ICS 683
Advanced Computer Vision

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Lecture 10

- Models of Reflectance
- Phong's Shading Model
- Photometric Stereo

- A significant part of the materials in this lecture notes are from the similar courses offered by George C. Stockman, David Forsyth, and Francesc Moreno-Noguer. I would like to thank the instructors for the slides and content used in this lecture.

Announcements

- Homework assignment #3
  - Due: **Tuesday, October 15**

- Tuesday, **October 8**: study day
  - Class and office hours are canceled

Previously…

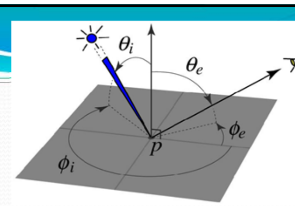
- Radiance: Power emitted from a surface patch, in watts per square meter per steradian (W m^{-2} sr^{-1}) (i.e. power per unit foreshortened area per unit solid angle emitted from the surface)

$$L = \frac{P}{dA \cos\theta \, d\omega} \Rightarrow P = L \, dA \cos\theta \, d\omega$$

- Irradiance: Power falling on a surface patch, in watts per square meter (W m^{-2}) (i.e. incident power per unit area *not foreshortened*)

$$E = \frac{P}{dA} = \frac{L dA \cos\theta \, d\omega}{dA} = L \cos\theta \, d\omega$$

The BRDF (I)



- Model of local reflection that tells how bright a surface appears when viewed from one direction while light falls on it from another
- Bidirectional Reflectance Distribution Function (BRDF)
  - **ratio of the radiance in the outgoing direction to the incident irradiance**

$$\rho_{bd}(P,\theta_e,\phi_e,\theta_i,\phi_i) = \frac{L_e(P,\theta_e,\phi_e)}{E_i(P,\theta_i,\phi_i)} = \frac{L_e(P,\theta_e,\phi_e)}{L_i(P,\theta_i,\phi_i)\cos\theta_i d\omega}$$

  - Function of incoming light direction (θ_i, ϕ_i) and outgoing light direction (θ_e, ϕ_e)

Figure credit: S. Seitz

The BRDF (II)

- Units: inverse steradians (sr^{-1})
- *Helmholtz reciprocity principle*
  - Reversing the path of light produces the same reflectance (i.e. symmetric in incoming and outgoing directions)

$$\rho_{bd}(x,\theta_e,\phi_e,\theta_i,\phi_i)=\rho_{bd}(x,\theta_i,\phi_i,\theta_e,\phi_e)$$

- Radiance leaving a surface in a particular direction:
  - Integrate irradiance from every incoming direction scaled by BRDF

$$L_e(P,\theta_e,\phi_e)=\int_{\Omega}\rho_{bd}(P,\theta_e,\phi_e,\theta_i,\phi_i,)L_i(P,\theta_i,\phi_i)\cos\theta_i d\omega$$

(Ω: incoming hemisphere)

BRDF Databases

- CUReT (Columbia-Utrecht) (http://www.cs.columbia.edu/CAVE/software/curet/)
  - 61 samples
  - BRDF was measured for each sample by recording images of the sample under 205 different combinations of viewing and illumination directions

BRDF Databases

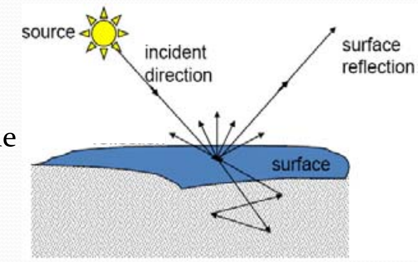
- MERL BRDF Database (http://www.merl.com/brdf/)
  - Reflectance functions of 100 different materials

Models of Reflectance

- We need to look at models for the physics of illumination and reflection that will
  1. help computer vision algorithms extract information about the 3D world,
  2. help computer graphics algorithms render realistic images of model scenes.
- Physics-based vision is the subarea of computer vision that uses physical models to understand image formation in order to better analyze real-world images.

Surface Reflectance

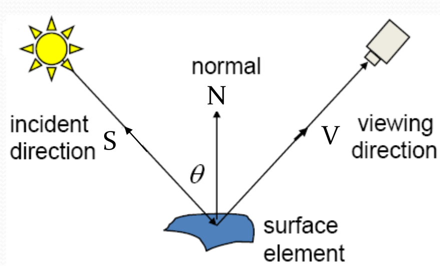
- **Diffuse reflection** (scattering)
  - Re-emit uniformly in all direction
  - Color strongly dependent on the nature of surface
  - Matte appearance



- **Specular reflection**
  - Mirror-like, produce highlights, highly directional
  - Same color as incident light (assume independent to material)
  - Glossy appearance, dominant for metals
- **Albedo** of a surface: ratio of the total reflected power to the total incident power. It is also called ***reflection coefficient***.

Diffuse Surface Reflection

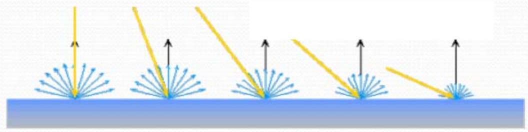
- **Lambertian surfaces** (or **ideal diffuse surfaces**) reflects light uniformly in all directions
- Surface appears equally bright from ALL directions! (independent of *V* when *S* is fixed)



  - e.g. cotton cloth, carpets, matte paper, matte paints, etc.
- For such surfaces, radiance leaving the surface is independent of viewing angle (therefore, BRDF is independent of angle, too)

Diffuse Surface Reflection

- Brightness does depend on direction of illumination



- **Lambert's Law:** The intensity of light energy (I_d) reflected from an ideal diffuse surfaces is proportional to the cosine of the angle θ between the surface normal (N) and the illumination direction (S)

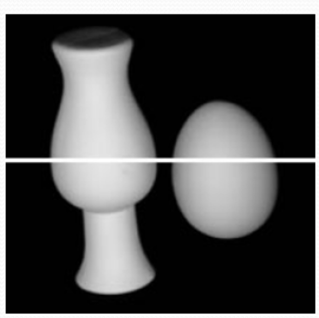
$$I_d = \rho_d L_i \cos\theta = \rho_d L_i (N \cdot S)$$

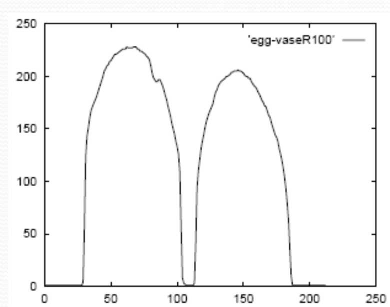
diffuse albedo

  - N and S are unit vectors

Figure credit: S. Seitz

Example Matte Objects

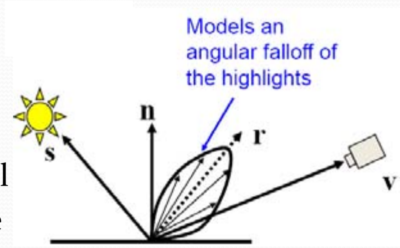




Diffuse reflection from Lambertian objects and a plot of intensities across the highlighted row. The intensities are closely related to the object shape.

Specular Surfaces

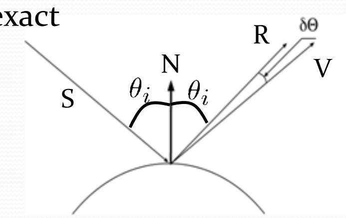
- Radiation arriving along a direction leaves along the specular direction (source direction reflected about normal
- Some fraction is absorbed, some reflected



- On real surfaces, energy usually goes into a lobe of directions
- The wavelength composition of the reflected light is similar to that of the source and independent of the surface color
- A ***highlight*** on an object is a bright spot caused by the specular reflection of a light source

Specular Reflection

- There are very few cases where the exact shape of the specular lobe matters
  - R: ray of reflection
  - V: viewing direction
  - α: shininess parameter



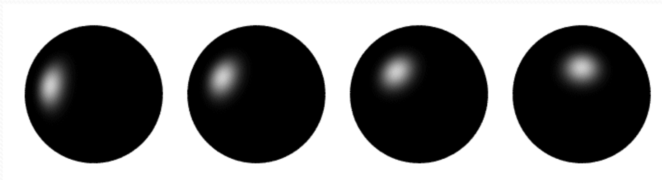
$\boxed{R = 2(-S \cdot N)N + S}$

- Phong's model of specular reflection
  - The intensity of reflected light energy falls off with $\cos^{\alpha}(\delta\theta)$

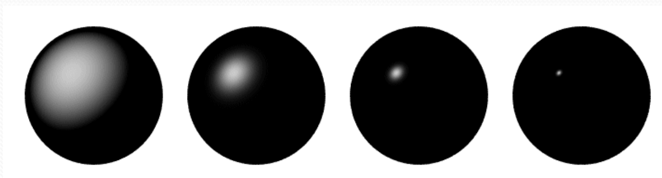
$$I_s = \rho_s L_i \cos^{\alpha}(\delta\theta) = \rho_s L_i (R \cdot V)^{\alpha}$$

specular albedo

Specular Reflection



Moving the light source



Changing α

Slide credit: S. Seitz

Lambertian + Specular

- Many real surfaces combine diffuse and glossy component of reflection
- In order to approximate these surfaces
  $\rightarrow$ Models combining Diffuse and Specular reflection (e.g. Phong)

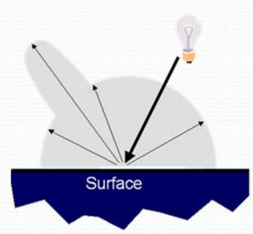
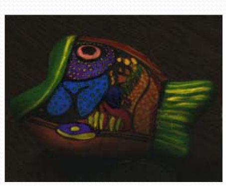
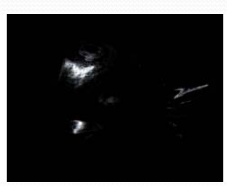


Figure credit: M. Pollefeys

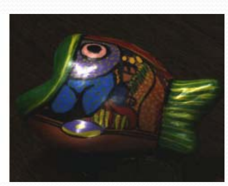
Observed Image Color = $\alpha \times$ Diffuse Ref. Color + $\beta \times$ Specular Ref. Color



diffuse



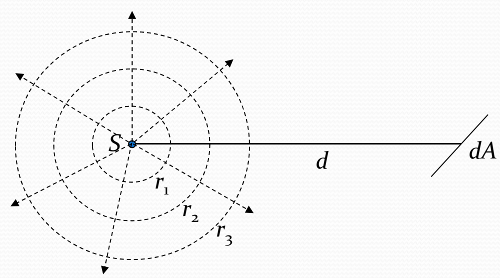
specular



diffuse+specular

Darkening with Distance

- The intensity of light energy reaching a surface decreases with the distance of that surface from the light source
  - The intensity of light received by any object surface decreases with the square of its distance from the source



Shading Models

- Local shading model
  - Surface shading is due only to sources visible at each point
  - Advantages
    - Often easy to manipulate, expressions easy
    - Supports quite simple theories of how shape information can be extracted from shading
- Global shading model
  - Surface shading is due to radiance reflected from other surfaces as well as from sources
  - Advantage: usually very accurate
  - Disadvantage: extremely difficult to infer anything from shading values

Phong's Shading Model

- Reasonable realism, reasonable computing
- Popular shading model used in graphics – modern computer games use more complicated models though
- Phong's shading model accounts for
  - (a) ambient light
  - (b) diffuse reflection
  - (c) specular reflection
  - (d) darkening with distance
- **Ambient light** is steady state light energy everywhere in the scene resulting from many light sources and the inter-reflections off many surfaces

Phong's Shading Model

ambient reflectivity

intensity of light source *m*

reflection ray

viewing direction

$$I_\lambda(x,y) = I_{a\lambda}\rho_{a\lambda} + \sum_{m=1}^{s}\left(\frac{1}{cd_m^2} I_{m\lambda}\left[\rho_{d\lambda}(N\cdot S_m) + \rho_{s\lambda}(R_m\cdot V)^n\right]\right)$$

ambient illumination

distance from the surface

diffuse reflectivity

specular reflectivity

Photometric Stereo (Shape from Shading)

- Can we reconstruct the shape of an object based on shading cues? Yes! (Woodham, 1980)
- Given multiple images of the same surface under different known lighting conditions, we can recover the surface shape

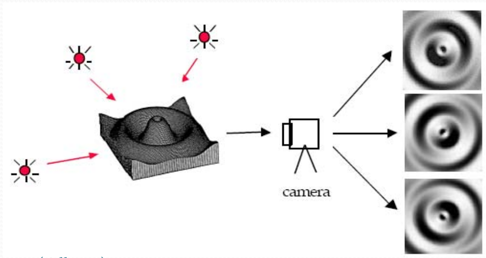


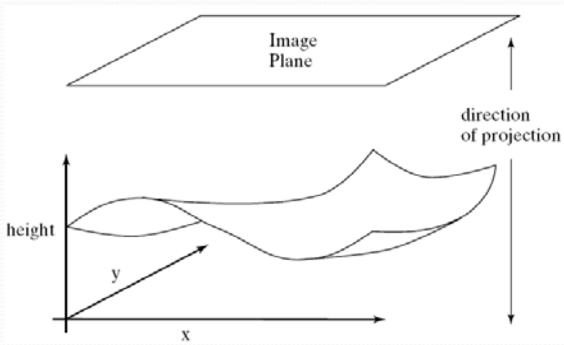
Figure credit: S. Park

Photometric Stereo

- Assume:
  - Lambertian object (or the specular component has been identified and removed)
  - Local shading model (each point on a surface receives light only from sources visible at that point)
  - Set of point sources that are infinitely distant (light source directions are known)
  - Set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
  - Orthographic projection
- **Goal**: Reconstruct object shape and albedo

Recovering the Surface Shape

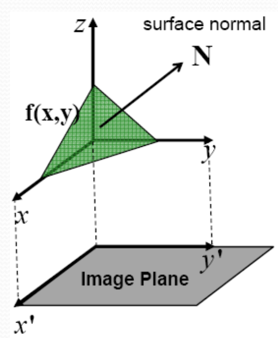
- Projection model for surface recovery



$z = f(x, y)$, surface is set of points $(x, y, f(x, y))$

Surface Orientation (I)

- **Surface:** $s = (x, y, f(x, y))$

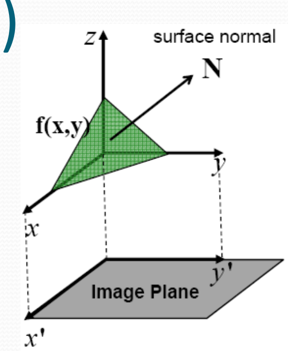


$$\begin{cases} f_x = \dfrac{\partial f}{\partial x} & \text{: slope in the x-direction} \\ f_y = \dfrac{\partial f}{\partial y} & \text{: slope in the y-direction} \end{cases}$$

$$\Rightarrow \begin{cases} \partial x \cdot f_x & \text{: change in height for small step of length } \partial x \text{ in the } x\text{-direction} \\ \partial y \cdot f_y & \text{: change in height for small step of length } \partial y \text{ in the } y\text{-direction} \end{cases}$$

First small step in vector form: $(\partial x, 0, \partial x f_x)$, and $(0, \partial y, \partial y f_y)$

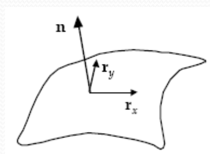
Surface Orientation (II)



- **Tangent vectors:** $\frac{\partial s(x,y)}{\partial x} = (1, 0, f_x)$

$$\frac{\partial s(x,y)}{\partial y} = (0, 1, f_y)$$

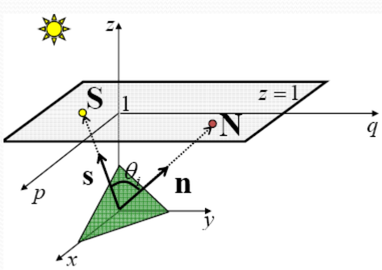
- **Surface normal**:



$$\mathbf{N}(x,y) = \frac{\partial s}{\partial x} \times \frac{\partial s}{\partial y}$$
$$= (-f_x, -f_y, 1)$$
$$= (p, q, 1)$$

Gradient Space

- $z = 1$ plane is called the Gradient Space (pq plane)
  - Its components p and q are the surface slopes in the x- and y- direction
  - Every point on it corresponds to a particular surface orientation



unit normal vector:

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p,q,1)}{\sqrt{p^2 + q^2 + 1}}$$

unit source vector:

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_s, q_s, 1)}{\sqrt{p_s^2 + q_s^2 + 1}}$$

Reflectance Map

- Relates image intensity $I(x, y)$ to surface orientation (p, q) for GIVEN source direction and surface reflectance
- Tool used in developing method for recovering shape from images
- Example: Lambertian case

$$I = \rho_d L_i \cos\theta_i = \rho_d L_i (\mathbf{n} \cdot \mathbf{s})$$

Let $\rho_d L_i = 1$ then $\boxed{I = \cos\theta_i = \mathbf{n} \cdot \mathbf{s}}$

Reflectance Map: Lambertian Case (I)

$$I = \cos\theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2 + q^2 + 1}\sqrt{p_s^2 + q_s^2 + 1}} = \boxed{R(p, q)}$$

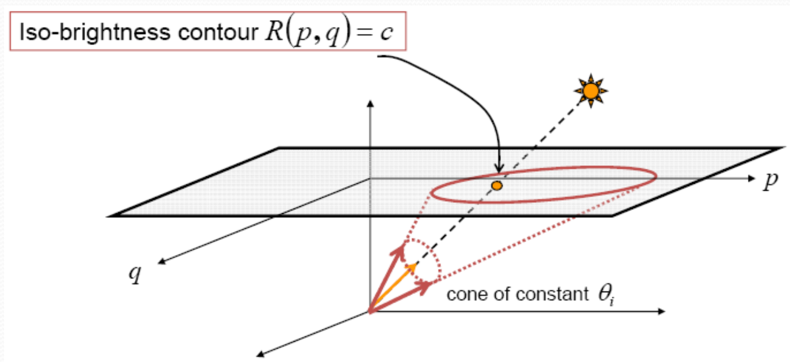
Reflectance Map (Lambertian)

- Contours of constant brightness are conic sections in the *pq*-plane:

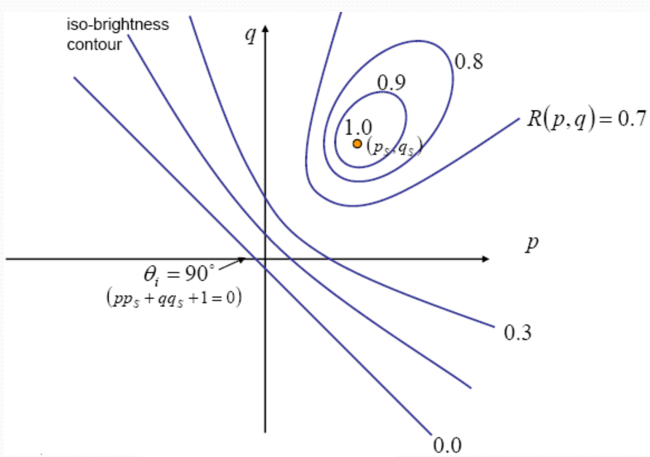
$$R(p, q) = c \Rightarrow (pp_s + qq_s + 1)^2 = c^2 (p^2 + q^2 + 1)(p_s^2 + q_s^2 + 1)$$

Reflectance Map: Lambertian Case (II)

$$(pp_s + qq_s + 1)^2 = c^2(p^2 + q^2 + 1)(p_s^2 + q_s^2 + 1)$$



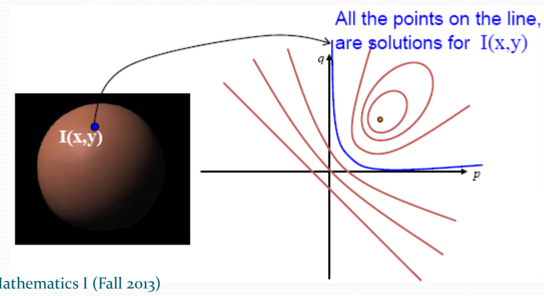
Reflectance Map: Lambertian Case (III)



Note: $R(p, q)$ is maximum when $(p, q) = (p_s, q_s)$

Shape from a Single Image?

- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?
- Given $R(p, q)$ ((p_s, q_s) and surface reflectance) can we determine (p, q) uniquely for each image point?



NO!

Photometric Stereo: Concept

- Solution: Take more images with varying direction of illumination while holding the viewing direction constant (i.e. no change in imaging geometry)
  - Pixel (x, y) in the images corresponds to the same object point

    ⇒ corresponds to the same gradient (p, q)