

P	$::=$	$\overline{R} \ \overline{C} \ e$	<i>program</i>
R	$::=$	$\mathbf{m} \leq \mathbf{m}'$	<i>mode order</i>
C	$::=$	$\mathbf{class} \ c \ \Delta \ \mathbf{extends} \ T\{\overline{F} \ \overline{M} \ A\}$	<i>class</i>
F	$::=$	$T \ \mathbf{fd} = e$	<i>field</i>
M	$::=$	$T \ \mathbf{md}(\overline{T} \ \overline{x})\{e\}$	<i>method</i>
A	$::=$	e	<i>attributor</i>
e	$::=$	$\mathbf{x} \mid e.\mathbf{fd} \mid \mathbf{new} \ c\langle\iota\rangle \mid e.\mathbf{md}(\overline{e})$	<i>expressions</i>
		$(T)e \mid \mathbf{snapshot} \ e \ [\eta, \eta] \mid e \triangleright \eta$	
		$\{\overline{m} : e\}^T$	

Figure 1. Syntax

T	$::=$	$c\langle\iota\rangle \mid \mathbf{mcase}\langle T \rangle$	<i>programmer type</i>
ι	$::=$	$\overline{\eta} \mid ? , \overline{\eta}$	<i>object mode parameter list</i>
η	$::=$	$\mathbf{m} \mid \mathbf{mt} \mid \top \mid \perp$	<i>static mode</i>
μ	$::=$	$\eta \mid ?$	<i>mode</i>
\mathbf{mt}			<i>mode type variable</i>
$?$			<i>dynamic mode type</i>
ω	$::=$	$\eta \leq \mathbf{mt} \leq \eta'$	<i>constrained mode</i>
Δ	$::=$	$? \rightarrow \omega, \overline{\Omega} \mid \Omega$	<i>class mode parameter list</i>
Ω	$::=$	$\overline{\omega}$	<i>constrained mode list</i>
τ	$::=$	$T \mid \exists \omega, \tau \mid \mathbf{modev}$	<i>type</i>
K	$::=$	$\eta \leq \eta'$	<i>constraints</i>

Figure 2. Type Elements

(WF-Class)	$\frac{\mathbf{class} \ c \ \Omega' \dots \in P \quad \mathbf{eparam}(\Omega') = \iota' \quad \mathbf{cons}(\Omega') = K' \quad K \models K'\{\eta/\iota'\}}{K \vdash_{\text{wft}} c\langle\overline{\eta}\rangle}$
(WF-ClassDyn)	$\frac{\mathbf{class} \ c \ ? \rightarrow \omega, \Omega' \dots \in P \quad \mathbf{eparam}(? \rightarrow \omega, \Omega') = \iota' \quad \mathbf{cons}(\Omega') = K' \quad K \models K'\{\eta/\iota'\}}{K \vdash_{\text{wft}} c\langle?, \overline{\eta}\rangle}$
(WF-Top)	$K \vdash_{\text{wft}} \mathbf{Object}\langle\eta\rangle$
(WF-MCase)	$\frac{K \vdash_{\text{wft}} c\langle\iota\rangle}{K \vdash_{\text{wft}} \mathbf{mcase}\langle c\langle\iota\rangle \rangle}$

Figure 3. Type Well-Formedness

(WF-Empty)	$P \vdash_{\text{wfe}} \epsilon$
(WF-ESpec)	$\frac{P \vdash_{\text{wfe}} \Omega \quad \eta \leq \eta'}{P \vdash_{\text{wfe}} \Omega, \eta \leq \mathbf{mt} \leq \eta'}$
(WF-TSpec)	$\frac{P \vdash_{\text{wfe}} \omega, \Omega}{P \vdash_{\text{wfe}} ? \rightarrow \omega, \Omega}$

Figure 4. Environment Well-Formedness

(FD-Object)	$\mathbf{fields}(\mathbf{Object}\langle\eta\rangle) = \bullet$
(FD-Class)	$\frac{\mathbf{class} \ c \ \Delta \ \mathbf{extends} \ T_0\{\overline{\tau} \ \overline{\mathbf{fd}} \ \overline{M} \ A\} \quad \mathbf{eparam}(\Delta) = \iota' \quad \mathbf{fields}(T\{\iota/\iota'\}) = \overline{\tau_0} \ \overline{\mathbf{fd}_0} = \overline{e_0}}{\mathbf{fields}(c\langle\iota\rangle) = \overline{\tau_0} \ \overline{\mathbf{fd}_0} = \overline{e_0}, \tau\{\iota/\iota'\} \ \mathbf{fd} = e\{\iota/\iota'\}}$
(MT-Class)	$\frac{\mathbf{class} \ c \ \Delta \ \mathbf{extends} \ T\{\overline{F} \ \overline{M} \ A\} \quad T' \ \mathbf{md}(\overline{T} \ \overline{e}) \ e \in \overline{M} \quad \mathbf{eparam}(\Delta) = \iota'}{\mathbf{mtype}(\mathbf{md}, c\langle\iota\rangle) = (\overline{T} \rightarrow T')\{\iota/\iota'\}}$
(MT-Super)	$\frac{\mathbf{class} \ c \ \Delta \ \mathbf{extends} \ T\{\overline{F} \ \overline{M} \ A\} \quad \mathbf{md} \notin \overline{M} \quad \mathbf{eparam}(\Delta) = \iota'}{\mathbf{mtype}(\mathbf{md}, c\langle\iota\rangle) = \mathbf{mtype}(\mathbf{md}, T\{\iota/\iota'\})}$
Override	$\frac{\mathbf{mtype}(\mathbf{md}, T) = \overline{T'} \rightarrow T'_0 \quad K \vdash T_0 <: T'_0}{\mathbf{override}(\mathbf{md}, T, K, \overline{T} \rightarrow T_0)}$
(MB-Class)	$\frac{\mathbf{class} \ c \ \Delta \ \mathbf{extends} \ T\{\overline{F} \ \overline{M} \ A\} \quad \mathbf{eparam}(\Delta) = \iota' \quad \tau \ \mathbf{md}(\overline{\tau} \ \overline{x})\{e\} \in \overline{M}}{\mathbf{mbody}(\mathbf{md}, c\langle\iota\rangle) = \overline{x}.e\{\iota/\iota'\}}$
(MB-Super)	$\frac{\mathbf{class} \ c \ \Delta \ \mathbf{extends} \ c'\{\overline{F} \ \overline{M} \ A\} \quad \mathbf{eparam}(\Delta) = \iota' \quad \mathbf{md} \notin \overline{M}}{\mathbf{mbody}(\mathbf{md}, c\langle\iota\rangle) = \mathbf{mbody}(\mathbf{md}, c'\{\iota/\iota'\})}$
(AB-Class)	$\frac{\mathbf{class} \ c \ \Delta \ \mathbf{extends} \ c'\{\overline{F} \ \overline{M} \ A\} \quad \mathbf{eparam}(\Delta) = \iota' \quad A = e}{\mathbf{abody}(c\langle\iota\rangle) = e\{\iota/\iota'\}}$

Figure 5. FJ Functions

(T-Program)	$\frac{R \ \text{form a lattice} \quad \emptyset \vdash e \quad \overline{C} \ \text{OK}}{R \ \overline{C} \ e \ \text{OK}}$
(T-Class)	$\frac{\iota = \mathbf{iparam}(\Delta) \quad K = \mathbf{cons}(\Delta) \quad \mathbf{this} : c\langle\iota\rangle; K \vdash A : \mathbf{modev} \quad \mathbf{this}(\Delta) = \mathbf{mode}(T') \quad \overline{x} : \overline{T}; \mathbf{this} : c\langle\iota\rangle; K \vdash e : T \text{ for each } T \ \mathbf{md}(\overline{T} \ \overline{x})\{e\} \in \overline{M} \quad \mathbf{class} \ c \ \Delta \ \mathbf{extends} \ T'\{\overline{F} \ \overline{M} \ A\} \ \text{FJ OK}}{\mathbf{class} \ c \ \Delta \ \mathbf{extends} \ T'\{\overline{F} \ \overline{M} \ A\} \ \text{OK}}$
(T-Attributor)	$\frac{A = e \quad \Delta; \mathbf{this} : c\langle\iota\rangle \vdash e : \mathbf{modev}}{A \ \text{OK IN } c, \Delta}$
(T-Method)	$\frac{\mathbf{class} \ c \ \Delta \ \mathbf{extends} \ T\{\dots\} \ \text{OK} \quad \iota = \mathbf{iparam}(\Delta) \quad K = \mathbf{cons}(\Delta) \quad \overline{x} : \overline{\tau}; \mathbf{this} : c\langle\iota\rangle; K \vdash e : \tau \quad \mathbf{override}(\mathbf{md}, c\langle\iota\rangle, K, \overline{\tau} \rightarrow \tau)}{\tau \ \mathbf{md}(\overline{\tau} \ \overline{x})\{e\} \ \text{OK IN } c \ \Delta}$

Figure 6. Class Typing

(T-Var)	$\frac{}{\Gamma; K \vdash x : \Gamma(x)}$
(T-New)	$\frac{\begin{array}{l} \iota = ?, \iota' \text{ iff } \mathbf{class} \ c \ \Delta' \dots \in P \text{ and } \mathbf{ethis}(\Delta') = ? \\ \iota \neq ?, \iota' \text{ iff } \mathbf{class} \ c \ \Delta' \dots \in P \text{ and } \mathbf{ethis}(\Delta') \neq ? \\ K \models \mathbf{cons}(\Delta') \end{array}}{\Gamma; K \vdash \mathbf{new} \ c(\iota) : c(\iota)}$
(T-Cast)	$\frac{\Gamma; K \vdash e : c(\iota)}{\Gamma; K \vdash (T)e : T}$
(T-Msg)	$\frac{\begin{array}{l} \Gamma; K \vdash e : T \quad \mathbf{mtype}(\mathbf{md}, T) = \overline{T} \rightarrow T' \\ \Gamma; K \vdash \bar{e} : \overline{T} \quad \Gamma; K \vdash \mathbf{this} : T_{this} \\ K \models \{\mathbf{mode}(T) \leq \mathbf{mode}(T_{this})\} \quad \mathbf{mode}(T) \neq ? \end{array}}{\Gamma; K \vdash e.\mathbf{md}(\bar{e}) : T'}$
(T-Field)	$\frac{\begin{array}{l} \Gamma; K \vdash e : T \quad \Gamma; K \vdash \mathbf{this} : T_{this} \quad \mathbf{fields}(T) = \overline{T} \ \overline{\mathbf{fd}} \\ K \models \{\mathbf{mode}(T) \leq \mathbf{mode}(T_{this})\} \quad \mathbf{mode}(T) \neq ? \end{array}}{\Gamma; K \vdash e.\mathbf{fd}_i : T_i}$
(T-Snapshot)	$\frac{\Gamma; K \vdash e : c(\iota) \quad \omega = \eta_1 \leq \mathbf{mt} \leq \eta_2}{\Gamma; K \vdash \mathbf{snapshot} \ e[\eta_1, \eta_2] : \exists \omega. c(\mathbf{mt}, \iota)}$
(T-MCCase)	$\frac{\bar{\mathbf{m}} = \mathbf{modes}(P) \quad \Gamma; K \vdash e_i : T \text{ for all } i}{\Gamma; K \vdash \{\bar{\mathbf{m}} : \bar{e}\}^T : \mathbf{mcase}(T)}$
(T-ElimCase)	$\frac{\Gamma; K \vdash e : \mathbf{mcase}(T) \quad \eta \in \mathbf{modes}(P) \text{ or } \eta \text{ appears in } K}{\Gamma; K \vdash e \triangleright \eta : T}$
(T-ModeValue)	$\frac{\mathbf{m} \in \mathbf{modes}(P)}{\Gamma; \Delta \vdash \mathbf{m} : \mathbf{modev}}$
(T-Sub)	$\frac{\Gamma; K \vdash e : \tau \quad K \vdash \tau <: \tau'}{\Gamma; K \vdash e : \tau'}$

Figure 7. Expression Typing

(S-Dynamic)	$\frac{}{K \vdash c(\mu; \bar{\eta}) <: c(\iota; \bar{\eta})}$
(S-Mcase)	$\frac{K \vdash \tau <: \tau'}{K \vdash \mathbf{mcase}(\tau) <: \mathbf{mcase}(\tau')}$
(S-Exists)	$\frac{\begin{array}{l} \omega = \eta_1 \leq \mathbf{mt} \leq \eta_2 \\ K = \{\eta_1 \leq \mathbf{mt}, \mathbf{mt} \leq \eta_2\} \cup K' \quad \mathbf{mt} \text{ does not appear in } K' \end{array}}{K \vdash \exists \omega. \tau <: \tau}$
(S-Class)	$\frac{\begin{array}{l} \mathbf{class} \ c \ \Delta \text{ extends } T \dots \in P \\ \mathbf{eparam}(\Delta) = \iota' \quad K = \mathbf{cons}(\Delta) \end{array}}{K \vdash c(\iota) <: T\{\iota/\iota'\}}$

Figure 8. Subtyping (reflexivity and transitivity rules are omitted.)

(M-Sub)	$\frac{\{\eta \leq \mathbf{mt}, \mathbf{mt} \leq \eta', \eta \leq \eta'\} \in K}{K \models \{\eta \leq \mathbf{mt}, \mathbf{mt} \leq \eta', \eta \leq \eta'\}}$
---------	---

Figure 9. Submoding

e	$::=$	$\dots \mid \mathbf{check}(e, \mathbf{m}, \mathbf{m}')$	<i>runtime expressions</i>
\mathbf{E}	$::=$	$\begin{array}{l} \mid \mathbf{let} \ x = e \ \mathbf{in} \ e \\ \mid \odot \mid \mathbf{E}.\mathbf{md}(\bar{e}) \mid o.\mathbf{md}(\dots, o, \mathbf{E}, e, \dots) \\ \mid (T)\mathbf{E} \mid \mathbf{E}.\mathbf{fd} \\ \mid \mathbf{snapshot} \ \mathbf{E}[\mathbf{m}, \mathbf{m}'] \\ \mid \{\dots \mathbf{m} : v; \mathbf{m} : \mathbf{E}; \mathbf{m} : e \dots\} \mid \mathbf{E} \triangleright \mu \\ \mid \mathbf{check}(\mathbf{E}, \mathbf{m}, \mathbf{m}') \\ \mid \mathbf{obj}(\alpha, c(\iota), \dots v, \mathbf{E}, e \dots) \\ \mid \mathbf{let} \ x = \mathbf{E} \ \mathbf{in} \ e \end{array}$	<i>evaluation context</i>

Figure 10. Run-Time Elements

(T-Obj)	$\frac{\Gamma; K \vdash \bar{e} : \bar{\tau} \quad \mathbf{fields}(c(\iota)) = \bar{\tau} \ \overline{\mathbf{fd}} = \bar{e}}{\Gamma; K \vdash \mathbf{obj}(\alpha, c(\iota), \bar{e}) : c(\iota)}$
(T-Check)	$\frac{\Gamma; K \vdash e : \mathbf{modev}}{\Gamma; K \vdash \mathbf{check}(e, \mathbf{m}_1, \mathbf{m}_2) : \mathbf{modev}}$
(T-Let)	$\frac{\Gamma; K \vdash e_1 : \tau_1 \quad \Gamma, \mathbf{x} : \tau_1; K \vdash e_2 : \tau}{\Gamma; K \vdash \mathbf{let} \ x = e_1 \ \mathbf{in} \ e_2 : \tau}$

Figure 11. Auxiliary Run-time Expression Typing

(R-New)	$\text{new } c\langle\iota\rangle$	$\xRightarrow{\text{m}}$	$\text{obj}(\alpha, c\langle\iota\rangle, \text{init}(P, c))$	if α is <i>fresh</i>
(R-Cast)	$(\tau_0) o$	$\xRightarrow{\text{m}}$	o	if $\tau <: \tau_0$
(R-Msg)	$o.\text{md}(\overline{v}')$	$\xRightarrow{\text{m}}$	$\mathbf{E}_{a'}[e\{\overline{v}'/\overline{x}\}\{o/\text{this}\}]$	if $\mu \leq \text{m}, \text{m}' = \text{emode}(o)$
(R-Field)	$o.\text{fd}_i$	$\xRightarrow{\text{m}}$	v_i	if $\mu \leq \text{m}$
(R-Snapshot1)	$\text{snapshot } o \text{ } [\text{m}', \text{m}']$	$\xRightarrow{\text{m}}$	$\text{let } x = \text{check}(e_a\{o/\text{this}\}, \text{m}', \text{m}'')$ $\text{in obj}(\alpha', c\langle x, \iota \rangle, \overline{v},)$	if $\mu = ?, \text{class } c \cdots \{ \cdots A \} \in P, \alpha' \text{ is fresh, } \text{abody}(c\langle?, \iota \rangle) = e_a$
(R-Snapshot2)	$\text{snapshot } o \text{ } [\text{m}', \text{m}']$	$\xRightarrow{\text{m}}$	$\text{let } x = \text{check}(\mu, \text{m}', \text{m}'')$ $\text{in obj}(\alpha', c\langle x, \iota \rangle, \overline{v},)$	if $\mu = \text{m}''', \text{class } c \cdots \{ \cdots A \} \in P, \alpha' \text{ is fresh}$
(R-Check)	$\text{check}(v, \text{m}', \text{m}'')$	$\xRightarrow{\text{m}}$	v	if $\text{m}' \leq \text{emode}(v) \leq \text{m}''$
(R-McaseProj)	$\{\overline{m} : \overline{v}\}^T \triangleright \text{m}_j$	$\xRightarrow{\text{m}}$	v_j	
(R-Let)	$\text{let } x = v \text{ in } e$	$\xRightarrow{\text{m}}$	$e\{v/x\}$	
(R-Context)	$\mathbf{E}_m[e_1]$	$\xRightarrow{\text{m}'}$	$\mathbf{E}_m[e_2]$	if $e_1 \xRightarrow{\text{m}'} e_2$

for all rules: $o = \text{obj}(\alpha, T, \overline{v},), \text{mbody}(\text{md}, T) = \overline{x}.e, T = c\langle\mu, \iota\rangle$

Figure 12. Reduction Rules

$\text{modes}(P)$	\triangleq	$\overline{\text{m} \leq \text{m}'}$	
$\text{mode}(c\langle\iota\rangle)$	\triangleq	μ	if $\iota = \mu, \overline{\eta}$
$\text{attr}(c\langle\iota\rangle)$	\triangleq	$A\{\iota/\text{eparam}(\Delta)\}$	if $\text{class } c \Delta \text{ extends } \tau \{ \overline{F} \overline{M} A \} \in P$
$\text{eparam}(\overline{\eta \leq \text{mt} \leq \eta'})$	\triangleq	$\overline{\text{mt}}$	
$\text{eparam}(\omega \rightarrow \omega, \Omega)$	\triangleq	$\text{mt} \cup \text{eparam}(\Omega)$	if $\omega = \eta \leq \text{mt} \leq \eta'$
$\text{ethis}(\Omega)$	\triangleq	mt	if $\text{eparam}(\Omega) = \text{mt}$
$\text{init}(P, c)$	\triangleq	$\text{init}(c') \cup \overline{e\{\iota/\text{eparam}(\Delta)\}}$	if $\text{class } \Delta c \text{ extends } c' \overline{\tau \text{fd} = e} \in P$
$\text{init}(P, c)$	\triangleq	ϵ	if $c = \text{Object}$
$\text{eargs}(c\langle\iota\rangle)$	\triangleq	ι	
$\text{eargs}(\exists\omega.\tau)$	\triangleq	$\text{eargs}(\tau)$	
$\text{cons}(\eta \leq \text{mt} \leq \eta')$	\triangleq	$\bigcup\{\eta \leq \text{mt}, \text{mt} \leq \eta'\}$	
$\text{cons}(\omega \rightarrow \omega, \Omega)$	\triangleq	$\{\eta \leq \text{mt}, \text{mt} \leq \eta'\} \cup \text{cons}(\Omega)$	if $\omega = \eta \leq \text{mt} \leq \eta'$

We require $\overline{\text{m}}$ as a lattice. We use \perp and \top to represent the bottom and top of $\overline{\text{m}}$ respectively.

We define $\text{init}(P, c)$ as $\text{init}(P, c') \cup \overline{e}$ if $\text{class } c \text{ extends } c' \overline{\tau \text{fd} = e} \in P$ or ϵ if $c = \text{Object}$.

Figure 13. Compile Functions

$\text{emode}(\text{m})$	\triangleq	m
$\text{emode}(\text{obj}(c\langle\iota\rangle, \overline{v},))$	\triangleq	$\text{mode}(c\langle\iota\rangle)$

Figure 14. Runtime Functions