Ay190 – Worksheet 12
Anthony Alvarez
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# 1 Poisson Equation

We will consider the Poisson equation.

$$\nabla^2\Phi = 4\pi G\rho$$

## 1.1 Reading Data

First we read the data in and determine which columns are which. We determine the first column is simply a count. Column 2 is the mass, 3 is the radius, 4 is temerature, 5 is density, 6 is radial velocity, 7 is electron fraction and 8 is rotational velocity (constant and 0 for a non rotating star).

Next we make a plot of $\rho(r)$ as you can see in 1. Note that we restirct our selves to $r < 10^9 cm$.

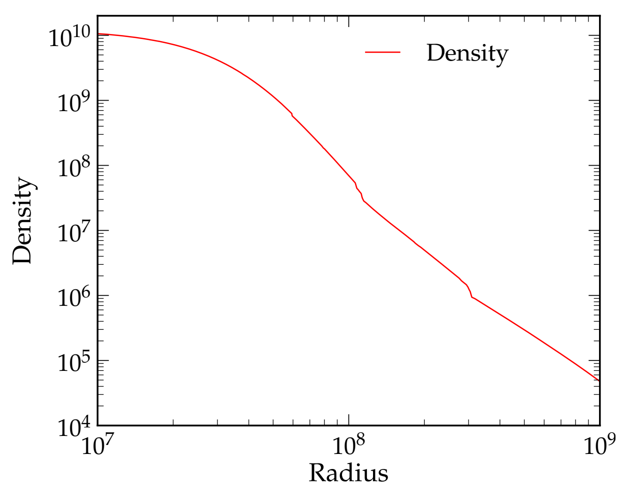


Figure 1: Plot of $\rho(r)$ for the inner part of the star.

## 1.2 Equidistant Grid

We use linear interpolation to put the data onto an equidistant grid from $10^7 cm$ to $10^9 cm$. We can see that this 2 is very similar, if not indistinguishable from 1.

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## 1.3 Constant Density $\Phi$

First we decompose the Poisson equation into two coupled first order ODEs for $\Phi$ and $\Phi' = u$. Then we use forward Euler integrator to find the value for $\Phi$ given the boundary conditions at $r = 0$ are that $\Phi' = 0$ since there should be no gravitational force at the center of the star.

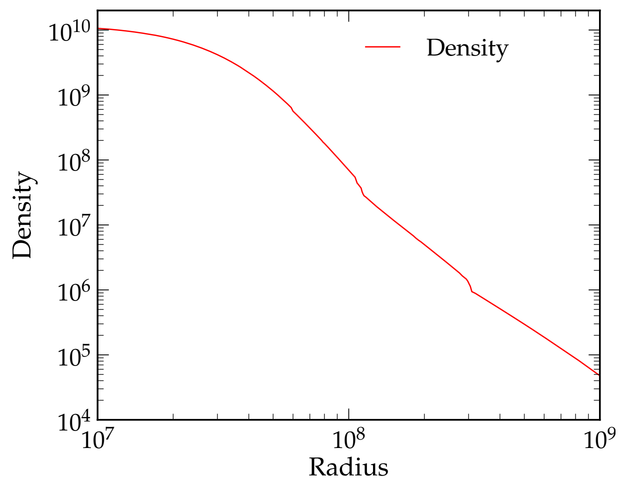


Figure 2: Plot of linearly interpolated $\rho(r)$ for the inner part of the star.

Since poisson equation only determies $\Phi$ upto a constant we adjust the outer boundary condition to match the known analytic $\Phi$ for a constant density. We can see that there is good agreement between the analytic value and the forward Euler integrated value in 3.

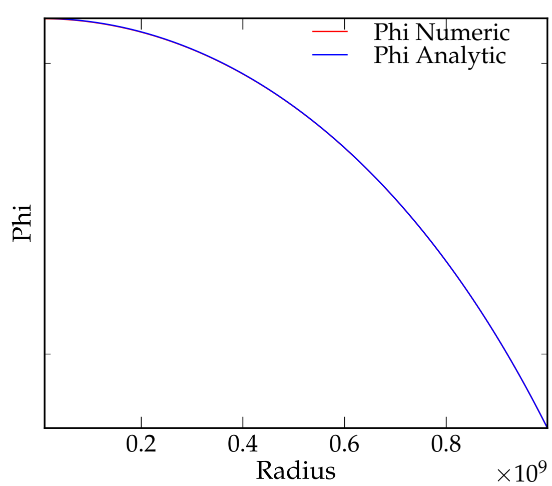


Figure 3: $-\Phi$ for a cosntant density star. We graph -$\Phi$ to get it on a log plot.

We also find that the convergance for the Forward Euler integrator is 1.00, just like we would expect. With this confirmed we feel free to move on to the actual supernova data.

### 1.4 Supernova Collapse $\Phi$

Now that we have verified our forward Euler on a case where we know the analytic answer we are able to implement it on a new problem. We see in 4 that the value of $\Phi(r = 10^9 cm)$ rouhly converges to $5e18$ which makes sense.

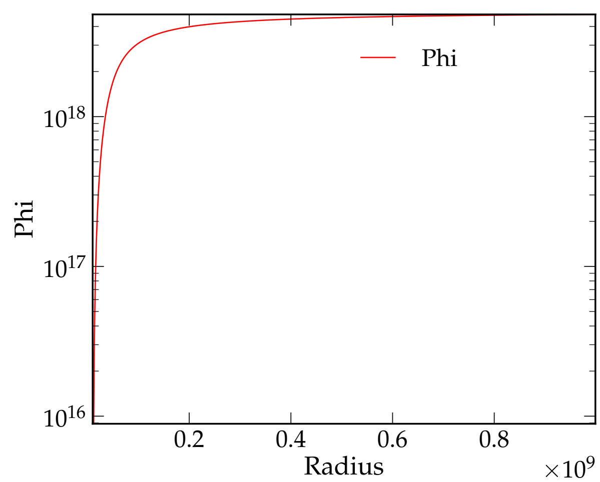


Figure 4: $-\Phi$ for a Supernova Colapse star. We graph -$\Phi$ to get it on a log plot.