

1 N-body Simulation

We first begin by implementing RK4 scheme for n-body simulation that accounts for gravitational interactions between all n objects. We also make a function that calculates the total energy of the system given a state vector.

1.1 Sun and Earth System

When running this simulation on the Sun and Earth system we find that the Earth's orbit remains bound with constant energy $-2.64 \cdot 10^{40} \text{erg}$. We also find that the Earth's orbital radius changes slightly thought the year, as it should, but it always comes back to the same value after a year. Our simulation doesn't seem to be leaking or gaining energy as the total energy in the system is very constant. Additionally, smaller stepsizes yield smaller errors as we would expect ¹. Lower resolutions seem to have increasing errors while the highest resolution I ran had relatively constant error.

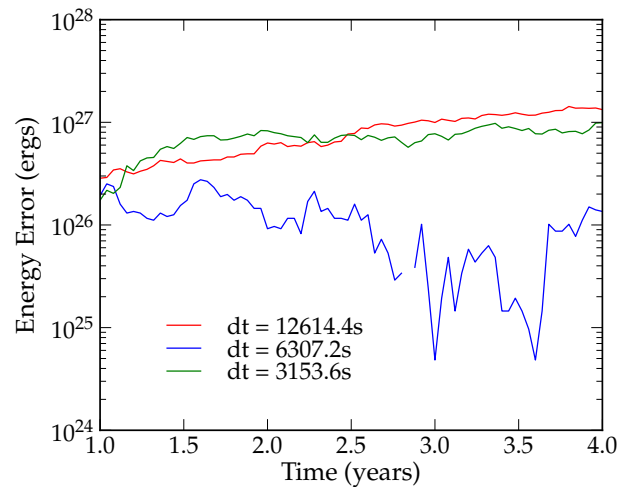


Figure 1: Energy Error for Earth Sun System.

1.2 Central Black Hole System

For the central black hole system we simulated first for 10 years just to make sure that the simulation was working correctly. We notice that at around 5 years there is a large spike in the error ². This is caused by a star coming very close to the central black hole, where we would expect large derivatives and inaccurate numerical integration.

We isolated just the first 4 years before the bumb to see if there were effects we couldn't see ³. We did not notice anything out of the ordinary.

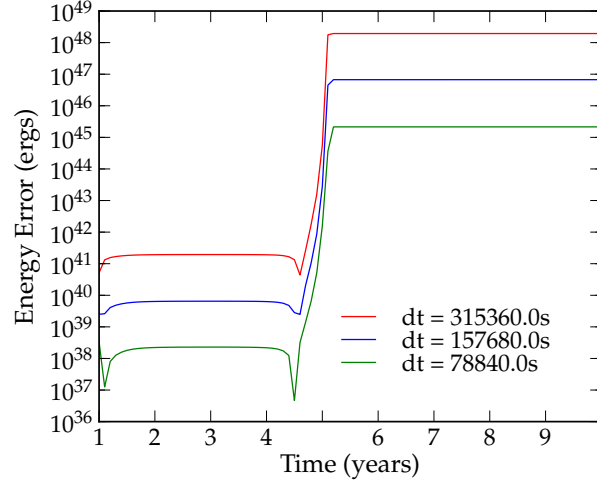


Figure 2: Energy Error for Central black hole system

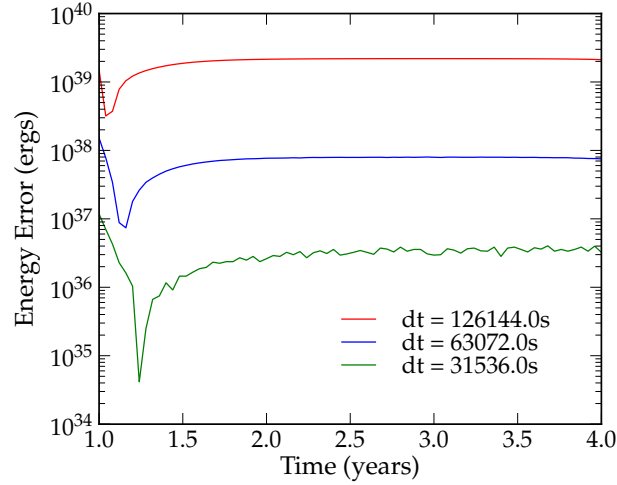


Figure 3: Energy Error for Central black hole system

We then ran the population of stars for 100 years and noted that for low resolutions the system becomes unbounded after 30 years. We increased the resolution to 10000 points for 100 years and then compared the final positions to the data obtained from

http://astro.uchicago.edu/cosmus/projects/UCLS_GCG/

. We found that the positions we calculated $(-0.1456, 0.067, -0.112)$ were vastly different than the final position calculated by uChicago $(0.218, -0.452, -0.234)$. This is expected as small numerical errors, even rounding will compound over many iterations of code.