Supersymmetry in a Chain of Majorana Fermions

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Introduction

Motivation

- Tricritical Ising (TCI) model in 2D described by CFT

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Method

Density Matrix Renormalization Group (DMRG)

Model

Call OF (O'Brien and Fendley) model

$$\mathcal{H} = 2\lambda_I \mathcal{H}_I + \lambda_3 \mathcal{H}_3 + \lambda_c \mathcal{H}_c$$

with

$$\mathcal{H}_I = i \sum_a \gamma_a \gamma_{a+1} \xrightarrow{\text{JW}} -\sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^z$$

$$\mathcal{H}_3 = -\sum_a \gamma_{a-2}\gamma_{a-1}\gamma_{a+1}\gamma_{a+2} \xrightarrow{\text{JW}} \sum_i \sigma_i^z \sigma_{i+1}^x \sigma_{i+2}^x + \sigma_i^x \sigma_{i+1}^x \sigma_{i+2}^z$$

$$\mathcal{H}_c = -i \sum_a \gamma_a \gamma_{a+2} \xrightarrow{\text{JW}} \sum_i \sigma_i^x \sigma_{i+1}^y - \sigma_i^y \sigma_{i+1}^x$$

where

- JW is Jordan-Wigner transformation
- γ_a is a Majorana fermion operator satisfying $\gamma_a = \gamma_a^\dagger$ and $\{\gamma_a, \gamma_b\} = 2\delta_{ab}$
- from now on, $\lambda_c = 0$

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Ground state search

Find MPS $|\psi\rangle$ minimizing

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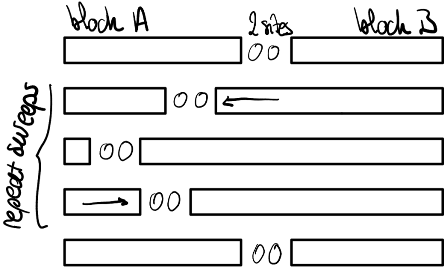
Algorithm

- 2-site update by applying $\mathcal{H}$ as MPO, diagonalize with Lanczos (or improved) and then truncate to χ (bond dimension) by SVD
- Sweep through until convergence criteria



Results – Central charge

Transverse-Field Ising

- TFI model

$$\mathcal{H} = -J\sum_i \sigma_i^x \sigma_{i+1}^x - h\sum_i \sigma_i^z$$

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Central charge

For open boundary conditions, entanglement entropy given by Cardy-Calabrese formula

$$S(l) = \frac{c}{6} \ln \left[\frac{2L}{\pi} \sin \frac{\pi l}{L} \right] + \text{const}$$

on bond l (in MPS language) for system of length L

Results – Central charge

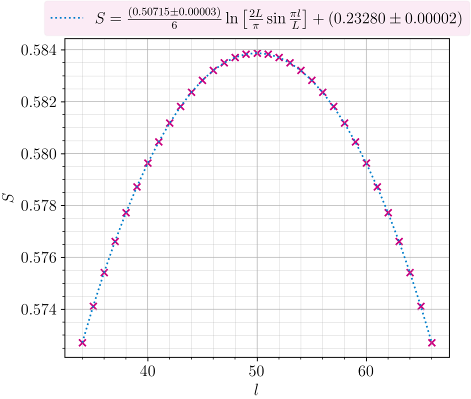

Figure: TFI with $J = h$, $L = 100$, $\chi = 100$

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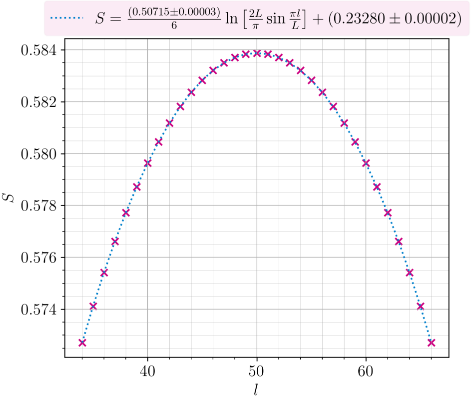

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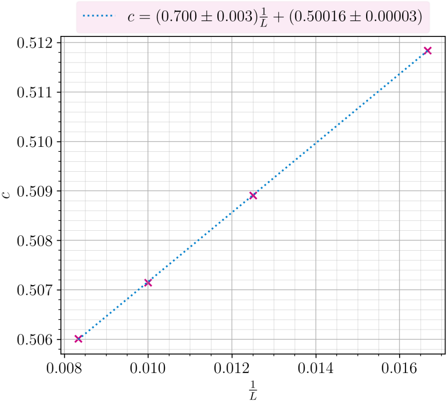

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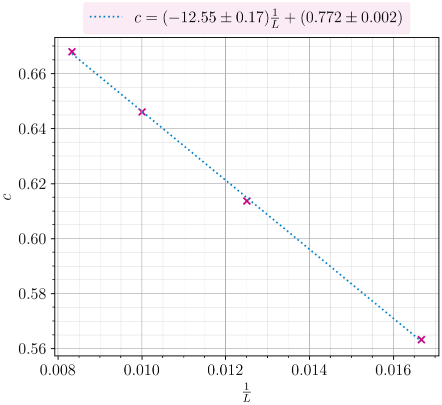

Figure: OF with $\lambda_3/\lambda_I \simeq 0.856$, $\chi = 100$

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- Good way to extrapolate ?
- Seems not as we approach $\lambda_3/\lambda_I \simeq 0.856$ $\rightarrow$ need to go to larger L (difficult)

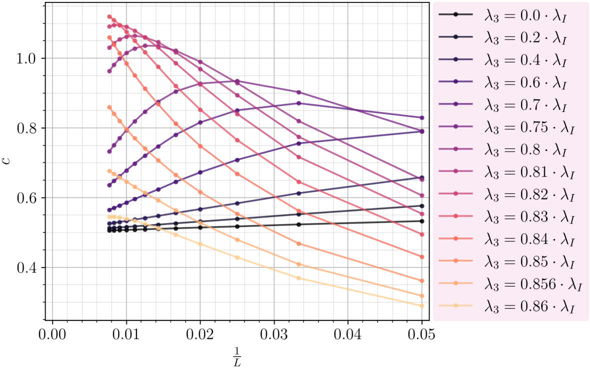

Figure: OF with $\chi = 100$

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Excited spectrum

- Get excited energies directly through diagonalization of the effective Hamiltionian in 2-site update, follow N. Chepiga and F. Mila Phys. Rev. B **96**, 054425 (2017)
- Works very well for critical TFI (expect slope 2)

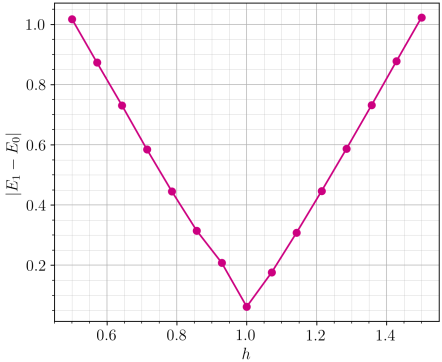

Figure: TFI with $J = 1$, $L = 50$, $\chi = 50$

Conclusion

Periodic boundary conditions

- However ratios need PBCs energies

$$R_1 = \frac{A_0^- - P_0^+}{P_1^+ - P_0^+},\ R_2 = \frac{P_0^- - P_0^+}{P_1^+ - P_0^+},\ R_3 = \frac{P_1^- - P_0^+}{P_1^+ - P_0^+}$$

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- Proving 3-fold degeneracy of ground state at $\lambda_I = \lambda_3$ need PBCs too
- DMRG with MPS as a loop is not suited (generalized eigenvalue problem + large χ)
- Can fold MPS and reformulate MPO (finalization...)