McMaster University

MATH 3MB3 Introduction to Mathematical Modelling

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Final Report:

Conservation and Wildlife Management and Bobcat Population

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Introduction

The bobcat (Lynx rufus) is one of the most widespread predators in North America ranging from Mexico to Canada. Despite their sparse population density, they have a significant impact on their surrounding ecosystem, earning them the title of a *"keystone species"*. They are able to have such a significant influence because they are an apex predator. This means they are at the top of their food chain and have no natural predators. Thus they play a crucial role in regulating prey populations and maintaining the balance of the ecosystem's food chain and supply.

Ecosystems are a delicately balanced web of relationships among diverse organisms, where each organism, from the smallest bacteria to the largest predator, plays a vital role in keeping the ecosystem healthy. These organisms collectively benefit from each other and a shared environment. It is important to understand the population dynamics of each species in an ecosystem as their relationships are dependent on each other.

Preserving ecological balance in an ecosystem is crucial as even a minor disruption can trigger effects with far-reaching consequences. One possible disruption that is most relevant to this research paper is the phenomenon of overpopulation or underpopulation. Overpopulation can result in the extinction of other species or the depletion of resources in the environment. On the other hand, underpopulation can result in dwindling populations of other species that rely on it.

Historically, the bobcat has faced threats of endangerment resulting from habitat loss, and overhunting. Recognizing the pivotal role of the bobcat in maintaining ecological balance, conservation efforts must be prioritized to ensure its long-term survival.

On the flip side, because the bobcat has no natural predators, there is a concern about overpopulation. It might be necessary to increase hunting efforts to maintain balance within the ecosystem. However, it is imperative to understand the thresholds of sustainable hunting practices before the population is at risk of endangerment.

By employing modeling techniques our research aims to provide insights that can guide conservation and population management strategies to ensure the health of the ecosystem long-term. Thus understanding the population dynamics of bobcats within an ecosystem is our primary motivation for this research paper. We frame our experiment by posing the research question; How does the introduction of a constant number of young bobcats and hunting at both a constant and proportional rate affect the long-term population growth of the bobcats?

Base Model

This model simulates the age-structured dynamics of a bobcat population. We are considering factors like survival and reproduction rates across different age classes. The model omits factors like environmental changes, events where a large amount of the population is killed, environment availability, and diseases.

Assumptions:

- The 16 age classes are the only age classes
- The environment is stable
- They have a consistent food supply
- The population is closed (no migrations in or out of the population)
- Only tracking bobcats that can give birth

These assumptions allow us to simplify the real-world system to a mathematical model that is defined by only a few values.

Model:

$$P_1(N+1) = r_1P_1(N) + r_2P_2(N) + r_3P_3(N) + \dots + r_{16}P_{16}(N)$$

$$P_2(N+1) = s_1P_1(N)$$

$$P_3(N+1) = s_2P_2(N)$$

$$\dots$$

$$P_{16}(N+1) = s_{15}P_{15}(N)$$

The first equation $P_1(N+1)$ calculates the total amount of bobcats that are born in the year N+1. The equations $P_2(N+1) \dots P_{16}(N+1)$ calculate the population of bobcats at age class x that survives into the following year (N+1) to become age class x+1.

Using these equations we can create the following matrix model:

$$\begin{bmatrix} P_1(N+1) \\ P_2(N+1) \\ P_3(N+1) \\ P_4(N+1) \\ P_5(N+1) \\ P_6(N+1) \\ P_7(N+1) \\ P_8(N+1) \\ P_9(N+1) \\ P_{10}(N+1) \\ P_{11}(N+1) \\ P_{12}(N+1) \\ P_{13}(N+1) \\ P_{14}(N+1) \\ P_{15}(N+1) \\ P_{16}(N+1) \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & r_7 & r_8 & r_9 & r_{10} & r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ s_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & s_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{15} & 0 \end{bmatrix} \begin{bmatrix} P_1(N) \\ P_2(N) \\ P_3(N) \\ P_4(N) \\ P_5(N) \\ P_6(N) \\ P_7(N) \\ P_8(N) \\ P_9(N) \\ P_{10}(N) \\ P_{11}(N) \\ P_{12}(N) \\ P_{13}(N) \\ P_{14}(N) \\ P_{15}(N) \\ P_{16}(N) \end{bmatrix}$$

State Variables: P_x(N) is the state variable. It represents the population of bobcats of the age class x, as a function of time.

Parameters: s_x and r_x are out parameter values. They correspond to the survival rate of age class x and reproduction rate of age class x respectively. The parameter values are represented as percentages of the population.

The parameter values we use are given by the following table

Age Class	Survival Worst Case	Survival Best Case	Reproduction Worst Case	Reproduction Best Case
1	0.32	0.34	0.60	0.63
2	0.68	0.71	0.60	0.63
3	0.68	0.71	1.15	1.20
4	0.68	0.71	1.15	1.20
5	0.68	0.71	1.15	1.20
⋮	⋮	⋮	⋮	⋮
16	0.68	0.71	1.15	1.20

Modeling Choices: This is a discrete-time model as the population dynamics are updated annually rather than continuously. This choice aligns with the discrete nature of annual reproductive cycles and simplifies the modeling process.

Analysis of Base Model

For a 16×16 matrix calculation, it is tedious for us to do it by hand, instead, we use R code to create a matrix then we plug in the parameters (S: survival rate and R: reproduction rate) for the best and worst case scenario. First, we calculate the determinants for the best and worst case and since both are not equal to 0, then $x^* = 0$ can be the only possible fixed point for both cases. We also found that both cases have at least one eigenvalue ≥ 1 this further proves that the model has no particular value is stable.

Intervention

To address the declining bobcat population, a proposed solution involves introducing 300 bobcats annually, comprising 300 newborns, 300 one-year-olds, and 300 two-year-olds. This strategy aims to reverse the trend by boosting their numbers gradually. Studies suggest a promising outcome, with projections indicating a substantial rise in bobcat population over time. The most optimistic scenario forecasts a peak of around 2.8 billion bobcats, while even the least favorable outcome predicts a population of 250 million after 50 years—both figures significantly higher than current estimates. However, this potential surge in population raises concerns about overpopulation and its potential repercussions on ecosystems. It underscores the critical importance of closely monitoring and managing bobcat populations to maintain ecological balance. Careful observation and intervention are necessary to prevent any unintended consequences that may arise from such a significant population increase. Balancing conservation efforts with ecological sustainability becomes paramount in ensuring the well-being of both bobcats and their habitats. Therefore, implementing effective strategies for population control and habitat management is essential for the long-term survival of bobcats.

Model Extension

Hunting is a common wildlife management strategy employed to control animal populations, especially when they become overly abundant and pose risks to ecosystem balance. In the case of bobcats, if their population exceeds the carrying capacity of their habitat, they may overconsume their prey and disrupt the ecosystem.

In the first scenario, we assume a constant hunting rate (h) where adult bobcats aged 3 and older are hunted each year. This means a fixed number of adult bobcats are removed from the population annually, regardless of the population size.

Effect on Population Dynamics: Constant hunting creates a steady pressure on the bobcat population. If the hunting rate exceeds the population growth rate, the population will decline over time until it reaches a new equilibrium where the birth rate equals the hunting rate.

In the second scenario, we assume the hunting rate (h) is proportional to the current bobcat population. This means that the higher the population, the more bobcats are hunted each year.

Effect on Population Dynamics: Proportional hunting introduces a feedback mechanism. As the population grows, more bobcats are hunted, which reduces the population growth rate. Conversely, if the population declines, fewer bobcats are hunted, allowing the population to recover. This creates a self-regulating mechanism that can potentially stabilize the population around a certain level.

Assumptions

- **Hunting Efficiency:** We assume that hunting is effective and that all hunted bobcats are successfully removed from the population.
- **Population Structure:** We maintain the age structure model with 16 age classes, assuming a steady-state population distribution among these classes.

- **Constant Reproductive Parameters:** We assume that the reproductive parameters remain constant over time and are not influenced by hunting pressure.

<u>Equations</u>

Constant rate:

$$P_1(N+1) = r_1P_1(N) + r_2P_2(N) + r_3P_3(N) + \dots + r_{16}P_{16}(N)$$

$$P_2(N+1) = s_1P_1(N)$$

$$P_3(N+1) = s_2P_2(N) - h * P_2(N)$$

$$\dots$$

$$P_{16}(N+1) = s_{15}P_{15}(N) - h* P_{15}(N)$$

$$\begin{bmatrix} P_1(N+1) \\ P_2(N+1) \\ P_3(N+1) \\ P_4(N+1) \\ P_5(N+1) \\ P_6(N+1) \\ P_7(N+1) \\ P_8(N+1) \\ P_9(N+1) \\ P_{10}(N+1) \\ P_{11}(N+1) \\ P_{12}(N+1) \\ P_{13}(N+1) \\ P_{14}(N+1) \\ P_{15}(N+1) \\ P_{16}(N+1) \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & r_7 & r_8 & r_9 & r_{10} & r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ s_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_2-h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s_3-h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_4-h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_5-h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_6-h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & s_7-h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_8-h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_9-h & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{10}-h & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{11}-h & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{12}-h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{13}-h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{14}-h & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s_{15}-h & 0 \end{bmatrix}$$

Proportional to population:

$$P_1(N+1) = r_1P_1(N) + r_2P_2(N) + r_3P_3(N) + \dots + r_{16}P_{16}(N)$$

$$P_2(N+1) = s_1P_1(N)$$

$$P_3(N+1) = s_2P_2(N) - a* P_3(N) - b* P_4(N) - c* P_5(N) - d* P_6(N) - e* P_7(N) - f* P_8(N) - g* P_9(N) - i* P_{10}(N) - j* P_{11}(N) - k* P_{12}(N) - l* P_{13}(N) - m* P_{14}(N) - n* P_{15}(N) - o* P_{16}(N)$$

$$\dots$$

$$P_{16}(N+1) = s_{15}P_{15}(N) - a* P_3(N) - b* P_4(N) - c* P_5(N) - d* P_6(N) - e* P_7(N) - f* P_8(N) - g* P_9(N) - i* P_{10}(N) - j* P_{11}(N) - k* P_{12}(N) - l* P_{13}(N) - m* P_{14}(N) - n* P_{15}(N) - o* P_{16}(N)$$

$$
\begin{bmatrix} P_1(N+1) \\ P_2(N+1) \\ P_3(N+1) \\ P_4(N+1) \\ P_5(N+1) \\ P_6(N+1) \\ P_7(N+1) \\ P_8(N+1) \\ P_9(N+1) \\ P_{10}(N+1) \\ P_{11}(N+1) \\ P_{12}(N+1) \\ P_{13}(N+1) \\ P_{14}(N+1) \\ P_{15}(N+1) \\ P_{16}(N+1) \end{bmatrix} = \begin{bmatrix}
r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & r_7 & r_8 & r_9 & r_{10} & r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\
s_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & s_2 & -a & -b & -c & -d & -e & -f & -g & -i & -j & -k & -l & -m & -n & -o \\
0 & 0 & s_3-a & -b & -c & -d & -e & -f & -g & -i & -j & -k & -l & -m & -n & -o \\
0 & 0 & -a & s_4-b & -c & -d & -e & -f & -g & -i & -j & -k & -l & -m & -n & -o \\
0 & 0 & -a & -b & s_5-c & -d & -e & -f & -g & -i & -j & -k & -l & -m & -n & -o \\
0 & 0 & -a & -b & -c & s_6-d & -e & -f & -g & -i & -j & -k & -l & -m & -n & -o \\
0 & 0 & -a & -b & -c & -d & s_7-e & -f & -g & -i & -j & -k & -l & -m & -n & -o \\
0 & 0 & -a & -b & -c & -d & -e & s_8-f & -g & -i & -j & -k & -l & -m & -n & -o \\
0 & 0 & -a & -b & -c & -d & -e & -f & s_9-g & -i & -j & -k & -l & -m & -n & -o \\
0 & 0 & -a & -b & -c & -d & -e & -f & -g & s_{10}-i & -j & -k & -l & -m & -n & -o \\
0 & 0 & -a & -b & -c & -d & -e & -f & -g & -i & s_{11}-j & -k & -l & -m & -n & -o \\
0 & 0 & -a & -b & -c & -d & -e & -f & -g & -i & -j & s_{12}-k & -l & -m & -n & -o \\
0 & 0 & -a & -b & -c & -d & -e & -f & -g & -i & -j & -k & s_{13}-l & -m & -n & -o \\
0 & 0 & -a & -b & -c & -d & -e & -f & -g & -i & -j & -k & -l & s_{14}-m & -n & -o \\
0 & 0 & -a & -b & -c & -d & -e & -f & -g & -i & -j & -k & -l & -m & s_{15}-n & -o
\end{bmatrix}
$$

Results

The analyses we performed on the extension included simulating a graph up to 50 years for both the constant and proportional hunting rate to observe the long term growth patterns. For the proportional rate we modeled three different percentages of the total population to hunt. These simulations are shown in figures 5, 6, 7, and 8.

Discussion

To evaluate the effectiveness of conservation and management programs, we framed our experiment by posing the research question: How does the introduction of a constant number of young bobcats and hunting at both a constant and proportional rate affect the long-term population growth of the bobcats?

In the base model, in the best case scenario, we found that the bobcat population shows negligible growth up to the 30 year mark, with a total population of about less a million, followed by exponential growth towards the 50 year mark, with a total population of about 12 million. The worst case scenario had a comparable trend, with the total population being lower than the best case, about 60,000 in 30 years and 1,000,000 in 50 years.

In the intervention strategy, by introducing 300 bobcats aged 0, 1, and 2 annually the total population also experiences exponential growth after the 30 year mark, growing up to a total of approximately 2.8 billion bobcats in 50 years in the best case and 250 million in the worst case.

In the hunting extension, by hunting at a constant rate each year, the population experiences exponential growth after the 20 year mark, increasing up to 100,000 in 50 years in the best case and around 10,000 in the worst case.

By hunting at a rate proportional to the total population, in the best case we modeled hunting 12% of the population, 13% of the population, and 14% of the population. In all cases the population dipped slightly from the initial value and then experienced a constant increase, to about 2500 in 50 years for 12%, 2100 for 13%, and a slight decrease to 1900 for 14%. In the worst case, we modeled hunting various percentages of the population as well and observed a similar trend with the population experiencing slower and slower constant increase the bigger the hunting proportion gets and a decrease when the proportion is big enough.

Comparing the best case scenarios of all cases, by the annual intervention strategy, the total bobcat population 50 years increased to 2.8 billion, which is about 200x more than the base model. By hunting at a constant rate annually, the total population went up to only 100,000, about 120x less than the base model. By hunting at a rate proportional to the population annually the total population decreased even more, with a smaller population the bigger the percentage of the population that is being hunted. By hunting 12% of the population annually, the population increases to only 2500 in 50 years, about 5000x less than the base model.

To answer our research question according to our results and apply them in the real world context, our results show the effectiveness of conservation and management strategies. When a population is endangered, it can be sustained by adding young individuals through breeding and conservation programs, greatly contributing to its growth. Conversely, when a population is overabundant and could potentially overconsume and endanger their prey, management strategies such as hunting can effectively control population growth. Hunting at a rate proportional to the total population can even more significantly curb the population growth, however, hunting too big of a percentage of the population can lead to extinction, therefore it should be carefully implemented. These findings underscore the importance of proactive conservation measures to maintain the balance of ecosystems that are dependent on a certain population. By mitigating population fluctuations we can safeguard not only that species itself but also other species that are affected by them in the ecosystem.

Moving on to the limitations of this model, One factor we have omitted is the bobcats that cannot give birth. By tracking only bobcats that can give birth, the only parameters we have are s_x and r_x, and including bobcats that cannot give birth could make the model more complicated by adding more parameters. Another assumption we have made that can greatly affect the model is that the population is closed and there are no migrations in or out of the population. Like the previous assumption, accounting for migrations in and out of the population would require additional parameters in our model, further complicating it. Some other assumptions that we have made include a stable environment and consistent food supply, and without this assumption the survival rate s_x could be much lower.

Furthermore, we have additional assumptions for the hunting extension; that hunting is effective and that all hunted bobcats are successfully removed from the population, that we maintain the population distribution among the 16 age classes, and that reproductive parameters are not influenced by hunting pressure. The strongest assumption here is a bout all age classes being equally removed from the population and maintaining their distribution since in the real world it is much more likely that bobcats of ages too young or old would not be hunted. Without this assumption, the changes in age distribution could affect and change the pattern of the population growth.

Although this research provides valuable insights into how conservation and management programs can control population fluctuations, there are always further possible directions for future research and more cases to consider. These considerations could just be looking at a more complicated model that relaxes the assumptions we made, or go into further detail, about other external factors. We've read about habitat loss being one of the biggest causes of endangerment, however we did not account for this in our model. This could open up many more directions for research, such as researching the effect on the bobcats' habitat by natural disasters, as well as by human intervention.

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Appendix: R code

```r
library(Matrix)
library(ggplot2)
library(matrixcalc)

M_best <- matrix(c(
  0.63, 0.63, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20,
  0.34, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
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  0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
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  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
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  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
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  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00
), nrow = 16, ncol = 16, byrow = TRUE)

M_worst <- matrix(c(
  0.60, 0.60, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15,
  0.32, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68,0.00), nrow = 16, ncol = 16


init_vals1 <- matrix(c(10,10,10,10,10,10,10,10,10,10,10,10,10,10,10,10), nrow = 1)

total_best <- function(T = 50, init_vals = init_vals1) {
  total_p <- rep(0, T+1)
  tvec <- 0:T
  init_vals <- as.vector(init_vals)

  for (i in tvec) {
    if (i == 0) {
      total_p[i+1] <- sum(init_vals)
```

```r
  for (i in tvec) {
    if (i == 0) {
      total_p[i+1] <- sum(init_vals)
    } else {
      x_n <- matrix.power(M_best, i) %*% init_vals
      total_p[i+1] <- sum(x_n)
    }
  }

  return(total_p)
}
total_p <- total_best()
number <- 0:50

ggplot(data = data.frame(number = number, total_p = total_p), aes(x = number, y = total_p)) +
  geom_point(size = 5) +
  labs(title = "Total population in the Best Case", x = "Years", y = "Population")



#total worst case
total_worst <- function(T = 50, init_vals = init_vals1) {
  i <- 0
  total_p <- rep(0, T+1)
  tvec <- 0:T
  init_vals <- as.vector(init_vals)
  for (i in tvec) {
    if (i == 0) {
      total_p[i+1] <- sum(init_vals)
    } else {
      x_n <- matrix.power(M_worst, i) %*% init_vals
      total_p[i+1] <- sum(x_n)
    }
  }
  return(total_p)
}
total_p <- total_worst()
number <- 0:50
ggplot(data = data.frame(number = number, total_p = total_p), aes(x = number, y = total_p)) +
  geom_point(size = 5) +
  labs(title = "Total population in the Worst Case", x = "Years", y = "Population")

#best case
population_b <- function(times = 5, init_vals = init_vals1){
  x_n <- matrix.power(M_best, times) %*% t(init_vals)
  return(x_n)
}

number <- 1:17
number <- number[-1]
#x_n <- unlist(x_n)
case1 <- population_b()
```

```r
}
total_p <- total_worst()
number <- 0:50
ggplot(data = data.frame(number = number, total_p = total_p), aes(x = number, y = total_p)) +
  geom_point(size = 5) +
  labs(title = "Total population in the worst Case", x = "Years", y = "Population")

#best case
population_b <- function(times = 5, init_vals = init_vals1){
  x_n <- matrix.power(M_best, times) %*% t(init_vals)
  return(x_n)
}

number <- 1:17
number <- number[-1]
#x_n <- unlist(x_n)
case1 <- population_b()
case2 <- population_b(times  = 10)
case3 <- population_b(times  =20)
df <- data.frame(number = number, case1 = case1, case2 = case2, case3 = case3)

ggplot(df, aes(x = number)) +
  geom_point(aes(y = case1, color = "After 5 Years"), size = 3) +
  geom_point(aes(y = case2, color = "After 10 Years"), size = 3) +
  geom_point(aes(y = case3, color = "After 20 Years"), size = 3) +
  scale_color_manual(values = c("After 5 Years" = "blue", "After 10 Years" = "red", "After 20 Years" = "green")) +
  guides(color = guide_legend(title = "Time")) +
  labs(title = "Best Case", x = "Ages", y = "Population") +
  theme(legend.position = c(0.9, 0.9))

#worst case
population_w <- function(times = 5, init_vals = init_vals1){
  x_n <- matrix.power(M_worst, times) %*% t(init_vals)
  return(x_n)
}
number <- 1:17
number <- number[-1]
wcase1 <- population_w()
wcase2 <- population_w(times  = 10)
wcase3 <- population_w(times  = 20)
df <- data.frame(number = number, wcase1 = wcase1, wcase2 = wcase2, wcase3 = wcase3)

ggplot(df, aes(x = number)) +
  geom_point(aes(y = wcase1, color = "After 5 Years"), size = 5) +
  geom_point(aes(y = wcase2, color = "After 10 Years"), size = 5) +
  geom_point(aes(y = wcase3, color = "After 20 Years"), size = 5) +
  scale_color_manual(values = c("After 5 Years" = "blue", "After 10 Years" = "red", "After 20 Years" = "green")) +
  guides(color = guide_legend(title = "Time")) +
  labs(title = "Worst Case", x = "Ages", y = "Population") +
  theme(legend.position = c(0.9, 0.9))
```

Figure 1: Total population in the Best Case:

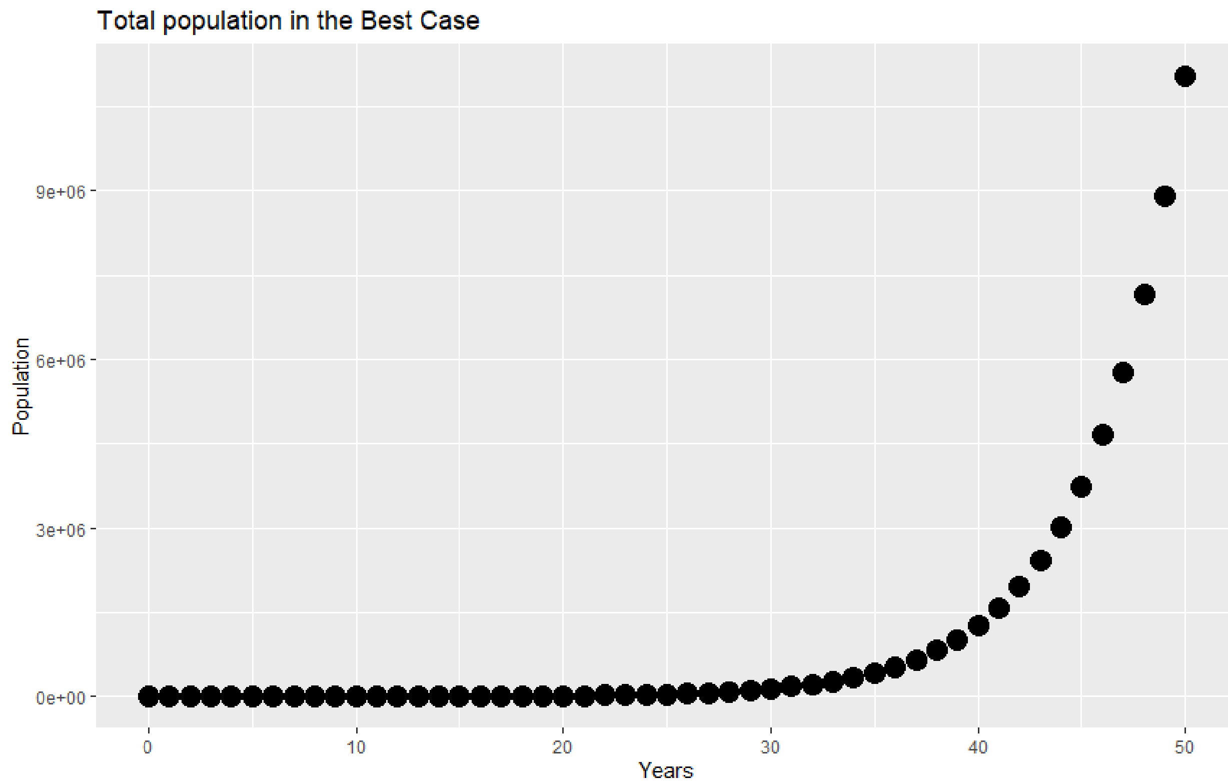


Figure 2: Total population in the Worst Case:

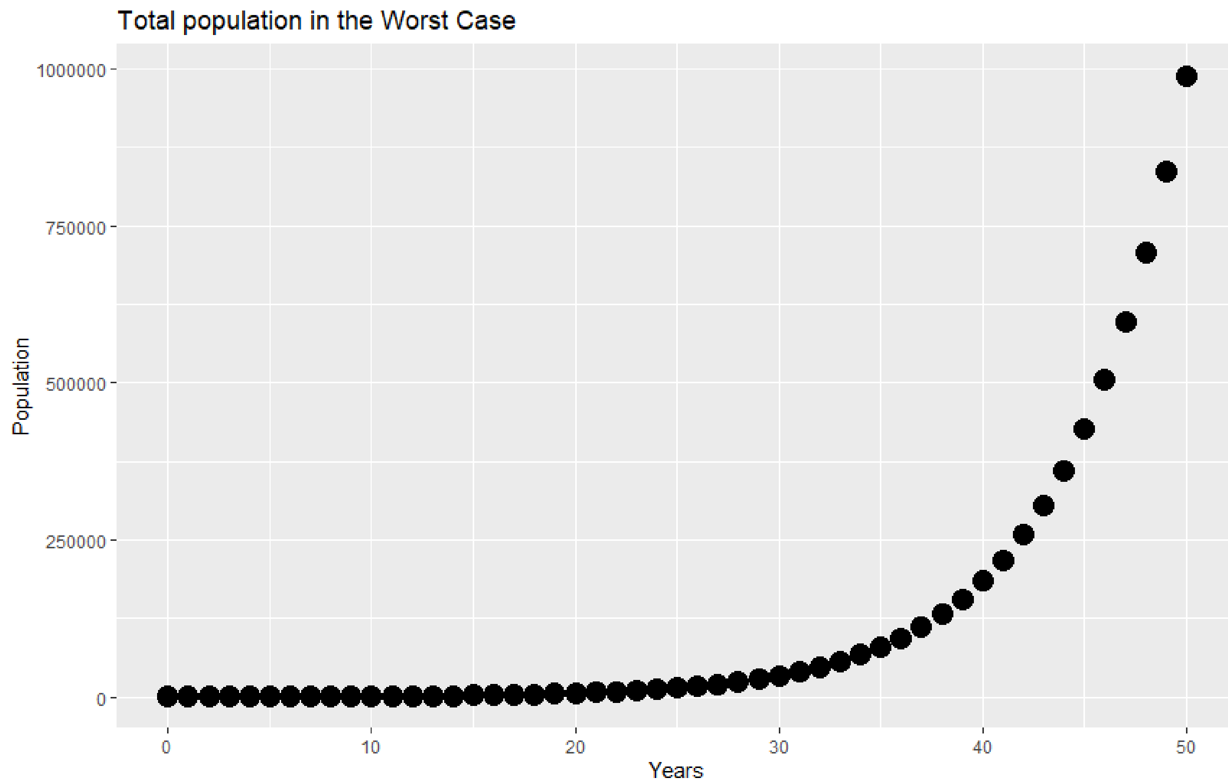


Figure 3: Age Classes Population in Best Case

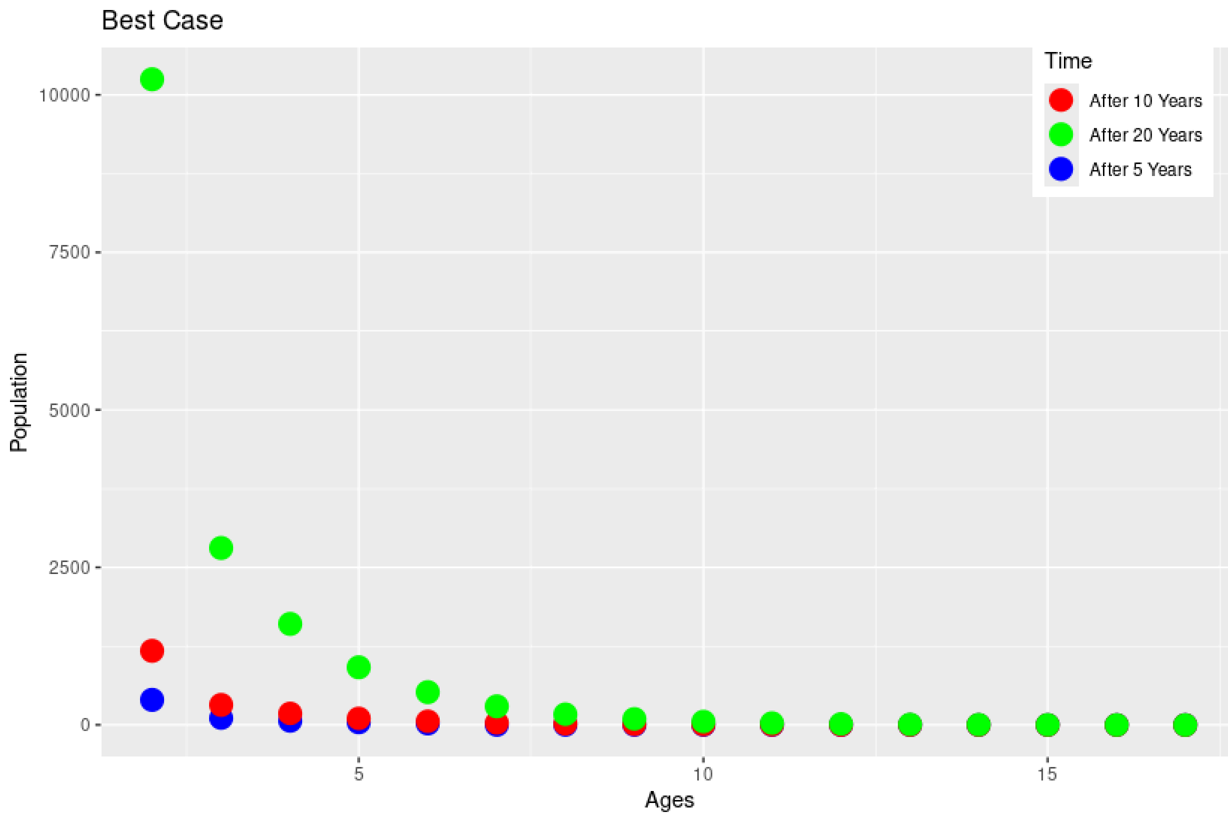
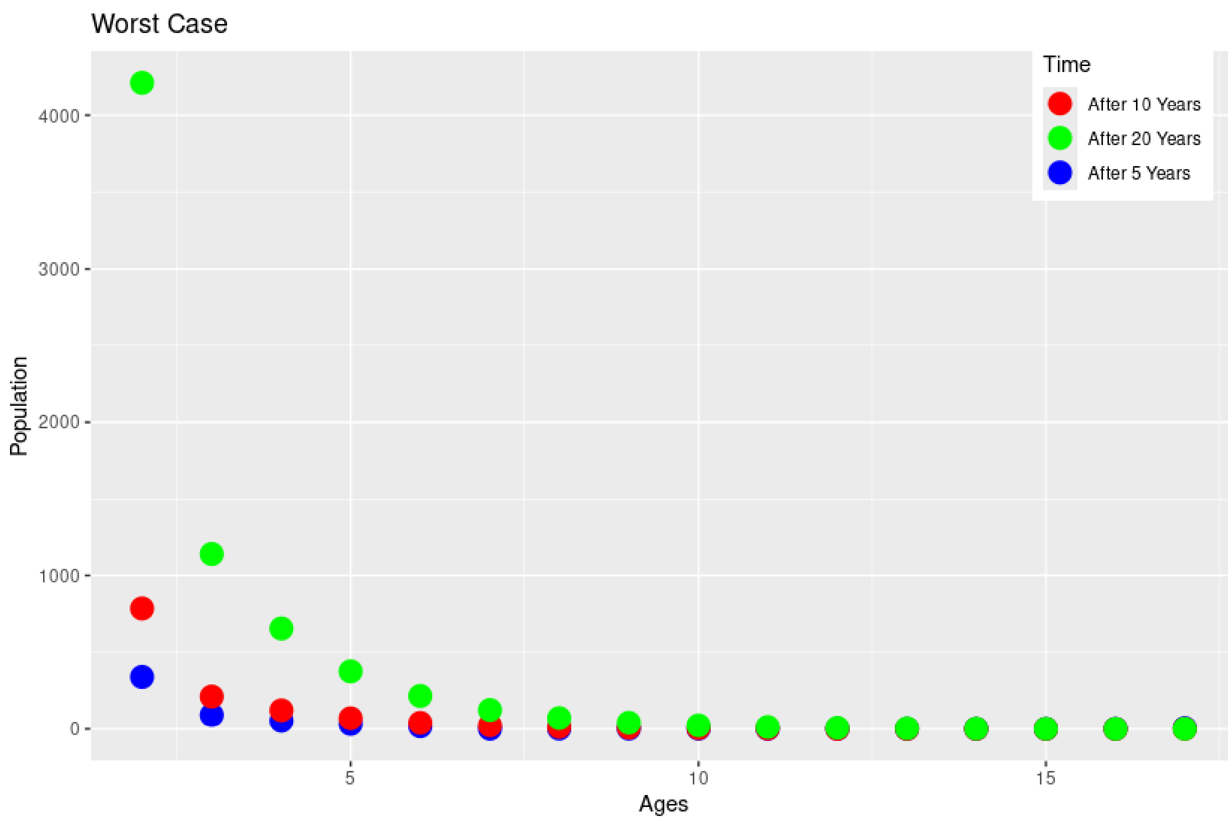


Figure 4: Age Classes Population in Worst Case



Model Analysis (Best case)

```r
library(Matrix)

M_best <- matrix(c(
  0.63, 0.63, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20,
  0.34, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00),
  nrow = 16, byrow = TRUE)

# Create identity matrix I_b
I_b <- diag(16)

# Calculate matrix A_b
A_b <- I_b - M_best

# Print determinant of A_b
cat("Determinant:", det(A_b), "\n")

# Calculate eigenvalues and eigenvectors of M_best
eig_b <- eigen(M_best)$values
P_b <- eigen(M_best)$vectors

# Print eigenvalues
cat("Eigenvalues:", "\n")
print(eig_b)
```

```
Determinant: -0.8348336
Eigenvalues:
 [1]  1.2414402+0.0000000i  0.6389141+0.2911959i  0.6389141-0.2911959i  0.4679897+0.5124445i
 [5]  0.4679897-0.5124445i  0.2398663+0.6462321i  0.2398663-0.6462321i -0.0127441+0.6811687i
 [9] -0.0127441-0.6811687i -0.2576225+0.6165321i -0.2576225-0.6165321i -0.4621207+0.4651450i
[13] -0.4621207-0.4651450i -0.5975540+0.2496598i -0.5975540-0.2496598i -0.6448978+0.0000000i
> 
```

Model Analysis (Worst Case)

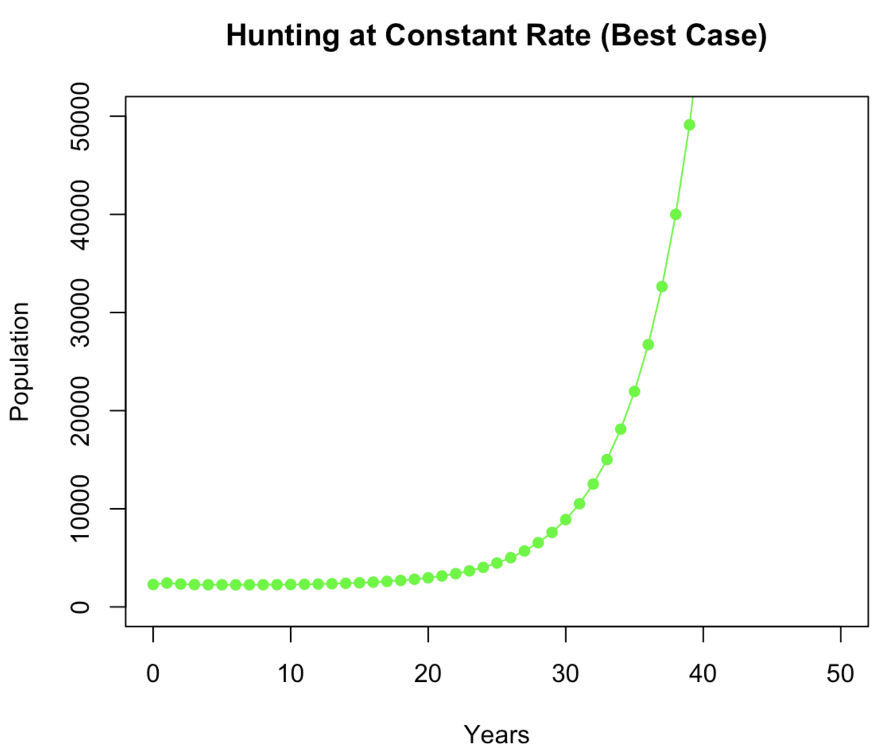
```r
library(Matrix)

# Define matrix M_worst
M_worst <- matrix(c(
  0.60, 0.60, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15,
  0.32, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00),
  nrow = 16, byrow = TRUE)

# Create identity matrix I_b
I_b <- diag(16)

# Calculate matrix A_b
A_b <- I_b - M_worst

# Print determinant of A_b
cat("Determinant:", det(A_b), "\n")

# Calculate eigenvalues and eigenvectors of M_worst
eig_b <- eigen(M_worst)$values
P_b <- eigen(M_worst)$vectors

# Print eigenvalues
cat("Eigenvalues:", "\n")
print(eig_b)
```

```
Determinant: -0.5704655
Eigenvalues:
 [1]  1.1828518+0.0000000i  0.6117292+0.2789076i  0.6117292-0.2789076i  0.4480233+0.4905486i
 [5]  0.4480233-0.4905486i  0.2296965+0.6184589i  0.2296965-0.6184589i -0.0120604+0.6517932i
 [9] -0.0120604-0.6517932i -0.2464271+0.5899047i -0.2464271-0.5899047i -0.4421333+0.4450492i
[13] -0.4421333-0.4450492i -0.5717348+0.2388734i -0.5717348-0.2388734i -0.6170387+0.0000000i
>
```

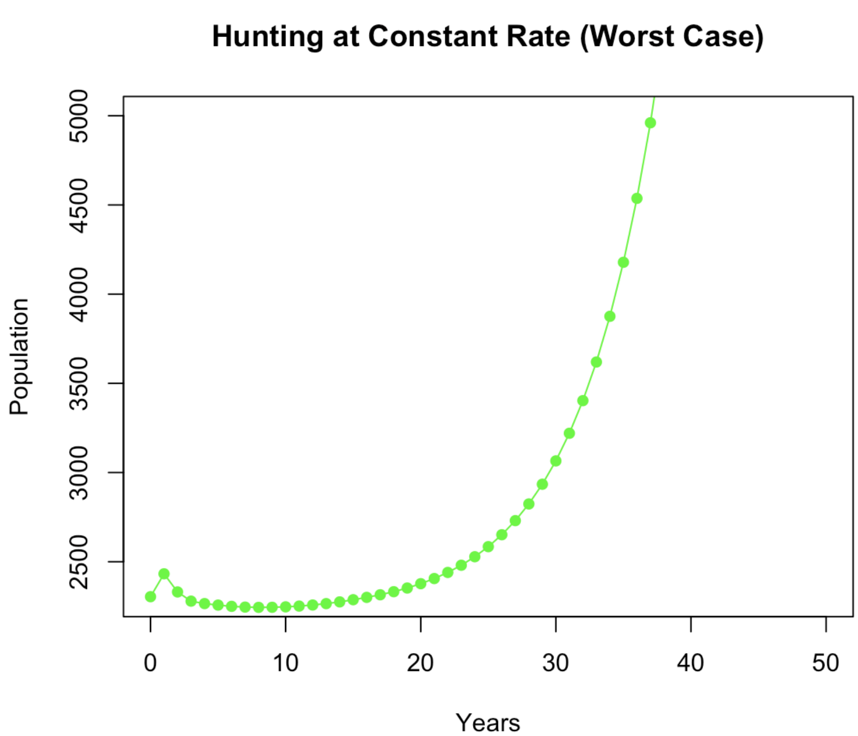
Hunting Extension (Constant Rate)

```r
initial_values <- matrix(c(100,100,100,100,100,100,100,100,100,100,100,100,100,100,100,100), nrow = 1)
hunt_values <- matrix(c(350,100,20,20,20,20,10,10,0,0,0,0,0,0,0,0), nrow = 1)

hunt_best_constant <- function(T = 50, initial_values, M_best){
  i <- 0
  total_population <- numeric(T+1)

  while (i <= T){
    if (i == 0){
      population <- M_best %*% t(initial_values)
      population <- population - t(hunt_values)
      total_population[i+1] <- sum(population)
    } else {
      population <- M_best %*% population
      population <- population - t(hunt_values)
      total_population[i+1] <- sum(population)
    }
    i <- i + 1
  }
  return(total_population)
}

M_best <- matrix(c(
  0.63, 0.63, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20,
  0.34, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00),
  nrow = 16, byrow = TRUE)

total_population <- hunt_best_constant(T = 50, initial_values = initial_values, M_best = M_best)

number_of_years <- 0:50
plot(number_of_years, total_population, type = 'o', pch = 16, col = 'green', ylim = c(0, 50000), xlim = c(0, 50),
     xlab = 'Years', ylab = 'Population', main = 'Hunting at Constant Rate under Best Case')
```

Figure 5: Hunting at a Constant Rate in the Best Case



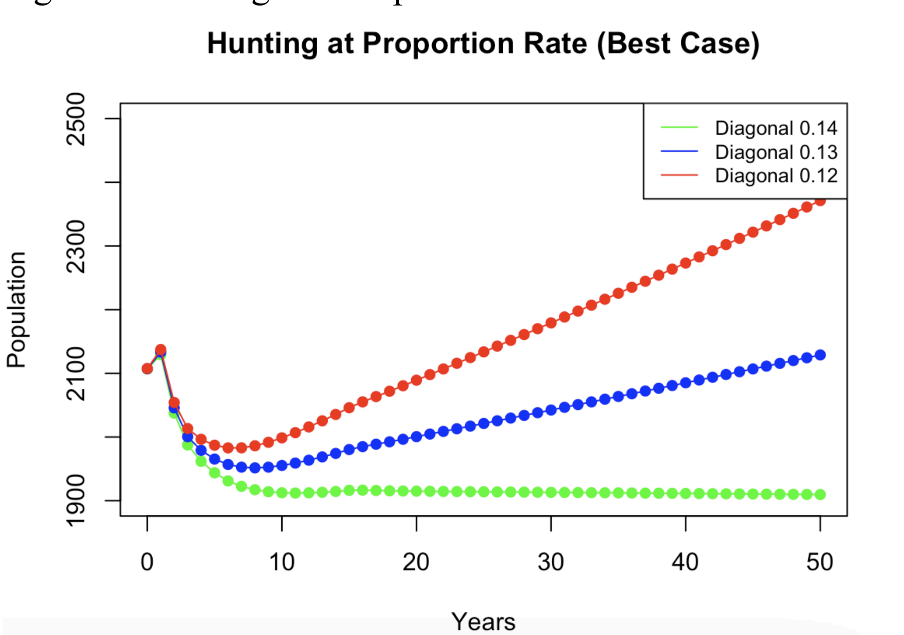
```r
M_worst <- matrix(c(
  0.60, 0.60, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15,
  0.32, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00),
  nrow = 16, byrow = TRUE)

init_vals <- matrix(c(100,100,100,100,100,100,100,100,100,100,100,100,100,100,100,100), nrow = 1)
hunt <- matrix(c(260,50,20,20,20,20,10,10,0,0,0,0,0,0,0,0), nrow = 1)
hunt_worst_constant <- function(T = 50, initial_values){
  i <- 0
  total_p <- numeric(T+1)

  while (i <= T){
    if (i == 0){
      population <- M_worst %*% t(initial_values)
      population <- population - t(hunt)
      total_p[i+1] <- sum(population)
    } else {
      population <- M_worst %*% population
      population <- population - t(hunt)
      total_p[i+1] <- sum(population)
    }
    i <- i + 1
  }
  return(total_p)
}
total_p <- hunt_worst_constant(T = 50, initial_values = init_vals)
number <- 0:50
plot(number, total_p, type = 'o', pch = 16, col = 'green', ylim = c(2300, 5000), xlim = c(0, 50),
     xlab = 'Years', ylab = 'Population', main = 'Hunting at Constant Rate under Worst Case')
```
Figure 6: Hunting at a Constant Rate in the Worst Case



Hunting Extension (Proportion Rate)

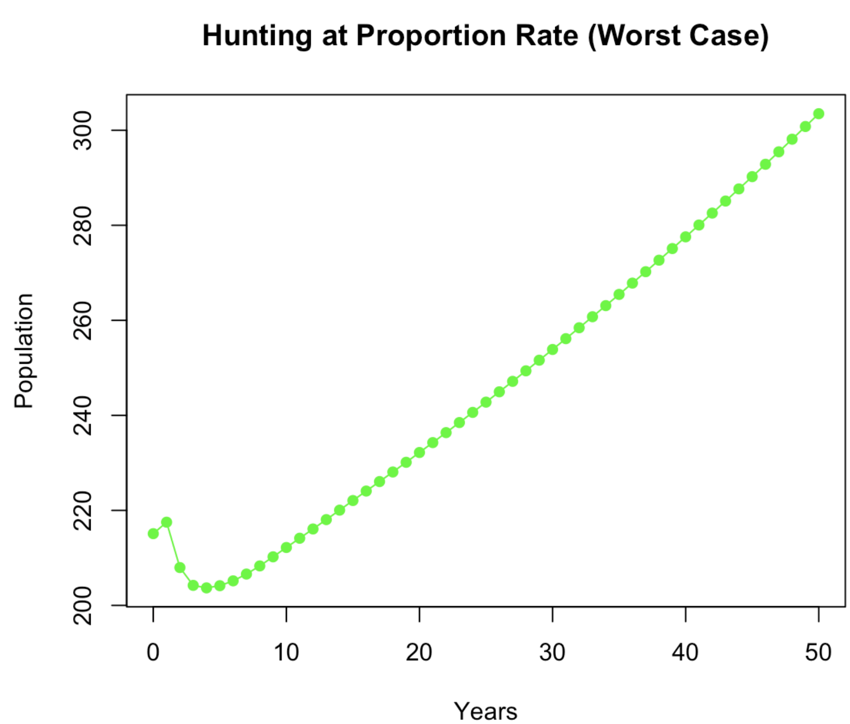
```r
D <- matrix(0, nrow = 16, ncol = 16)
diag_values <- c(0.4, 0.14, rep(0, 14))
diag(D) <- diag_values

hunt_best_prop <- function(T = 50, initial_values, M_best, D) {
  i <- 0
  total_population <- numeric(T+1)

  while(i <= T){
    if (i == 0) {
      population <- M_best %*% t(initial_values)
      population <- population - D %*% population
      total_population[i+1] <- sum(population)
    }
    else {
      population <- M_best %*% population
      population <- population - D %*% population
      total_population[i+1] <- sum(population)
    }
    i <- i + 1
  }
  return(total_population)
}

total_population <- hunt_best_prop(T = 50, initial_values = initial_values, M_best = M_best, D = D)

diag_values_2 <- c(0.4, 0.13, rep(0, 14))
diag_values_3 <- c(0.4, 0.12, rep(0, 14))

total_population_2 <- hunt_best_prop(T = 50, initial_values = initial_values, M_best = M_best, D = diag(diag_val
total_population_3 <- hunt_best_prop(T = 50, initial_values = initial_values, M_best = M_best, D = diag(diag_val

number_of_years <- 0:50

plot(number_of_years, total_population, type='o', pch = 16, col = "green", ylim = c(1900, 2500),
     xlab = 'Years', ylab = 'Population', main = 'Hunting at Proportion Rate (Best Case)')
lines(number_of_years, total_population_2, type='o', pch = 16, col = "blue")
lines(number_of_years, total_population_3, type='o', pch = 16, col = "red")
legend("topright", legend=c("Diagonal 0.14", "Diagonal 0.13", "Diagonal 0.12"), col=c("green", "blue", "red"), l
```

Figure 7: Hunting at a Proportional Rate in the Best Case



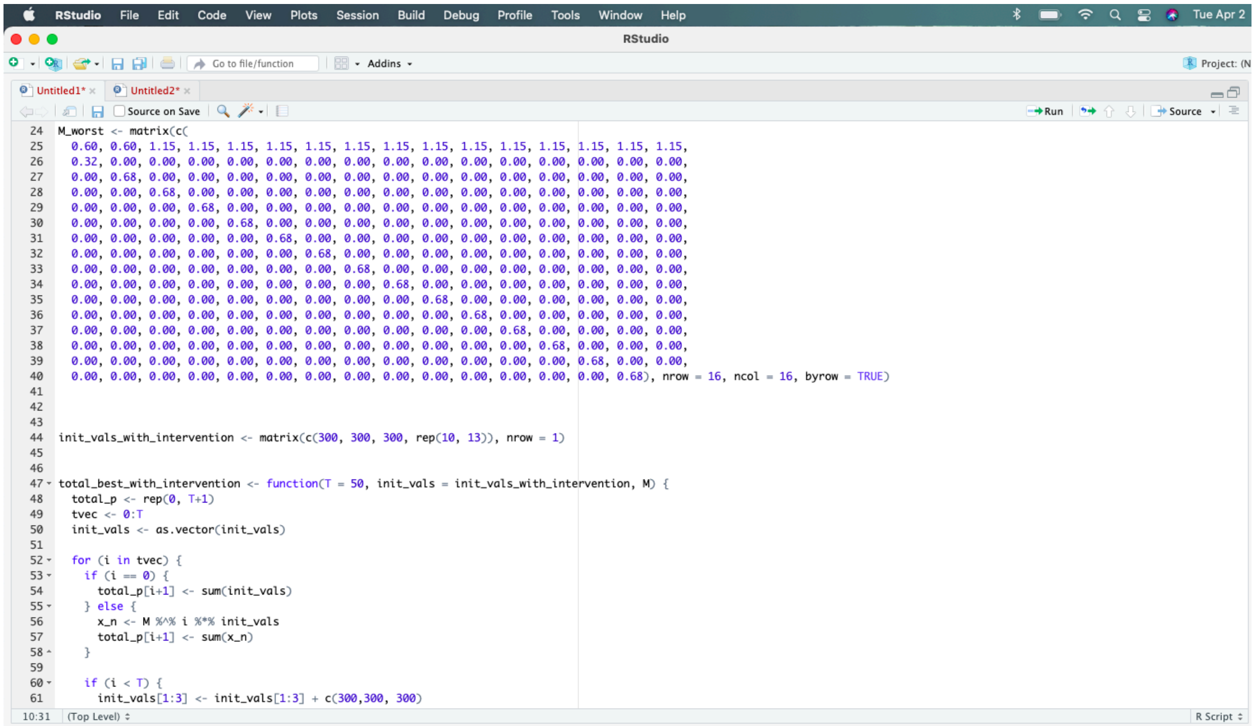
```r
D <- matrix(0, nrow = 16, ncol = 16)
prop <- matrix(c(0.30,0.1,rep(0,8),0.1,0.1,0.1,0.1,0.1,0.1), nrow = 1)
diag(D) <- prop

hunt_worst_prop <- function(T = 50, initial_values, M_worst, D){
  i <- 0
  total_population <- numeric(T+1)

  while (i <= T){
    if (i == 0){
      population <- M_worst %*% t(initial_values)
      population <- population - D %*% population
      total_population[i+1] <- sum(population)
    } else {
      population <- M_worst %*% population
      population <- population - D %*% population
      total_population[i+1] <- sum(population)
    }
    i <- i + 1
  }

  return(total_population)
}

init_vals <- matrix(c(10,10,10,10,10,10,10,10,10,10,10,10,10,10,10,10), nrow = 1)

total_population <- hunt_worst_prop(T = 50, initial_values = init_vals, M_worst = M_worst, D = D)
print(total_population)
number <- 0:50
total_p <- hunt_worst_prop(T = 50, initial_values = init_vals, M_worst = M_worst, D = D)
plot(number, total_p, type = 'o', pch = 16, col = 'green', xlab = 'Years', ylab = 'Population',
     main = 'Hunting at Proportion Rate under Worst Case')
```

Figure 8: Hunting at a Proportional Rate in the Worst Case

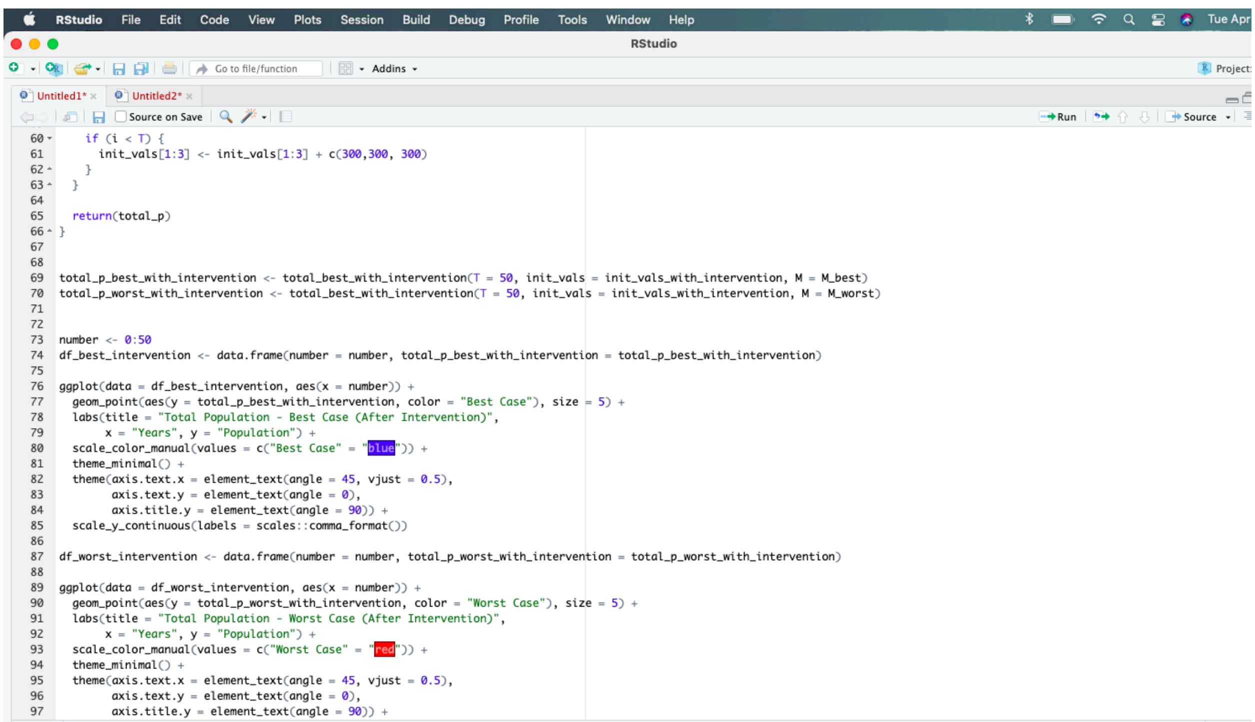


Intervention

```r
library(Matrix)
library(ggplot2)
library(matrixcalc)

M_best <- matrix(c(
  0.63, 0.63, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20, 1.20,
  0.34, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.71, 0.00
), nrow = 16, ncol = 16, byrow = TRUE)

M_worst <- matrix(c(
  0.60, 0.60, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15,
  0.32, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00,
```



```r
M_worst <- matrix(c(
  0.60, 0.60, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15, 1.15,
  0.32, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68, 0.00, 0.00,
  0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.68), nrow = 16, ncol = 16, byrow = TRUE)



init_vals_with_intervention <- matrix(c(300, 300, 300, rep(10, 13)), nrow = 1)


total_best_with_intervention <- function(T = 50, init_vals = init_vals_with_intervention, M) {
  total_p <- rep(0, T+1)
  tvec <- 0:T
  init_vals <- as.vector(init_vals)

  for (i in tvec) {
    if (i == 0) {
      total_p[i+1] <- sum(init_vals)
    } else {
      x_n <- M %^% i %*% init_vals
      total_p[i+1] <- sum(x_n)
    }

    if (i < T) {
      init_vals[1:3] <- init_vals[1:3] + c(300,300, 300)
```



```r
    if (i < T) {
      init_vals[1:3] <- init_vals[1:3] + c(300,300, 300)
    }
  }

  return(total_p)
}


total_p_best_with_intervention <- total_best_with_intervention(T = 50, init_vals = init_vals_with_intervention, M = M_best)
total_p_worst_with_intervention <- total_best_with_intervention(T = 50, init_vals = init_vals_with_intervention, M = M_worst)


number <- 0:50
df_best_intervention <- data.frame(number = number, total_p_best_with_intervention = total_p_best_with_intervention)

ggplot(data = df_best_intervention, aes(x = number)) +
  geom_point(aes(y = total_p_best_with_intervention, color = "Best Case"), size = 5) +
  labs(title = "Total Population - Best Case (After Intervention)",
       x = "Years", y = "Population") +
  scale_color_manual(values = c("Best Case" = "blue")) +
  theme_minimal() +
  theme(axis.text.x = element_text(angle = 45, vjust = 0.5),
        axis.text.y = element_text(angle = 0),
        axis.title.y = element_text(angle = 90)) +
  scale_y_continuous(labels = scales::comma_format())

df_worst_intervention <- data.frame(number = number, total_p_worst_with_intervention = total_p_worst_with_intervention)

ggplot(data = df_worst_intervention, aes(x = number)) +
  geom_point(aes(y = total_p_worst_with_intervention, color = "Worst Case"), size = 5) +
  labs(title = "Total Population - Worst Case (After Intervention)",
       x = "Years", y = "Population") +
  scale_color_manual(values = c("Worst Case" = "red")) +
  theme_minimal() +
  theme(axis.text.x = element_text(angle = 45, vjust = 0.5),
        axis.text.y = element_text(angle = 0),
        axis.title.y = element_text(angle = 90)) +
```

Figure 9: Total Population After Intervention in The Best Case

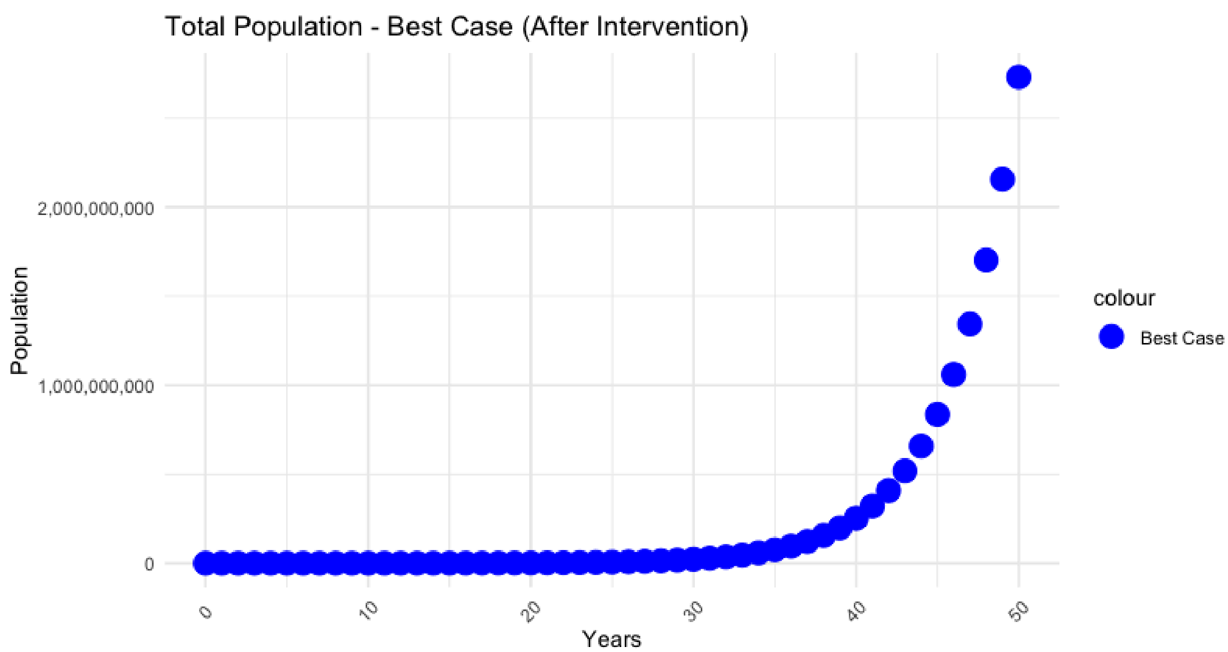
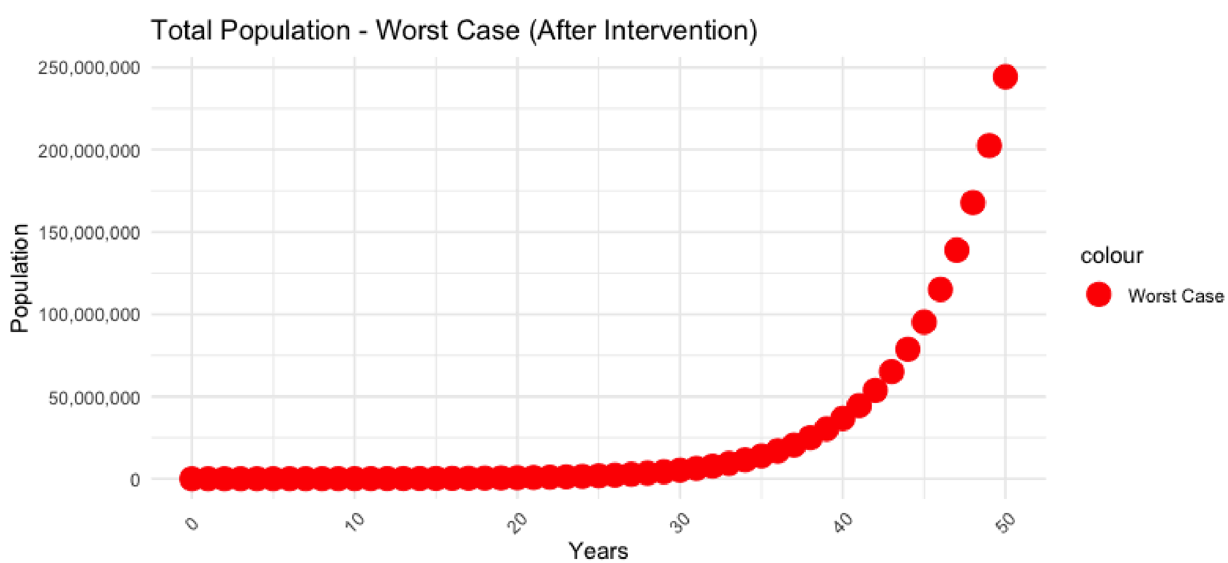


Figure 10: Total Population After Intervention in The Worst Case



Appendix: Individual Contributions

Afsah: Discussion, Results

Ahmed: Intervention, Model extension.

Anthony: Hunting Extension, Base Model, Simulations

Johanna: Introduction, research, Base Model

Ian: Simulations, Conclusion

Chris: Simulations, Base Model Code