Anthony Yoon
Min Seo Kim
Sam Konkel
Pratyush Sharma

Todo list

Anthony Yoon
Min Seo Kim
Sam Konkel
Pratyush Sharma

Problem 1. Explaining lower real interest rates.

Problem 2. Transitional Dynamics in Solow Growth Model

(1)

Solution: Given the given parameters, we can see that:

$$k_t = \frac{K_t}{A_t N_t} \quad y_t = \frac{y_t}{A_t N_t}$$

Thus

$$\begin{aligned} K_{t+1} &= K_t(1-\delta) + I_t \\ K_{t+1} &= K_t(1-\delta) + s(A_t N_t)^{1-\alpha} K_t^{\alpha} \\ K_{t+1} - K_t &= s(A_t N_t)^{1-\alpha} K_t^{\alpha} - \delta K_t \\ \frac{\Delta K_t}{K_t} &= \frac{s(A_t N_t)^{1-\alpha} K_t^{\alpha}}{K_t} - \delta \\ \frac{\Delta K_t}{K_t} &= \frac{s K_t^{\alpha-1}}{(A_t N_t)^{\alpha-1}} - \delta \\ \frac{\Delta K_t}{K_t} &= s k_t^{\alpha-1} - \delta \end{aligned}$$

Note that $K_t = A_t N_t k_t$ This implies:

$$\begin{aligned} \frac{\Delta K_t}{K_t} &= s k_t^{\alpha-1} - \delta \\ \frac{\Delta k_t A_t N_t}{k_t A_t N_t} &= s k_t^{\alpha-1} - \delta \\ \frac{\Delta k_t}{k_t} + \frac{\Delta A_t}{A_t} + \frac{\Delta N_t}{N_t} &= s k_t^{\alpha-1} - \delta \\ \frac{\Delta k_t}{k_t} + g + n &= s k_t^{\alpha-1} - \delta \\ \Delta k_t &= s k_t^{\alpha} - (n+g+\delta) k_t \end{aligned}$$

□

(2)

Solution: At steady state, $\Delta k_t = 0$ For notational state, let $x = k_{ss}$. This implies that

$$\begin{aligned} 0 &= s x^{\alpha} - (n+g+\delta) k_t \\ (n+g+\delta) x &= s x^{\alpha} \\ \frac{n+g+\delta}{s} &= x^{\alpha-1} \\ x &= \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

□

Anthony Yoon
Min Seo Kim
Sam Konkel
Pratyush Sharma

(3)

Solution: We are interested in the following optimization problem:

$$\max \quad k_t^{\alpha} - (n+g+\delta)k_t$$

Taking the first order deriative with respect to k_t allows to see:

$$\alpha k_t^{\alpha-1} - (n+g+\delta) = 0 \implies k_{gr} = \left(\frac{\alpha}{n+g+\delta}\right)^{\frac{1}{\alpha-1}}$$

□

(4)

Solution: Code for the simulation:

```
% Setting parameters
s = 0.4;
delta = 0.06;
n = 0.02;
g = 0.02;
alpha = 1/3;
z = 100; % Number of iterations

% x axis creation
X = 0:1:z;
X = X';

K = zeros(z+1, 1);
A = zeros(z+1, 1);
N = zeros(z+1, 1);
k = zeros(z+1,1);
y = zeros(z+1,1);
Y = zeros(z+1,1);

% setting values
A(1) = 1;
K(1) = 1;
N(1) = 1;
Y(1) = K(1)^alpha * (A(1) * N(1))^(1-alpha);

% Time iteration
for i = 1:(z+1)
    A(i + 1) = A(i) * (1 + g);
    N(i + 1) = N(i) * (1 + n);
    K(i + 1) = K(i) * (1 - delta) + s * (A(i) * N(i))^(1 - alpha) *
       K(i)^alpha;
    k(i) = (K(i) / (A(i) * N(i)));
```

```
    Y(i) = K(i)^alpha * (A(i) * N(i))^(1-alpha);
    y(i) = Y(i) / (A(i) * N(i));
end

figure;

subplot(2, 2, 1);
plot(X, y);
title('Plot of y vs X');
grid on;

subplot(2, 2, 2);
plot(X, Y);
title('Plot of Y vs X');
grid on;

subplot(2, 2, 3);
plot(X, k);
title('Plot of k vs X');
grid on;

subplot(2, 2, 4);
plot(X, K(1:101));
title('Plot of K vs X');
grid on;
```

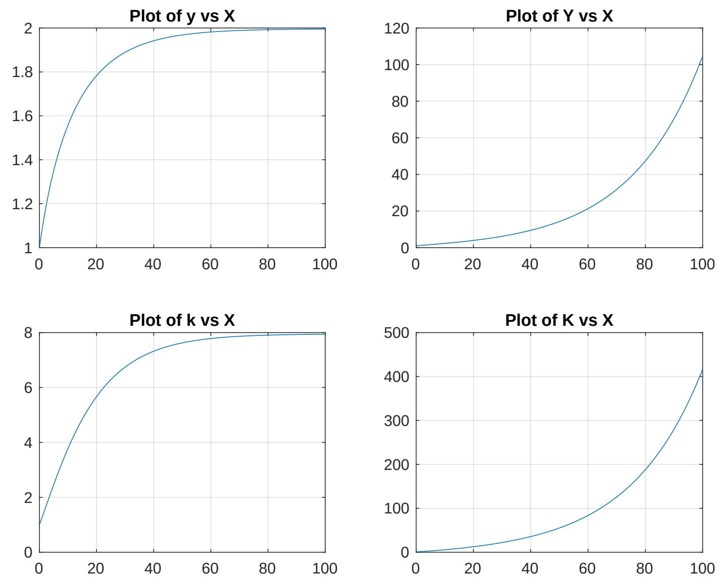

Figure 1: Figure Econ 20210 Problem 3 Question 5

Note that k and y are appraoch the steady state behaviors and Y and K approach infinity, which resemble the Inada conditions. □

(5)

Solution:

```
% Problem 3 Q5

% Setting values
k_steady_state = (s / (n+g+delta))^(1.5);
k_1 = zeros(z, 1);
```

```
k_1(1) = k_steady_state;
s = 0.35;

for i=1:z
    k_1(i+1) = s * k_1(i)^alpha - (n + g + delta) * k_1(i) + k_1(i);
end

figure;

plot(X,k_1)
```

INSERT GRAPH □

Problem 3. Cookie Eating - Part 1

(1)

Solution: We can see the law of depreciation is:

$$W_{t+1} = W_t - c_t \quad \text{s.t} \quad W_0 > 0$$

□

(2)

Solution: Note that $W_{t+1} = W_t - c_t$ and thus $W_t = W_{t-1} - c_{t-1}$. This implies that via a recursive arguement:

$$W_{t+1} = W_t - c_t \implies W_{t+1} = W_0 - \sum_{t=1}^{T} c_t$$

such that $W_{t+1} \geq 0$ □

(3)

Solution: The Langrangian is as follows:

$$L = s - \lambda \left(W_{t+1} - W_0 + \sum_{0}^{t} c_t \right)$$

with the following FOCs:

$$[c_i] \quad \left(\frac{\partial u}{\partial c}\bigg|_{c_i} \right) \cdot \beta^i + \lambda \leq 0$$

$$[\lambda] \quad W_{t+1} \leq W_0 - \sum_{t=1}^{T} c_t$$

Note that W_{t+1} has to be 0, as no utility is derived from W_{t+1} period. □

(4)

Solution: From a $[c_{i+1}]$ and $[c_i]$, we see that:

$$u'(c_{t+1}) = \frac{\lambda}{\beta^{t+1}} \quad u'(c_t) = \frac{\lambda}{\beta^t}$$

This implies

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{\frac{\lambda}{\beta^t}}{\frac{\lambda}{\beta^{t+1}}} = \beta \iff u'(c_t) = \beta u'(c_{t+1})$$

□

(5)

Solution: From (4), we see that

$$\beta c_t = c_{t+1} \iff \frac{c_t}{\beta} = c_{t-1}$$

This implies that using the $[\lambda]$ condition, we are innterested in solving:

$$W_0 = \sum_{i=0}^{t} \beta^{-i} c_t$$

which is equivalent to

$$c_t \sum_{i=0}^{t} = W_0 \iff c_t = \frac{W}{\sum_{i=0}^{t} \beta^{-i}}$$

□ Do final part

Problem 4. Crusoe's Intratemporal Choice

(1)

Solution:

□