# Exercise 1: Introduction To Simulation Methods

1. Let $X \sim \mathcal{U}(0,1)$. Show graphically that the sample mean $\bar{X} = \dfrac{1}{N}\sum_{i=1}^{N} X_i$ converges to a normal distribution $\mathcal{N}(\mu, \sigma^2)$. What is $(\mu, \sigma^2)$ as a function of $N$?
2. Let $X \sim \mathcal{E}(1)$ and $Y \sim \mathcal{G}(3,2)$. Let $(\bar{X}, \hat{\sigma}^2_X)$ and $(\bar{Y}, \hat{\sigma}^2_Y)$ be the sample mean and variance of $X$ and $Y$ respectively. Show graphically that

$$Z = \frac{\bar{X} - \bar{Y}}{\sqrt{\dfrac{\hat{\sigma}^2}{N_X}}}$$

converges to the standard normal distribution as $N_X \to \infty$ and $N_Y \to \infty$.

3. Let $X \sim \mathcal{C}$, where $\mathcal{C}$ denotes the standard Cauchy distribution. Show graphically that the sample mean $\bar{X} = \dfrac{1}{N}\sum_{i=1}^{N} X_i$ does not converge to a normal distribution as $N \to \infty$.