P	$::=$	$R\ \overline{C}\ e$	*program*
R	$::=$	$\mathtt{m} \leq \mathtt{m}'$	*mode order*
C	$::=$	$\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\ \}$	*class*
F	$::=$	$T\ \mathtt{fd} = e$	*field*
M	$::=$	$T\ \mathtt{md}(\overline{T}\ \overline{\mathtt{x}})\{e\}$	*method*
A	$::=$	e	*attributor*
e	$::=$	$\mathtt{x} \mid e.\mathtt{fd} \mid \textbf{new}\ \mathtt{c}\langle\iota\rangle \mid e.\mathtt{md}(\overline{e})$	*expressions*
	$\mid$	$(T)e \mid \textbf{snapshot}\ e\ [\eta,\eta] \mid e \triangleright \eta$	
	$\mid$	$\{\overline{\mathtt{m} : e}\}^{T}$	

Figure 1. Syntax

T	$::=$	$\mathtt{c}\langle\iota\rangle \mid \textbf{mcase}\langle T\rangle$	*programmer type*
ι	$::=$	$\overline{\eta} \mid ?, \overline{\eta}$	*object mode parameter list*
η	$::=$	$\mathtt{m} \mid \mathtt{mt} \mid \top \mid \bot$	*static mode*
μ	$::=$	$\eta \mid ?$	*mode*
$\mathtt{mt}$			*mode type variable*
$?$			*dynamic mode type*
ω	$::=$	$\eta \leq \mathtt{mt} \leq \eta'$	*constrained mode*
Δ	$::=$	$? \rightarrow \omega, \Omega \mid \Omega$	*class mode parameter list*
Ω	$::=$	$\overline{\omega}$	*constrained mode list*
τ	$::=$	$T \mid \exists\omega.\tau \mid \mathtt{modev}$	*type*
$\mathtt{K}$	$::=$	$\overline{\eta \leq \eta'}$	*constraints*

Figure 2. Type Elements

$$\text{(WF-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Omega' \cdots \in P\\ \mathtt{eparam}(\Omega') = \iota' \qquad \mathtt{cons}(\Omega') = \mathtt{K}' \qquad \mathtt{K} \models \mathtt{K}'\{\eta/\iota'\}\end{array}}{\mathtt{K} \vdash_{\mathtt{wft}} \mathtt{c}\langle\overline{\eta}\rangle}$$

$$\text{(WF-ClassDyn)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ ? \rightarrow \omega, \Omega' \cdots \in P \qquad \mathtt{eparam}(? \rightarrow \omega, \Omega') = \iota'\\ \mathtt{cons}(\Omega') = \mathtt{K}' \qquad \mathtt{K} \models \mathtt{K}'\{\eta/\iota'\}\end{array}}{\mathtt{K} \vdash_{\mathtt{wft}} \mathtt{c}\langle ?, \overline{\eta}\rangle}$$

$$\text{(WF-Top)}\ \mathtt{K} \vdash_{\mathtt{wft}} \mathtt{Object}\langle\eta\rangle$$

$$\text{(WF-MCase)}\ \frac{\mathtt{K} \vdash_{\mathtt{wft}} T}{\mathtt{K} \vdash_{\mathtt{wft}} \textbf{mcase}\langle T\rangle}$$

Figure 3. Type Well-Formedness

$$\text{(WF-Empty)}\ P \vdash_{\mathtt{wfe}} \epsilon$$

$$\text{(WF-ESpec)}\ \frac{P \vdash_{\mathtt{wfe}} \Omega \qquad \eta \leq \eta'}{P \vdash_{\mathtt{wfe}} \Omega, \eta \leq \mathtt{mt} \leq \eta'}$$

$$\text{(WF-TSpec)}\ \frac{P \vdash_{\mathtt{wfe}} \omega, \Omega}{P \vdash_{\mathtt{wfe}} ? \rightarrow \omega, \Omega}$$

Figure 4. Environment Well-Formedness

$$\text{(FD-Object)}\ \mathtt{fields}(\mathtt{Object}\langle\eta\rangle) = \bullet$$

$$\text{(FD-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{T\ \mathtt{fd} = e}\ \overline{M}\ A\}\\ \mathtt{eparam}(\Delta) = \iota' \qquad \mathtt{fields}(\mathtt{c}'\{\iota/\iota'\}) = \overline{T_0}\ \overline{\mathtt{fd}_0} = \overline{e_0}\end{array}}{\mathtt{fields}(\mathtt{c}\langle\iota\rangle) = \overline{T_0}\ \overline{\mathtt{fd}_0} = \overline{e_0},\ \overline{T\{\iota/\iota'\}}\ \overline{\mathtt{fd}} = \overline{e\{\iota/\iota'\}}}$$

$$\text{(MT-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\}\\ T\ \mathtt{md}(\overline{T}\ \overline{\mathtt{x}})\ \{\ e\ \}\ \in \overline{M} \qquad \mathtt{eparam}(\Delta) = \iota'\end{array}}{\mathtt{mtype}(\mathtt{md}, \mathtt{c}\langle\iota\rangle) = (\overline{T} \rightarrow T)\{\iota/\iota'\}}$$

$$\text{(MT-Super)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\}\\ \mathtt{md} \notin \overline{M} \qquad \mathtt{eparam}(\Delta) = \iota'\end{array}}{\mathtt{mtype}(\mathtt{md}, \mathtt{c}\langle\iota\rangle) = \mathtt{mtype}(\mathtt{md}, \mathtt{c}'\{\iota/\iota'\})}$$

$$\text{Override}\ \frac{\mathtt{mtype}(\mathtt{md}, T) = \overline{T'} \rightarrow T_0' \qquad \mathtt{K} \vdash T_0 <: T_0'}{\mathtt{override}(\mathtt{md}, T, \mathtt{K}, \overline{T} \rightarrow T_0)}$$

$$\text{(MB-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\}\\ \mathtt{eparam}(\Delta) = \iota' \qquad T\ \mathtt{md}(\overline{T}\ \overline{\mathtt{x}})\{e\} \in \overline{M}\end{array}}{\mathtt{mbody}(\mathtt{md}, \mathtt{c}\langle\iota\rangle) = \overline{\mathtt{x}}.e\{\iota/\iota'\}}$$

$$\text{(MB-Super)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\}\\ \mathtt{eparam}(\Delta) = \iota' \qquad \mathtt{md} \notin \overline{M}\end{array}}{\mathtt{mbody}(\mathtt{md}, \mathtt{c}\langle\iota\rangle) = \mathtt{mbody}(\mathtt{md}, \mathtt{c}'\{\iota/\iota'\})}$$

$$\text{(AB-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\}\\ \mathtt{eparam}(\Delta) = \iota' \qquad A = e\end{array}}{\mathtt{abody}(\mathtt{c}\langle\iota\rangle) = e\{\iota/\iota'\}}$$

Figure 5. FJ Functions

$$\text{(T-Program)}\ \frac{R\ \text{form a latice} \qquad \varnothing \vdash e \qquad \overline{C}\ \text{OK}}{R\ \overline{C}\ e\ \text{OK}}$$

$$\text{(T-Class)}\ \frac{\begin{array}{c}\iota = \mathtt{iparam}(\Delta)\\ \mathtt{K} = \mathtt{cons}(\Delta) \qquad \mathtt{ithis}(\Delta) = \mathtt{mode}(\mathtt{c}') \qquad \overline{M}\ \text{OK IN}\ \mathtt{c}, \Delta\\ A\ \text{OK IN}\ \mathtt{c}, \Delta \qquad \textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\}\ \text{FJ OK}\end{array}}{\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\}\ \text{OK}}$$

$$\text{(T-Attributor)}\ \frac{\begin{array}{c}\iota = \mathtt{iparam}(\Delta)\\ \mathtt{K} = \mathtt{cons}(\Delta) \qquad A = e \qquad \Delta; \textbf{this} : \mathtt{c}\langle\iota\rangle \vdash e : \mathtt{modev}\end{array}}{A\ \text{OK IN}\ \mathtt{c}, \Delta}$$

$$\text{(T-Method)}\ \frac{\begin{array}{c}\iota = \mathtt{iparam}(\Delta)\\ \mathtt{K} = \mathtt{cons}(\Delta) \qquad \overline{x} : \overline{T}; \textbf{this} : \mathtt{c}\langle\iota\rangle; \mathtt{K} \vdash e : T\\ \mathtt{override}(\mathtt{md}, \mathtt{c}\langle\iota\rangle, \mathtt{K}, \overline{T} \rightarrow T)\end{array}}{T\ \mathtt{md}(\overline{T}\ \overline{x})\{\ e\ \}\ \text{OK IN}\ \mathtt{c}\ \Delta}$$

Figure 6. Class Typing

$$\text{(T-Var)}\ \frac{}{\Gamma;\mathtt{K}\vdash \mathtt{x} : \Gamma(\mathtt{x})}$$

$$\text{(T-New)}\ \frac{\begin{array}{c}\iota = ?, \iota' \text{ iff } \textbf{class}\ \mathtt{c}\ \Delta' \cdots \in P \text{ and } \mathtt{ethis}(\Delta') = ?\\ \iota \neq ?, \iota' \text{ iff } \textbf{class}\ \mathtt{c}\ \Delta' \cdots \in P \text{ and } \mathtt{ethis}(\Delta') \neq ?\\ \mathtt{K} \models \mathtt{cons}(\Delta')\end{array}}{\Gamma;\mathtt{K}\vdash\ \textbf{new}\ \mathtt{c}\langle\iota\rangle : \mathtt{c}\langle\iota\rangle}$$

$$\text{(T-Cast)}\ \frac{\Gamma;\mathtt{K}\vdash e : \mathtt{c}\langle\iota\rangle}{\Gamma;\mathtt{K}\vdash (T)e : T}$$

$$\text{(T-Msg)}\ \frac{\begin{array}{c}\Gamma;\mathtt{K}\vdash\ e : T \qquad \mathtt{mtype}(\mathtt{md}, T) = \overline{T} \rightarrow T'\\ \Gamma;\mathtt{K}\vdash \overline{e} : \overline{T} \qquad \Gamma;\mathtt{K}\vdash\ \textbf{this} : T_{this}\\ \mathtt{K}\models\{\mathtt{mode}(T) \leq \mathtt{mode}(T_{this})\} \qquad \mathtt{mode}(T) \neq ?\end{array}}{\Gamma;\mathtt{K}\vdash e.\mathtt{md}(\overline{e}) : T'}$$

$$\text{(T-Field)}\ \frac{\begin{array}{c}\Gamma;\mathtt{K}\vdash e : T \qquad \Gamma;\mathtt{K}\vdash \textbf{this} : T_{this} \qquad \mathtt{fields}(T) = \overline{T}\ \overline{\mathtt{fd}}\\ \mathtt{K}\models\{\mathtt{mode}(T) \leq \mathtt{mode}(T_{this})\} \qquad \mathtt{mode}(T) \neq ?\end{array}}{\Gamma;\mathtt{K}\vdash e.\mathtt{fd}_i : T_i}$$

$$\text{(T-Snapshot)}\ \frac{\Gamma;\mathtt{K}\vdash e : \mathtt{c}\langle ?, \iota\rangle \qquad \omega = \eta_1 \leq \mathtt{mt} \leq \eta_2}{\Gamma;\mathtt{K}\vdash \textbf{snapshot}\ e\ [\eta_1, \eta_2] : \exists\omega.\mathtt{c}\langle\mathtt{mt}, \iota\rangle}$$

$$\text{(T-MCase)}\ \frac{\overline{\mathtt{m}} = \mathtt{modes}(P) \qquad \Gamma;\mathtt{K}\vdash e_i : T \text{ for all } i}{\Gamma;\mathtt{K}\vdash \{\overline{\mathtt{m} : e}\}^T : \textbf{mcase}\langle T\rangle}$$

$$\text{(T-ElimCase)}\ \frac{\Gamma;\mathtt{K}\vdash e : \textbf{mcase}\langle T\rangle \qquad \eta \in \mathtt{modes}(P) \text{ or } \eta \text{ appears in } \mathtt{K}}{\Gamma;\mathtt{K}\vdash e \triangleright \eta : T}$$

$$\text{(T-ModeValue)}\ \frac{\mathtt{m} \in \mathtt{modes}(P)}{\Gamma;\Delta\vdash \mathtt{m} : \mathtt{modev}}$$

$$\text{(T-Sub)}\ \frac{\Gamma;\mathtt{K}\vdash e : \tau \qquad \mathtt{K}\vdash \tau <: \tau'}{\Gamma;\mathtt{K}\vdash e : \tau'}$$

Figure 7. Expression Typing

$$\text{(S-Dynamic)}\ \frac{}{\mathtt{K}\vdash \mathtt{c}\langle\mu;\overline{\eta}\rangle <: \mathtt{c}\langle ?;\overline{\eta}\rangle}$$

$$\text{(S-Mcase)}\ \frac{\mathtt{K}\vdash \tau <: \tau'}{\mathtt{K}\vdash \textbf{mcase}\langle\tau\rangle <: \textbf{mcase}\langle\tau'\rangle}$$

$$\text{(S-Exists)}\ \frac{\begin{array}{c}\omega = \eta_1 \leq \mathtt{mt} \leq \eta_2\\ \mathtt{K} = \{\eta_1 \leq \mathtt{mt}, \mathtt{mt} \leq \eta_2\} \cup \mathtt{K}' \qquad \mathtt{mt} \text{ does not appear in } \mathtt{K}'\end{array}}{\mathtt{K}\vdash \exists\omega.\tau <: \tau}$$

$$\text{(S-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}' \cdots \in P\\ \mathtt{eparam}(\Delta) = \iota' \qquad \mathtt{K} = \mathtt{cons}(\Delta)\end{array}}{\mathtt{K}\vdash \mathtt{c}\langle\iota\rangle <: \mathtt{c}'\{\iota/\iota'\}}$$

Figure 8. Subtyping (reflexivity and transitivity rules are omitted.)

$$\text{(M-Sub)}\ \frac{\{\eta \leq \mathtt{mt}, \mathtt{mt} \leq \eta', \eta \leq \eta'\} \in \mathtt{K}}{\mathtt{K}\models\{\eta \leq \mathtt{mt}, \mathtt{mt} \leq \eta', \eta \leq \eta'\}}$$

Figure 9. Submoding

$$\begin{array}{lcll}
e & ::= & \cdots \mid \textbf{check}(e, \mathtt{m}, \mathtt{m}', e) & \textit{runtime expressions}\\
 & \mid & \textbf{let}\ \mathtt{x} = e\ \textbf{in}\ e & \\
\mathbf{E} & ::= & \odot \mid \mathbf{E}.\mathtt{md}(\overline{e}) \mid o.\mathtt{md}(\ldots, o, \mathbf{E}, e, \ldots) & \textit{evaluation context}\\
 & \mid & (T)\mathbf{E} \mid \mathbf{E}.\mathtt{fd} & \\
 & \mid & \textbf{snapshot}\ \mathbf{E}\ [\mathtt{m}, \mathtt{m}'] & \\
 & \mid & \{\ldots\mathtt{m} : v; \mathtt{m} : \mathbf{E}; \mathtt{m} : e\ldots\} \mid \mathbf{E} \triangleright \mu & \\
 & \mid & \textbf{check}(\mathbf{E}, \mathtt{m}, \mathtt{m}', \mathbf{E}) & \\
 & \mid & \mathtt{obj}(\alpha, \mathtt{c}\langle\iota\rangle, \ldots v, \mathbf{E}, e\ldots) & \\
 & \mid & \textbf{let}\ \mathtt{x} = \mathbf{E}\ \textbf{in}\ e &
\end{array}$$

Figure 10. Run-Time Elements

$$\text{(T-Obj)}\ \frac{\Gamma;\mathtt{K}\vdash \overline{e} : \overline{T} \qquad \mathtt{fields}(\mathtt{c}\langle\iota\rangle) = \overline{T}\ \overline{\mathtt{fd}} = \overline{e}}{\Gamma;\mathtt{K}\vdash \mathtt{obj}(\alpha, \mathtt{c}\langle\iota\rangle, \overline{e}) : \mathtt{c}\langle\iota\rangle}$$

$$\text{(T-Check)}\ \frac{\Gamma;\mathtt{K}\vdash e_1 : \mathtt{modev} \qquad \mathtt{m} = \mathtt{emode}(e_1) \qquad \Gamma;\mathtt{K}\vdash e_2 : \mathtt{c}\langle\mu, \iota\rangle}{\Gamma;\mathtt{K}\vdash \textbf{check}(e_1, \mathtt{m}_1, \mathtt{m}_2, e_2) : \mathtt{c}\langle\mathtt{m}, \iota\rangle}$$

Figure 11. Auxiliary Run-time Expression Typing

(R-New)	**new** $\mathtt{c}\langle\iota\rangle$	$\xrightarrow{\mathtt{m}}$	$\mathtt{obj}(\alpha, \mathtt{c}\langle\iota\rangle, \mathtt{init}(P, \mathtt{c}))$	if α is $fresh$
(R-Cast)	$(\tau_0)\, o$	$\xrightarrow{\mathtt{m}}$	o	if $\tau <: \tau_0$
(R-Msg)	$o.\mathtt{md}(\overline{v}')$	$\xrightarrow{\mathtt{m}}$	$\mathbf{E}_{\mathtt{m}'}[\, e\{\overline{v}'/\overline{\mathtt{x}}\}\{o/\mathbf{this}\}\,]$	if $\mu \le \mathtt{m}, \mathtt{m}' = \mathtt{emode}(o)$
(R-Field)	$o.\mathtt{fd}_i$	$\xrightarrow{\mathtt{m}}$	v_i	if $\mu \le \mathtt{m}$
(R-Snapshot1)	**snapshot** $o\ [\mathtt{m}_1, \mathtt{m}_2]$	$\xrightarrow{\mathtt{m}}$	$\mathbf{check}(e_a\{o/\mathbf{this}\}, \mathtt{m}_1, \mathtt{m}_2, o)$	if $\mu = ?$, **class** $\mathtt{c}\ \cdots\ \{\ \cdots\ A\ \} \in P$, $\mathtt{abody}(\mathtt{c}\langle ?, \iota\rangle) = e_a$
(R-Snapshot2)	**snapshot** $o\ [\mathtt{m}_1, \mathtt{m}_2]$	$\xrightarrow{\mathtt{m}}$	$\mathbf{check}(\mathtt{m}', \mathtt{m}_1, \mathtt{m}_2, o)$	if $\mu = \mathtt{m}'$, **class** $\mathtt{c}\ \cdots\ \{\ \cdots\ A\ \} \in P$
(R-Check)	$\mathbf{check}(v, \mathtt{m}_1, \mathtt{m}_2, o)$	$\xrightarrow{\mathtt{m}}$	$\mathtt{obj}(\alpha', \mathtt{c}\langle\mathtt{m}', \iota\rangle, \overline{v})$	if $\mathtt{emode}(v) = \mathtt{m}', \mathtt{m}_1 \le \mathtt{m}' \le \mathtt{m}_2, \alpha'$ is $fresh$
(R-McaseProj)	$\{\overline{\mathtt{m} : v}\}^T \rhd \mathtt{m}_j$	$\xrightarrow{\mathtt{m}}$	v_j	
(R-Context)	$\mathbf{E}_{\mathtt{m}}[\, e_1\,]$	$\xrightarrow{\mathtt{m}'}$	$\mathbf{E}_{\mathtt{m}}[\, e_2\,]$	if $e_1 \xrightarrow{\mathtt{m}'} e_2$

for all rules: $o = \mathtt{obj}(\alpha,\ T, \overline{v},), \mathtt{mbody}(\mathtt{md},\ T) = \overline{\mathtt{x}}.e,\ T = \mathtt{c}\langle\mu, \iota\rangle$

Figure 12. Reduction Rules

$\mathtt{modes}(P)$	$\triangleq$	$\overline{\mathtt{m} \le \mathtt{m}'}$	
$\mathtt{mode}(\mathtt{c}\langle\overline{\iota}\rangle)$	$\triangleq$	μ	if $\iota = \mu, \overline{\eta}$
$\mathtt{attr}(\mathtt{c}\langle\iota\rangle)$	$\triangleq$	$A\{\iota/\mathtt{eparam}(\Delta)\}$	if **class** $\mathtt{c}\ \Delta$ **extends** $\tau\ \{\ \overline{F}\ \overline{M}\ A\}\ \in\ P$
$\mathtt{eparam}(\overline{\eta \le \mathtt{mt} \le \eta'})$	$\triangleq$	$\overline{\mathtt{mt}}$	
$\mathtt{eparam}(? \to \omega, \Omega)$	$\triangleq$	$\mathtt{mt} \cup \mathtt{eparam}(\Omega)$	if $\omega = \eta \le \mathtt{mt} \le \eta'$
$\mathtt{ethis}(\Omega)$	$\triangleq$	$\mathtt{mt}$	if $\mathtt{eparam}(\Omega) = \mathtt{mt}$
$\mathtt{init}(P, \mathtt{c})$	$\triangleq$	$\mathtt{init}(\mathtt{c}') \cup \overline{e\{\iota/\mathtt{eparam}(\Delta)\}}$	if **class** $\Delta\ \mathtt{c}$ **extends** $\mathtt{c}'\ \overline{\tau\ \mathtt{fd} = e} \in P$
$\mathtt{init}(P, \mathtt{c})$	$\triangleq$	ϵ	if $\mathtt{c} = \mathtt{Object}$
$\mathtt{eargs}(\mathtt{c}\langle\iota\rangle)$	$\triangleq$	ι	
$\mathtt{eargs}(\exists\omega.\tau)$	$\triangleq$	$\mathtt{eargs}(\tau)$	
$\mathtt{cons}(\eta \le \mathtt{mt} \le \eta')$	$\triangleq$	$\bigcup\{\eta \le \mathtt{mt}, \mathtt{mt} \le \eta'\}$	
$\mathtt{cons}(? \to \omega, \Omega)$	$\triangleq$	$\{\eta \le \mathtt{mt}, \mathtt{mt} \le \eta'\} \cup \mathtt{cons}(\Omega)$	if $\omega = \eta \le \mathtt{mt} \le \eta'$

We require $\overline{\mathtt{m}}$ as a lattice. We use $\perp$ and $\top$ to represent the bottom and top of $\overline{\mathtt{m}}$ respectively.
We define $\mathtt{init}(P, \mathtt{c})$ as $\mathtt{init}(P, \mathtt{c}') \cup \overline{e}$ if **class** $\mathtt{c}$ **extends** $\mathtt{c}'\ \overline{\tau\ \mathtt{fd} = e} \in P$ or ϵ if $\mathtt{c} = \mathtt{Object}$.

Figure 13. Compile Functions

$\mathtt{emode}(\mathtt{m})$	$\triangleq$	$\mathtt{m}$
$\mathtt{emode}(\mathtt{obj}(\mathtt{c}\langle\iota\rangle, \overline{v},))$	$\triangleq$	$\mathtt{mode}(\mathtt{c}\langle\iota\rangle)$

Figure 14. Runtime Functions