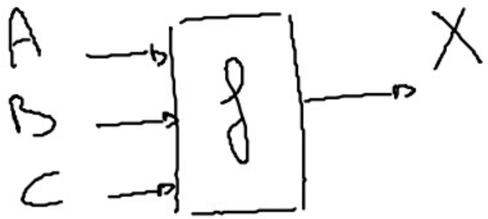
$$X = f(A, B, C)$$



$n = 3$

	A	B	C	X
m_0	0	0	0	1
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	0
m_6	1	1	0	0
m_7	1	1	1	1

$$X = \overbrace{\bar{A}.\bar{B}.\bar{C}}^{\substack{m_0 \\ 0\ 0\ 0}} + \overbrace{\bar{A}B.\bar{C}}^{\substack{m_2 \\ 0\ 1\ 0}} + \overbrace{\bar{A}BC}^{\substack{m_3 \\ 0\ 1\ 1}} + \overbrace{ABC}^{\substack{m_7 \\ 1\ 1\ 1}}$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}B(\underbrace{\bar{C} + C}_{1}) + ABC$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}B + ABC$$

$$\boxed{X + YZ = (X+Y)(X+Z)}$$

$$(a \bullet b) + (a \bullet \bar{b}) = a$$

$$(a + b) \bullet (a + \bar{b}) = a$$

$$a \bullet (\bar{a} + b) = a \bullet b$$

$$a + (\bar{a} \bullet b) = a + b$$

$$ab + a\bar{b} = a(\overbrace{b + \bar{b}}^{1 =}) = a$$

$$(a+b)(a+\bar{b}) = a + \underbrace{b.\bar{b}}_{= 0} = a$$

$$a.(\bar{a} + b) = \underbrace{a.\bar{a}}_{= 0} + ab = ab$$

$$a + \bar{a}.b = (\underbrace{a + \bar{a}}_{= 1})(a+b) = a + b.$$

15. $X + X \cdot Z = X$

16. $X \cdot (X + Y) = X$

17. $(X + Y) \cdot (X + Z) = X + Y \cdot Z$

18. $X + \overline{X} \cdot Y = X + Y$

19. $X \cdot Y + Y \cdot Z + \overline{X} \cdot Z = X \cdot Y + \overline{X} \cdot Z$

15. $X + X . Z = X(\underbrace{1 + Z}_{=1}) = X$

16. $X . (X + Y) = X . X + X . Y = X + XY = X(\underbrace{1 + Y}_{=1}) = X$

17. $(X + Y)(X + Z) = X . X + X . Z + Y . X + Y . Z$

$= X + XZ + X . Y + YZ$

$= X(\underbrace{1 + Z + Y}_{=1}) + Y . Z$

$= X + Y . Z$

15. $X + X \cdot Z = X$

16. $X \cdot (X + Y) = X$

17. $(X + Y) \cdot (X + Z) = X + Y \cdot Z \leftarrow$

18. $X + \overline{X} \cdot Y = X + Y$

19. $X \cdot Y + Y \cdot Z + \overline{X} \cdot Z = X \cdot Y + \overline{X} \cdot Z$

$$18.\quad X + \bar{X}.Y = (\underbrace{X + \bar{X}}_{=1}).(X + Y) = X + Y$$

$$19.\quad XY + Y.Z + \bar{X}Z = XY + YZ.(X + \bar{X}) + \bar{X}Z$$

$$= XY + XYZ + \bar{X}ZY + \bar{X}Z$$

$$= XY(\underbrace{1 + Z}_{=1}) + \bar{X}Z(\underbrace{Y + 1}_{=1})$$

$$= XY + \bar{X}Z$$

Exercice 1

Établir les tables de vérité des fonctions suivantes, puis les écrire sous les deux formes canoniques :

1. $F_1 = XY + YZ + XZ$

2. $F_2 = X + YZ + \overline{Y}\,\overline{Z}\,T$

3. $F_3 = (X + Y)(\overline{X} + Y + Z)$

4. $F_4 = (\overline{X} + \overline{Z})(X + \overline{T} + Z)Y\overline{Z}$

5. $F_5 = (\overline{X}Y + X\overline{Y})\overline{Z} + (\overline{X}\,\overline{Y} + XY)Z$

6. $F_6 = \overline{X} + YZ$

7. $F_7 = \overline{X}\,\overline{Y}\,Z + X\overline{Y}\,Z + X\overline{Y}\,\overline{Z} + XY\overline{Z} + XYZ$

8. $F_8 = (\overline{X} + \overline{Y} + Z)(X + \overline{Y} + Z)(X + \overline{Y} + \overline{Z})(X + Y + \overline{Z})(X + Y + Z)$

1. $F_1 = XY + YZ + XZ$

$n = 3$

SOP

La table de verité

maxterm	minterm	X Y Z	XY	YZ	XZ	F_1
$M_0 = \overline{m_0} = X + Y + Z$	$m_0 = \bar{X}\bar{Y}\bar{Z}$	0 0 0	0	0	0	0
$M_1 = \overline{m_1} = X + Y + \bar{Z}$	$m_1 = \bar{X}\bar{Y}Z$	0 0 1	0	0	0	0
$M_2 = \overline{m_2} = X + \bar{Y} + Z$	$m_2 = \bar{X}Y.\bar{Z}$	0 1 0	0	0	0	0
	m_3	0 1 1	0	1	0	1
$M_4 = \overline{m_4} = \bar{X} + Y + Z$	$m_4 = X\bar{Y}\bar{Z}$	1 0 0	0	0	0	0
	m_5	1 0 1	0	0	1	1
	m_6	1 1 0	1	0	0	1
$M_7 = \overline{m_7} = \bar{X} + \bar{Y} + \bar{Z}$	$m_7 = X.Y.Z$	1 1 1	1	1	1	1

$F_1 = m_3 + m_5 + m_6 + m_7$

$$F_1 = \bar{X}YZ + X\bar{Y}Z + XY\bar{Z} + XYZ$$

1ère forme canonique

$\Sigma\Pi$

$$\overline{F_1} = m_0 + m_1 + m_2 + m_4$$

$$F_1 = \overline{\overline{F_1}} = \overline{m_0 + m_1 + m_2 + m_4}$$

$$F_1 = \overline{m_0} \cdot \overline{m_1} \cdot \overline{m_2} \cdot \overline{m_4}$$

$$F_1 = M_0 \cdot M_1 \cdot M_2 \cdot M_4$$

$$F_1 = (X+Y+Z)\cdot(X+Y+\overline{Z})\cdot(X+\overline{Y}+Z)\cdot(\overline{X}+Y+Z)$$

2ème forme canonique

$\Pi\Sigma$.

$$m_0 = \bar{X} \cdot \bar{Y} \cdot \bar{Z}$$

$$M_0 = \overline{m_0} = \overline{\bar{X} \cdot \bar{Y} \cdot \bar{Z}} = \bar{\bar{X}} + \bar{\bar{Y}} + \bar{\bar{Z}}$$

$$= X + Y + Z$$

$$m_5 = \underset{1}{X} \cdot \underset{0}{\bar{Y}} \cdot \underset{1}{Z}$$

$$M_5 = \overline{m_5} = \overline{X \cdot \bar{Y} \cdot Z} = \bar{X} + \bar{\bar{Y}} + \bar{Z}$$

$$= \bar{X} + Y + \bar{Z}$$

4. $F_4 = (\overline{X}+\overline{Z})(X+\overline{T}+Z)Y\overline{Z}$ $= (\overset{0}{\cancel{\overline{X}X}} + \overline{X}\,\overline{T} + \overline{X}Z + \overline{Z}.X + \overline{Z}.\overline{T} + \overset{0}{\cancel{\overline{Z}Z}}).Y\overline{Z}$

$$= \overline{X}\,\overline{T}Y\overline{Z} + \overset{0}{\cancel{\overline{X}ZY\overline{Z}}} + \overline{Z}XY\overline{Z} + \overline{Z}\,\overline{T}Y\overline{Z}$$

$$\boxed{F_4 = \overline{X}Y\overline{Z}\,\overline{T} + XY\overline{Z} + Y\overline{Z}\,\overline{T}}$$

$$= \overline{X}Y\overline{Z}\,\overline{T} + XY\overline{Z}(T+\overline{T}) + (X+\overline{X})Y\overline{Z}\,\overline{T}$$

$$= \overline{X}Y\overline{Z}\,\overline{T} + XY\overline{Z}T + XY\overline{Z}\,\overline{T} + XY\overline{Z}\,\overline{T} + \overline{X}Y\overline{Z}\,\overline{T}$$

$$= \underset{0\,1\,0\,0}{\overline{X}Y\overline{Z}\,\overline{T}} + \underset{1\,1\,0\,1}{XY\overline{Z}T} + \underset{1\,1\,0\,0}{XY\overline{Z}\,\overline{T}}$$

$$= m_4 + m_{13} + m_{12}$$

$$F_4 = \overset{0}{\bar{X}}\,\overset{1}{Y}\,\overset{0}{\bar{Z}}\,\overset{0}{\bar{T}} + XY\bar{Z} + Y\bar{Z}\bar{T}$$

	X	Y	Z	T	$\bar{X}Y\bar{Z}\bar{T}$	$XY\bar{Z}$	$Y\bar{Z}\bar{T}$	F_4
m_0	0	0	0	0	0	0	0	0
m_1	0	0	0	1	0	0	0	0
m_2	0	0	1	0	0	0	0	0
m_3	0	0	1	1	0	0	0	0
m_4	0	1	0	0	1	0	1	1
	0	1	0	1	0	0	0	0
⋮	0	1	1	0	0	0	0	0
	0	1	1	1	0	0	0	0
m_8	1	0	0	0	0	0	0	0
m_9	1	0	0	1	0	0	0	0
m_{10}	1	0	1	0	0	0	0	0
m_{11}	1	0	1	1	0	0	0	0
m_{12}	1	1	0	0	0	1	1	1
m_{13}	1	1	0	1	0	1	0	1
m_{14}	1	1	1	0	0	0	0	0
m_{15}	1	1	1	1	0	0	0	0

$$F_4 = m_4 + m_{12} + m_{13}$$

$$= \bar{X}Y\bar{Z}\bar{T} + XY\bar{Z} + Y\bar{Z}\bar{T}$$

$$(\overset{3}{1}\overset{2}{1}\overset{1}{0}\overset{0}{0})_2 = 2^3 + 2^2 = 8 + 4 = 12$$

$$1101 = 2^3 + 2^2 + 2^0 = 13$$

8. $F_8 = (\overline{X} + \overline{Y} + Z)(X + \overline{Y} + Z)(X + \overline{Y} + \overline{Z})(X + Y + \overline{Z})(X + Y + Z)$

$$\boxed{a + bc = (a+b)(a+c)}$$

$M=3$

$$= \left(Z + (\bar{X} + \bar{Y})(X + \bar{Y})\right)\left(\bar{Z} + (X + \bar{Y})\cdot(X+Y)\right)\cdot(X + Y + Z)$$

$$= \left(Z + (\bar{Y} + \overset{0}{\cancel{X\cdot\bar{X}}})\right)\left(\bar{Z} + (X + \cancel{\bar{Y}Y})\right)(X + Y + Z)$$

$$= (Z + \bar{Y})(\bar{Z} + X)(X + Y + Z)$$

$$= (\overset{0}{\cancel{Z\bar{Z}}} + ZX + \bar{Y}\bar{Z} + \bar{Y}X)(X + Y + Z)$$

$$= (ZX + \bar{Y}\bar{Z} + X\bar{Y}(Z + \bar{Z}))(X + Y + Z)$$

$$= (XZ + \bar{Y}\bar{Z} + XZ\bar{Y} + X\bar{Y}\bar{Z})(X + Y + Z)$$

$$F_8 = \left(XZ(\underbrace{1+\bar{Y}}_{1}) + \bar{Y}\bar{Z}(\underbrace{1+X}_{1}) \right)(X+Y+Z)$$

$$= \left(XZ + \bar{Y}\bar{Z} \right)(X+Y+Z)$$

$$= \underline{XZ} + XYZ + \underline{XZ} + X\bar{Y}\bar{Z} + \overset{0}{\cancel{Y\bar{Y}}}\bar{Z} + \bar{Y}\overset{0}{\cancel{\bar{Z}Z}}$$

$$= XZ + XYZ + X\bar{Y}\bar{Z}$$

$$= X(Y+\bar{Y})Z + XYZ + X\bar{Y}\bar{Z}$$

$$F_8 = \underline{XY}Z + X\bar{Y}Z + \underline{XY}Z + X\bar{Y}\bar{Z}$$

$$F_8 = \underset{1\;0\;0}{X\bar{Y}\bar{Z}} + \underset{1\;0\;1}{X\bar{Y}Z} + \underset{1\;1\;1}{XYZ}$$

$$= m_4 + m_5 + m_7$$

1. $F_1 = XY + YZ + XZ$

$\underline{\underline{n=3}}$

La table de verité

X	Y	Z	XY	YZ	XZ	F_1	
0	0	0	0	0	0	0	
0	0	1	0	0	0	0	
0	1	0	0	0	0	0	
0	1	1	0	1	0	1	
1	0	0	0	0	0	0	
1	0	1	0	0	1	1	
1	1	0	1	0	0	1	
1	1	1	1	1	1	1	

Tableau de Karnaugh

X \ YZ	00	01	11	10
0	0	0	1	0
1	0	1	1	1

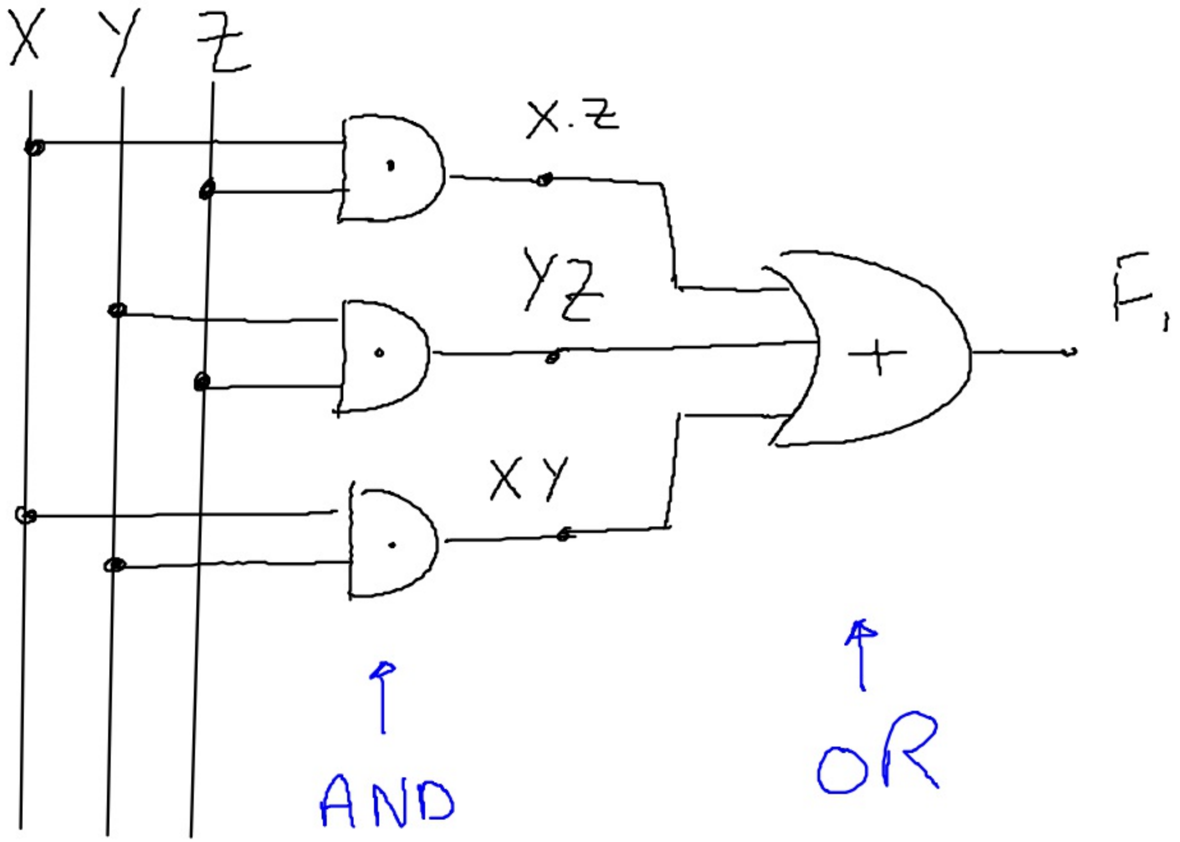
YZ

$X.Z$

XY

$$F_1 = XZ + YZ + XY$$

$F_1 = XZ + YZ + XY$



a	b	F
0	0	0
0	1	1
1	0	0
1	1	1

Table de vérité

a \ b	0	1
0	0	1
1	0	1

$\rightarrow b$

Tableau de Karnaugh

$$\boxed{F = b}$$

$$F = \bar{a}b + a.b = b(\underbrace{\bar{a}+a}_{=1}) = b$$

Exercice 2

Complémenter les expressions suivantes (sans simplification) :

1. $F_1 = \overline{X}\,\overline{Y} + XY + \overline{X}Y$
2. $F_2 = X(\overline{Y}\,\overline{Z} + YZ) + \overline{X}Y\overline{Z} + \overline{X}\,\overline{Y}Z$
3. $F_3 = X\overline{Y} + Z\overline{T} + \overline{X}\,\overline{Y} + \overline{Z}\,\overline{T}$
4. $F_4 = X\overline{Y}Z\overline{T} + \overline{X}YT + \overline{X}\,\overline{Z} + (Z + T)(X\overline{Y} + Z)$
5. $F_5 = (X + Y)(\overline{X} + Z)$
6. $F_6 = (\overline{X} + \overline{Y}\,\overline{Z}T)(XY + Z + \overline{T})(\overline{X} + \overline{Y} + Z)$

1. $F_1 = \overline{X}\,\overline{Y} + XY + \overline{X}Y$

$$\overline{F_1} = \overline{\overline{X}\,\overline{Y} + XY + \overline{X}\,Y}$$

$$= \left(\overline{\overline{X}\cdot\overline{Y}}\right)\cdot\left(\overline{XY}\right)\cdot\left(\overline{\overline{X}\,Y}\right)$$

$$= \left(\overline{\overline{X}} + \overline{\overline{Y}}\right)\cdot\left(\overline{X} + \overline{Y}\right)\cdot\left(\overline{\overline{X}} + \overline{Y}\right)$$

$$= \left(X + Y\right)\cdot\left(\overline{X} + \overline{Y}\right)\cdot\left(X + \overline{Y}\right)$$

4. $F_4 = X\overline{Y}\,Z\overline{T} + \overline{X}YT + \overline{X}\,\overline{Z} + (Z+T)(X\overline{Y} + Z)$

$$\overline{F_4} = \overline{X\overline{Y}Z\overline{T} + \overline{X}YT + \overline{X}\overline{Z} + (Z+T)(X\overline{Y}+Z)}$$

$$= \left(\overline{X\overline{Y}Z\overline{T}}\right) \cdot \left(\overline{\overline{X}YT}\right) \cdot \left(\overline{\overline{X}\overline{Z}}\right) \cdot \left(\overline{(Z+T)\cdot(X\overline{Y}+Z)}\right)$$

$$= \left(\overline{X} + \overline{\overline{Y}} + \overline{Z} + \overline{\overline{T}}\right) \cdot \left(\overline{\overline{X}} + \overline{Y} + \overline{T}\right) \cdot \left(\overline{\overline{X}} + \overline{\overline{Z}}\right) \cdot \left(\left(\overline{Z+T}\right) + \left(\overline{X\overline{Y}+Z}\right)\right)$$

$$= \left(\overline{X} + Y + \overline{Z} + T\right) \cdot \left(X + \overline{Y} + \overline{T}\right) \cdot \left(X + Z\right) \cdot \left(\overline{Z}\overline{T} + \left(\overline{X\overline{Y}} \cdot \overline{Z}\right)\right)$$

$$= \left(\overline{X} + Y + \overline{Z} + T\right) \cdot \left(X + \overline{Y} + \overline{T}\right) \cdot \left(X + Z\right) \cdot \left(\overline{Z} \cdot \overline{T} + \left(\overline{X} + Y\right) \cdot \overline{Z}\right)$$