# 1D Euler Equations with a Shock

Write a program to compute the solution to the one dimensional Euler equations:

$$\frac{\partial}{\partial} \begin{bmatrix} \rho \\ \rho u \\ \rho(e + \frac{1}{2}u^2) \end{bmatrix} + \frac{\partial}{\partial x} \begin{bmatrix} \rho u \\ \rho u^2 + p \\ (\rho u(e + \frac{1}{2}u^2) + pu) \end{bmatrix} = 0 \tag{1}$$

or

$$\frac{\partial \mathbf{f}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0. \tag{2}$$

where the pressure is given by the ideal gas relation $p = (\gamma - 1)\rho e$. The sound speed is given by $c^2 = \gamma p/\rho$. Use the split Lax-Wendroff method discussed in class

$$\mathbf{f}^*_{j+1/2} = 0.5\left(\mathbf{f}^n_j + \mathbf{f}^n_{j+1}\right) - 0.5\frac{\Delta t}{h}\left(\mathbf{F}^n_{j+1} - \mathbf{F}^n_j\right) \tag{3}$$

$$\mathbf{f}^{n+1}_j = \mathbf{f}^n_j - \frac{\Delta t}{h}\left(\mathbf{F}^*_{j+1/2} - \mathbf{F}^*_{j-1/2}\right) \tag{4}$$

The initial condition consists of a shock tube with two constant states, the left and the right state: $p_L = 10^5$, $p_R = 10^4$, $\rho_L = 1$; $\rho_R = 0.125$; $u_R = u_L = 0$.

The tube is 10 meter long and you should assume that the initial separation between the two states is in the middle. The exact solution at time 0.005 is given in the notes

Run your program for several different spatial resolution and time steps. In general you are going to find that the solution is not very good. Large "wiggles" arise–even when the time step is small– and if the "wiggles" become negative, bad things will happen! To improve the results, add artificial viscous fluxes of the form:

$$\mathbf{F_{AV}} = -\alpha \Delta x^2 \rho \begin{bmatrix} 0 \\ 1 \\ u \end{bmatrix} \left|\frac{\partial u}{\partial x}\right| \frac{\partial u}{\partial x} \tag{5}$$

where $\alpha$ is an adjustable coefficient. Experiment with the coefficient. Generally, the stability requirement become more restrictive once we add artificial viscosity, so we want the coefficient to be as small as possible

Your report should include a printout of your code and plots of the solution at one or more times for a few different resolutions. You can also compare with the results in the lecture notes. Remember to be professional and concise.