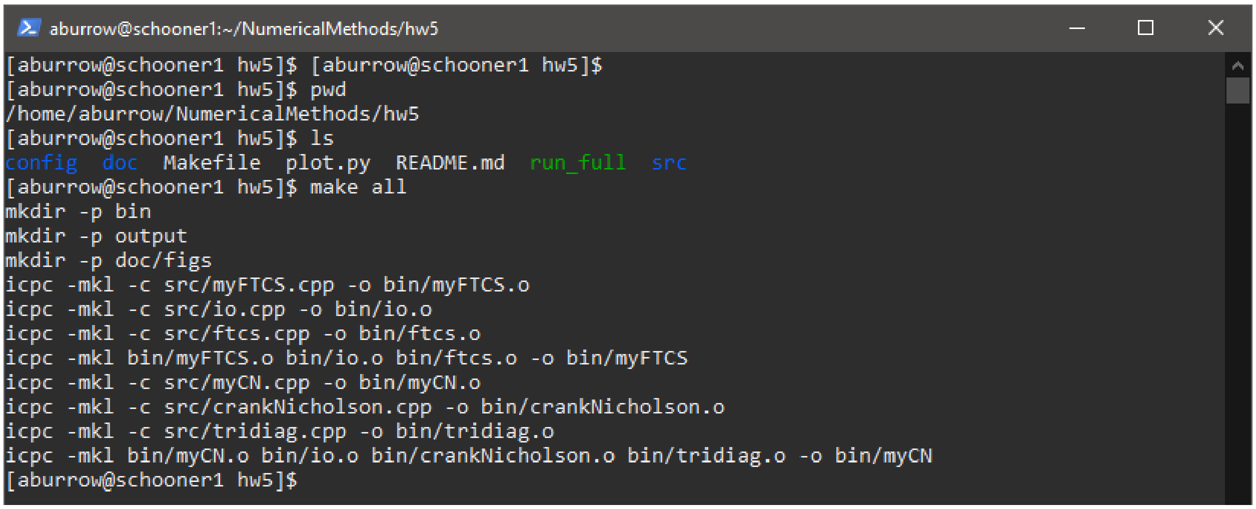
Assignment 5: Results/Description

Problem 1

For this problem I have written two programs (`myFTCS` and `myCN`) to solve the following problem. These may be compiled all together with a `make all` as shown below.



```
aburrow@schooner1:~/NumericalMethods/hw5
[aburrow@schooner1 hw5]$ [aburrow@schooner1 hw5]$
[aburrow@schooner1 hw5]$ pwd
/home/aburrow/NumericalMethods/hw5
[aburrow@schooner1 hw5]$ ls
config  doc  Makefile  plot.py  README.md  run_full  src
[aburrow@schooner1 hw5]$ make all
mkdir -p bin
mkdir -p output
mkdir -p doc/figs
icpc -mkl -c src/myFTCS.cpp -o bin/myFTCS.o
icpc -mkl -c src/io.cpp -o bin/io.o
icpc -mkl -c src/ftcs.cpp -o bin/ftcs.o
icpc -mkl bin/myFTCS.o bin/io.o bin/ftcs.o -o bin/myFTCS
icpc -mkl -c src/myCN.cpp -o bin/myCN.o
icpc -mkl -c src/crankNicholson.cpp -o bin/crankNicholson.o
icpc -mkl -c src/tridiag.cpp -o bin/tridiag.o
icpc -mkl bin/myCN.o bin/io.o bin/crankNicholson.o bin/tridiag.o -o bin/myCN
[aburrow@schooner1 hw5]$
```

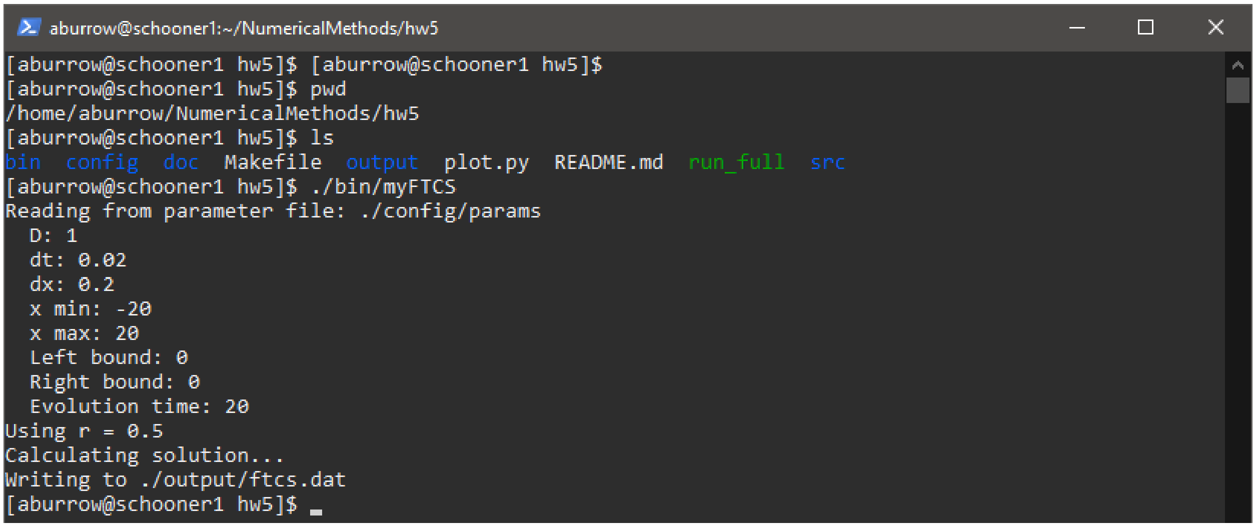
This generates executables `myFTCS` and `myCN` in the "./bin" directory, which may be run one at a time. There is also a `plot.py` in the root which may be run to generate plots from the output of the executables. However, to run all these programs at once, I have also included a `run_full` bash script that may be executed for convenience.

In this problem, I use (a) the FTCS method and (b) the Crank-Nicolson method to solve the diffusion equation

$$\frac{\partial n}{\partial t} = \frac{\partial^2 n}{\partial x^2}.$$

(a)

Below I run the program that demonstrates solving our differential equation using the FTCS method:



```
aburrow@schooner1:~/NumericalMethods/hw5
[aburrow@schooner1 hw5]$ [aburrow@schooner1 hw5]$
[aburrow@schooner1 hw5]$ pwd
/home/aburrow/NumericalMethods/hw5
[aburrow@schooner1 hw5]$ ls
bin  config  doc  Makefile  output  plot.py  README.md  run_full  src
[aburrow@schooner1 hw5]$ ./bin/myFTCS
Reading from parameter file: ./config/params
  D: 1
  dt: 0.02
  dx: 0.2
  x min: -20
  x max: 20
  Left bound: 0
  Right bound: 0
  Evolution time: 20
Using r = 0.5
Calculating solution...
Writing to ./output/ftcs.dat
[aburrow@schooner1 hw5]$
```

This was equation was solved over $t \in [0, 20]$ with the boundary conditions given in the problem. First, I solved this using $\Delta t = 0.2$ ($r = 0.5$), and the result is shown in Figure 1. We see clear diffusion in this solution, so the solver works as intended.

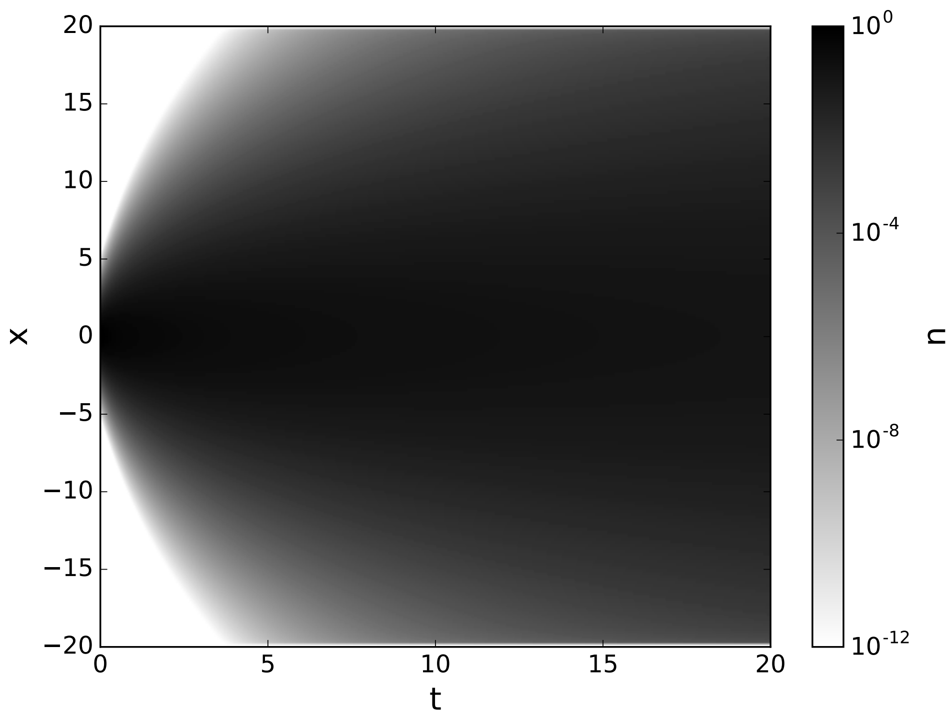


Figure 1: Solution using the FTCS method with $r = 0.5$.

Next, I change my time step to be $\Delta t = 0.266$ ($r = 0.665 > 0.5$), and the result is given in Figure 2. We see clearly that a divergence has occured as expected, due to the stability condition of the FTCS method. The solution apparently becomes completely unstable at about t = 2.5.

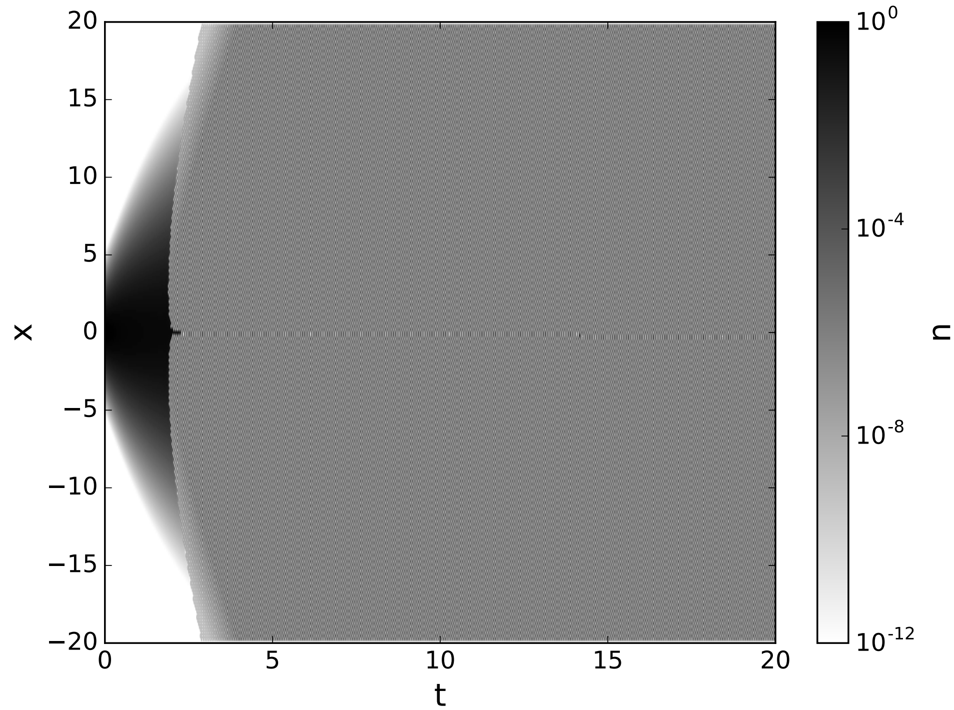
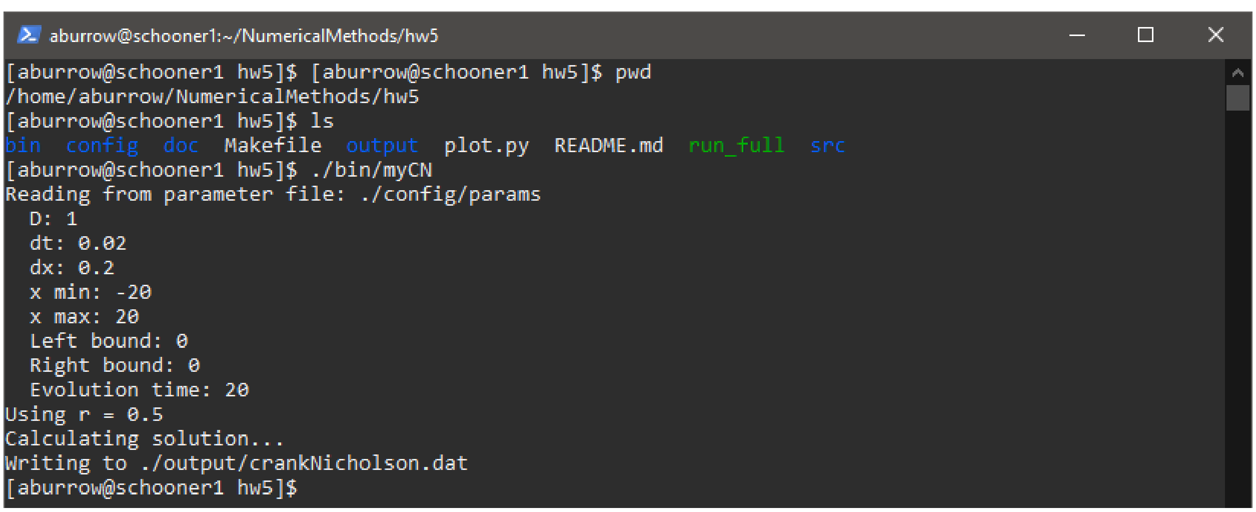


Figure 2: Solution using the FTCS method with $r > 0.5$.

(b)

Below I run the program that demonstrates solving our differential equation using the Crank-Nicolson method:



```
aburrow@schooner1:~/NumericalMethods/hw5
[aburrow@schooner1 hw5]$ [aburrow@schooner1 hw5]$ pwd
/home/aburrow/NumericalMethods/hw5
[aburrow@schooner1 hw5]$ ls
bin  config  doc  Makefile  output  plot.py  README.md  run_full  src
[aburrow@schooner1 hw5]$ ./bin/myCN
Reading from parameter file: ./config/params
  D: 1
  dt: 0.02
  dx: 0.2
  x min: -20
  x max: 20
  Left bound: 0
  Right bound: 0
  Evolution time: 20
Using r = 0.5
Calculating solution...
Writing to ./output/crankNicholson.dat
[aburrow@schooner1 hw5]$
```

For this method I use the same boundary conditions as before over the same time period. Again I calculate the solution with $\Delta t = 0.2$, and the result (shown in **??**) is the same as that in Figure 1.

Figure 3: Solution using the Crank-Nicolson method with $r = 0.5$.

However, when I use $\Delta t = 0.266$ to calculate the solution (shown in **??**), we do not see any unstability, unlike for the FTCS case. Instead, we find the intended solution. I was not able to find any reasonable value of Δt that led to an unstable solution with the Crank-Nicolson method. In general, this should not happen, as it is proven to have no

Figure 4: Solution using the Crank-Nicolson method with $r > 0.5$.