Anthony Yoon
Min Seo Kim
Sam Konkel
Pratyush Sharma

Class: ECON 20210   Assignment: 2

Problem 1. Explaining lower real interest rates.

Problem 2. Transitional Dynamics in Solow Growth Model

(a)

Solution: Given the given parameters, we can see that:

$$k_t = \frac{K_t}{A_t N_t} \quad y_t = \frac{y_t}{A_t N_t}$$

Thus

$$K_{t+1} = K_t(1-\delta) + I_t$$
$$K_{t+1} = K_t(1-\delta) + s(A_t N_t)^{1-\alpha} K_t^{\alpha}$$
$$K_{t+1} - K_t = s(A_t N_t)^{1-\alpha} K_t^{\alpha} - \delta K_t$$
$$\frac{\Delta K_t}{K_t} = \frac{s(A_t N_t)^{1-\alpha} K_t^{\alpha}}{K_t} - \delta$$
$$\frac{\Delta K_t}{K_t} = \frac{s K_t^{\alpha-1}}{(A_t N_t)^{\alpha-1}} - \delta$$
$$\frac{\Delta K_t}{K_t} = s k_t^{\alpha-1} - \delta$$

Note that $K_t = A_t N_t k_t$ This implies:

$$\frac{\Delta K_t}{K_t} = s k_t^{\alpha-1} - \delta$$
$$\frac{\Delta k_t A_t N_t}{k_t A_t N_t} = s k_t^{\alpha-1} - \delta$$
$$\frac{\Delta k_t}{k_t} + \frac{\Delta A_t}{A_t} + \frac{\Delta N_t}{N_t} = s k_t^{\alpha-1} - \delta$$
$$\frac{\Delta k_t}{k_t} + g + n = s k_t^{\alpha-1} - \delta$$
$$\Delta k_t = s k_t^{\alpha} - (n + g + \delta) k_t$$

$\square$

(b)

Solution: At steady state, $\Delta k_t = 0$ For notational state, let $x = k_{ss}$. This implies that

$$0 = s x^{\alpha} - (n + g + \delta) k_t$$
$$(n + g + \delta) x = s x^{\alpha}$$
$$\frac{n + g + \delta}{s} = x^{\alpha-1}$$
$$x = \left(\frac{s}{n + g + \delta} \right)^{\frac{1}{1-\alpha}}$$

$\square$

(c)

Solution: We are interested in the following optimization problem:

$$\max \quad k_t^\alpha - (n + g + \delta)k_t$$

Taking the first order deriative with respect to k_t allows to see:

$$\alpha k_t^{\alpha-1} - (n + g + \delta) = 0 \implies k_{gr} = \left(\frac{\alpha}{n + g + \delta}\right)^{\frac{1}{\alpha-1}}$$

□

(d)

Solution: Code for the simulation:

```
% Setting parameters
s = 0.4;
delta = 0.06;
n = 0.02;
g = 0.02;
alpha = 1/3;
z = 100; % Number of iterations

% x axis creation
X = 0:1:z;
X = X';

K = zeros(z+1, 1);
A = zeros(z+1, 1);
N = zeros(z+1, 1);
k = zeros(z+1,1);
y = zeros(z+1,1);
Y = zeros(z+1,1);

% setting values
A(1) = 1;
K(1) = 1;
N(1) = 1;
Y(1) = K(1)^alpha * (A(1) * N(1))^(1-alpha);

% Time iteration
for i = 1:(z+1)
    A(i + 1) = A(i) * (1 + g);
    N(i + 1) = N(i) * (1 + n);
    K(i + 1) = K(i) * (1 - delta) + s * (A(i) * N(i))^(1 - alpha) *
        K(i)^alpha;
    k(i) = (K(i) / (A(i) * N(i)));
```

```matlab
    Y(i) = K(i)^alpha * (A(i) * N(i))^(1-alpha);
    y(i) = Y(i) / (A(i) * N(i));
end

figure;

subplot(2, 2, 1);
plot(X, y);
title('Plot of y vs X');
grid on;

subplot(2, 2, 2);
plot(X, Y);
title('Plot of Y vs X');
grid on;

subplot(2, 2, 3);
plot(X, k);
title('Plot of k vs X');
grid on;

subplot(2, 2, 4);
plot(X, K(1:101));
title('Plot of K vs X');
grid on;
```
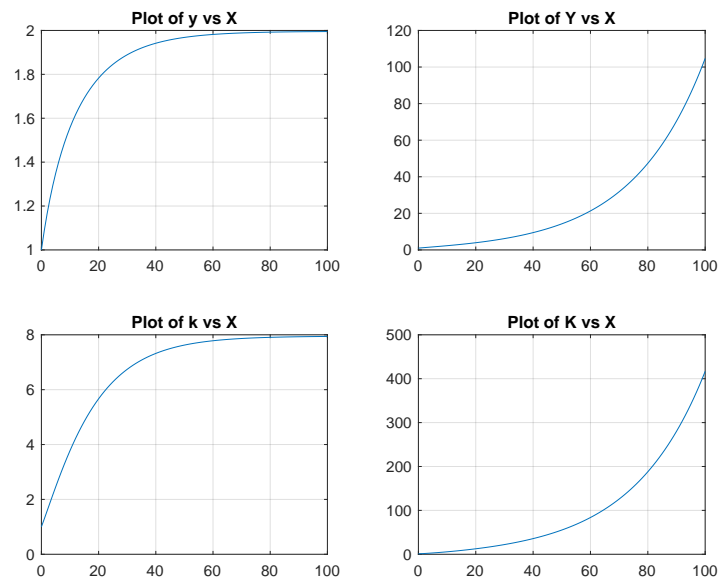
Figure 1: Figure Econ 20210 Problem 3 Question 5

Note that k and y are appraoch the steady state behaviors and Y and K approach infinity, which resemble the Inada conditions. □

(e)

Solution:

```
% Problem 3 Q5

% Setting values
k_steady_state = (s / (n+g+delta))^(1.5);
k_1 = zeros(z, 1);
```

```
k_1(1) = k_steady_state;
s = 0.35;

for i=1:z
    k_1(i+1) = s * k_1(i)^alpha - (n + g + delta) * k_1(i) + k_1(i);
end

figure;

plot(X,k_1)
```

$\square$

Problem 3. Cookie Eating - Part 1

[0]

Problem 4. Crusoe's Intratemporal Choice

(a)

Solution:

$\square$