Ferromagnetism in Hubbard model for TP4 2021-2022

September 2021

Topics:

1. Lieb-Mattis Ferrimagnetism theorem: It discusses that $E(S_{tot}+1) > E(S_{tot})$ for $S_{tot} \geq |S^A_{tot} - S^B_{tot}|$ where A and B are sub-lattices (**Ref**: J. Math. Phys. 3, 749 (1962)). Special case is anti-ferromagnetism when $S^A_{tot} = S^B_{tot}$.

2. We discuss Stoner criterion and a very rudimentary mean-field argument which leads to Hubbard model phase diagram where FM phase also appears for intermediate values of U/t.

   - Stoner criterion:
   $$U\rho(E_F) \geq 1$$
   - Kanamori criterion:
   $$U_{eff}\rho(E_F) = 1$$

   (**Ref:** Fazekas chapter 7, exercise 7.3, Fazekas chapter 8 and Arovas et.al.)

3. Thouless-Nagaoka-Tasaki theorem:

   - Proof of weaker statement of Nagaoka theorem i.e. among the possible ground states, we have $2S_{tot}+1$ states with $S_{tot} = S_{max.}$ i.e ferromagnetic states.
   - Proof of the stronger statement of Nagaoka theorem i.e. ground state has $S_{tot} = S_{max.}$ and is unique upto trivial $2S_{tot}+1$ states.
   - Comment on "connectivity" of the *super lattice* constructed out of configurations.
   - An example : Exercise 8.1 from Fazekas book.

   (**Ref:** H. Tasaki - Phys. Rev. B 40, 9192 (1989), Y. Nagaoka - Phys. Rev. vol 147, num 1(1966), P. Fazekas - Exercise 8.1)

4. Flat-band ferromagnetism and ring-exchange mechanism (Exercise 8.2 which introduces a toy model, Section 8.3 of Fazekas book, Section 11.3.1 from Tasaki's chapter on Ferromagnetism).