Ay190 – Worksheet 3
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Date: January 22, 2014

1 Integration via Newton Cotes Formulae

1.1

Integrating the function $f(x) = sin\ x$ from 0 to π using the midpoint rule, the trapezoidal rule, and Simpson's rule. We use this integral to demonstrate the effectiveness of these numerical integration methods because it is easy to calculate analytically. We know that the answer should be 2 which we use to determine the absolute error.

We can see in figure figure 1 that the midpoint rule, though it should be a bit worse than the trapezoidal rule, it actually performs better because it on average doesn't over or underestimate any linear term in a function.

Additionally we note that simpsons rule converges much more quickly as the error term is proportional to h^{TK}.

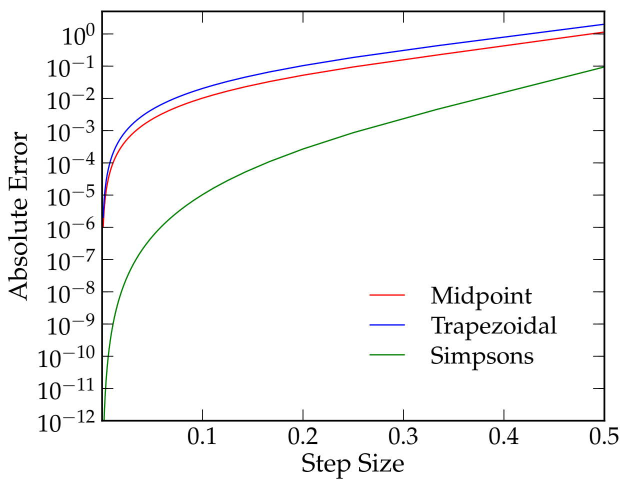

Figure 1: For $sin(x)$ the Midpoint, Trapezoidal, and Simpsons rule all converge.

1.2

While we have shown that these methods converge for $sin(x)$ but this may be a fluke. We examine the function $x\ sin(x)$ in figure 2.

2 Gaussian Quadrature

When performing Gaussian-Laguerre quadrature the accuracy is determined by the number of nodes that are used. To determine the convergance of the process we take the Gaussian-Laguerre

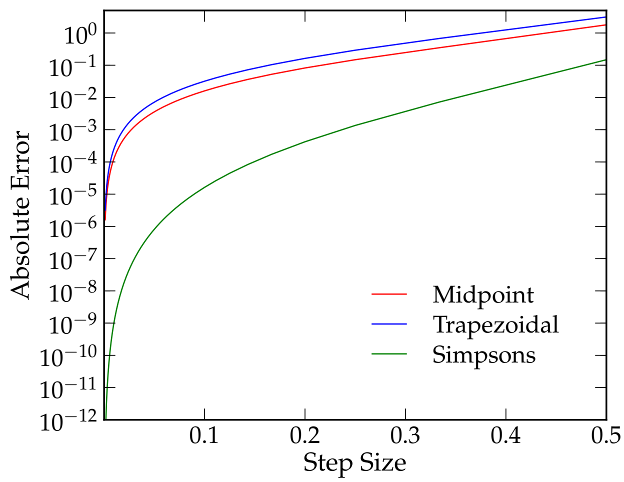

Figure 2: For $x\ sin(x)$ the Midpoint, Trapezoidal, and Simpsons rule all converge.

with the most nodes and treat that as the 'correct' answer.

By subtracting it from the lower node estimations we get some determination of error. We can see from figure 3 that the error gets exponentially smaller and thus can conlude that the Gaussian-Laguerre method does infact converge. It does converge to the value of 1.52 10^{37} when we include the constant $\frac{8\pi(k_bT)^3}{(2\pi\hbar c)^3}$

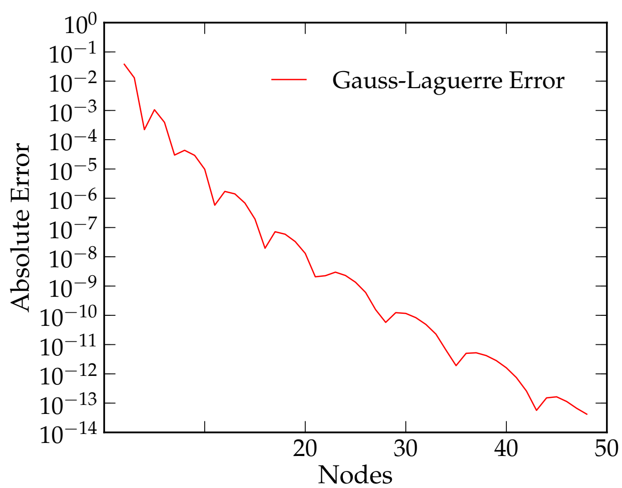

Figure 3: Guass-Laguerre error convergance with increasing nodes.

2.1 Gauss-Legendre Quadrature

By binning the integral into segments and then utilizing the Gaussian-Lagendre integration on each bin we can calculate an estimation of a derivative of n_e. Note that when we sum the derivative multiplied by the bin size we recover the estimated value of n_e.

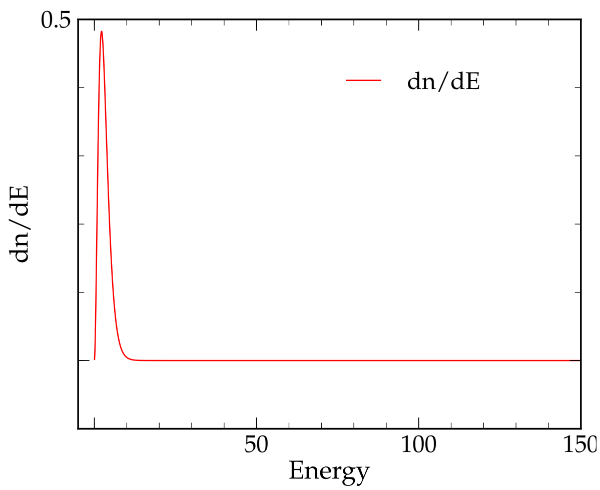

Figure 4: Estimation of the derivative of n_e using the Gaussian-Lagendre integration.