Ay190 – Worksheet 10
Anthony Alvarez
Date: February 18, 2014

1 Advection Equation

We will consider the advection equation.

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0$$

We use it to advect a Gaussian

$$\Psi_0 = \Psi(x, t = 0) = e^{-(x-x_0)^2/(2\sigma^2))}$$

. Where $x_0 = 30$ and $\sigma^2 = 15$. We initially choose a positive velocity $v = 0.1$ in a $[0, 100]$ domain.

1.1 Moving Analytic Gaussian

As you can see in myadvect.py I have implemented the moving analytic Gaussian.

What I notice happening is it slowly moves to the right as it updates. Since, it is the anlaytic version it does not change shape or height as no errors accumulate.

1.2 Upwind Advection

When we implement the upwind advection we notice that it is stable for $0 \leq \alpha = \frac{v\delta t}{\delta x}$. Se can see in 1 and in 2 that the upwind scheme (Shown in blue) flattens and spreads out over time. This contribues to the error which can be seen in 3. We can see that although this system accumulates error it is still in some sense stable.

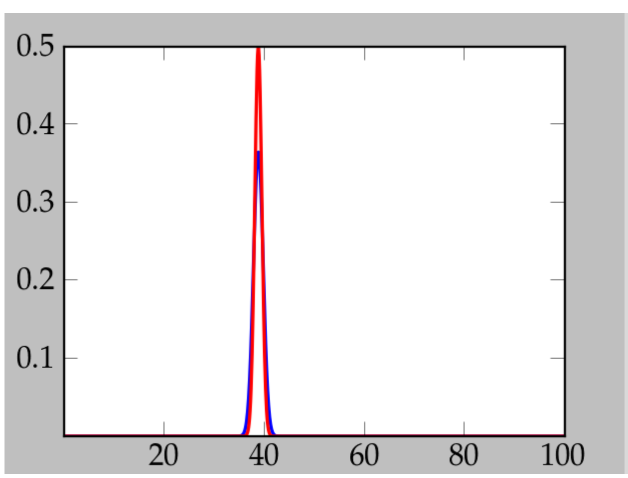

Figure 1: Upwind scheme after a small amount of time. Upwind in Blue, Analytic in Red.

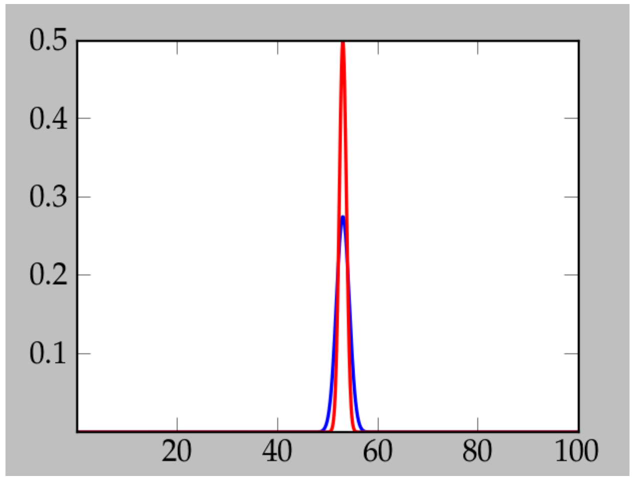

Figure 2: Upwind scheme after a longer time. Upwind in Blue, Analytic in Red.

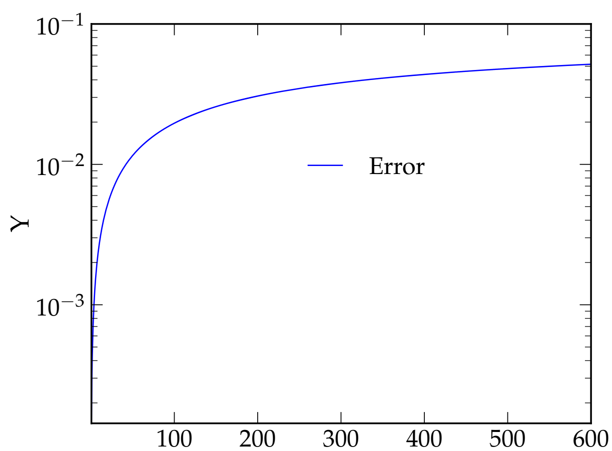

Figure 3: Error in upwind scheme for $\alpha < 1$ over time.

When we implement the same upwind scheme with $\alpha > 1$ we can see that this system is highly unstable, the error is completely unbounded and grows out of control as we can see in 4

When we try a Gaussian with a σ five times smaller we find that the upwind scheme performs equivalently visually, no plots shown. But that it does accumulate errors more wuickly as we can see in 5. This is probably due to the increased sharpness of the curve.

1.3 FTCS

Se can see in 6 that a small instability quickly develops. Later the instability grows and spawns a smaller instability further left of the true peak as we can see in 7. Eventually the left most instability grows to eclipse everything as we can see in 8. These massive instabilites contribue to the error which can be seen in 4. We can see that this system accumulates a massive amount of error and is in no sense stable.

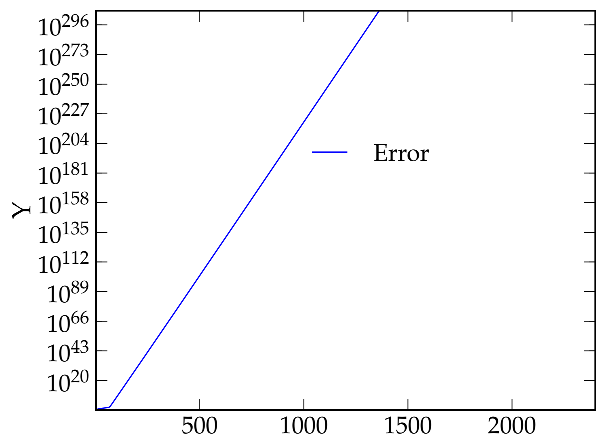

Figure 4: Error in upwind scheme for $\alpha > 1$. The error seems totally unbounded.

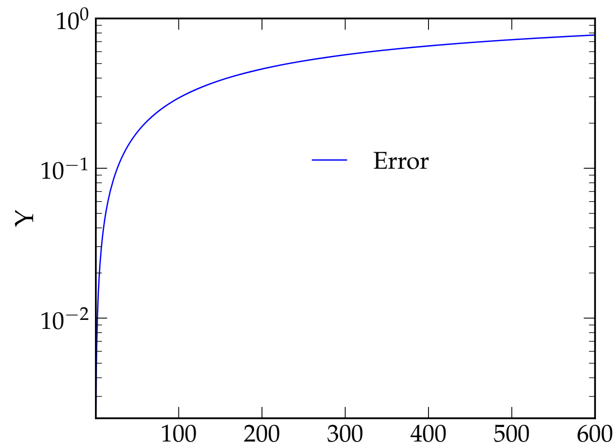

Figure 5: Error in upwind scheme for $\alpha < 1, \sigma = \frac{\sqrt{15}}{5}$

1.4 Lax-Friedrich Method

We implement the Lax-Friedrich method and as we can see in 9 and 10. The Lax-Friedrich method has a similar flattening process as the upwind method seen in 1 and 2. At least visually these methods produce very similar results.

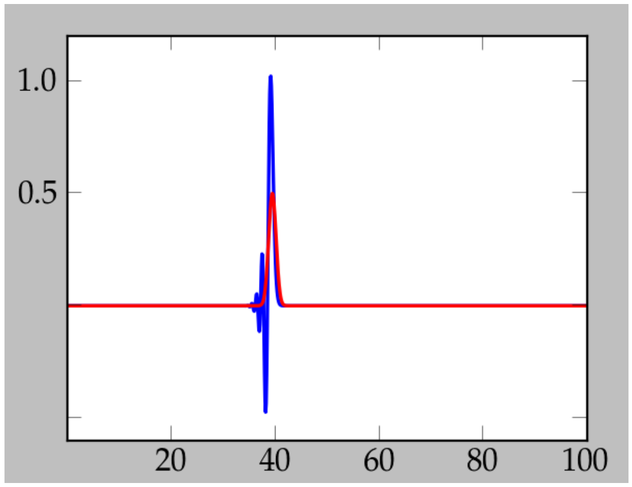

Figure 6: FTCS scheme after a small amount of time. Notice thes small instability. FTCS in Blue, Analytic in Red.

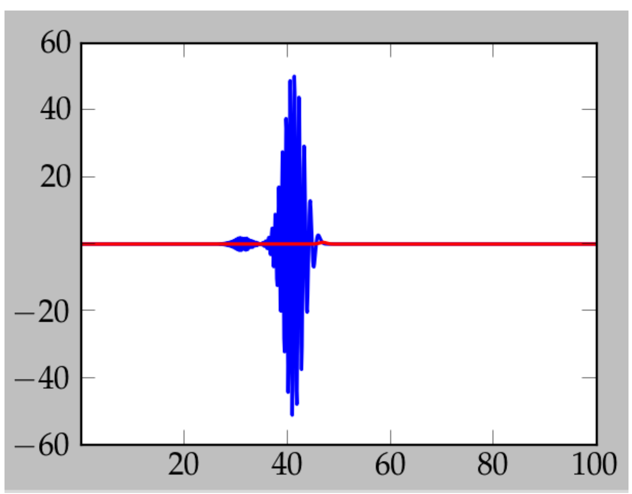

Figure 7: FTCS scheme after a longer time. The instability grows significantly. FTCS in Blue, Analytic in Red.

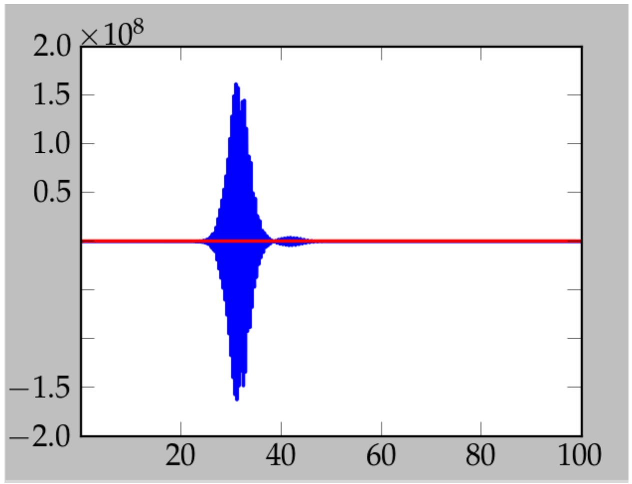

Figure 8: FTCS scheme after a longer time. The instability now dominates completely. FTCS in Blue, Analytic in Red.

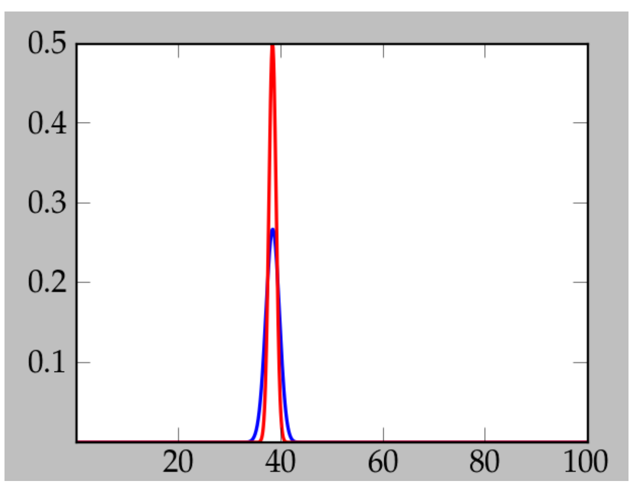

Figure 9: Lax-Friedrich method scheme after a short time. The numerical result flattens out slightly. Lax-Friedrich method in Blue, Analytic in Red.

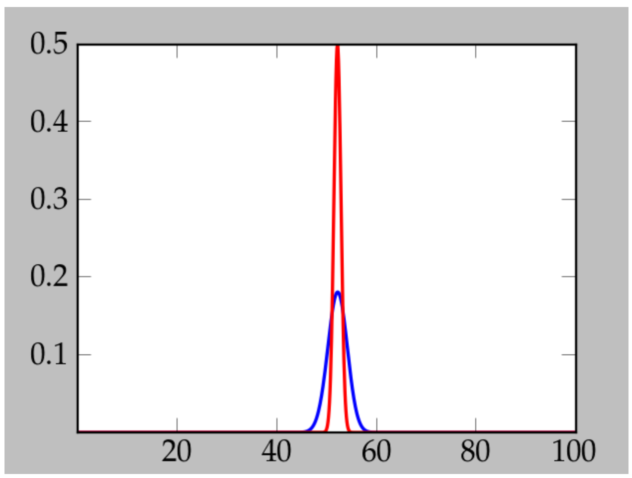

Figure 10: Lax-Friedrich method scheme after a longer time. The numerical result flattens out slightly. Lax-Friedrich method in Blue, Analytic in Red.