P	::=	$R\ \overline{C}\ e$	*program*
R	::=	$\mathtt{m} \le \mathtt{m}'$	*mode order*
C	::=	**class** $\mathtt{c}\ \Delta$ **extends** $\mathtt{c}'\{\overline{F}\ \overline{M}\ A\}$	*class*
F	::=	$T\ \mathtt{fd} = e$	*field*
M	::=	$T\ \mathtt{md}(\overline{T\ \mathtt{x}})\{e\}$	*method*
A	::=	e	*attributor*
e	::=	$\mathtt{x} \mid e.\mathtt{fd} \mid$ **new** $\mathtt{c}\langle\iota\rangle \mid e.\mathtt{md}(\overline{e})$	*expressions*
	$\mid$	$(T)e \mid$ **snapshot** $e\ [\eta, \eta] \mid e \triangleright \eta$	
	$\mid$	$\{\overline{\mathtt{m} : e}\}^T$	

Figure 1. Syntax

T	::=	$\mathtt{c}\langle\iota\rangle \mid \textbf{mcase}\langle T\rangle$	*programmer type*
ι	::=	$\overline{\eta} \mid ?, \overline{\eta}$	*object mode parameter list*
η	::=	$\mathtt{m} \mid \mathtt{mt} \mid \top \mid \bot$	*static mode*
μ	::=	$\eta \mid ?$	*mode*
$\mathtt{mt}$			*mode type variable*
$?$			*dynamic mode type*
ω	::=	$\eta \le \mathtt{mt} \le \eta'$	*constrained mode*
Δ	::=	$? \to \omega, \Omega \mid \Omega$	*class mode parameter list*
Ω	::=	$\overline{\omega}$	*constrained mode list*
τ	::=	$T \mid \exists\omega.\tau \mid \mathtt{modev}$	*type*
υ	::=	$\mathtt{c}\langle\iota\rangle \mid \exists\omega.\tau$	*object type*
$\mathtt{K}$	::=	$\overline{\eta \le \eta'}$	*constraints*

Figure 2. Type Elements

$$\text{(WF-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Omega\ \textbf{extends}\ \mathtt{c}'\ \cdots \in P \qquad \mathtt{eparam}(\Omega) = \iota' \\ \mathtt{cons}(\Omega) = \mathtt{K}' \qquad \mathtt{K} \models \mathtt{K}'\{\overline{\eta}/\iota'\} \qquad \mathtt{K} \vdash_{\mathtt{wft}} \mathtt{c}'\langle\overline{\eta}\rangle\end{array}}{\mathtt{K} \vdash_{\mathtt{wft}} \mathtt{c}\langle\overline{\eta}\rangle}$$

$$\text{(WF-ClassDyn)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ ? \to \omega, \Omega\ \textbf{extends}\ \mathtt{c}'\cdots \in P \\ \mathtt{eparam}(? \to \omega, \Omega) = ?, \iota' \\ \mathtt{cons}(\Omega) = \mathtt{K}' \qquad \mathtt{K} \models \mathtt{K}'\{\overline{\eta}/\iota'\} \qquad \mathtt{K} \vdash_{\mathtt{wft}} \mathtt{c}'\langle ?, \overline{\eta}\rangle\end{array}}{\mathtt{K} \vdash_{\mathtt{wft}} \mathtt{c}\langle ?, \overline{\eta}\rangle}$$

$$\text{(WF-Top)}\ \mathtt{K} \vdash_{\mathtt{wft}} \mathtt{Object}\langle\eta\rangle$$

$$\text{(WF-Exist)}\ \frac{\begin{array}{c}\omega = \eta_1 \le \mathtt{mt} \le \eta_2 \qquad \mathtt{K} = \mathtt{K}' \cup \{\eta_1 \le \mathtt{mt}, \mathtt{mt} \le \eta_2\} \\ \mathtt{mt} \notin \mathtt{K}' \qquad \mathtt{K}' \vdash_{\mathtt{wft}} \mathtt{c}\langle ?, \iota\rangle \qquad \mathtt{K} \succ_{\mathtt{wft}} \mathtt{c}\langle \mathtt{mt}, \iota\rangle\end{array}}{\mathtt{K} \vdash_{\mathtt{wft}} \exists\omega.\mathtt{c}\langle ?, \iota\rangle}$$

$$\text{(WF-MCase)}\ \frac{\mathtt{K} \vdash_{\mathtt{wft}} T}{\mathtt{K} \vdash_{\mathtt{wft}} \textbf{mcase}\langle T\rangle}$$

Figure 3. External Type Well-Formedness

$$\text{(WF-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\ \cdots \in P \qquad \mathtt{iparam}(\Delta) = \iota' \\ \mathtt{cons}(\Delta) = \mathtt{K}' \qquad \mathtt{K} \models \mathtt{K}'\{\overline{\eta}/\iota'\} \qquad \mathtt{K} \succ_{\mathtt{wft}} \mathtt{c}'\langle\overline{\eta}\rangle\end{array}}{\mathtt{K} \succ_{\mathtt{wft}} \mathtt{c}\langle\overline{\eta}\rangle}$$

Figure 4. Internal Type Well-Formedness

$$\text{(WF-Empty)}\ P \vdash_{\mathtt{wfe}} \epsilon$$

$$\text{(WF-ESpec)}\ \frac{P \vdash_{\mathtt{wfe}} \Omega \qquad \eta \le \eta'}{P \vdash_{\mathtt{wfe}} \Omega, \eta \le \mathtt{mt} \le \eta'}$$

$$\text{(WF-TSpec)}\ \frac{P \vdash_{\mathtt{wfe}} \omega, \Omega}{P \vdash_{\mathtt{wfe}} ? \to \omega, \Omega}$$

Figure 5. Environment Well-Formedness

$$\text{(FD-Object)}\ \mathtt{fields}(\mathtt{Object}\langle\eta\rangle) = \bullet$$

$$\text{(FD-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{T\ \mathtt{fd} = e}\ \overline{M}\ A\} \\ \mathtt{eparam}(\Delta) = \iota' \qquad \mathtt{fields}(\mathtt{c}'\langle\iota\rangle) = \overline{T_0}\ \overline{\mathtt{fd}_0} = \overline{e_0}\end{array}}{\mathtt{fields}(\mathtt{c}\langle\iota\rangle) = \overline{T_0}\ \overline{\mathtt{fd}_0} = \overline{e_0},\ \overline{T\{\iota/\iota'\}}\ \overline{\mathtt{fd}} = \overline{e\{\iota/\iota'\}}}$$

$$\text{(MT-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\} \\ T\ \mathtt{md}(\overline{T\ \mathtt{x}})\ \{\ e\ \} \in \overline{M} \qquad \mathtt{eparam}(\Delta) = \iota'\end{array}}{\mathtt{mtype}(\mathtt{md}, \mathtt{c}\langle\iota\rangle) = (\overline{T} \to T)\{\iota/\iota'\}}$$

$$\text{(MT-Super)}\ \frac{\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\} \qquad \mathtt{md} \notin \overline{M}}{\mathtt{mtype}(\mathtt{md}, \mathtt{c}\langle\iota\rangle) = \mathtt{mtype}(\mathtt{md}, \mathtt{c}'\langle\iota\rangle)}$$

$$\text{(MB-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\} \\ \mathtt{iparam}(\Delta) = \iota' \qquad T\ \mathtt{md}(\overline{T\ \mathtt{x}})\{e\} \in \overline{M}\end{array}}{\mathtt{mbody}(\mathtt{md}, \mathtt{c}\langle\iota\rangle) = \overline{\mathtt{x}}.e\{\iota/\iota'\}}$$

$$\text{(MB-Exist)}\ \frac{\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\} \qquad \omega = \eta_1 \le \mathtt{mt} \le \eta_2}{\mathtt{mbody}(\mathtt{md}, \exists\omega.\mathtt{c}\langle ?, \iota\rangle) = \mathtt{mbody}(\mathtt{md}, \mathtt{c}\langle \mathtt{mt}, \iota\rangle)}$$

$$\text{(MB-Super)}\ \frac{\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\} \qquad \mathtt{md} \notin \overline{M}}{\mathtt{mbody}(\mathtt{md}, \mathtt{c}\langle\iota\rangle) = \mathtt{mbody}(\mathtt{md}, \mathtt{c}'\langle\iota\rangle)}$$

$$\text{(AB-Class)}\ \frac{\begin{array}{c}\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\} \\ \mathtt{eparam}(\Delta) = \iota' \qquad A = e\end{array}}{\mathtt{abody}(\mathtt{c}\langle\iota\rangle) = e\{\iota/\iota'\}}$$

Figure 6. FJ Functions

$$\text{(T-Program)}\ \frac{R \text{ form a latice} \qquad \varnothing \vdash e \qquad \overline{C}\ \text{OK}}{R\ \overline{C}\ e\ \text{OK}}$$

$$\text{(T-Class)}\ \frac{\begin{array}{c}\iota = \mathtt{iparam}(\Delta) \\ \mathtt{K} = \mathtt{cons}(\Delta) \qquad \mathtt{ithis}(\Delta) = \mathtt{mode}(\mathtt{c}') \qquad \overline{M}\ \text{OK IN}\ \mathtt{c}, \Delta \\ A\ \text{OK IN}\ \mathtt{c}, \Delta \qquad \overline{F} = \overline{T}\ \overline{\mathtt{fd}} = \overline{e} \qquad \varnothing; \mathtt{K} \vdash \overline{e} : \overline{T}\end{array}}{\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\{\overline{F}\ \overline{M}\ A\}\ \text{OK}}$$

$$\text{(T-Attributor)}\ \frac{\begin{array}{c}\iota = \mathtt{iparam}(\Delta) \qquad \mathtt{K} = \mathtt{cons}(\Delta) \\ A = e \qquad \mathtt{K} \succ_{\mathtt{wft}} \mathtt{c}\langle\iota\rangle \qquad \mathtt{K}; \textbf{this} : \mathtt{c}\langle\iota\rangle \vdash e : \mathtt{modev}\end{array}}{A\ \text{OK IN}\ \mathtt{c}, \Delta}$$

$$\text{(T-Method)}\ \frac{\begin{array}{c}\iota = \mathtt{iparam}(\Delta) \qquad \mathtt{K} = \mathtt{cons}(\Delta) \\ \mathtt{K} \succ_{\mathtt{wft}} \mathtt{c}\langle\iota\rangle \qquad \overline{x} : \overline{T}; \textbf{this} : \mathtt{c}\langle\iota\rangle; \mathtt{K} \vdash e : T\end{array}}{T\ \mathtt{md}(\overline{T\ x})\{\ e\ \}\ \text{OK IN}\ \mathtt{c}\ \Delta}$$

Figure 7. Class Typing

$$\text{(T-Var)}\ \frac{}{\Gamma;\mathtt{K}\vdash \mathtt{x} : \Gamma(\mathtt{x})}$$

$$\text{(T-New)}\ \frac{\begin{array}{c}\iota = ?, \iota' \text{ iff } \textbf{class}\ \mathtt{c}\ \Delta\cdots\in P \text{ and } \mathtt{ethis}(\Delta) = ?\\ \iota \neq ?, \iota' \text{ iff } \textbf{class}\ \mathtt{c}\ \Delta\cdots\in P \text{ and } \mathtt{ethis}(\Delta) \neq ? \qquad \mathtt{K}\models\mathtt{cons}(\Delta)\end{array}}{\Gamma;\mathtt{K}\vdash\ \textbf{new}\ \mathtt{c}\langle\iota\rangle : \mathtt{c}\langle\iota\rangle}$$

$$\text{(T-Cast)}\ \frac{\Gamma;\mathtt{K}\vdash e : T'}{\Gamma;\mathtt{K}\vdash (T)e : T}$$

$$\text{(T-Msg)}\ \frac{\begin{array}{c}\Gamma;\mathtt{K}\vdash\ e : \upsilon\\ \mathtt{mtype}(\mathtt{md},\upsilon) = \overline{T}\rightarrow T \qquad \Gamma;\mathtt{K}\vdash\overline{e} : \overline{T} \qquad \Gamma;\mathtt{K}\vdash\ \textbf{this} : T_{this}\\ \mathtt{K}\models\{\mathtt{mode}(\upsilon)\leq\mathtt{mode}(T_{this})\} \qquad \mathtt{mode}(\upsilon)\neq ?\end{array}}{\Gamma;\mathtt{K}\vdash e.\mathtt{md}(\overline{e}) : T}$$

$$\text{(T-Field)}\ \frac{\begin{array}{c}\Gamma;\mathtt{K}\vdash e : \upsilon \qquad \Gamma;\mathtt{K}\vdash \textbf{this} : T_{this} \qquad \mathtt{fields}(\upsilon) = \overline{T}\,\overline{\mathtt{fd}}\\ \mathtt{K}\models\{\mathtt{mode}(\upsilon)\leq\mathtt{mode}(T_{this})\} \qquad \mathtt{mode}(\mathtt{c}\langle\iota\rangle)\neq ?\end{array}}{\Gamma;\mathtt{K}\vdash e.\mathtt{fd}_i : T_i}$$

$$\text{(T-Snapshot1)}\ \frac{\begin{array}{c}\Gamma;\mathtt{K}'\vdash e : \mathtt{c}\langle ?,\iota\rangle\\ \omega = \eta_1\leq\mathtt{mt}\leq\eta_2 \qquad \mathtt{K} = \mathtt{K}'\cup\{\eta_1\leq\mathtt{mt},\mathtt{mt}\leq\eta_2\}\end{array}}{\Gamma;\mathtt{K}\vdash \textbf{snapshot}\ e\ [\eta_1,\eta_2] : \exists\omega.\mathtt{c}\langle ?,\iota\rangle}$$

$$\text{(T-MCase)}\ \frac{\overline{\mathtt{m}} = \mathtt{modes}(P) \qquad \Gamma;\mathtt{K}\vdash e_i : T \text{ for all } i}{\Gamma;\mathtt{K}\vdash\{\overline{\mathtt{m}:e}\}^{T} : \textbf{mcase}\langle T\rangle}$$

$$\text{(T-ElimCase)}\ \frac{\Gamma;\mathtt{K}\vdash e : \textbf{mcase}\langle T\rangle \qquad \eta\in\mathtt{modes}(P) \text{ or } \eta \text{ appears in } \mathtt{K}}{\Gamma;\mathtt{K}\vdash e \rhd \eta : T}$$

$$\text{(T-ModeValue)}\ \frac{\mathtt{m}\in\mathtt{modes}(P)}{\Gamma;\mathtt{K}\vdash\mathtt{m} : \mathtt{modev}}$$

$$\text{(T-Sub)}\ \frac{\Gamma;\mathtt{K}\vdash e : \tau \qquad \mathtt{K}\vdash\tau <: \tau'}{\Gamma;\mathtt{K}\vdash e : \tau'}$$

Figure 8. Expression Typing

$$\text{(S-Dynamic)}\ \mathtt{K}\vdash\mathtt{c}\langle\mu;\overline{\eta}\rangle <: \mathtt{c}\langle ?;\overline{\eta}\rangle$$

$$\text{(S-Mcase)}\ \frac{\mathtt{K}\vdash\tau <: \tau'}{\mathtt{K}\vdash\textbf{mcase}\langle\tau\rangle <: \textbf{mcase}\langle\tau'\rangle}$$

$$\text{(S-Class)}\ \frac{\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \mathtt{c}'\cdots\in P \qquad \mathtt{K}\models\mathtt{cons}(\Delta)}{\mathtt{K}\vdash\mathtt{c}\langle\iota\rangle <: \mathtt{c}'\langle\iota\rangle}$$

Figure 9. Subtyping (reflexivity and transitivity rules are omitted.)

$$\text{(M-Sub)}\ \frac{\{\eta\leq\mathtt{mt},\mathtt{mt}\leq\eta',\eta\leq\eta'\}\in\mathtt{K}}{\mathtt{K}\models\{\eta\leq\mathtt{mt},\mathtt{mt}\leq\eta',\eta\leq\eta'\}}$$

Figure 10. Submoding

e	$::=$	$\ldots$	*runtime expressions*
	$\mid$	$\textbf{check}(e,\mathtt{m},\mathtt{m}',e)$	
	$\mid$	$\mathtt{obj}(\alpha,\mathtt{c}\langle\iota\rangle,\overline{e})$	
	$\mid$	$\mathtt{cl}(\mathtt{m},e)$	
$\mathbf{E}$	$::=$	$\odot \mid \mathbf{E}.\mathtt{md}(\overline{e}) \mid o.\mathtt{md}(\ldots,o,\mathbf{E},e,\ldots)$	*evaluation context*
	$\mid$	$(T)\mathbf{E} \mid \mathbf{E}.\mathtt{fd}$	
	$\mid$	$\textbf{snapshot}\ \mathbf{E}\ [\mathtt{m}_1,\mathtt{m}_2]$	
	$\mid$	$\{\ldots\mathtt{m}:v;\mathtt{m}:\mathbf{E};\mathtt{m}:e\ldots\} \mid \mathbf{E}\rhd\mu$	
	$\mid$	$\textbf{check}(\mathbf{E},\mathtt{m},\mathtt{m}',\mathbf{E})$	
	$\mid$	$\mathtt{obj}(\alpha,\mathtt{c}\langle\iota\rangle,\ldots v,\mathbf{E},e\ldots)$	
	$\mid$	$\mathtt{cl}(\mathtt{m},\mathbf{E})$	

Figure 11. Run-Time Elements

$$\text{(T-Obj)}\ \frac{\Gamma;\mathtt{K}\vdash\overline{e} : \overline{T} \qquad \mathtt{fields}(\upsilon) = \overline{T}\,\overline{\mathtt{fd}} = \overline{e}}{\Gamma;\mathtt{K}\vdash\mathtt{obj}(\alpha,\upsilon,\overline{e}) : \upsilon}$$

$$\text{(T-Check)}\ \frac{\begin{array}{c}\Gamma;\mathtt{K}'\vdash e_1 : \mathtt{modev} \qquad \Gamma;\mathtt{K}'\vdash e_2 : \mathtt{c}\langle ?,\iota\rangle\\ \omega = \mathtt{m}_1\leq\mathtt{mt}\leq\mathtt{m}_2 \qquad \mathtt{K} = \mathtt{K}'\cup\{\eta_1\leq\mathtt{mt},\mathtt{mt}\leq\eta_2\}\end{array}}{\Gamma;\mathtt{K}\vdash\textbf{check}(e_1,\mathtt{m}_1,\mathtt{m}_2,e_2) : \exists\omega.\mathtt{c}\langle ?,\iota\rangle}$$

$$\text{(T-Closure)}\ \frac{\Gamma;\mathtt{K}\vdash e : \tau}{\Gamma;\mathtt{K}\vdash\mathtt{cl}(\mathtt{m},e) : \tau}$$

Figure 12. Auxiliary Run-time Expression Typing

(R-New)	$\textbf{new}\ \mathtt{c}\langle\iota\rangle$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	$\mathtt{obj}(\alpha, \mathtt{c}\langle\iota\rangle, \mathtt{init}(P, \mathtt{c}))$	if α is $fresh$
(R-Cast)	$(\tau_0)\,o$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	o	if $\tau <: \tau_0$
(R-Msg)	$o.\mathtt{md}(\overline{v}')$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	$\mathtt{cl}(\mathtt{m}', e\{\overline{v}'/\overline{\mathtt{x}}\}\{o/\textbf{this}\})$	if $\mu \leq \mathtt{m}, \mathtt{m}' = \mathtt{emode}(o)$
(R-Field)	$o.\mathtt{fd}_i$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	v_i	if $\mu \leq \mathtt{m}$
(R-Snapshot1)	$\textbf{snapshot}\ o\ [\mathtt{m}_1, \mathtt{m}_2]$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	$\textbf{check}(e_a\{o/\textbf{this}\}, \mathtt{m}_1, \mathtt{m}_2, o)$	if $\mu = ?, \textbf{class}\ \mathtt{c}\ \cdots\ \{\ \cdots\ A\ \} \in P, \alpha'$ is $fresh$, $\mathtt{abody}(\mathtt{c}\langle ?, \iota\rangle) = e_a$
(R-Snapshot2)	$\textbf{snapshot}\ o\ [\mathtt{m}_1, \mathtt{m}_2]$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	o	if $\mu = \mathtt{m}', \textbf{class}\ \mathtt{c}\ \cdots\ \{\ \cdots\ A\ \} \in P, \mathtt{m}_1 \leq \mathtt{m}' \leq \mathtt{m}_2$
(R-Check)	$\textbf{check}(\mathtt{m}', \mathtt{m}_1, \mathtt{m}_2, o)$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	$\mathtt{obj}(\alpha', \exists \mathtt{m}'.\mathtt{c}\langle ?, \iota\rangle, \overline{v})$	if $\mathtt{m}_1 \leq \mathtt{m}' \leq \mathtt{m}_2, \alpha'$ is $fresh$
(R-McaseProj)	$\{\overline{\mathtt{m} : v}\}^T \rhd \mathtt{m}_j$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	v_j	
(R-Closure1)	$\mathtt{cl}(\mathtt{m}', e)$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	$\mathtt{cl}(\mathtt{m}', e')$	if $e \stackrel{\mathtt{m}'}{\Longrightarrow} e'$
(R-Closure2)	$\mathtt{cl}(\mathtt{m}', v)$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	v	
(R-Context)	$\mathbf{E}[\,e_1\,]$	$\stackrel{\mathtt{m}}{\Longrightarrow}$	$\mathbf{E}[\,e_2\,]$	if $e_1 \stackrel{\mathtt{m}}{\Longrightarrow} e_2$

for all rules: $o = \mathtt{obj}(\alpha,\ T, \overline{v},), \mathtt{mbody}(\mathtt{md},\ T) = \overline{\mathtt{x}}.e,\ T = \mathtt{c}\langle\mu, \iota\rangle$

Figure 13. Reduction Rules

$\mathtt{modes}(P)$	$\triangleq$	$\overline{\mathtt{m} \leq \mathtt{m}'}$	
$\mathtt{mode}(\mathtt{c}\langle\iota\rangle)$	$\triangleq$	μ	if $\iota = \mu, \overline{\eta}$
$\mathtt{mode}(\exists\omega.\mathtt{c}\langle ?, \iota\rangle)$	$\triangleq$	$\mathtt{mt}$	if $\omega = \eta_1 \leq \mathtt{mt} \leq \eta_2$
$\mathtt{attr}(\mathtt{c}\langle\iota\rangle)$	$\triangleq$	$A\{\iota/\mathtt{eparam}(\Delta)\}$	if $\textbf{class}\ \mathtt{c}\ \Delta\ \textbf{extends}\ \tau\ \{\ \overline{F}\ \overline{M}\ A\}\ \in\ P$
$\mathtt{eparam}(\overline{\eta \leq \mathtt{mt} \leq \eta'})$	$\triangleq$	$\overline{\mathtt{mt}}$	
$\mathtt{eparam}(? \rightarrow \omega, \Omega)$	$\triangleq$	$?, \mathtt{eparam}(\Omega)$	
$\mathtt{iparam}(\overline{\eta \leq \mathtt{mt} \leq \eta'})$	$\triangleq$	$\overline{\mathtt{mt}}$	
$\mathtt{iparam}(? \rightarrow \omega, \Omega)$	$\triangleq$	$\mathtt{mt}, \mathtt{iparam}(\Omega)$	if $\omega = \eta \leq \mathtt{mt} \leq \eta'$
$\mathtt{ethis}(\Omega)$	$\triangleq$	$\mathtt{mt}$	if $\mathtt{eparam}(\Omega) = \mathtt{mt}$
$\mathtt{init}(P, \mathtt{c})$	$\triangleq$	$\mathtt{init}(\mathtt{c}') \cup \overline{e\{\iota/\mathtt{eparam}(\Delta)\}}$	if $\textbf{class}\ \Delta\ \mathtt{c}\ \textbf{extends}\ \mathtt{c}'\ \overline{\tau\ \mathtt{fd} = e} \in P$
$\mathtt{init}(P, \mathtt{c})$	$\triangleq$	ϵ	if $\mathtt{c} = \mathtt{Object}$
$\mathtt{eargs}(\mathtt{c}\langle\iota\rangle)$	$\triangleq$	ι	
$\mathtt{eargs}(\exists\omega.\tau)$	$\triangleq$	$\mathtt{eargs}(\tau)$	
$\mathtt{cons}(\eta \leq \mathtt{mt} \leq \eta')$	$\triangleq$	$\bigcup\{\eta \leq \mathtt{mt}, \mathtt{mt} \leq \eta'\}$	
$\mathtt{cons}(? \rightarrow \omega, \Omega)$	$\triangleq$	$\{\eta \leq \mathtt{mt}, \mathtt{mt} \leq \eta'\} \cup \mathtt{cons}(\Omega)$	if $\omega = \eta \leq \mathtt{mt} \leq \eta'$

We write $\exists\mathtt{m}.\mathtt{c}\langle ?, \iota\rangle$ as shorthand for $\exists\omega.\mathtt{c}\langle ?, \iota\rangle$ if $\omega = \mathtt{m} \leq \mathtt{mt} \leq \mathtt{m}$.
We require $\overline{\mathtt{m}}$ as a lattice. We use $\perp$ and $\top$ to represent the bottom and top of $\overline{\mathtt{m}}$ respectively.
We define $\mathtt{init}(P, \mathtt{c})$ as $\mathtt{init}(P, \mathtt{c}') \cup \overline{e}$ if $\textbf{class}\ \mathtt{c}\ \textbf{extends}\ \mathtt{c}'\ \overline{\tau\ \mathtt{fd} = e} \in P$ or ϵ if $\mathtt{c} = \mathtt{Object}$.

Figure 14. Compile Functions

$\mathtt{emode}(\mathtt{m})$	$\triangleq$	$\mathtt{m}$
$\mathtt{emode}(\mathtt{obj}(\mathtt{c}\langle\iota\rangle, \overline{v},))$	$\triangleq$	$\mathtt{mode}(\mathtt{c}\langle\iota\rangle)$

Figure 15. Runtime Functions