1D Chain of Majorana Fermions

Anthony REY

École Polytechnique Fédérale de Lausanne

June 7, 2022

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Method
Using DRMG, with open and periodic boundary conditions

Model

Use O'Brien and Fendley (OF) model

$$\mathcal{H} = 2\lambda_I \mathcal{H}_I + \lambda_3 \mathcal{H}_3 + \lambda_c \mathcal{H}_c$$

with

$$\mathcal{H}_I = i \sum_a \gamma_a \gamma_{a+1} \xrightarrow{\text{JW}} - \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^z$$

$$\mathcal{H}_3 = - \sum_a \gamma_{a-2} \gamma_{a-1} \gamma_{a+1} \gamma_{a+2} \xrightarrow{\text{JW}} \sum_i \sigma_i^z \sigma_{i+1}^x \sigma_{i+2}^x + \sigma_i^x \sigma_{i+1}^x \sigma_{i+2}^z$$

$$\mathcal{H}_c = -i \sum_a \gamma_a \gamma_{a+2} \xrightarrow{\text{JW}} \sum_i \sigma_i^x \sigma_{i+1}^y - \sigma_i^y \sigma_{i+1}^x$$

where

- JW is Jordan-Wigner transformation
- γ_a is a Majorana fermion operator satisfying $\gamma_a = \gamma_a^\dagger$ and Clifford algebra $\{\gamma_a, \gamma_b\} = 2\delta_{ab}$
- from now on, $\lambda_c = 0$

Method

Transverse-Field Ising

- TFI model

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Central charge

For OBCs, entanglement entropy given by Cardy-Calabrese formula

$$S(\ell) = \frac{c}{6} \ln \left[\frac{2L}{\pi} \sin \frac{\pi \ell}{L} \right] + \text{const}$$

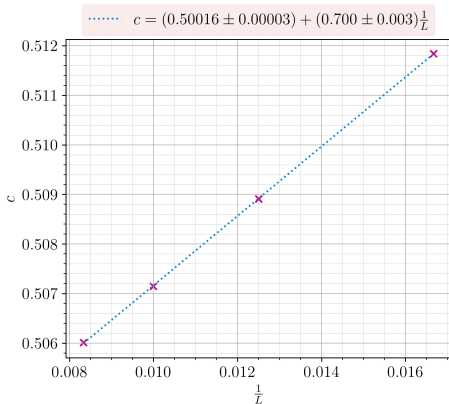on bond ℓ for system of length L

Results – free OBCs

Figure: TFI with $J = h$, $\chi = 100$

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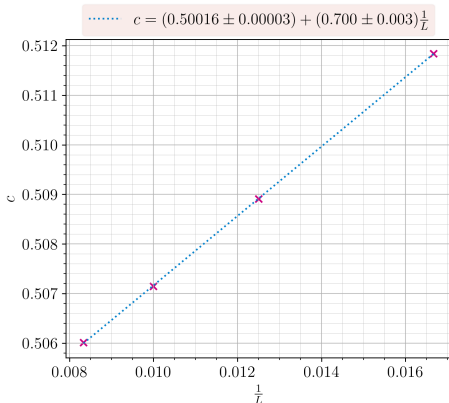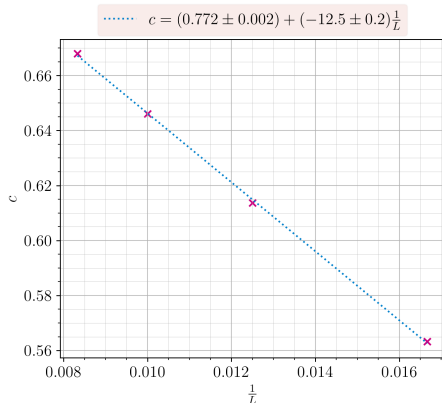

Figure: TFI with $J = h$, $\chi = 100$

Figure: OF with $\lambda_3/\lambda_I = 0.856$, $\chi = 100$

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  $\lambda_3/\lambda_I \in [0, 0.856[$
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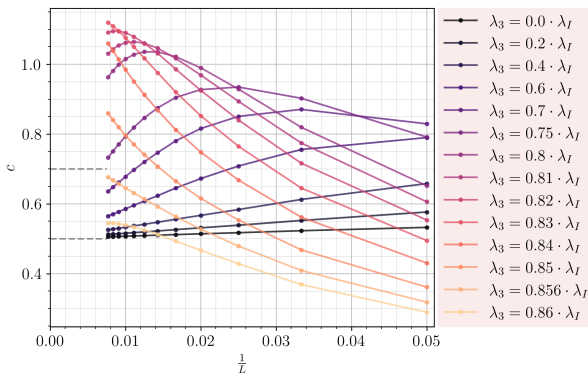- is gapped for $\lambda_3/\lambda_I > 0.856$
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Figure: OF with λ_3/λ_I varied, $\chi = 100$

Legend:
- $\lambda_3 = 0.0 \cdot \lambda_I$
- $\lambda_3 = 0.2 \cdot \lambda_I$
- $\lambda_3 = 0.4 \cdot \lambda_I$
- $\lambda_3 = 0.6 \cdot \lambda_I$
- $\lambda_3 = 0.7 \cdot \lambda_I$
- $\lambda_3 = 0.75 \cdot \lambda_I$
- $\lambda_3 = 0.8 \cdot \lambda_I$
- $\lambda_3 = 0.81 \cdot \lambda_I$
- $\lambda_3 = 0.82 \cdot \lambda_I$
- $\lambda_3 = 0.83 \cdot \lambda_I$
- $\lambda_3 = 0.84 \cdot \lambda_I$
- $\lambda_3 = 0.85 \cdot \lambda_I$
- $\lambda_3 = 0.856 \cdot \lambda_I$
- $\lambda_3 = 0.86 \cdot \lambda_I$

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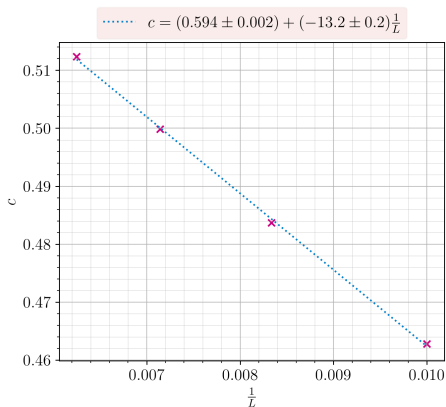- But results even worse for TCI point

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$$c = (0.594 \pm 0.002) + (-13.2 \pm 0.2)\tfrac{1}{L}$$

Figure: OF with $\lambda_3/\lambda_I = 0.856$, $\chi = 100$ and $h_{\text{pin}} = -100$ ([++] BCs)
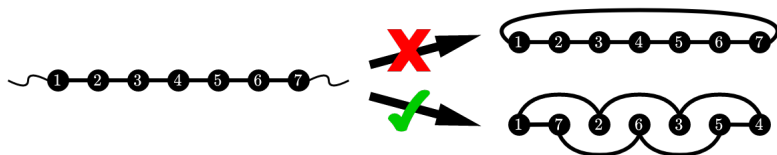
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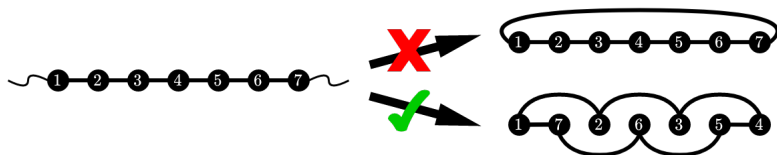
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- For PBCs, central charge recovered by

$$S(\ell) = \frac{c}{3} \ln \left[\frac{L}{\pi} \sin \frac{\pi\ell}{L} \right] + \text{const}$$

Results – PBCs

$$c = (0.699586 \pm 0.000008) + (2.61 \pm 0.02)\frac{1}{L^2}$$
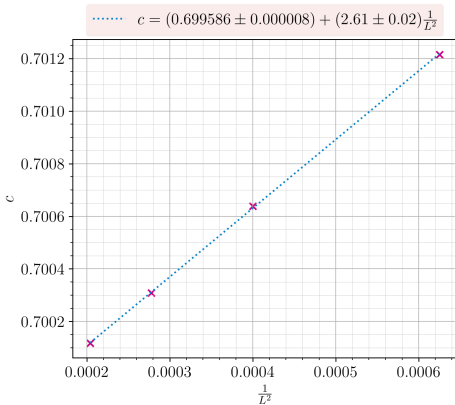
Figure: OF with $\lambda_3/\lambda_I = 0.856$ and variance $\sim 10^{-4}$
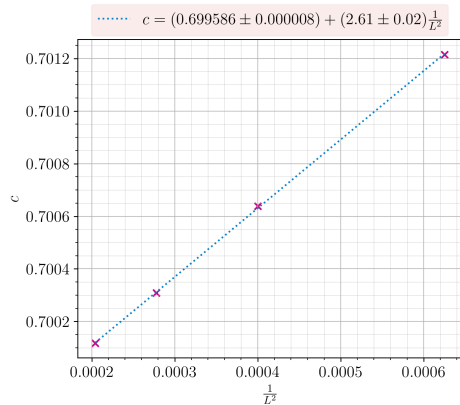
Results – PBCs

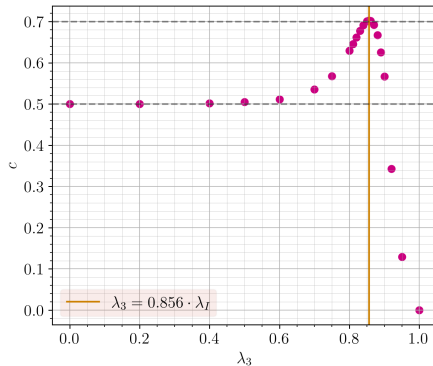Figure: OF with $\lambda_3/\lambda_I = 0.856$ and variance $\sim 10^{-4}$

Figure: OF with λ_3/λ_I varied, $L = 30$ and $\chi = 100$

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- Split $\mathcal{H}$ in sector with $\pm$ of $\mathcal{F}$, use projectors $\mathcal{F}^\pm = \frac{1}{2}[\mathbb{1} \pm \mathcal{F}]$ and run DMRG with $\tilde{\mathcal{H}} = \mathcal{F}^\pm \mathcal{H} \mathcal{F}^\pm$

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CFT	$R_1 = \frac{A_0^- - P_0^+}{P_1^+ - P_0^+}$	$R_2 = \frac{P_0^- - P_0^+}{P_1^+ - P_0^+}$	$R_3 = \frac{P_1^- - P_0^+}{P_1^+ - P_0^+}$
Ising	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{9}{8}$
TCI	$\frac{7}{2}$	$\frac{3}{8}$	$\frac{35}{8}$

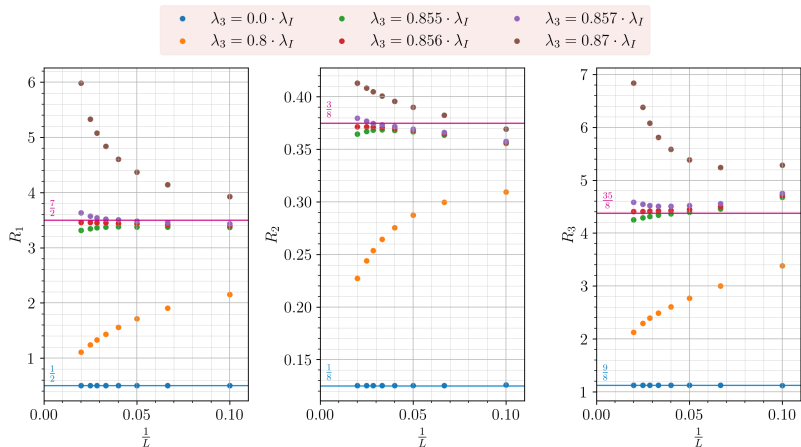Table: Universal ratios of energy computed from corresponding CFT.

Results – Ratios

Figure: Extrapolation of ratios with λ_3/λ_I varied and variance $\sim 10^{-4}$
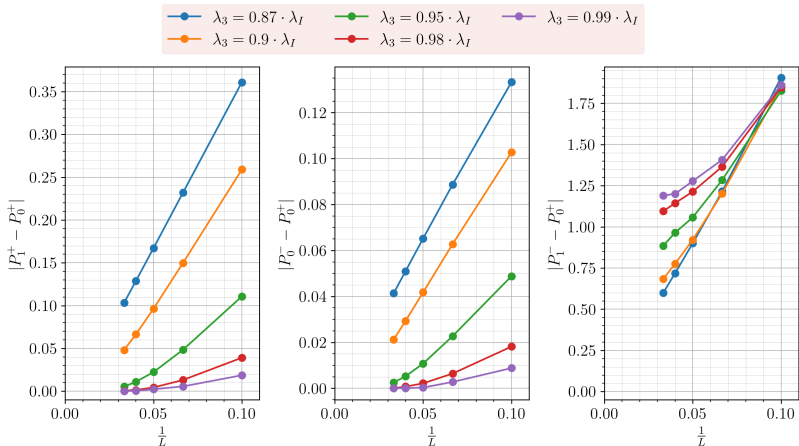
Results – Degeneracy in gapped phase

Figure: Extrapolation of the differences in energies with $\lambda_3/\lambda_I > 0.856$ varied and variance $\sim 10^{-4}$

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Main achievements

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- Phase diagram for OF model for $\lambda_3 \in [0, \lambda_I]$ and location of TCI CFT point at $\lambda_3/\lambda_I = 0.856$

Further work

Introduce $\lambda_c \neq 0$ and complete phase diagram, but not done due to lack of because of struggles with TCI central charge