IND E 250 HW 1

Homework 1 Problems:
3.4, 3.15, 3.22, 3.36, 3.42, 3.52 (bonus)

Problem 3.4

Excersize 1 *You are about to borrow $10,000 from a bank at an interest rate of 10% compounded annually. You are required to make 5 equal annual repayments in the amount of $2,638 per year, with the first repayment occuring at the end of year 1. Show the interest payment and principal payment in each year.*

1. Finding Year 1 Principal and Interest Payments

$$\text{Interest} = \text{Remaining Balance} * i$$
$$= \$10,000 * 0.1$$
$$\text{Interest} = \$1,000$$
$$\text{Principal} = \$2,638 - \text{Interest}$$
$$= \$2,638 - \$1,000$$
$$\text{Principal} = \$1,638$$
$$\text{Remaining} = \$10,000 - \text{Principal}$$
$$= \$10,000 - \$1,638$$
$$\text{Remaining} = \$8,362$$

2. Finding Year 2 Principal and Interest Payments

$$\text{Interest} = \text{Remaining Balance} * i$$
$$= \$8,362 * 0.1$$
$$\text{Interest} = \$836.2$$
$$\text{Principal} = \$2,638 - \text{Interest}$$
$$= \$2,638 - \$836.2$$
$$\text{Principal} = \$1,801.8$$
$$\text{Remaining} = \$8,362 - \text{Principal}$$
$$= \$8,362 - \$1,801.8$$
$$\text{Remaining} = \$6560.2$$

3. Finding Year 3 Principal and Interest Payments

$$\text{Interest} = \text{Remaining Balance} * i$$
$$= \$6560.2 * 0.1$$
$$\text{Interest} = \$656.02$$
$$\text{Principal} = \$2,638 - \text{Interest}$$
$$= \$2,638 - \$656.02$$
$$\text{Principal} = \$1,981.98$$
$$\text{Remaining} = \$6560.2 - \text{Principal}$$
$$= \$6560.2 - \$1,981.98$$
$$\text{Remaining} = \$4578.22$$

4. Finding Year 4 Principal and Interest Payments

$$\text{Interest} = \text{Remaining Balance} * i$$
$$= \$4578.22 * 0.1$$
$$\text{Interest} = \$457.822$$
$$\text{Principal} = \$2,638 - \text{Interest}$$
$$= \$2,638 - \$457.822$$
$$\text{Principal} = \$2180.178$$
$$\text{Remaining} = \$4578.22 - \text{Principal}$$
$$= \$4578.22 - \$2180.178$$
$$\text{Remaining} = \$2398.042$$

5. Finding Year 5 Principal and Interest Payments

$$\text{Interest} = \text{Remaining Balance} * i$$
$$= \$2398.042 * 0.1$$
$$\text{Interest} = \$239.80$$
$$\text{Principal} = \$2,638 - \text{Interest}$$
$$= \$2,638 - \$239.80$$
$$\text{Principal} = \$2380.20$$
$$\text{Remaining} = \$2398.042 - \text{Principal}$$
$$= \$2398.042 - \$2380.20$$
$$\text{Remaining} = -\$0.158 \approx 0$$

Problem 3.15

Excersize 2 *You bought 300 shares of GE stock at \$3,200 on April 30, 2009. Your intention is to keep the stock until it doubles in value. If you expect 8% annual growth for GE stock, how long do you anticipate holding onto the stock? Compare your answer with the solution obtained by the rule of 72/*

Want to find how long we hold for until the future value is worth twice the time you bought it - thus:

$$P = 3200 \quad i = 8\% \quad F = 2P \quad \text{Find N}$$

1. Setting up equation for compound interest/uniform payment series:
$$F = 2P = P(1 + i)^N$$

2. Solving for N
$$2P = P(1 + i)^N$$
$$2 = (1 + i)^N$$
$$log(2) = log((1 + i)^N)$$
$$log(2) = N * log((1 + i))$$
$$N = \frac{log(2)}{log(1 + i)}$$
$$N = 9.00647 \approx 9 \text{ years}$$

3. Comparing to the rule of 72:
$$N \cong \frac{72}{i * 100}$$
$$N = \frac{72}{0.08 * 100}$$
$$N = 9$$

The results from the rule of 72 and uniform payment series are the same.

Problem 3.22

Excersize 3 *How much invested now at 6% would be just sufficient to provide 3 payments, with the first payment in the amount of $8,000 occurring 2 years after, then $6,000 5 years since, and finally $4,000 7 years since?*

Have to find present worth given the future worth of the 3 payments.

1. Converting future worth to present worth:
$$P = \$8,000(P/F, 6\%, 2) + \$6,000(F/P, 6\%, 5) + \$4,000(F/P, 6\%, 7)$$

2. Setting up equations:
$$P = F(1 + i)^{-N}$$
$$P = \$8,000(1 + 0.06)^{-2} + \$6,000(1 + 0.06)^{-5} + \$4,000(1 + 0.06)^{-7}$$
$$P = \$14,263.75$$

Problem 3.36

Excersize 4 *You are considering buying a piece of industrial equipment to automate a part of your production process. This automation will save labor costs by as much as $35,000 per year over 10 years. The equipment costs $200,000. Should you purchase the equipment if your interest rate is 12%?*

In this case, you're looking to see if future annuity "payments" will be greater then the present payment. To compare between the two, you must find the present worth of the future annuity payments.

1. Converting annuity worth to present worth:
$$P = \$35,000(P/A, 12\%, 10)$$
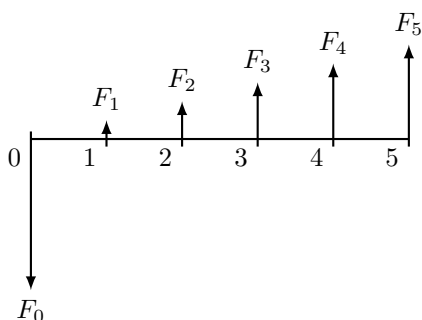
2. Setting up equation and finding P:
$$P = A\left(\frac{(1 + i)^N - 1}{i(1 + i)^N}\right)$$
$$P = \$35,000\left(\frac{(1 + 0.12)^{10} - 1}{0.12(1 + 0.12)^{10}}\right)$$
$$P = \$197,757.81 < \$200,000 \text{ - You should not buy the equipment.}$$

Problem 3.42

Excersize 5 *How much do you have to deposit now in your savings account that earns a 6% annual interest if you want to withdraw the annual payment series in the figure below?*

Where:

$$F_0 \text{ is P, } F_1 = \$1,000, F_2 = \$1,250, F_2 = \$1,500, F_3 = \$1,750, \text{ and } F_4 = \$2,000$$

Firstly, this looks like a linear gradient series. Knowing that all gradient series are made of the base annunity and gradient annunity:

1. Setting up composition for annunities:
$$P = P_1 + P_2,$$

   Where P_1 represents the base annunity and P_2 represents the gradient annunity.

2. Converting base annunity to present worth:

$$P_1 = \$1,000(P/A, 6\%, 5) \qquad P_1 = F_1 \left(\frac{(1+i)^N - 1}{i(1+i)^N} \right)$$

$$P_1 = \$1,000 \left(\frac{(1+0.06)^5 - 1}{0.06(1+0.06)^5} \right)$$

$$P_1 = \$4,212.36$$

3. Converting gradient annunity to present worth:

$$P_2 = \$250(P/G, 6\%, 5)$$

$$P_2 = G \left(\frac{(1+i)^N - iN - 1}{i^2(1+i)^N} \right)$$

$$P_2 = 250 \left(\frac{(1+0.06)^5 - 0.06 * 5 - 1}{0.06^2(1+0.06)^5} \right)$$

$$P_2 = \$1,983.64$$

4. Adding base and gradient annunities together:

$$P = P_1 + P_2$$
$$P = \$4,212.36 + \$1,983.64$$
$$P = \$6,196$$

IND E 250 HW 2

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Problem xxx