

# A Simple Taylor Rule to Understand How the Federal Reserve Sets Interest Rates

The **Taylor Rule** is a guideline suggesting how a central bank (such as the U.S. Federal Reserve) might adjust short-term interest rates.

Think of the economy as a large room with a **thermostat**, and **interest rates** are the thermostat's *temperature dial*. When the "room" (the economy) gets too hot, you lower the thermostat setting; when it's too cold, you raise it. The **Taylor Rule** helps decide how to adjust that dial, based on two key factors:

1. The gap between **actual inflation** and **target inflation**
2. The gap between **actual output** and **potential output**

Mathematically, in its original form, the **Taylor Rule** can be written as:

$$i_t = r^* + \pi_t + 0.5(\pi_t - \pi^*) + 0.5(\text{Output Gap}_t),$$

where:

- $i_t$  = **nominal interest rate** (our thermostat dial).
- $r^*$  = **long-run real interest rate** (think of this as a baseline setting).
- $\pi_t$  = **current inflation**.
- $\pi^*$  = **target (desired) inflation**.

Here's how this formula fits our "**thermostat**" analogy:

1. **Inflation Gap** ( $\pi_t - \pi^*$ ):
  - If actual inflation  $\pi_t$  is higher than the target  $\pi^*$ , it's like the room is *too hot*. The formula says to **increase**  $i_t$  (the interest rate), which "cools" the economy by making borrowing more expensive and slowing spending.
2. **Output Gap**:
  - If **actual output** is above **potential output**, it's like the room is overheated. The rule again prescribes increasing  $i_t$  so the economy doesn't overheat further.
  - If **actual output** is below **potential**, it's like the room is *too cold*. Lower  $i_t$  (the thermostat) to warm up the economy—cheaper borrowing can stimulate growth.

Although real-world policy can be more complex, the Taylor Rule offers a simple, intuitive lens for analyzing monetary policy decisions. This lab will guide you in applying time-series methods to **quarterly** U.S. macroeconomic data—retrieved from FRED—to see how well the Taylor Rule can explain and forecast interest rates behavior. You will build, test, and compare models on a **training set (up to 2022)** and a **test set (after 2022)**, evaluating forecasts using RMSE and related metrics.

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## Task 1: Data Retrieval and Initial EDA (Quarterly Frequency)(10 Points)

### 1. Obtain Data (Quarterly) (2 Points)

- Collect quarterly data from 1980Q1 through the latest available quarter (e.g., 2023Q4). You will use a *training* period (e.g., 1980Q1–2022Q4) and reserve the rest (2023Q1 onward) as the *test* set.
  - Required series:
    - Federal Funds Effective Rate (e.g., FEDFUNDS)
    - Inflation measure (e.g., Personal Consumption Expenditures Index (PCEPI))
    - Real GDP (GDPC1) and Potential Real GDP (e.g., GDPPOT from CBO)
- Hint: Use the FRED API (with your free API key) to query these series programmatically.

### 2. Construct Variables (2 Points)

- Inflation: Percentage change in PCE.
- Output Gap:

$$\frac{\text{Real GDP} - \text{Potential Real GDP}}{\text{Potential Real GDP}} \times 100.$$

### 3. Initial Plots and Summaries (6 Points)

- Plot each series over time.
- Provide summary statistics (mean, variance, etc.).
- Briefly comment on notable historical events or patterns (e.g., high inflation periods, recessions, near-zero interest rates).

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## Task 2: Preprocessing (12 Points)

We will use *quarterly* data throughout. Please break your analysis into the following **four** subsections:

### 2.1: Frequency Alignment (Resampling) (2 Points)

- Resampling or Converting Frequencies:
  - If any of your raw data is at a different frequency (e.g., monthly), you can **resample** to quarterly. For instance, you might take the **quarterly average** or **quarter-end values**.
  - Provide a brief explanation of how you performed resampling (e.g.`aggregate()` in R).
- Ensure all series end up in the same **quarterly** date index (e.g., 1980Q1, 1980Q2, etc.).

### 2.2: Outlier Detection & Treatment (5 Points)

1. Use Your Plots from Task 1 to visually spot any extreme spikes.
2. Possible Methods to detect outliers:
  - Z-score Threshold (e.g., if absolute Z-score > 3).
  - Hampel Filter.

- Percentile-based Winsorizing (e.g., 1st/99th percentile).
3. Select One Method and Explain Why
    - Justify your choice. Maybe you prefer winsorizing because it retains all points but caps extremes.
  4. Implement the chosen method on your series:
    - If you `winsorize`, specify the cutoffs.
    - If you remove data, state how many points you removed.
  5. Explain how you treat (or keep) these outliers in your final dataset.

### 2.3: Seasonal Adjustment (5 Points)

Even with quarterly data, there might be seasonal patterns, especially in GDP series.

1. Detection
  - Produce seasonal plots.
  - Run a statistical test for seasonality (e.g., the QS test).
2. Different Methods for adjustment:
  - X-13ARIMA-SEATS (commonly used)
  - STL Decomposition
  - Multiplicative/Additive Seasonal Method
3. Select One
  - State which method you use (e.g., “We apply STL because it handles complex seasonal patterns”).
4. Seasonally Adjust the data (if necessary) and show the “before vs. after” comparison.

## Task 3: Stationarity Testing & Potential Structural Breaks (13 Points)

### 3.1: Stationarity Testing (8 Points)

One of the most crucial steps in time-series modeling is **checking for stationarity**, which often involves **unit root tests**. However, if there is reason to believe the data has **structural breaks** (abrupt changes in the time series behavior at certain points), *standard* tests like the Augmented Dickey-Fuller (ADF) may be misleading.

When performing an ADF (Augmented Dickey-Fuller) test, we must decide which *deterministic elements* (constant, trend) to include in the test equation.

1. `type = “none”`
  - No constant or trend.
  - This equation is the most restrictive. If the data actually have a nonzero mean or an upward/downward drift, ignoring those can affect the test.
2. `type = “drift”`
  - Allows for a constant (“intercept”) in the test equation.
  - Commonly used when we suspect the series has a nonzero average.
3. `type = “trend”`

- Allows for both a constant and a linear time trend.
- Used if the data appear to show a *systematic* upward or downward trend.

A common approach is to start with the most general case (**trend**) and see if the data statistically support including a trend. If we don't find evidence for a trend, we might move to **drift**. If we still find no evidence for an intercept, we end with **none**. This is sometimes called a *general-to-specific* approach.

The “augmented” part of the ADF test means we include *lagged* terms of the variable to account for potential autocorrelation. In practice, we often let a procedure pick the best number of lags using an *information criterion* (such as **AIC** or **BIC**). These criteria trade off model fit (likelihood) against model complexity (number of parameters).

### 3.2: Stationarity with structural break (5 Points)

If your series underwent a major shift (e.g., policy change, macro-economic event) at some point in time, a standard ADF test may struggle. Perron (1989) highlights that traditional ADF tests can be biased toward not rejecting the null of a unit root when there's a break in level or trend.

- Employ unit root tests that allow for an **endogenous** structural break, such as the **Zivot & Andrews** test to control for possible structural breaks.

The '`uroot`' or '`aTSA`' packages may provide a Zivot-Andrews test.

For instance, in package '`aTSA`', you have:

```
library(aTSA)
za_test <- ZivotAndrews(my_ts, max.lag = 5)
za_test
```

## Task 4: Estimating the Basic Taylor Rule (OLS) (10 Points)

Recall the simplified Taylor Rule:

$$i_t = \alpha_0 + \alpha_1(\pi_t - \pi^*) + \alpha_2(\text{Output Gap}_t) + \varepsilon_t,$$

where  $\pi^*$  could be 2%. Train this model on data from **1980Q1 to 2022Q4** and reserve the rest (2023Q1 onward) as the **test** set.

1. **Interpretation of  $\alpha_1$** 
  - If  $\alpha_1 > 1$ , it means the interest rate moves *more* than one-for-one with inflation (the “Taylor principle”).
  - Compare your estimated  $\alpha_1$  with 1. Discuss whether your result aligns with theory.
2. **Model Diagnostics**
  - Check whether residuals exhibit autocorrelation (ACF plots, Durbin-Watson, etc.).
  - Evaluate in-sample (train) RMSE.
3. **Forecast Performance**
  - Generate forecasts for the **test period** (2023Q1–2024Q4) using the fitted OLS model.
  - Calculate out-of-sample RMSE. Compare train vs. test performance briefly.

## Task 5: Cointegration and Error Correction Model (ECM) (15 Points)

Next, investigate whether there is a **long-run equilibrium** (in the sense of cointegration) among  $\{i_t, (\pi_t - \pi^*), \text{Output Gap}_t\}$ . Conceptually, if the Federal Reserve consistently adjusts the federal funds rate in response to changes in inflation and the output gap, you might expect a mean-reverting relationship in the long run (e.g., the gap between the actual interest rate and its “Taylor-implied” level might be stationary).

### 5.1: Cointegration Test (5 Points)

- Run an **Engle-Granger** two-step test:
  - Test the residuals of the estimated Basic Taylor Rule for a unit root (e.g., ADF).
- If the residuals are stationary, you conclude there's a long-run equilibrium relationship.

### 5.2: Estimating the ECM via OLS (5 Points)

An Error Correction Model (ECM) is a way to capture both:

- Long-Run Equilibrium: Some economic theory tells us that variables have a stable long-run relationship (cointegration).
- Short-Run Dynamics: In the short run, the variables might deviate from their equilibrium, but they will tend to move back (“correct”) over time.

$$\Delta i_t = \gamma_0 + \gamma_1 \Delta(\pi_t - \pi^*) + \gamma_2 \Delta(\text{Output Gap}_t) + \phi \left[ \underbrace{i_{t-1} - \beta_1(\pi_{t-1} - \pi^*) - \beta_2(\text{Output Gap}_{t-1})}_{\text{equilibrium error at } t-1} \right] + \epsilon_t.$$

where:

- $\Delta i_t = i_t - i_{t-1}$ : The *change* (first difference) in the interest rate.
- $\Delta(\pi_t - \pi^*)$ : The *change* in the inflation gap.
- $\Delta(\text{Output Gap}_t)$ : The *change* in the output gap.
- $i_{t-1} - \beta_1(\pi_{t-1} - \pi^*) - \beta_2(\text{Output Gap}_{t-1})$ :  
The **equilibrium error**—how far  $i_{t-1}$  was from the long-run “equilibrium level.”
- $\phi$ : The speed of adjustment (often negative).

You can estimate the ECM in two stages:

1. **Estimate the Long-Run Regression** (levels) and extract residuals.
2. **Estimate the ECM** (in differences + lagged residual) by OLS.

### 5.3: Compare ECM Forecasts (5 Points)

- Train the ECM on 1980Q1–2022Q4, forecast 2023Q1–2024Q4.
- Report RMSE and compare to the basic OLS-based Taylor Rule.

## Task 6: ARIMA Modeling of Taylor Rule Errors (10 Points)

Often, the **residuals** ( $\hat{\varepsilon}_t$ ) from the Taylor Rule have patterns (autocorrelation) left unexplained by the simple equation. We can fit an **ARIMA** model to these residuals.

### Combined Forecast

Finally, to **improve your forecast** of the interest rate, you can **combine**:

1. The **Taylor Rule** prediction (OLS model).
2. The **ARIMA** forecast of the residual (i.e., the unexplained part).

In other words, your final predicted interest rate is:

$$\hat{i}_t^{\text{combined}} = \hat{i}_t^{\text{Taylor}} + \hat{\varepsilon}_t^{\text{ARIMA}}.$$

Then, in a **comparison table**, report the **in-sample (train)** and **out-of-sample (test)** RMSE for:

- The **simple Taylor Rule** (OLS)
  - The **ECM model**
  - The **Taylor Rule + ARMA** combined model
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## Task 7: VAR Modeling (15 Points)

Now consider  $\{i_t, (\pi_t - \pi^*), \text{Output Gap}_t\}$  as a **system**. Vector Autoregression (VAR) captures *feedback* among these variables.

### 7.1: Estimate a VAR Model (10 Points)

1. **Estimate a VAR**
  - Use data from 1980Q1–2022Q4.
  - Choose a lag order (p) by AIC/BIC or other criteria.
  - Check if differencing is needed (based on stationarity tests).
2. **Forecasting**
  - Generate multi-step forecasts for 2023Q1–2024Q4.
  - Compute RMSE for train and test sets for all equations. Compare the Fed funds rate RMSEs with previous models.
3. **Granger Causality**
  - Test whether  $(\pi_t - \pi^*)$  and Output Gap<sub>t</sub> *Granger-cause*  $i_t$ , etc.

### Task 7.2: Impulse Response Function (IRF) (5 Points)

**Impulse Response Functions (IRFs)** help us understand how **one variable** in a VAR system **responds** over time when there's a *sudden, one-time change* (a “shock”) in **another** variable.

1. **Definition**

- An **IRF** shows how  $\{i_t, (\pi_t - \pi^*), \text{Output Gap}_t\}$  each *evolve* over several periods **after** a *one-time* shock in (for example)  $(\pi_t - \pi^*)$ .
- **Why Do We Care?** In economic terms: *If inflation suddenly increases by X%, how does the Fed Funds Rate react over the next few quarters?* Does it rise quickly, does it overshoot, etc.?

## 2. Implementation in R (Example)

```
library(vars)

# Suppose 'var_model' is a VAR(2) with variables c("inflation", "interest_rate", "output_gap")
# We'll see how a shock to 'inflation' affects 'interest_rate' over the next 8 quarters

irf_result <- irf(
  var_model,
  impulse = "inflation",      # the variable that gets the shock
  response = "interest_rate", # the variable we observe reacting
  n.ahead = 8,                # how many periods (quarters) ahead
  boot = TRUE,                # use bootstrapped confidence intervals
  ci = 0.90                   # confidence interval (e.g., 90%)
)

plot(irf_result)
```

- By default, `irf()` usually applies either a 1-unit or 1-standard-deviation shock (depending on your settings).

## Final Deliverable

Your final submission should include:

- A main report (in PDF) that is well-organized and accessible to a moderately informed audience.
- Any additional or technical appendices if desired (e.g., extra analysis).
- Reproducible code (this .Rmd) so another analyst could replicate your results.