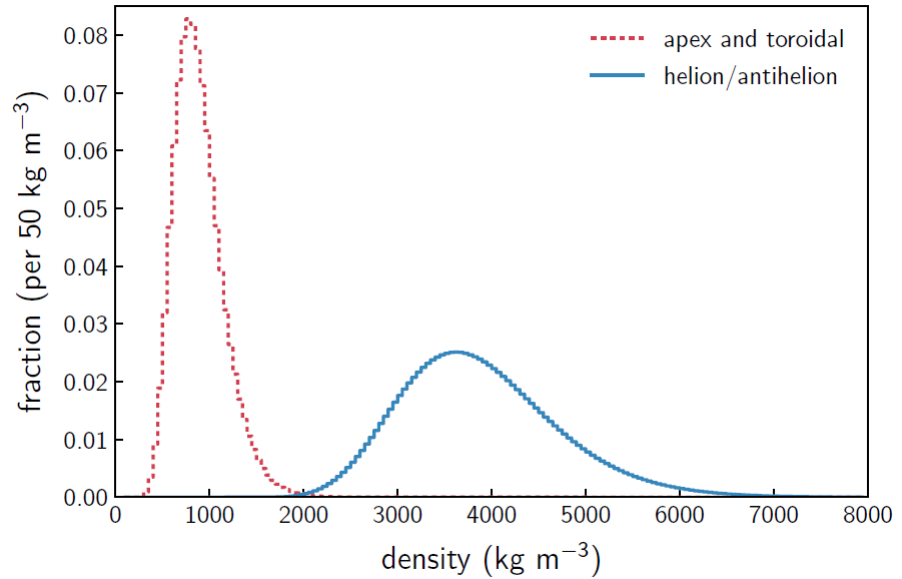


Figure 7: Meteoroid density distribution according to the MEM3 User Guide. The apex and toroidal meteoroid sources constitute the low-density population, while the helion/antihelion source constitutes the high-density population. Each set of densities follows a log-normal distribution [c.f. Figure 11, [Moorhead et al., 2019](#)].

6.7 Meteoroid Projectile Speed and Angle Distribution

MEM3 gives the incoming meteoroid flux (in units of # per km² per year) in terms of the speed U and both azimuth θ and altitude ϕ angles for a location on the Moon. At the moment, since we are assuming azimuthally symmetric ejecta, we will sum over

```

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[1/2/2020 5:16:00 PM      Plot: "Graph5"]
Non-linear fit of dataset: Table3_2, using function: log10(a/((10^x)*s*2.506628275)*exp(-(x*ln(10)-m)^2/(2*s^2)))
Y standard errors: Unknown
Scaled Levenberg-Marquardt algorithm with tolerance = 0.0001
From x = 2.097 to x = 3.902
a = 53.839135327051 +/- 0.82189223780795
s = 0.294917539134767 +/- 0.000210957288819632
m = 6.7346464600191 +/- 0.00131007263737737
-----
Chi^2 = 0.305039664208057
R^2 = 0.999894109124399
-----
Iterations = 0
Status = success
-----

```

Figure 8: Non-linear fit of the low density profile in Figure 7 with Eq. 40 in SciDAVis, giving the constants for $a \rightarrow A$, $s \rightarrow \sigma$, and $m \rightarrow \mu_\delta$.

```

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[1/2/2020 5:19:18 PM      Plot: "Graph6"]
Non-linear fit of dataset: Table4_2, using function: log10(a/((10^x)*s*2.506628275)*exp(-(x*ln(10)-m)^2/(2*s^2)))
Y standard errors: Unknown
Scaled Levenberg-Marquardt algorithm with tolerance = 0.0001
From x = 2.096910013 to x = 3.901730692
a = 39.742506482046 +/- 1.58430260083343
s = 0.221066131940955 +/- 0.00039359322394943
m = 8.26026111215463 +/- 0.00352064330966803
-----
Chi^2 = 5.98458401110512
R^2 = 0.999293469696896
-----
Iterations = 0
Status = iteration is not making progress towards solution
-----

```

Figure 9: Non-linear fit of the high density profile in Figure 7 with Eq. 40 in SciDAVis, giving the constants for $a \rightarrow A$, $s \rightarrow \sigma$, and $m \rightarrow \mu_\delta$.

all θ azimuthal angles to simplify our calculation. In the future, we plan to incorporate azimuthal dependence for oblique impacts by including the azimuthal dependence of the ejecta blanket. The ϕ angle in MEM3 corresponds to our α angle, which is the impact angle with respect to the horizon. There are 36 ϕ bins³ and 40 speed bins, after integrating over the 72 θ bins for each ϕ bin.

6.8 Secondary Ejecta Distance, Speed, & Angle

We would like to relate the distance from the meteorite impact to the secondary ejecta impact site by the secondary ejecta speed v and angle γ from zenith. If we assume the Moon is a perfect sphere with no atmosphere, we can calculate this distance by following the elliptical path the ejecta makes. The semi-major axis and eccentricity of

³Half of the ϕ bins will always be zero, since they are below the horizon, so they can be ignored.

the elliptical orbit are given by⁴

$$\frac{a}{r_m} = \frac{1}{2 \left(1 - \frac{v^2}{v_{esc}^2} \right)}, \quad (43)$$

where $r_m = 1737.1$ km is the radius of the Moon and $v_{esc} = 2.38$ km/s is the Moon's escape velocity, and

$$e = \sqrt{\left(\frac{2v^2}{v_{esc}^2} - 1 \right)^2 \sin^2 \gamma + \cos^2 \gamma}, \quad (44)$$

where we employed the fact that the gravity of the Moon is $g = GM/r_m^2$ and the escape velocity is related by $v_{esc} = \sqrt{2gr_m}$. The third equation we need gives the location in the elliptical orbit by the angle β from the perilune, the semi-major axis a , and the eccentricity e by

$$r = \frac{a(1 - e^2)}{1 + e \cos \beta}. \quad (45)$$

Solving for $\cos \beta$ in Eq. 45, we have

$$\cos \beta = \frac{1}{e} \left(\frac{a(1 - e^2)}{r} - 1 \right). \quad (46)$$

In addition, we also need the equation for $\sin \beta$, which is given by (using a right triangle)

$$\sin \beta = \frac{1}{e} \sqrt{e^2 - \left[\frac{a(1 - e^2)}{r} - 1 \right]^2}, \quad (47)$$

so that $\tan \beta$ is

$$\tan \beta = \frac{\sqrt{e^2 - \left[\frac{a(1 - e^2)}{r} - 1 \right]^2}}{\frac{a(1 - e^2)}{r} - 1}. \quad (48)$$

We found that the distance the secondary ejecta travels is given by the arc length of Moon the orbit travels greater than the radius of the Moon:

$$D = 2(\pi - \beta)r_m, \quad (49)$$

or solving for the angle β ,

$$\beta = \pi - \frac{D}{2r_m}. \quad (50)$$

Using Eqs. 43 and 44, we can write

$$\frac{a}{r_m}(1 - e^2) = 2 \frac{v^2}{v_{esc}^2} \sin^2 \gamma, \quad (51)$$

so Eq. 48 becomes [c.f., Eq. (1) of [Vickery, 1986](#)]

$$\tan \left(\frac{D}{2r_m} \right) = \frac{2 \frac{v^2}{v_{esc}^2} \sin \gamma \cos \gamma}{1 - 2 \frac{v^2}{v_{esc}^2} \sin^2 \gamma} = \frac{\frac{v^2}{v_{esc}^2} \sin(2\gamma)}{\frac{v^2}{v_{esc}^2} [\cos(2\gamma) - 1] + 1} = \frac{2 \frac{v^2}{v_{esc}^2} \tan \gamma}{1 + (1 - 2 \frac{v^2}{v_{esc}^2}) \tan^2 \gamma}. \quad (52)$$

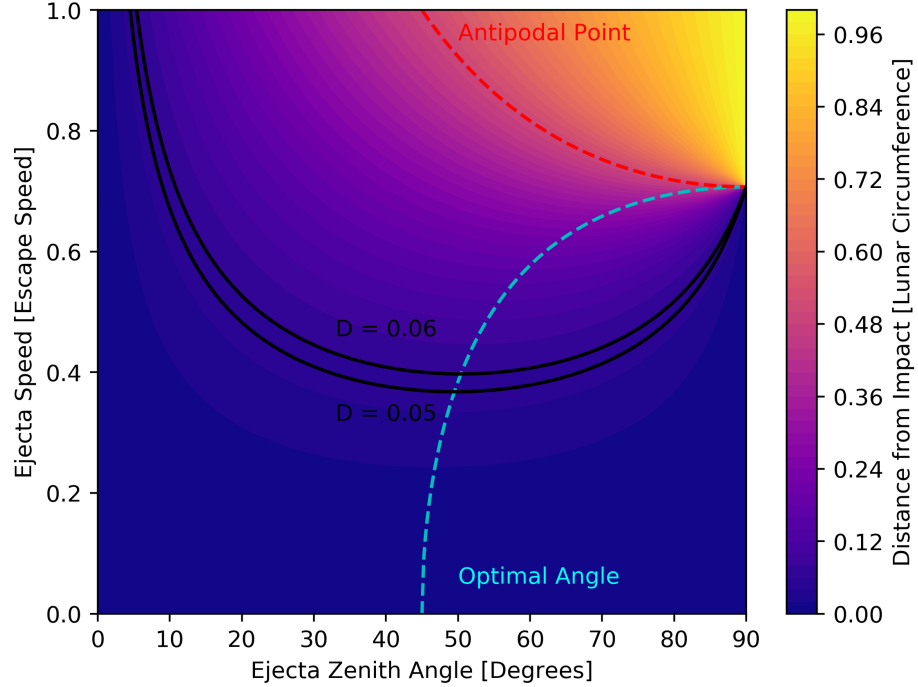


Figure 10: The color gradient shows the distance a projectile goes that has a given ejected speed and zenith angle. The cyan dashed line gives the optimal angle for a given speed to reach the furthest distance, i.e., Eq. (58). The red dashed line shows for which pairs of speeds and zenith angles are required to hit the antipodal point. As an example, all ejecta with speed and angle pairs between the two black curves will reach a location between 0.05 and 0.06 lunar circumference units away, using Eq. (55).

For $D \ll 2r_m$, the optimum angle that gives the smallest velocity needed is 45° . In other words, for a given velocity, the greatest distance is found by taking $\gamma = 45^\circ$. However, if the distance D is roughly the same order as the diameter of the Moon $2r_m$, ($D > 0.01 \times 2r_m$), then the optimal angle from zenith is greater than 45° , i.e. a shallower angle to the horizon. This is because at large velocities, the curvature of the Moon comes into play. For larger velocities, there are also angles γ that cannot reach a distance D . Allowable angles (in radians) that can travel a distance D are defined by

$$\gamma > \frac{D}{4r_m}. \quad (53)$$

⁴See Eqs. 4.30 and 4.32 from <http://www.braeunig.us/space/orbmech.htm>.

In other words, the maximum distance the ejecta can reach for a given angle is

$$D = 4\gamma r_m. \quad (54)$$

For example, from this equation we can conclude that for $\gamma < 45^\circ$, the ejecta will not reach the antipodal point, see Figure 10.

Solving for v we have

$$\frac{v}{v_{esc}} = \frac{+1}{\sqrt{\sin(2\gamma) \left(\cot\left(\frac{D}{2r_m}\right) + \tan\gamma \right)}} = \frac{+1}{\sqrt{1 + \sin(2\gamma) \cot\left(\frac{D}{2r_m}\right) - \cos(2\gamma)}}. \quad (55)$$

We can also solve for the zenith angle γ , given by

$$\cot\gamma = x^2 \cot\left(\frac{D}{2r_m}\right) \pm \sqrt{x^4 \cot^2\left(\frac{D}{2r_m}\right) + (2x^2 - 1)}, \quad (56)$$

where $x = v/v_{esc}$. Solving for the discriminant, the minimum x can be for a given distance D is

$$x_{min}^2 = \tan^2\left(\frac{D}{2r_m}\right) \left[\csc\left(\frac{D}{2r_m}\right) - 1 \right]. \quad (57)$$

Plugging into Equation 56, the optimal angle from zenith is given by

$$\cot\gamma_{opt} = \sec\left(\frac{D}{2r_m}\right) - \tan\left(\frac{D}{2r_m}\right). \quad (58)$$

In terms of x , we have

$$\cos(2\gamma_{opt}) = \frac{x^2}{x^2 - 1}. \quad (59)$$

Once $D > \pi r_m$, the optimal angle is $\gamma = 90^\circ$, i.e., parallel to the horizon. For small distances $D \ll 2r_m$, the optimal angle is $\gamma = 45^\circ$, as mentioned above.

6.8.1 Coriolis Force

The Coriolis force on secondary ejecta may also affect the ground path. To estimate the strength of the Coriolis force, the greatest speed due to the rotation of the Moon is at the equator, given by

$$v_c = \frac{2\pi r_m}{T} = \frac{2\pi * 1737.1 \text{ km}}{27.322 \text{ days}} = 4.62 \text{ m/s}. \quad (60)$$

Therefore, we can ignore the Coriolis force if the ejecta speed v is greater than roughly $\sim 10 - 15 \times v_c$, or about 46 m/s to 70 m/s. This translates into ejecta distances less than 3 km, which at those small distances the Coriolis force should not cause an effect anyways. So in general, we conclude that we can ignore the Coriolis force all together.

To quantify this conclusion, let us compute the Rossby number

$$R_o = \frac{v}{fL}. \quad (61)$$

If we assume an ejecta angle of 45° , then plotting D as a function of v in Eq. 52 shows that $D \rightarrow L \sim v^2$. Taking our example above for $v = 70$ m/s, $L = 3$ km, and $f = 2T$ to solve for A , we find that the Rossby number for secondary ejecta on the Moon is

$$R_o = \frac{A}{fv}, \quad (62)$$

where $A = 1.63 \text{ m/s}^2$,⁵ $f = 5.328 \times 10^{-6} \text{ rad/s}$, and v is in units of m/s. In order to have $R_o \sim 1$ (small R_o means the Coriolis forces cannot be ignored), we would need $v > 306 \text{ km/s}$, which far exceeds the escape speed. The smallest R_o can ever be is $R_o \sim 128$ when taking $v \rightarrow v_{esc}$. Therefore, we feel confident in our omission of the Coriolis force.

6.9 Distance and Bearing

Given two latitude-longitude points on a sphere, (ϕ_1, θ_1) and (ϕ_2, θ_2) , we can compute the distance and bearing following Chris Veness's webpage⁶.

The distance D is given by the equation

$$\tan\left(\frac{D}{2R_m}\right) = \sqrt{\frac{a}{1-a}}, \quad (63)$$

where a is given by

$$a = \sin^2(\Delta\phi/2) + \cos\phi_1 \cos\phi_2 \sin^2(\Delta\lambda/2), \quad (64)$$

for $\Delta\phi = \phi_2 - \phi_1$ and $\Delta\lambda = \lambda_2 - \lambda_1$. Eq. (63) is the shortest distance between two coordinate points. For the long-distance, use

$$\tan\left(\pi - \frac{D}{2R_m}\right) = \sqrt{\frac{a}{1-a}}. \quad (65)$$

The initial bearing θ (from due East) is given by the following equation (assuming the short-distance):

$$\tan\theta_{i(1,2)} = \frac{\sin\Delta\lambda \cos\phi_2}{\cos\phi_1 \sin\phi_2 - \sin\phi_1 \cos\phi_2 \cos\Delta\lambda}. \quad (66)$$

To find the final bearing (assuming the short-distance), swap $\phi_1 \longleftrightarrow \phi_2$ and $\lambda_1 \longleftrightarrow \lambda_2$ and reverse the angle such that

$$\theta_{f(1,2)} = (\theta_{i(2,1)} + \pi) \mod 2\pi. \quad (67)$$

In order to compute the initial and final bearing for the long-distance trajectory, add π and then mod by 2π to Eqs. (66) and (67). In other words, swap initial and final bearings $\theta_{i(1,2)} \longleftrightarrow \theta_{f(1,2)}$.

⁵Curiously, this is basically the acceleration due to gravity on the Moon.

⁶<https://www.movable-type.co.uk/scripts/latlong.html>

7 NASA SP-8013 Meteoroid Environment Model - 1969

The NASA SP-8013 Meteoroid Environment Model [[Cour-Palais, 1969](#)], is a document published in 1969 that describes the meteoroid and lunar ejecta environment of cometary origin with masses between 10^{-12} g and 1 g. The flux-mass models and the associated density and velocity characteristics are for engineering applications in the design of space vehicles for near-Earth orbit, cis-lunar, lunar orbit, and lunar surface missions.

Our aim is to provide an updated specification to NASA SP-8013 for the lunar impact ejecta environment. Until the update is finished, DSNE points to Figure 10 (shown here in Figure 11) of [Cour-Palais \[1969\]](#) for lunar ejecta. The results that follow from Sections 5 and 6 will be verified against NASA SP-8013. We already anticipate that our new environments will be more benign (e.g., [[Bjorkman and Christiansen, 2019](#), pointing out that 50 J is the critical energy]), however to what degree from our analysis is yet to be determined. We also plan to provide output in terms of penetrating flux as a function of critical kinetic energy, as comparison to Figure 5 of [Bjorkman and Christiansen \[2019\]](#), to aid in risk assessment of lunar impact ejecta.

Our ultimate goal in providing an updated environment definition is to have an output in the same format as MEM 3's igloo files (see Section 3.4.4 of [Moorhead et al. \[2019\]](#)). This will allow for current analysis tools already familiar with MEM 3 to ingest our new environments without modification to those tools.

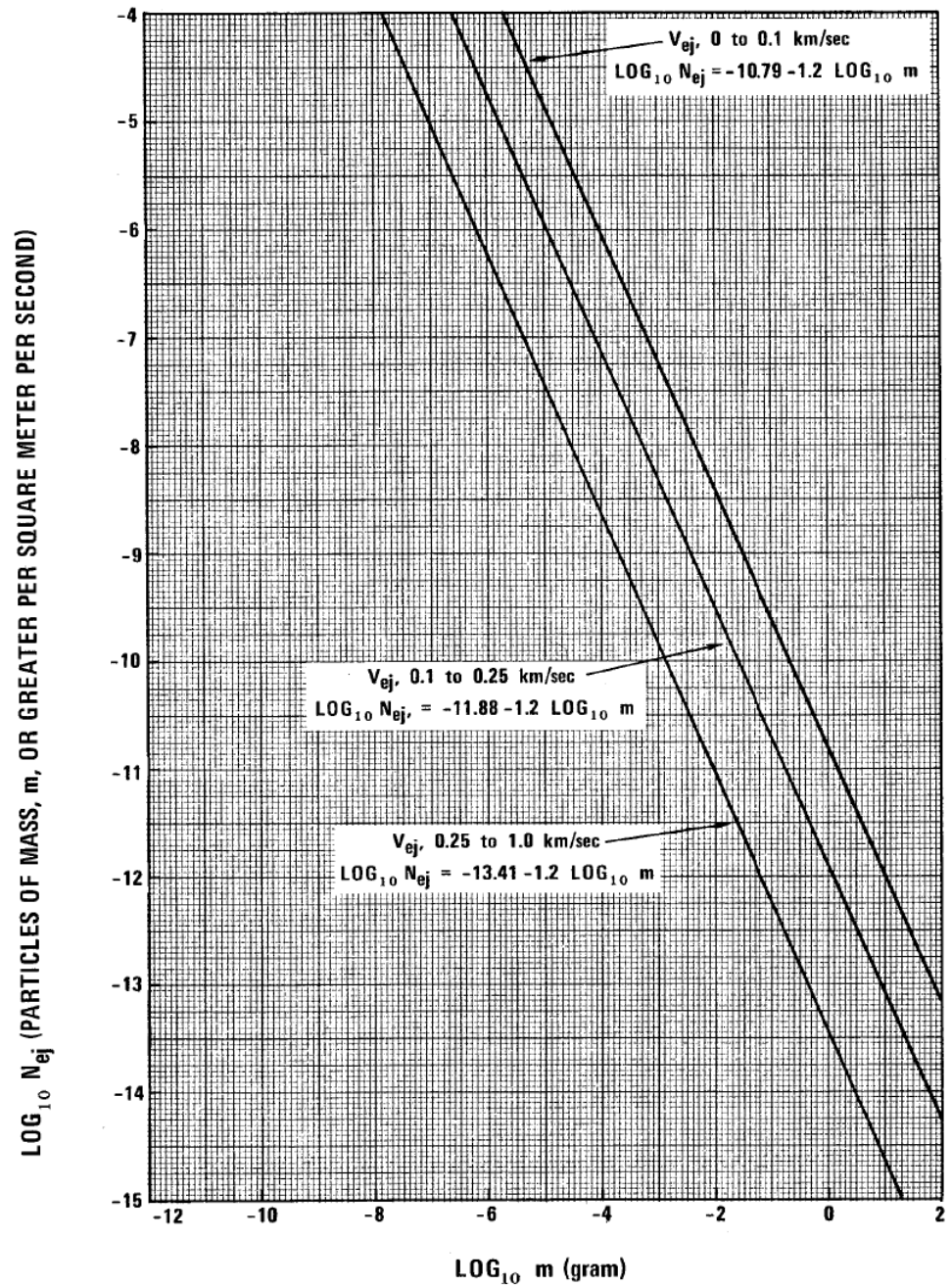


Figure 11: Average cumulative lunar ejecta flux-mass distribution for each of three ejecta velocity intervals [Cour-Palais, 1969].

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