Problem 1: $[10 + 10 = 20 \text{ points}]$

Consider the three axis manipulator shown in Figure 1. The first joint is rotational (Frame F_0, θ_1), the second primatic (Frame F_1, d_2), and the third is rotational (Frame F_2, θ_3). Note that x_0 is given, and the θ's are not shown in the diagram for clarity. Joint 2 slides and also carries joint 3 along with it. Frame F_3 represents the tool frame.

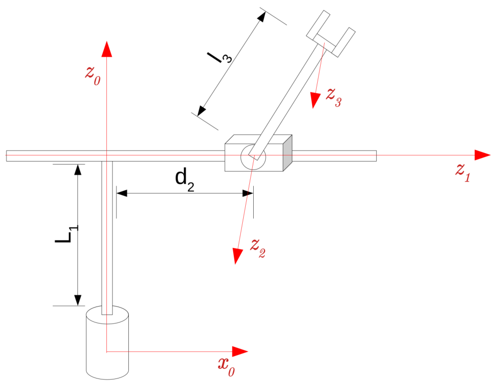

Figure 1: Figure for problem 1.

(a) **Draw the x-axes, the tool frame, and fill in the DH table. For axes perpendicular to the page, choose the positive direction as going into the page.**

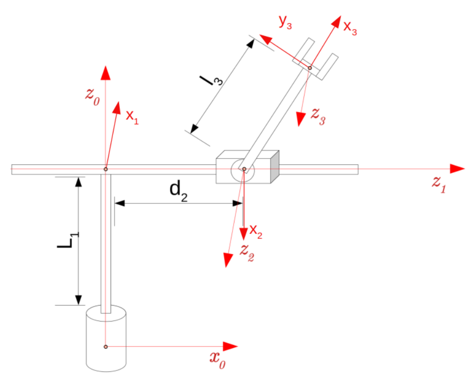

Figure 2: Figure 1 with x-axes and tool frame added.

Joint	θ_i	d_i	a_i	α_i
1	θ_1^*	L_1	0	$\frac{\pi}{2}$
2	$\frac{3\pi}{2}$	d_2^*	0	$\frac{\pi}{2}$
3	θ_3^*	0	l_3	0

Table 1: DH table for manipulator shown in Figure 1.

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(b) **Compute the tool transform T_{03}. Please note z_3 has intentionally been placed to come out of the paper.**

Let the homogenous transformation matrix $A_i = T_{(i-1)i}$ be a function of the joint variable q_i. It follows that $T_{ij} = A_{i+1}A_{i+2}...A_{j-1}A_j$. Using the DH convention, each of these homogenous transformations can be represented as follows:

$$A_i = Rot_{z,\theta_i}\ Trans_{z,d_i}\ Trans_{x,a_i}\ Rot_{x,\alpha_i} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where θ_i, d_i, a_i, and α_i are the i^{th} joint angle, link offset, link length, and link twist respectively. Note that all of these values can be found in Table 1 from part (a). Observe,

$$\begin{aligned} T_{03} &= A_1A_2A_3 \\ &= \begin{bmatrix} c\theta_1 & -s\theta_1 c\alpha_1 & s\theta_1 s\alpha_1 & a_1 c\theta_1 \\ s\theta_1 & c\theta_1 c\alpha_1 & -c\theta_1 s\alpha_1 & a_1 s\theta_1 \\ 0 & s\alpha_1 & c\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 c\alpha_2 & s\theta_2 s\alpha_2 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 c\alpha_2 & -c\theta_2 s\alpha_2 & a_2 s\theta_2 \\ 0 & s\alpha_2 & c\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 c\alpha_3 & s\theta_3 s\alpha_3 & a_3 c\theta_3 \\ s\theta_3 & c\theta_3 c\alpha_3 & -c\theta_3 s\alpha_3 & a_3 s\theta_3 \\ 0 & s\alpha_3 & c\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Substituting-in the values found in Table 1:

$$\begin{aligned} &= \begin{bmatrix} c\theta_1^* & -s\theta_1^* c\frac{\pi}{2} & s\theta_1^* s\frac{\pi}{2} & 0 \\ s\theta_1^* & c\theta_1^* c\frac{\pi}{2} & -c\theta_1^* s\frac{\pi}{2} & 0 \\ 0 & s\frac{\pi}{2} & c\frac{\pi}{2} & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\frac{3\pi}{2} & -s\frac{3\pi}{2} c\frac{\pi}{2} & s\frac{3\pi}{2} s\frac{\pi}{2} & 0 \\ s\frac{3\pi}{2} & c\frac{3\pi}{2} c\frac{\pi}{2} & -c\frac{3\pi}{2} s\frac{\pi}{2} & 0 \\ 0 & s\frac{\pi}{2} & c\frac{\pi}{2} & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3^* & -s\theta_3^* c(0) & s\theta_3^* s(0) & L_3 c\theta_3^* \\ s\theta_3^* & c\theta_3^* c(0) & -c\theta_3^* s(0) & L_3 s\theta_3^* \\ 0 & s(0) & c(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c\theta_1^* & 0 & s\theta_1^* & 0 \\ s\theta_1^* & 0 & -c\theta_1^* & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3^* & -s\theta_3^* & 0 & L_3 c\theta_3^* \\ s\theta_3^* & c\theta_3^* & 0 & L_3 s\theta_3^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c\theta_1^* & 0 & s\theta_1^* & 0 \\ s\theta_1^* & 0 & -c\theta_1^* & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ -c\theta_3^* & s\theta_3^* & 0 & -L_3 c\theta_3^* \\ s\theta_3^* & c\theta_3^* & 0 & L_3 s\theta_3^* + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} s\theta_1^* s\theta_3^* & s\theta_1^* c\theta_3^* & -c\theta_1^* & L_3 s\theta_1^* s\theta_3^* + d_2^* s\theta_1^* \\ -c\theta_1^* s\theta_3^* & -c\theta_1^* c\theta_3^* & -s\theta_1^* & -L_3 c\theta_1^* s\theta_3^* - d_2^* c\theta_1^* \\ -c\theta_3^* & s\theta_3^* & 0 & -L_3 c\theta_3^* + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Therefore,

$$T_{03} = \begin{bmatrix} s\theta_1^* s\theta_3^* & s\theta_1^* c\theta_3^* & -c\theta_1^* & L_3 s\theta_1^* s\theta_3^* + d_2^* s\theta_1^* \\ -c\theta_1^* s\theta_3^* & -c\theta_1^* c\theta_3^* & -s\theta_1^* & -L_3 c\theta_1^* s\theta_3^* - d_2^* c\theta_1^* \\ -c\theta_3^* & s\theta_3^* & 0 & -L_3 c\theta_3^* + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 2: [20 points]

Consider the DH table below. Each joint variable is marked with a "*" and the current position is given. Please draw the robot starting with the given base frame, shown in Figure 3. Show and label all frames, all joints and joint displacements, and the end-effector frame.

Joint	θ_i	d_i	a_i	α_i
1	$\theta_1^* = \frac{\pi}{2}$	3	0	$-\frac{\pi}{2}$
2	$\theta_2^* = \frac{\pi}{2}$	0	0	$\frac{\pi}{2}$
3	0	$d_3^* = 2$	0	0
4	$\theta_4^* = \frac{\pi}{2}$	0	0	$-\frac{\pi}{2}$
5	$\theta_5^* = 0$	2	0	0

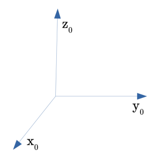

Figure 3: The starting axes orientations.

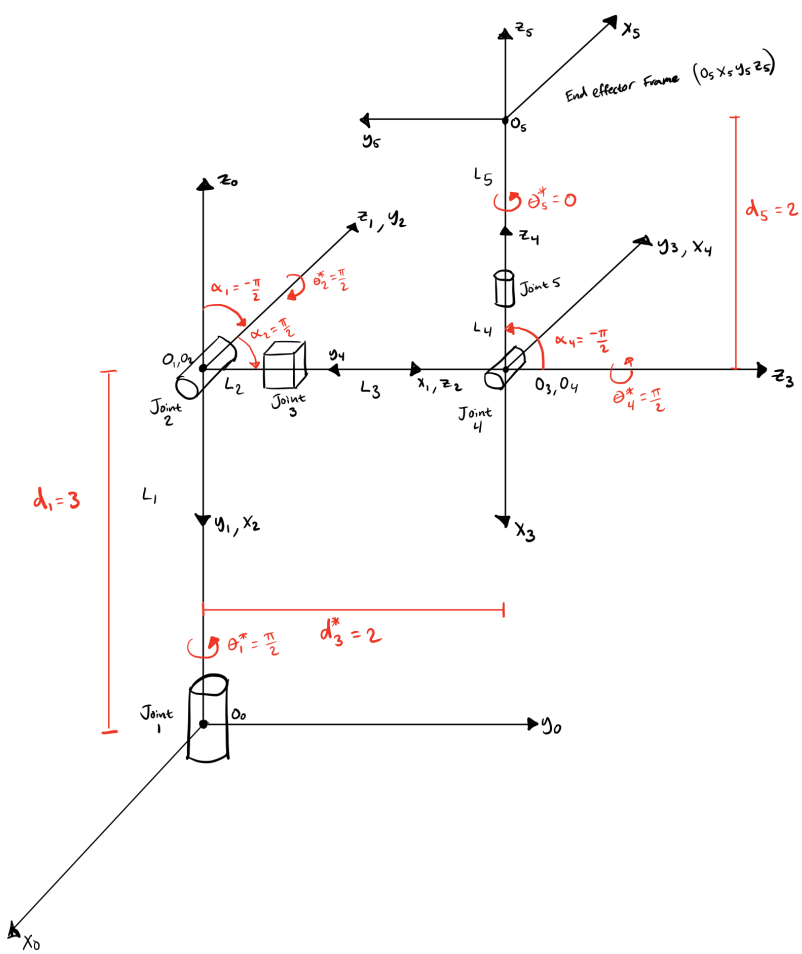

Figure 4: Robot corresponding to the provided DH table.

Problem 3: [20 points]

Find the DH table parameters for the CanadaArm manipulator shown in Figure 5. Also, find T_{04}. Note that the Z-directions are given in red arrows, F_0 is given, as is F_t. The circular joints are revolute, and the diamond-shape joints are prismatic.

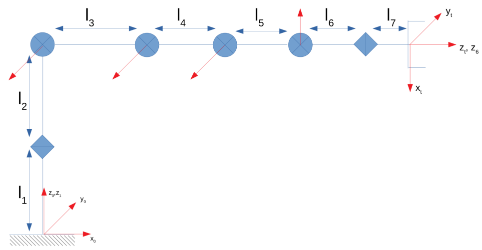

Figure 5: The CanadaArm.

Note that joint 1 in the table below is a fixed joint (i.e. neither revolute nor prismatic). It is included for the purpose of maintaining the convention that joint i connects link $i-1$ to link i. Furthermore, the value l_i represents the length of link i. When joint i is prismatic, d_i^* is used to represent the joint variable instead of l_i.

Joint	θ_i	d_i	a_i	α_i
1	0	l_1	0	0
2	0	$d_2^* = l_2$	0	$\frac{\pi}{2}$
3	$\theta_3^* = 0$	0	l_3	0
4	$\theta_4^* = 0$	0	l_4	0
5	$\theta_5^* = 0$	0	l_5	$\frac{-\pi}{2}$
6	$\theta_6^* = \frac{\pi}{2}$	0	0	$\frac{\pi}{2}$
7	$\frac{-\pi}{2}$	$l_6 + d_7^* = l_6 + l_7$	0	0

Table 2: DH table for manipulator configuration shown in Figure 5 with current position.

With the DH table in hand, we will now find T_{04}. Note that the origin of F_4 is located at the origin of joint 5.

Let the homogenous transformation matrix $A_i = T_{(i-1)i}$ be a function of the joint variable q_i. It follows that $T_{ij} = A_{i+1}A_{i+2}...A_{j-1}A_j$. Using the DH convention, each of these homogenous transformations can be represented as follows:

$$A_i = Rot_{z,\theta_i}\ Trans_{z,d_i}\ Trans_{x,a_i}\ Rot_{x,\alpha_i} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where θ_i, d_i, a_i, and α_i are the i^{th} joint angle, link offset, link length, and link twist respectively. Using the values found in table 2, then,

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$$
\begin{aligned}
T_{04} &= A_1A_2A_3A_4 \\
&= \begin{bmatrix} c\theta_1 & -s\theta_1 c\alpha_1 & s\theta_1 s\alpha_1 & a_1 c\theta_1 \\ s\theta_1 & c\theta_1 c\alpha_1 & -c\theta_1 s\alpha_1 & a_1 s\theta_1 \\ 0 & s\alpha_1 & c\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 c\alpha_2 & s\theta_2 s\alpha_2 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 c\alpha_2 & -c\theta_2 s\alpha_2 & a_2 s\theta_2 \\ 0 & s\alpha_2 & c\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad \begin{bmatrix} c\theta_3 & -s\theta_3 c\alpha_3 & s\theta_3 s\alpha_3 & a_3 c\theta_3 \\ s\theta_3 & c\theta_3 c\alpha_3 & -c\theta_3 s\alpha_3 & a_3 s\theta_3 \\ 0 & s\alpha_3 & c\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_4 & -s\theta_4 c\alpha_4 & s\theta_4 s\alpha_4 & a_4 c\theta_4 \\ s\theta_4 & c\theta_4 c\alpha_4 & -c\theta_4 s\alpha_4 & a_4 s\theta_4 \\ 0 & s\alpha_4 & c\alpha_4 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&= \begin{bmatrix} c(0) & -s(0)c(0) & s(0)s(0) & (0)c(0) \\ s(0) & c(0)c(0) & -c(0)s(0) & (0)s(0) \\ 0 & s(0) & c(0) & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c(0) & -s(0)c\frac{\pi}{2} & s(0)s\frac{\pi}{2} & (0)c(0) \\ s(0) & c(0)c\frac{\pi}{2} & -c(0)s\frac{\pi}{2} & (0)s(0) \\ 0 & s\frac{\pi}{2} & c\frac{\pi}{2} & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad \begin{bmatrix} c\theta_3^* & -s\theta_3^* c(0) & s\theta_3^* s(0) & l_3 c\theta_3^* \\ s\theta_3^* & c\theta_3^* c(0) & -c\theta_3^* s(0) & l_3 s\theta_3^* \\ 0 & s(0) & c(0) & (0) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_4^* & -s\theta_4^* c(0) & s\theta_4^* s(0) & l_4 c\theta_4^* \\ s\theta_4^* & c\theta_4^* c(0) & -c\theta_4^* s(0) & l_4 s\theta_4^* \\ 0 & s(0) & c(0) & (0) \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$
= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3^* & -s\theta_3^* & 0 & l_3 c\theta_3^* \\ s\theta_3^* & c\theta_3^* & 0 & l_3 s\theta_3^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_4^* & -s\theta_4^* & 0 & l_4 c\theta_4^* \\ s\theta_4^* & c\theta_4^* & 0 & l_4 s\theta_4^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3^* & -s\theta_3^* & 0 & l_3 c\theta_3^* \\ s\theta_3^* & c\theta_3^* & 0 & l_3 s\theta_3^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_4^* & -s\theta_4^* & 0 & l_4 c\theta_4^* \\ s\theta_4^* & c\theta_4^* & 0 & l_4 s\theta_4^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c\theta_3^* & -s\theta_3^* & 0 & l_3 c\theta_3^* \\ 0 & 0 & -1 & 0 \\ s\theta_3^* & c\theta_3^* & 0 & l_1 + l_3 s\theta_3^* + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_4^* & -s\theta_4^* & 0 & l_4 c\theta_4^* \\ s\theta_4^* & c\theta_4^* & 0 & l_4 s\theta_4^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c\theta_3^* c\theta_4^* - s\theta_3^* s\theta_4^* & -c\theta_3^* s\theta_4^* - s\theta_3^* c\theta_4^* & 0 & l_4 c\theta_3^* c\theta_4^* + l_3 c\theta_3^* - l_4 s\theta_3^* s\theta_4^* \\ 0 & 0 & -1 & 0 \\ s\theta_3^* c\theta_4^* + c\theta_3^* s\theta_4^* & c\theta_3^* c\theta_4^* - s\theta_3^* s\theta_4^* & 0 & l_4 s\theta_3^* c\theta_4^* + l_4 c\theta_3^* s\theta_4^* + l_3 s\theta_3^* + l_1 + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Problem 4: [20 points]

Create the table of DH parameters for the MOM manipulator shown in Figure 6. The first joint is revolute and is driven by the shoulder motor (1). The next two joints are prismatic and slide along the shoulder (8) and the forearm (5) respectively. Assume a standard spherical wrist. Find T_{03} and T_{06}.

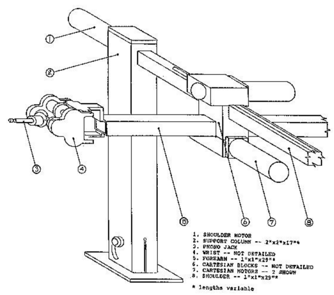

Figure 6: The MOM manipulator.

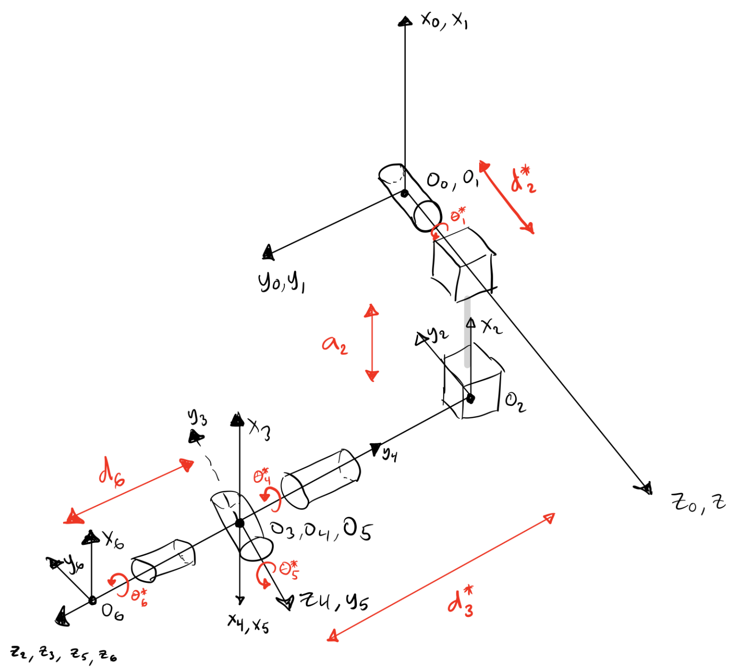

Figure 7: Reference Frames assigned to MOM manipulator in figure 6.

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Joint	θ_i	d_i	a_i	α_i
1	θ_1^*	0	0	0
2	0	d_2^*	$-a_2$	$\frac{-\pi}{2}$
3	0	d_3^*	0	0
4	θ_4^*	0	0	$\frac{-\pi}{2}$
5	θ_5^*	0	0	$\frac{\pi}{2}$
6	θ_6^*	d_6	0	0

Table 3: DH table for manipulator shown in figure 6 and 7.

With the DH table in hand, we will now find T_{03} and T_{06}.

First, begin by finding T_{03}:

Let the homogenous transformation matrix $A_i = T_{(i-1)i}$ be a function of the joint variable q_i. It follows that $T_{ij} = A_{i+1}A_{i+2}...A_{j-1}A_j$. Using the DH convention, each of these homogenous transformations can be represented as follows:

$$A_i = Rot_{z,\theta_i}\ Trans_{z,d_i}\ Trans_{x,a_i}\ Rot_{x,\alpha_i} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where θ_i, d_i, a_i, and α_i are the i^{th} joint angle, link offset, link length, and link twist respectively. Using the values found in table 3, then,

$$\begin{aligned} T_{03} &= A_1A_2A_3 \\ &= \begin{bmatrix} c\theta_1 & -s\theta_1 c\alpha_1 & s\theta_1 s\alpha_1 & a_1 c\theta_1 \\ s\theta_1 & c\theta_1 c\alpha_1 & -c\theta_1 s\alpha_1 & a_1 s\theta_1 \\ 0 & s\alpha_1 & c\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 c\alpha_2 & s\theta_2 s\alpha_2 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 c\alpha_2 & -c\theta_2 s\alpha_2 & a_2 s\theta_2 \\ 0 & s\alpha_2 & c\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 c\alpha_3 & s\theta_3 s\alpha_3 & a_3 c\theta_3 \\ s\theta_3 & c\theta_3 c\alpha_3 & -c\theta_3 s\alpha_3 & a_3 s\theta_3 \\ 0 & s\alpha_3 & c\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c\theta_1^* & -s\theta_1^* c(0) & s\theta_1^* s(0) & (0)c\theta_1^* \\ s\theta_1^* & c\theta_1^* c(0) & -c\theta_1^* s(0) & (0)s\theta_1^* \\ 0 & s(0) & c(0) & (0) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c(0) & -s(0)c\frac{-\pi}{2} & s(0)s\frac{-\pi}{2} & -a_2 c(0) \\ s(0) & c(0)c\frac{-\pi}{2} & -c(0)s\frac{-\pi}{2} & -a_2 s(0) \\ 0 & s\frac{-\pi}{2} & c\frac{-\pi}{2} & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c(0) & -s(0)c(0) & s(0)s(0) & (0)c(0) \\ s(0) & c(0)c(0) & -c(0)s(0) & (0)s(0) \\ 0 & s(0) & c(0) & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c\theta_1^* & -s\theta_1^* & 0 & 0 \\ s\theta_1^* & c\theta_1^* & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -a_2 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c\theta_1^* & -s\theta_1^* & 0 & 0 \\ s\theta_1^* & c\theta_1^* & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -a_2 \\ 0 & 0 & 1 & d_3^* \\ 0 & -1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c\theta_1^* & 0 & -s\theta_1^* & -a_2 c\theta_1^* - d_3^* s\theta_1^* \\ s\theta_1^* & 0 & c\theta_1^* & d_3^* c\theta_1^* - a_2 s\theta_1^* \\ 0 & -1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Next, realize that $T_{06} = T_{03}T_{36}$. Additionaly, since joints 4, 5, and 6 make up a standard spherical wrist, T_{36} is nothing more than an Euler angle transformation on the set of Euler angles $\theta_4^*, \theta_5^*, \theta_6^*$, a fact that will now be derived:

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$$
\begin{aligned}
T_{36} &= A_4 A_5 A_6 \\
&= \begin{bmatrix} c\theta_4 & -s\theta_4 c\alpha_4 & s\theta_4 s\alpha_4 & a_4 c\theta_4 \\ s\theta_4 & c\theta_4 c\alpha_4 & -c\theta_4 s\alpha_4 & a_4 s\theta_4 \\ 0 & s\alpha_4 & c\alpha_4 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_5 & -s\theta_5 c\alpha_5 & s\theta_5 s\alpha_5 & a_5 c\theta_5 \\ s\theta_5 & c\theta_5 c\alpha_5 & -c\theta_5 s\alpha_5 & a_5 s\theta_5 \\ 0 & s\alpha_5 & c\alpha_5 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_6 & -s\theta_6 c\alpha_6 & s\theta_6 s\alpha_6 & a_6 c\theta_6 \\ s\theta_6 & c\theta_6 c\alpha_6 & -c\theta_6 s\alpha_6 & a_6 s\theta_6 \\ 0 & s\alpha_6 & c\alpha_6 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c\theta_4^* & -s\theta_4^* c\frac{-\pi}{2} & s\theta_4^* s\frac{-\pi}{2} & (0)c\theta_4^* \\ s\theta_4^* & c\theta_4^* c\frac{-\pi}{2} & -c\theta_4^* s\frac{-\pi}{2} & (0)s\theta_4^* \\ 0 & s\frac{-\pi}{2} & c\frac{-\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_5^* & -s\theta_5^* c\frac{\pi}{2} & s\theta_5^* s\frac{\pi}{2} & (0)c\theta_5^* \\ s\theta_5^* & c\theta_5^* c\frac{\pi}{2} & -c\theta_5^* s\frac{\pi}{2} & (0)s\theta_5^* \\ 0 & s\frac{\pi}{2} & c\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_6^* & -s\theta_6^* c(0) & s\theta_6^* s(0) & (0)c\theta_6^* \\ s\theta_6^* & c\theta_6^* c(0) & -c\theta_6^* s(0) & (0)s\theta_6^* \\ 0 & s(0) & c(0) & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c\theta_4^* & 0 & -s\theta_4^* & 0 \\ s\theta_4^* & 0 & c\theta_4^* & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_5^* & 0 & s\theta_5^* & 0 \\ s\theta_5^* & 0 & -c\theta_5^* & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_6^* & -s\theta_6^* & 0 & 0 \\ s\theta_6^* & c\theta_6^* & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c\theta_4^* & 0 & s\theta_4^* & 0 \\ s\theta_4^* & 0 & -c\theta_4^* & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_5^* c\theta_6^* & -c\theta_5^* s\theta_6^* & -s\theta_5^* & -d_6 s\theta_5^* \\ s\theta_5^* c\theta_6^* & -s\theta_5^* s\theta_6^* & c\theta_5^* & d_6 c\theta_5^* \\ s\theta_6^* & c\theta_6^* & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c\theta_4^* c\theta_5^* c\theta_6^* - s\theta_4^* s\theta_6^* & -c\theta_4^* c\theta_5^* s\theta_6^* - s\theta_4^* c\theta_6^* & c\theta_4^* s\theta_5^* & c\theta_4^* s\theta_5^* d_6 \\ s\theta_4^* c\theta_5^* c\theta_6^* + c\theta_4^* s\theta_6^* & -s\theta_4^* c\theta_5^* s\theta_6^* + c\theta_4^* c\theta_6^* & s\theta_4^* s\theta_5^* & s\theta_4^* s\theta_5^* d_6 \\ -s\theta_5^* c\theta_6^* & s\theta_5^* s\theta_6^* & c\theta_5^* & c\theta_5^* d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

For the sake of clarity, let $c\theta_i^* = c_i$ and let $s\theta_i^* = s_i$. It follows that:

$$
\begin{aligned}
T_{06} &= T_{03} T_{36} \\
&= \begin{bmatrix} c\theta_1^* & 0 & -s\theta_1^* & -a_2 c\theta_1^* - d_3^* s\theta_1^* \\ s\theta_1^* & 0 & c\theta_1^* & d_3^* c\theta_1^* - a_2 s\theta_1^* \\ 0 & -1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_4^* c\theta_5^* c\theta_6^* - s\theta_4^* s\theta_6^* & -c\theta_4^* c\theta_5^* s\theta_6^* - s\theta_4^* c\theta_6^* & c\theta_4^* s\theta_5^* & c\theta_4^* s\theta_5^* d_6 \\ s\theta_4^* c\theta_5^* c\theta_6^* + c\theta_4^* s\theta_6^* & -s\theta_4^* c\theta_5^* s\theta_6^* + c\theta_4^* c\theta_6^* & s\theta_4^* s\theta_5^* & s\theta_4^* s\theta_5^* d_6 \\ -s\theta_5^* c\theta_6^* & s\theta_5^* s\theta_6^* & c\theta_5^* & c\theta_5^* d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_1 c_4 c_5 c_6 - c_1 s_4 s_6 + c_6 s_1 s_5 & -c_1 c_6 s_4 - c_1 c_4 c_5 c_6 - s_1 s_6 s_5 & c_1 c_4 s_5 - c_5 s_1 & -c_5 d_6 s_1 - d_3^* s_1 + c_1 c_4 d_6 s_5 - c_1 a_2 \\ s_1 c_4 c_5 c_6 - s_1 s_4 s_6 - c_1 c_6 s_5 & c_1 s_6 s_5 - s_1 c_6 s_4 - s_1 c_4 c_5 c_6 & c_1 c_5 + c_4 s_1 s_5 & c_4 d_6 s_1 s_5 - a_2 s_1 + c_1 c_5 d_6 + c_1 d_3^* \\ -c_4 s_6 - c_5 c_6 s_4 & -c_4 c_6 + c_5 s_4 s_6 & -s_4 s_5 & d_2^* - d_6 s_4 s_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Problem 5: [20 points]

Create the table of DH parameters for the Stanford manipulator shown in Figure 7.

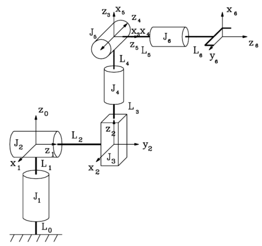

Figure 8: The Stanford Manipulator.

Note that o_0 is assumed to coincide with o_1. Additionally, x_0 is assumed to be aligned with x_1. These assumptions mean that $d_1 = 0$, and that in the current position $\theta_1^* = 0$.

Joint	θ_i	d_i	a_i	α_i
1	$\theta_1^* = 0$	0	0	$\frac{-\pi}{2}$
2	$\theta_2^* = 0$	L_2	0	$\frac{\pi}{2}$
3	0	$d_3^* = L_3 + L_4$	0	0
4	$\theta_4^* = 0$	0	0	$\frac{-\pi}{2}$
5	$\theta_5^* = \frac{-\pi}{2}$	0	0	$\frac{-\pi}{2}$
6	$\theta_6^* = 0$	$L_5 + L_6$	0	0

Table 4: DH table for stanford manipulator shown in figure 8 with current position.