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Class: ECON 20210 Assignment: 6

Problem 1a. *(T/F)* In order to generate persistence in business cycle fluctuations, we need to model a persistent process for productivity shock

Solution: **False**. While we can model a persistent shock to productivity to show that there are such in business cycle fluctuations, this does not guarntee that we can do so for all shocks. This only depends if we know that the shock itself is autoregressive in nature. If the shock is autoregressive, then we know that the shock will die sufficiently fast enough such that it follows growth facts. If not, it could have some unexpected behavior. □

Problem 1b. *(T/F)* The value function iteration algorithm for a stochastic model involves discretizing both the endogenous and exogenous state space and iterating on the value function for each grid on the state space.

Solution: **True**. This is a standard formulation of the value function iteration algorithm. □

Problem 1c. *(T/F)* Calculating correlation between raw data (e.g., real GDP vs consumption) would still give us a good sense of how the variables co-move along business cycle

Solution: **False**. This is because there is some noise within the data that must be removed before any proper analysis can be conducted. The noise being the clycial part of the data, which means that we have to use a filter like the HP filter to ensure that we conduct analysis only on the residual cylical data rather than entire data. □

Problem 3. Consider the same model environment as in lecture. Suppose that the wage distribution follows a uniform distribution w $u(\underline{w}, \bar{w})$. For parameters, use $\beta = 0.9$, $b = 0.3$, $\underline{w} = 0$, and $\bar{w} = 1$

Problem 3.1. Describe the value function iteration algorithm verbally. Describe each step.

Solution: We first

- Choose an error ϵ
- descritize the space of wage $w_i \in \{w_1, w_2, \ldots, w_n\}$
- Intialize the probabilities of each wage, a tuple of length n, where $\{\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\}$
- Pick a guess $V \in \mathbb{R}^n$
- Calculate $V =$ some equation
- Calculate $|V' - V|$, and if this is less than ϵ, end and return values.
- If else, continue to next iteration.

$\square$

Problem 3.2.

Solution:

```
% McCall Model - Baseline Solution + Parameter Analysis + Plot Saving
clear; clc; close all;

%% ==== BASELINE PARAMETERS ================================================
beta = 0.90;
b    = 0.30;
wMin = 0.0;   wMax = 1.0;
N    = 10000;

wGrid = linspace(wMin, wMax, N)';
Ve    = wGrid ./ (1 - beta);

%% ==== VALUE FUNCTION ITERATION ==========================================
Vu   = 0;   tol = 1e-10;   diff = Inf;

while diff > tol
    EV     = mean( max(Ve, Vu) );
    Vu_new = b + beta * EV;
    diff   = abs(Vu_new - Vu);
```

```
            Vu      = Vu_new;
        end

        wStar = (1 - beta) * Vu;
        Vw    = max(Vu, Ve);

        fprintf('\nBaseline Results:\n');
        fprintf('  Vu    = %.8f\n', Vu);
        fprintf('  w*    = %.8f\n\n', wStar);

        %% ==== FIGURE 1: Accept/Reject ============================================
        decisionRule = Ve >= Vu;

        figure(1); clf;
        plot(wGrid, decisionRule, '.', 'MarkerSize', 4); hold on;
        xline(wStar, 'r--', 'w*', 'LineWidth', 1.5, ...
              'LabelHorizontalAlignment', 'left', 'LabelVerticalAlignment', 'middle');
        xlabel('wage offer w'); ylabel('Accept? (1 = Yes, 0 = No)');
        title('Figure 1: Accept-Reject Decision Rule');
        ylim([-0.05, 1.05]); grid on;
        saveas(gcf, 'figure1_accept_rule.pdf');

        %% ==== FIGURE 2: Value Function ===========================================
        figure(2); clf;
        plot(wGrid, Vw, 'b', 'LineWidth', 1.4); hold on;
        xline(wStar, 'r--', 'w*', 'LineWidth', 1.5, ...
              'LabelHorizontalAlignment', 'left', 'LabelVerticalAlignment', 'middle');
        yline(Vu, 'k--', 'V_u', 'LineWidth', 1.2);
        xlabel('wage'); ylabel('value');
        title('Figure 2: Value Function V(w) = max{V_u, V_e(w)}');
        grid on;
        ylim([0.95*Vu, max(Vw)*1.05]);
        saveas(gcf, 'figure2_value_function.pdf');

        %% ==== COMPARATIVE STATICS ===============================================
        betaVec = linspace(0.5, 0.99, 60);
        bVec    = linspace(0.0, 0.5, 60);
        wStar_beta = zeros(size(betaVec));
        wStar_b    = zeros(size(bVec));

        for i = 1:length(betaVec)
            wStar_beta(i) = reserv_wage(betaVec(i), b, N, wMin, wMax
```

```
                );
        end

        for j = 1:length(bVec)
            wStar_b(j) = reserv_wage(beta, bVec(j), N, wMin, wMax);
        end

        %% ==== FIGURE 3: w* vs beta ==============================================
        figure(3); clf;
        plot(betaVec, wStar_beta, 'LineWidth', 1.6);
        xlabel('beta'); ylabel('reservation wage w*');
        title('Figure 3: w* vs beta (b fixed)');
        grid on;
        saveas(gcf, 'figure3_wstar_vs_beta.pdf');

        %% ==== FIGURE 4: w* vs b =================================================
        figure(4); clf;
        plot(bVec, wStar_b, 'LineWidth', 1.6);
        xlabel('b (unemployment benefit)'); ylabel('reservation wage w*');
        title('Figure 4: w* vs b (beta fixed)');
        grid on;
        saveas(gcf, 'figure4_wstar_vs_b.pdf');

        %% ==== FUNCTION: reservation wage ========================================
        function wStar = reserv_wage(beta, b, N, wMin, wMax)
            if nargin < 3, N = 10000; end
            if nargin < 4, wMin = 0.0; end
            if nargin < 5, wMax = 1.0; end

            w = linspace(wMin, wMax, N)';
            Ve = w ./ (1 - beta);
            Vu = 0; tol = 1e-10; diff = Inf;

            while diff > tol
                EV   = mean( max(Ve, Vu) );
                Vu_n = b + beta * EV;
                diff = abs(Vu_n - Vu);
                Vu   = Vu_n;
            end

            wStar = (1 - beta) * Vu;
        end
```

□

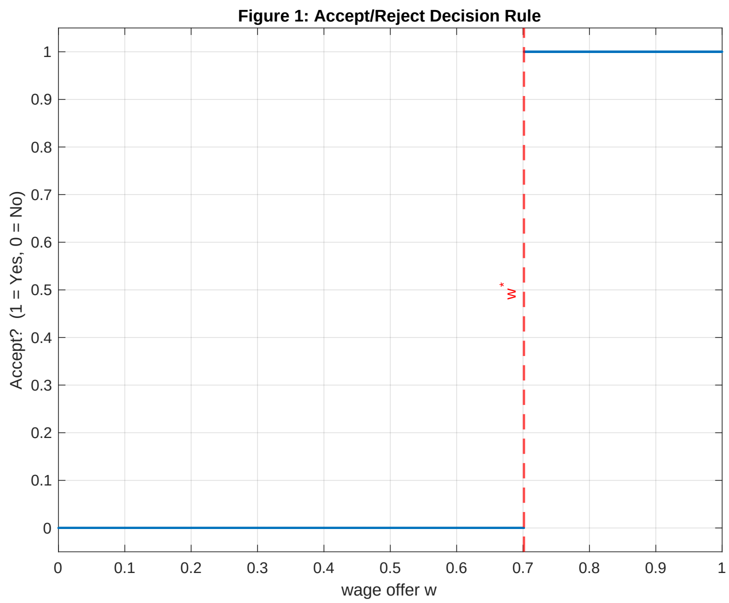

Figure 1: Figure 1

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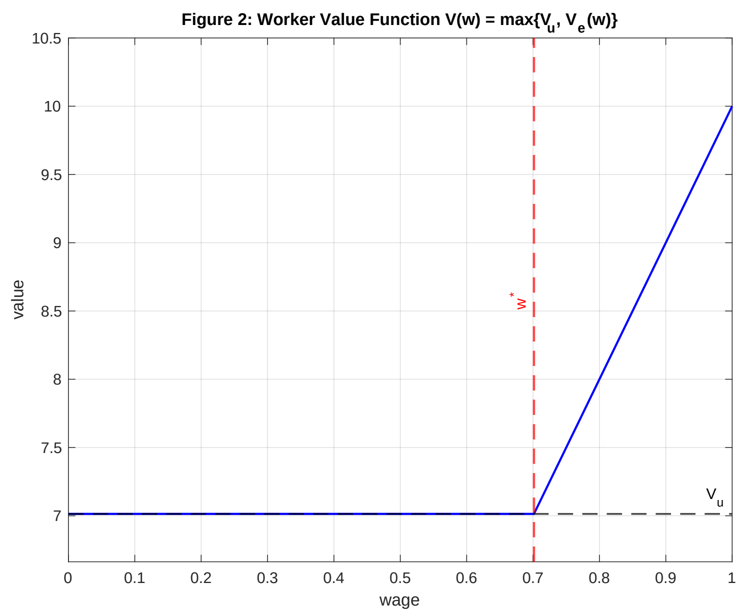

Figure 2: Figure 2

$W_R \approx 0.7$

Problem 3.3.

Solution:

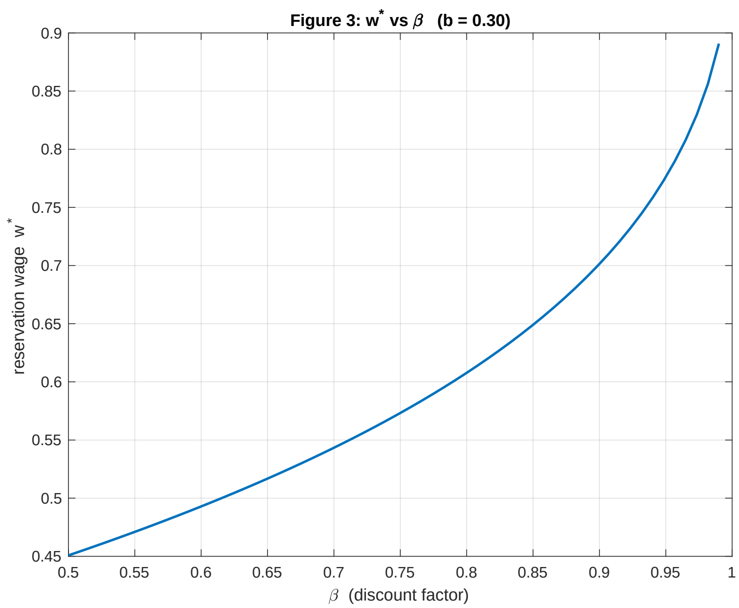

Figure 3: Figure 3

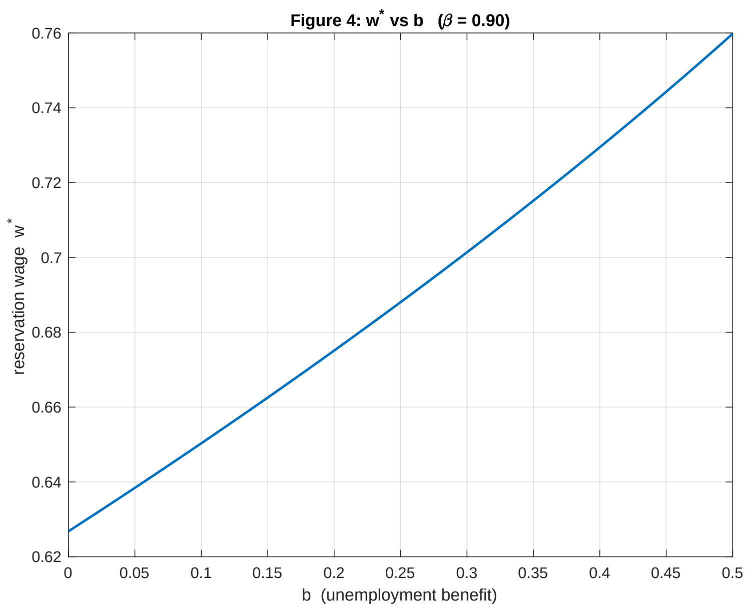

Figure 4: Figure 4

□

Problem 4.

Solution: Note:

$$\begin{aligned}
V(w) &= \max\{V_E(w), c + \beta E[(V(w'))]\} \\
&= \max\left\{\frac{w}{1-\beta}, c + \beta(\phi E(V(w')|\phi) + (1-\phi)E(V(w')|(1-\phi)))\right\} \\
&= \max\left\{\frac{w}{1-\beta}, c + \beta(\phi V(0) + (1-\phi)E(V(w')|(1-\phi)))\right\} \\
&= \max\left\{\frac{w}{1-\beta}, c + \beta(\phi V(0) + (1-\phi)\int V(w')dF(w'))\right\}
\end{aligned}$$

□