6 Supersymmetry in a Chain of Majorana Fermions

Technique: Density Matrix Renormalization Group (DMRG)

Model:

$$H = 2\lambda_I H_I + \lambda_3 H_3 \tag{10}$$

with

$$H_I = i \sum_a \gamma_a \gamma_{a+1}, \text{ and } H_3 = -\sum_a \gamma_{a-2}\gamma_{a-1}\gamma_{a+1}\gamma_{a+2}.$$

Here, γ_a is a Majorana fermion operator satisfying $\gamma_a = \gamma_a^\dagger$ and the Clifford algebra $\{\gamma_a, \gamma_b\} = 2\delta_{a,b}$.

Introduction: The tricritical Ising model in two dimensions can be described in the scaling limit by a conformal field theory (CFT) with a central charge $c = 7/10$. One of the remarkable properties of this CFT is the presence of supersymmetry . In this project, we propose to investigate numerically a chain of Majorana fermions where the supersymmetry not only emerges at the tricritical Ising point described by the corresponding CFT, but also manifests itself on the lattice [1]. The supersymmetry is preserved on the line of coexistence between ordered and disordered phases but breaks along the flow to the Ising point. The aim of the project is to provide numerical evidence for the presence of the tricritical point along the lines of Ref.[1]. The best suited method for such a task is the DMRG algorithm. In particular, the Jordan-Wigner transformation maps the model onto a quantum spin model with local interactions such that the MPO of the Hamiltonian can easily be constructed.

Objectives:

1. Implement the DMRG algorithm and benchmark it with the Ising model in a transverse field
$$H = -J \sum_j \sigma_j^z \sigma_{j+1}^z - h \sum_j \sigma_j^x \tag{11}$$
by computing the central charge at the critical point $h = J$.
2. Reformulate the Hamiltonian (10) in terms of Pauli matrices using the Jordan-Wigner transformation and write the corresponding MPO.
3. Find the ground-state at the frustration-free point $\lambda_I = \lambda_3$.
4. Compute the central charge at $\lambda_3/\lambda_I \approx 0.856$ and the critical exponent ν as λ_3/λ_I is increased towards this point.
5. Add the chiral term $H_c = -i \sum_a \gamma_a \gamma_{a+2}$ to the Hamiltonian (10) and attempt to recover the phase diagram of Ref.[1].

Group members: Ivo Maceira, Samuel Nyckees, Zakaria Jouini, Mithilesh Nayak

References

[1] E. O'Brien and P. Fendley, Phys. Rev. Lett. **120**, 206403 (2018).