ASTR 5900 Final Project: Hydro1D

Anthony Burrow and Sarah Stangl

1 Introduction and Goals

This project is aimed to model the infall and subsequent shockwave exhibited by core-collapse supernovae (CCSNe). In modern studies of all types of SNe (including Type Ia SNe, CCSNe, etc.), one of the most prominent goals involved is to achieve a better understanding of the so-called progenitor problem. To briefly summarize, we wish for the ability to understand the different possible scenarios of a supernova progenitor as well as to diagnose the scenario with observable features from the subsequent explosion. If this could be achieved, this would provide insight into stellar evolution as a whole and minimize a large source of uncertainty in many interesting predictions in astronomy made by using these SNe.

The goal of this project is only to model the dynamics of the collapse of a massive ($\sim 10\ M_\odot$) star. This is done using a 1D Lagrangian-frame hydrodynamical code that solves equations of mass conservation, momentum conservation, and energy conservation, with the addition of an equation of state. Our secondary and hopeful goal is to find a distinguishable shock event that should occur when a bulk of infalling material rapidly rebounds off of a highly dense and degenerate core of neutrons. This shockwave is what signifies that a CCSN occurs.

2 Methods

To generate the aforementioned model, we first assume a star of mass $10\ M_\odot$ coming directly from hydrostatic equilibrium. The star's collapse would be caused by a sudden decrease in pressure. The model is separated into a set number of zones, all with equal mass. Because this is a Lagrangian hydrocode, the zones always have the same constant mass.

The initial conditions (radius, density, pressure) are all solved for hydrostatic equilibrium through the Lane-Emden equation. In order to solve this equation we assume the star—which will be treated as a fully degenerate Fermi gas until nuclear degeneracy—can be modeled as an $n = 3$ polytrope. By doing this, we therefore assume an equation of state of $P = K_{4/3}\,\rho^{4/3}$, where P is pressure, ρ is density, and $K_{4/3} = 1.2 \times 10^{15}$ in CGS units (Arnett, 1966). This equation of state is for densities under nuclear degeneracy $\rho_{nuc} = 2.3 \times 10^{14}$ g cm^{-3}. We also assume an intial central density of $\rho_c = 1.0 \times 10^7$ g cm^{-3}.

Once these initial conditions have been set, the dynamics of density, pressure, and velocities are calculated by following the exact difference equations found in the Appendix of Arnett (1966). However, because of time constraints, we simplify the calculation heavily by not accounting for radiation or neutrinos as a source of energy. Because we assume a polytropic equation of state, our model also has zero temperature, and no temperature calculations are needed. From the initial conditions, we do 10,000 iterations of calculating zone boundary velocities (U_j), boundary positions (R_j), specific volume of each zone ($V_{j+1/2} = 1/\rho_{j+1/2}$), and zone pressure ($P_{j+1/2}$) in that order. These are calculated using simple difference equations that describe the equations of mass, momentum, and energy conservation, and they are all listed in the Appendix of Arnett (1966).

From an initial time step (we typically us $\Delta t^0 = 0.001$ second), each iteration calculates a new maximum time step that holds causality due to sound velocity. This is done in the same way as Colgate & H. (1964) by finding the maximum value (for any j^{th} zone) of

$$\Delta t^{n+1/2} = \frac{0.02 \cdot V_{j+1/2}^n \Delta t^{n-1/2}}{|V_{j+1/2}^n - V_{j-1/2}^n|},$$
$$\Delta t^n = \frac{1}{2}(\Delta t^{n+1/2} + \Delta t^{n-1/2}).$$

We found that this keeps causality well for our purposes; in other words, this means no negative densities, etc. appear due to over-correction to the boundary positions.

To reiterate, the pressure we assume is due solely to electron pressure, i.e.

$$P_j = P(e^-)_j = K_{4/3}\ (1/V_j)^{4/3}.$$

However, to model neutron degeneracy pressure, for any zone with densities above nuclear density ρ_{nuc}, this pressure is ammended to

$$P_j = P(e^-)_j + K_3\ (1/V_j)^3,$$

where

$$K_3 = \frac{K_0}{27\rho_c^2 m_n}$$

with $K_0 = 140$ MeV and $m_n = 1.674920 \times 10^{-24}$ g is the mass of a neutron. This is based on the cold nuclear pressure suggested by and $K_0 = K_0(0.33)$ found in Baron et al. (1985) with $\gamma = 3$.

Finally, to cause a collapse, we calculate our initial hydrostatic condition with $K'_{4/3} = 1.1 \cdot K_{4/3}$ within the equation of state, and then lower the pressure by using $K_{4/3}$ in our subsequent iterations instead.

3 Results

* check hydrostatic equilibrium (no K change)
* check P=0 case (freefall)
* collapse case (K changes by -10

4 Conclusion

References

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