Ay190 – Worksheet 2
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1 An Unstable Calculation

Consider the following sequence and recurrence relation

$$x_0 = 1;\ x1 = \frac{1}{3},\ x_{n+1} = \frac{13}{3}x_n - \frac{4}{3}x_{n-1},$$

which is equivalent to

$$x_n = \left(\frac{1}{3}\right)^n.$$

In the recurence relation at $n = 15$ we have quite a large error. With the recursive realtion we get that $\frac{1}{3}^{15} = 0.914373$ while the *true* value is 6.969210^{-8}. The absolute error is -0.9008 and the relative error is -4308627.0610.

2 Finite Difference Approximation and Convergence

2.1 Forward Differencing

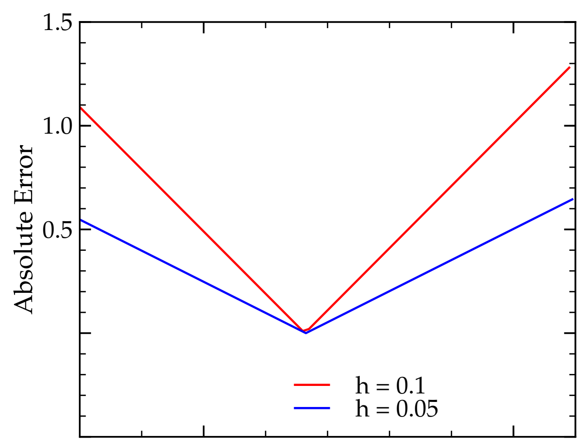

Figure 1: Absolute Error with Forward Differencing on the function $f(x) = x^3 - 5x^2 + x$.

In figure 1 we see that the forward differencing is first order convergent since by halving h we half the absolute error.

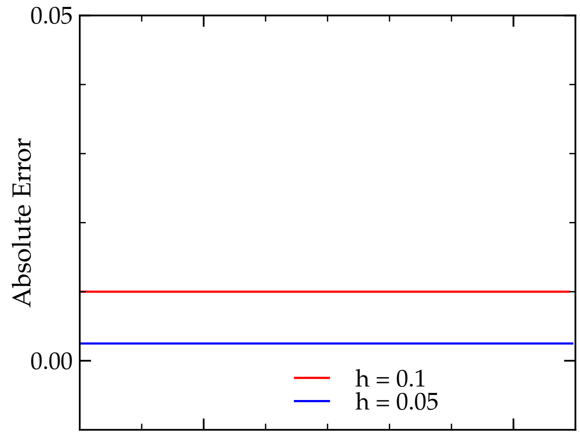

Figure 2: Absolute Error with Central Differencing on the function $f(x) = x^3 - 5x^2 + x$.

2.2 Central Differencing

In figure 2 we see that the central differencing is second order convergent since by halving h we quarter the absolute error. It's also interesting to note that for this particular function the central differencing scheme produces constant absolute error.

3 Second Derivative

$$f''(x) = \lim_{h\to 0} \frac{f'(x+h) - f'(x-h)}{2h}$$

We know that $f'(x)$ when calcualted using central diffferencing is second order. So we plug in

$$f'(x) \lim_{h\to 0} \frac{f(x+h) - f(x-h)}{2h}$$

giving

$$f''(x) = \lim_{h\to 0} \frac{(f(x+2h) - f(x)) - (f(x) + f(x-2h))}{4h^2}$$

$$f''(x) = \lim_{h\to 0} \frac{f(x+2h) - 2f(x) + f(x-2h)}{4h^2}$$

This is a second-order central finite difference approximation for the second derivative of the function $f(x)$ assuming a fixed step size h.

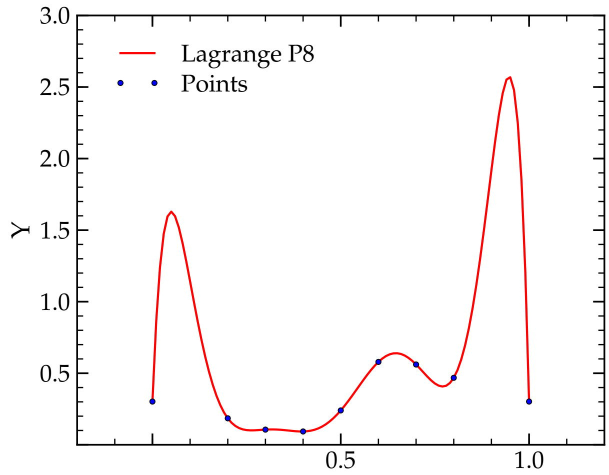

Figure 3: Lagrange interpolation polynomial $p_8(x)$ and the data together.

4 Interpolation: Cepheid Lightcurve

4.1 Lagrange Interpolation

In figure 3 we can see the Runge's phenomenon of non-convergence on the edges where the data gets a bit sparce.

4.2 Piecewise Linear & Quadratic Interpolation

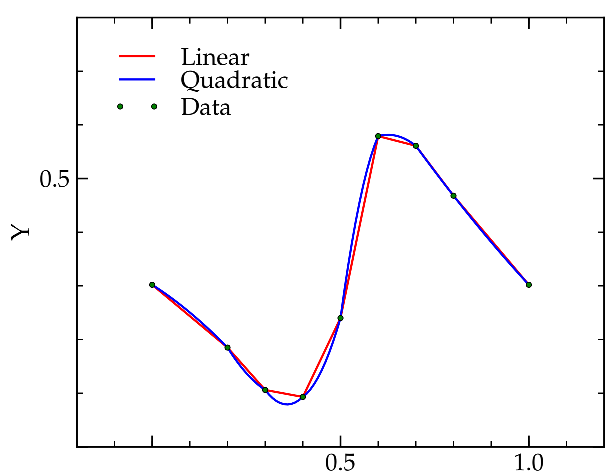

Figure 4: Piecewise linear and quadratic interpolation and the data together.

In figure 4 we can see the linear and quadratic model match the data better than the Lagrange interpolation scheme. Also we note that the quadratic model is smoother, as one would expect.

5 More Cepheid Lightcurve Interpolation

5.1 Piecewise Cubic Hermite & Natural Cubic Spline Interpolation

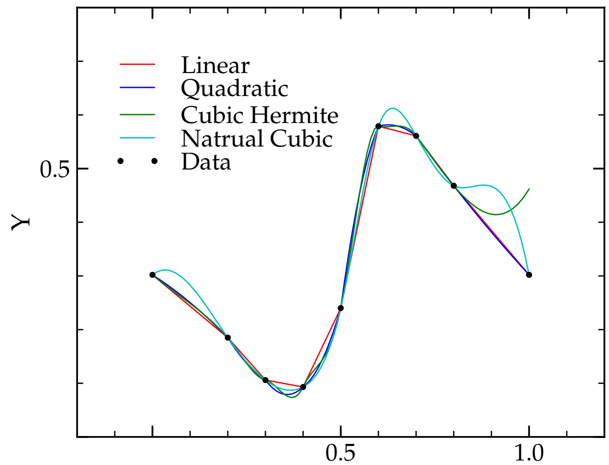

Figure 5: Piecewise Linear, Quadratic, Cubic Hermite, Natural Cubic Spline Interpolation & the data itself.

In figure 5 we can see that the Natural Cubic Spline is the smoothest of the interpolations. We also note that the Cubic Hermite has strange behavior in the last bin. This is due to the fact that we cannot numerically calculate the derivative on the right edge so this is infact not an interpolation but actually an extrapolation.