Honors Econ PSET 7

Anthony Yoon and Chimin Liu

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1

Yes, $y > h_{t+1}$. If the child has to give their parent some sort of compensation, then there must exists some sort of allocation or surplus seperate from h_{t+1} that allows this

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The descriptive setup is as follows:

$$\begin{aligned} \max\ & U(c_{t+1}, c_t) \\ s.t\ & c_t + n_{t+1}(h_t) = \omega_t \\ & c_{t+1} = \omega_{t+1} + n_{t+1}(y - h_{t+1}) \\ & c_{t+1} > 0 \quad c_t > 0 \quad h_{t+1} > 0 \quad h_t > 0 \quad n_{t+1} \geq 0 \quad \omega_{t+1} = 0 \end{aligned}$$

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Note that:

$$c_{t+1} = \omega_{t+1} + n_{t+1}(y - h_{t+1}) \iff n_{t+1} = \frac{c_{t+1}}{y - h_{t+1}}$$

thus, we can describe the canonical form as:

$$\begin{aligned} \max\ & U_{c_{t+1}, c_t} \\ s.t\ & c_t + \frac{h_t}{y - h_{t+1}} \cdot c_{t+1} = \omega_t \\ & c_{t+1} > 0 \quad c_t > 0 \quad h_{t+1} > 0 \quad h_t > 0 \end{aligned}$$

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The quantity $\frac{h_t}{y-h_{t+1}}$ represents the ratio between the cost of of raising a child now and the compensation the parent recieves later. If the cost of raising a child increase, h_t increases, then future consumption of c_{t+1} decreases. Or if the transfer from child to parent decreases, than future consumption c_{t+1} decreases. We can also see that since $h_t < y$, we see that this ratio is strictly positive. This condition forces the ratio to be positive, which allows us to see that consumption in the future period does not add to the endowment of the present, which sastifies the description of the problem

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The Langrangian is as follows:

$$L = \log(c_t) + \beta \log(c_{t+1}) + \lambda(\omega_t - c_t - \frac{h_t c_{t+1}}{y - h_{t+1}} \cdot h_t)$$

and the first order conditions are as follows:

$$\begin{aligned}
[c_t] \quad & \frac{1}{c_t} = \lambda \\
[c_{t+1}] \quad & \frac{\beta}{c_{t+1}} = \lambda \left(\frac{h_t}{y - h_{t+1}} \right) \\
[\lambda] \quad & c_t + \frac{h_t}{y - h_{t+1}} h_t = \omega_t
\end{aligned}$$

Equating λ yields:

$$\beta \frac{c_t}{c_{t+1}} = \frac{h_t}{y - h_{t+1}}$$

Using this ratio, we find that:

$$c_t + \frac{h_t}{y - h_{t+1}} h_t = \omega_t \implies c_t + \beta c_t = \omega_t \implies c_t^* = \frac{\omega_t}{1+\beta}$$

Additionally, to derive c_{t+1} we see that

$$\begin{aligned}
c_t + \frac{h_t}{y - h_{t+1}} h_t &= \omega_t \\
\frac{c_t h_{t+1}}{y - h_{t+1}} \beta^{-1} + \frac{h_t}{y - h_{t+1}} h_t &= \omega_t \\
c_{t+1} \left(\frac{h_t}{y - h_{t+1}} \right) (\beta^{-1} + 1) &= \omega_t \\
c_{t+1}^* = \omega_t \left(\frac{\beta}{\beta + 1} \right) \left(\frac{y - h_{t+1}}{h_t} \right)
\end{aligned}$$

Thus, $c_{t+1}^* = \omega_t \left(\frac{\beta}{\beta+1} \right) \left(\frac{y - h_{t+1}}{h_t} \right)$ and $c_t^* = \frac{\omega_t}{1+\beta}$.

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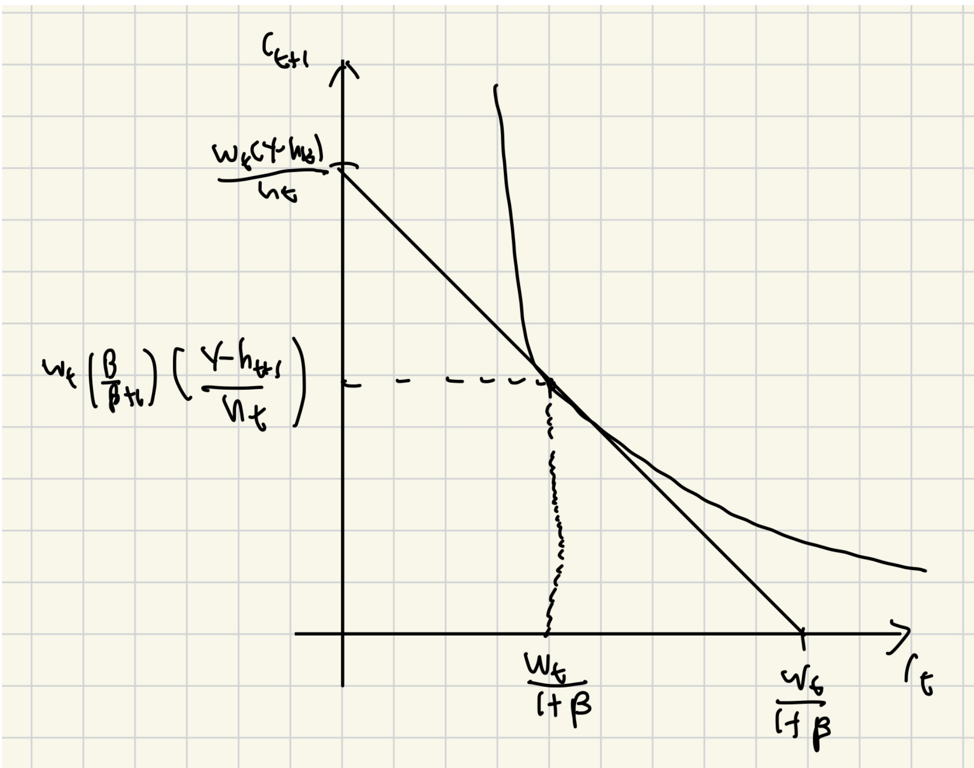

7

$$n^* = \frac{c_{t+1}}{y - h_{t+1}} \implies n^* = \frac{\beta}{h_t(\beta + 1)}\omega_t$$

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The derived expression shows that n^* and h_t are inversely related, as h_t is in the denominator. Thus, increasing h_t decreases the value of n^*. Intuitively, if the cost of raising a child a child increases, it would reduce the desire of having children because future consumption bcomes more limited as graphically, the slope of the budget constraint woudl flatten out and lower the c_{t+1} constraint.

9

Nothing happens, under ceteribus paribus conditions. The quantity $y - h_{t+1}$ is nowhere present in the expression for n^*. Intuitively, it is because the increase in future consumption cancels out the increase incentive to have more children.

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A consumer in the presemt has some sort of endowment ω_t. The optimal amount of children is based on current income. Additionally, we see that rasising the price of children reduces the number of children in the world. Additionally, we see that how much an individual values the future, or the β value, affects the number of children. If the individual does not value the future a lot, indicating a $\beta < 1$, then the children decreases. This value also decreases as β approaches 0. Conversely, as β increases, we see that n^* also increases.

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We see that there would be no children born. Because parents have to endowment in time period t, we see that they can neither consume any goods or provide for their children. Thus, n^* is dependent only on current income and not future income, so there will be 0 children since $\omega_t = 0$. Thus, there is no transfer between offspring and parent, which indicates that $c_{t+1} = \omega_{t+1}$ as endowment does not change. We also assume that there is no transfer possibe, as we are not in a capital market or a storage economy.

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If an individual does not have any future income, and also have a very low current income, the best way of spending income in the present a 'capital investment', as having children allows the individual to recieve some sort of artificial income from their kids later in life, akin to a return on investment. Hence, this is representing the ω_{t+1} in the descripitive setup.

24

1

Housing H is a unit where the individual does get any money from, rather the individual invest (flow) money h into or away from the unit H. Thus, H is a stock and h is an flow. We can see that H is a static unit itself, and the amount of money, denoted as h, that flows into the stock allows the value of H itself to be changed. Comparing them is meaningless, as these two are representations os different things entirely.

2

Period 1 is

$$A_1 + v_1 h_1 + p_1 c_1 = A_0(1+r) + y_1$$

and Period 2 is

$$v_2 h_2 + p_2 c_2 = A_1(1+r) + y_2$$

3

His lifetime utility maximation problem is given as

$$\begin{aligned} \max \quad & U(c_1, c_2, H_1, H_2) \\ s.t \quad & A_1 + v_1 h_1 + p_1 c_1 = A_0(1+r) + y_1 \\ & v_2 h_2 + p_2 c_2 = A_1(1+r) + y_2 \end{aligned}$$

4

We begin by denoting that:

$$v_2 h_2 + p_2 c_2 = A_1(1+r) + y_2 \implies A_1 = \frac{v_2 h_2 + p_2 c_2 - y_2}{1+r}$$

And thus, we find that we are left with one budget constraint in the following form:

$$\frac{v_2 h_2 + p_2 c_2 - y_2}{1+r} + v_1 h_1 + p_1 c_1 = A_0(1+r) + y_1$$

And note that

$$\begin{aligned} H_1 = (1-\delta)H_0 + h_1 &\iff H_1 - (1-\delta)H_0 = h_1 \\ H_2 = (1-\delta)H_1 + h_2 &\iff H_2 - (1-\delta)H_1 = h_1 \end{aligned}$$

Now for some algebra. Inserting the above terms and some rearranging the above yields:

$$\begin{aligned} \frac{v_2 h_2 + p_2 c_2 - y_2}{1+r} + v_1 h_1 + p_1 c_1 &= A_0(1+r) + y_1 \\ v_2 h_2 + p_2 c_2 - y_2 + (1+r) v_1 h_1 + (1+r) p_1 c_1 &= A_0(1+r)^2 + y_1(1+r) \\ v_2(H_2 - (1-\delta)H_1) + p_2 c_2 + (1+r) v_1 (H_1 - (1-\delta)H_0) + (1+r) p_1 c_1 &= A_0(1+r)^2 + y_1(1+r) + y_2 \end{aligned}$$

$$v_2 H_2 + p_2 c_2 - (1-\delta) v_2 H_1 + (1+r) v_1 H_1 + (1+r) p_1 c_1 = A_0(1+r)^2 + y_1(1+r) + y_2 + (1-\delta)(1+r) v_1 H_0$$

$$v_2 H_2 + H_1((1+r)v_1 - (1-\delta)v_2) + p_2 c_2 + (1+r) p_1 = A_0(1+r)^2 + y_1(1+r) + y_2 + (1-\delta)(1+r) v_1 H_0$$

Thus, we see that

- $\pi_1 = (1+r)p_1$
- $\pi_2 = p_2$
- $\tilde{\pi}_1 = ((1+r)v_1 - (1-\delta)v_2)$
- $\tilde{\pi}_2 = v_2$
- $S = A_0(1+r)^2 + y_1(1+r) + y_2 + (1-\delta)(1+r)v_1 H_0$

5

$S = A_0(1+r)^2 + y_1(1+r) + y_2 + (1-\delta)(1+r)v_1 H_0$. We see that

- $A_0(1+r)^2$ represents the amount of money in the original checking account after two rounds of interest
- $y_1(1+r)$ represents the income in time period 1 that is compounded once
- y_2 is the income in time period 1 that is never compounded.
- $(1-\delta)(1+r)v_1 H_0$ represents the money that is made off the original house accounting for interest rates and the depreciation of assets.

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π_1 and π_2 represnts the shadow price of consumption for their respective time periods. To be precise, π_1 is the shadow price of consumption in time period 1, but we have to account for the interest rate, and thus compound the price once. π_2 is the shadow price of consumption in time period 2, but we do not have to account for the interest rate.

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$\tilde{\pi}_1 = ((1+r)v_1 - (1-\delta)v_2), \tilde{\pi}_2 = v_2$. Both of these prices represent the cost of housing in their respective time periods. In $\tilde{\pi}_1$, $v_1(1+r)$ represents the amount of money that is invested into the house, acounting for the one time compounding. $-v_2(1-\delta)$ represents the depreciation of value in the house as time goes on, where this is the transition from $t = 1$ to $t = 2$

(a)

Yes. This is only true if and only if $(1+r)v_1 < (1-\delta)v_2$. This makes sense becaue if value that results in the house depreciating in value is greater than of the value of the investment into the house compounded once.

(b)

Yes. This is possible because the individual can choose to decrease the size of the house he owns and sells a portion of his house.

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(a)

Decreases in value. If $\delta' < \delta$, assuming that $(1+r)v_1$ is held constant, we see that:

$$\begin{gathered}
\delta' < \delta \\
1-\delta' > 1-\delta \\
v_2(1-\delta') > v_2(1-\delta) \\
-(1+r)v_1 - v_2(1-\delta') < -(1+r)v_1 - v_2(1-\delta)
\end{gathered}$$

(b)

Increase in value Assume that $(1-\delta)v_2$ is held constant.

$$\begin{gathered}
r' > r \\
1+r' > 1+r \\
v_1(1+r') > +v_1(1+r) \\
(1-\delta)v_2 + v_1(1+r') > v_1(1+r) + (1-\delta)v_2
\end{gathered}$$

9

The Langrangian is as follows:

$$L = \alpha \log(c_1) + (1-\alpha)\log(H_1) + \beta\alpha\log(c_2) + \beta(1-\alpha)\log(H_2) + \lambda(S - \pi_1 c_1 - \pi_2 c_2 - \tilde{\pi_1} H_1 + \tilde{\pi_2} H_2)$$

and the first order conditions are follows:

$$\begin{aligned}
[c_1] \quad & \frac{\alpha}{c_1} = \lambda\pi_1 \\
[c_2] \quad & \frac{\beta\alpha}{c_2} = \lambda\pi_2 \\
[H_1] \quad & \frac{1-\alpha}{H_1} = \tilde{\pi_1}\lambda \\
[H_2] \quad & \frac{\beta(1-\alpha)}{H_2} = \tilde{\pi_2}\lambda \\
[\lambda] \quad & \pi_1 c_1 + \pi_2 c_2 + \tilde{\pi_1} H_1 + \tilde{\pi_2} H_2 = S
\end{aligned}$$

Equating lambdas yields:

$$\frac{\alpha}{\pi_1 c_1} = \frac{\beta\alpha}{\pi_2 c_2} = \frac{1-\alpha}{\tilde{\pi_1} H_1} = \frac{\beta(1-\alpha)}{H_2\tilde{\pi_2}}$$

We now aim to solve for c_1. Note that:

$$\begin{aligned}
\frac{\alpha}{\pi_1 c_1} &= \frac{\beta\alpha}{\pi_2 c_2} \\
c_2 &= \frac{\beta\pi_1 c_1}{\pi_2}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\alpha}{\pi_1 c_1} &= \frac{1-\alpha}{\tilde{\pi_1} H_1} \\
H_1 &= \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\pi_1}{\tilde{\pi_1}}\right) c_1
\end{aligned}$$

and

$$\begin{aligned}
\frac{\alpha}{\pi_1 c_1} &= \frac{\beta(1-\alpha)}{H_2\tilde{\pi_2}} \\
H_2 &= \beta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\pi_1}{\tilde{\pi_2}}\right) c_1
\end{aligned}$$

and using the $[\lambda]$ condition, we see that

$$\begin{aligned}
\pi_1 c_1 + \pi_2 c_2 + \tilde{\pi_1} H_1 + \tilde{\pi_2} H_2 &= S \\
\pi_1 c_1 + \pi_2\left(\frac{\beta\pi_1 c_1}{\pi_2}\right) + \tilde{\pi_1}\left(\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\pi_1}{\tilde{\pi_1}}\right) c_1\right) + \tilde{\pi_2}\left(\beta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\pi_1}{\tilde{\pi_2}}\right) c_1\right) &= S \\
\pi_1 c_1\left(1 + \beta + \frac{1-\alpha}{\alpha} + \beta\left(\frac{1-\alpha}{\alpha}\right)\right) &= S \\
\pi_1 c_1\left(\frac{\beta+1}{\alpha}\right) &= S \\
c_1 &= \left(\frac{S}{\pi_1}\right)\left(\frac{\alpha}{\beta+1}\right)
\end{aligned}$$

We then use the relation derived from the lambdas to get

- $c_1^* = \left(\frac{S}{\pi_1}\right)\left(\frac{\alpha}{\beta+1}\right)$

- $c_1^* = \left(\frac{\alpha\beta}{\beta+1}\right)\left(\frac{S}{\pi_2}\right)$
- $H_1^* = \left(\frac{1-\alpha}{\beta+1}\right)\left(\frac{S}{\tilde{\pi_1}}\right)$
- $H_2^* = \left(\frac{\beta(1-\alpha)}{\beta+1}\right)\left(\frac{S}{\tilde{\pi_2}}\right)$

10

Note that

$$H_1^* = \frac{1-\alpha}{\beta+1} \cdot \frac{A_0(1+r)^2 + y_1(1+r) + y_2 + (1-\delta)(1+r)v_1 H_0}{((1+r)v_1 - (1-\delta)v_2)}$$

and note that:

$$\frac{\partial H_2^*}{\partial S} = \frac{\beta(1-\alpha)}{(\beta+1)\tilde{\pi_2}} \neq 0$$

Which implies that there exists an endowment and income effect. There is no exact way to tell if the consumption of H^* will change, but we can see that if the magnitude of $1-\delta$ is large, than the denominator increases at a faster rate than that of the numerator, implying that H^* decreases, Conversely, if $1-\delta$ is small, the numerator increaes at faster rate than that of the numerator, impylying that H^* increases.

- **Substitution Effect**: v_1 increasing will cause the consumer to substitute towards consumption
- **Endowment Effect**: Note that a higher v_1 value implies that there one's endowment will greater in value, thus allowing the consumer to consume more. In this case, John now has more money for him to spend towards consumption and investing in his house.
- **Income Effect**: The direction of this depends on whether $\tilde{\pi_1}$ is positive or negative. $\beta, \beta+1, 1-\alpha$ are all values that are strictly positive, and thus, $\tilde{\pi_1}$ is the determining factor here. We see that the good is inferior if the value that results from the depreciation of the hosue's value from $t=1$ to $t=2$ is greater than of the value of the value invested house compounded once $(1+r)v_1$, and by reversing which value is greater, we find that the H_1 is normal under the reversal of the aforementioned conditions.

11

The problem would change because now $h_1 \geq 0$ and $h_2 \geq 0$. And thus, we see that logically $H_2 \geq H_1(1-\delta)$ and $H_1 \geq H_0(1-\delta)$. Thus, we see that the size of the house will be monotonically increasing, which means that even if the rate of depreciation is high, John would still invest in his house and so he could be worse off because he cannot decrease the size of his house even when it reduces his utility.

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1

The expected value of utility for dying is:

$$p(U(c_{t+1})) + (1-p)B$$

Thus, the problem can be setup as:

$$\begin{aligned} \max \quad & \log(c_t) + \beta p \log(c_{t+1}) + \beta(1-p)B \\ s.t \quad & c_t + b_t = \omega_t \\ & c_{t+1} = \omega_{t+1} + b_t(1+r) \end{aligned}$$

2

The descripitive form is:

$$\begin{aligned} \max \quad & \log(c_t) + \beta p \log(c_{t+1}) + \beta(1-p)B \\ s.t \quad & c_t + \frac{c_{t+1}}{1+r} = \omega_t + \frac{\omega_{t+1}}{1+r} \end{aligned}$$

Where the Langrangian is as follows:

$$L = \log(c_t) + \beta p \log(c_{t+1}) + \beta(1-p)B + \lambda(\omega_t + \frac{\omega_{t+1}}{1+r} - (c_t + \frac{c_{t+1}}{1+r}))$$

where the first order conditions are

$$\begin{aligned} [c_t] \quad & \frac{1}{c_t} = \lambda \\ [c_{t+1}] \quad & \frac{\beta p}{c_{t+1}} = \frac{\lambda}{1+r} \\ [\lambda] \quad & c_t + \frac{c_{t+1}}{1+r} = \omega_t + \frac{\omega_{t+1}}{1+r} \end{aligned}$$

Equating the λs yields

$$\frac{\beta p}{c_{t+1}} = \frac{1}{c_t(1+r)}$$

We can use this ratio in conjuction with the $[\lambda]$ condition to find that:

$$c^*_{t+1} = \left(\frac{\beta p(1+r)}{\beta p + 1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right) \quad c^*_t = \left(\frac{1}{\beta p + 1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right)$$

and the indirect utility function being:

$$V(c_t, c_{t+1}) = \log\left(\left(\frac{1}{\beta p + 1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right)\right) + \beta p \log\left(\left(\frac{\beta p(1+r)}{\beta p + 1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right)\right) + \beta(1-p)B$$

and the amount that he saves is noted as:

$$b^*_t = \omega_t - c^*_t = \omega_t - \left(\frac{1}{\beta p + 1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right)$$

3

Since p is in the denominator, increasing p increases the magnitude of c_t, which decreases the value of b_t. Conversely, decreasing p decreases the magnitude of c_t, which increases the value of b_t

4

We will use EV as a measure, as decreasing the chance of dying indicates that the consumer is now on the final indifference curve, which means we can use EV to show how much this improvement is the individual. Since we are in the future, we will be using c_{t+1}, we assume duality conditions to make $c^m_{t+1} = c^h_{t+1}$ and $p_o < p_f$. We see that EV is in the form

$$EV = -\int_{p_o}^{p_f} c_{t+1} dp$$

And thus

$$
\begin{aligned}
EV &= -\int_{p_o}^{p_f} c_{t+1} dp \\
&= \int_{p_f}^{p_o} c_{t+1} dp \\
&= \int_{p_f}^{p_o} \left(\frac{\beta p(1+r)}{\beta p+1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right) dp \\
&= (1+r)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right)\int_{p_f}^{p_o}\left(\frac{\beta p}{\beta p+1}\right) dp \\
&= (1+r)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right)\int_{p_f}^{p_o}\left(1-\frac{1}{\beta p+1}\right) dp \\
&= (1+r)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right)\left(p_o - \frac{\ln(\beta p_o+1)}{\beta} - p_f + \frac{\ln(\beta p_f+1)}{\beta}\right)
\end{aligned}
$$

People who have high β values inherently value this more, as they value the future more than other individuals, so an increase in the probability of survival benefits them the most.

5

Note:

$$\frac{\partial \omega_t}{\partial p} = \frac{\frac{\partial V}{\partial p}}{\frac{\partial V}{\partial \omega_t}}$$

By the envelope theorem, we see that

$$\frac{\partial V}{\partial \omega_{t+1}} = \frac{\lambda^*}{1+r} = \frac{\beta p+1}{(1+r)\omega_t + \omega_{t+1}}$$

$$\frac{\partial V}{\partial p} = \left.\frac{\partial L}{\partial p}\right|_* = \beta(\log(c_{t+1}^*) - B) = \beta\left(\left(\frac{\beta p(1+r)}{\beta p+1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right) - B\right)$$

and

$$\frac{\partial V}{\partial \omega_t} = \left.\frac{\partial \mathcal{L}}{\partial \omega_x}\right|_* = \lambda^* = (c_t^*)^{-1} = (\beta p+1)\left(\frac{1+r}{(1+r)\omega_t + \omega_{t+1}}\right)$$

Thus, we get that

$$\frac{\partial \omega_t}{\partial p} = \frac{\beta\left(\left(\frac{\beta p(1+r)}{\beta p+1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right) - B\right)}{(\beta p+1)\left(\frac{1+r}{(1+r)\omega_t+\omega_{t+1}}\right)}$$

6

Note:

$$\frac{\partial \omega_{t+1}}{\partial p} = \frac{\frac{\partial V}{\partial p}}{\frac{\partial V}{\partial \omega_{t+1}}}$$

and by the envelope theorem we see that

$$\frac{\partial V}{\partial \omega_{t+1}} = \frac{\lambda^*}{1+r} = \frac{\beta p+1}{(1+r)\omega_t + \omega_{t+1}}$$

Thus,

$$\frac{\partial \omega_{t+1}}{\partial p} = \frac{\beta\left(\left(\frac{\beta p(1+r)}{\beta p+1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right) - B\right)}{\frac{\beta p+1}{(1+r)\omega_t+\omega_{t+1}}}$$

7

They differ by a factor of $1+r$ so $\frac{\partial \omega_t}{\partial p}(1+r) = \frac{\partial \omega_{t+1}}{\partial p}$, which represents the compounding of the money in the current, as the money now is compounded and future money isn't.

8

Using the derived idea from **Question 5**, we know that

$$\frac{\partial \omega_t}{\partial p} = \frac{\beta\left(\left(\frac{\beta p(1+r)}{\beta p+1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right) - B\right)}{(\beta p+1)\left(\frac{1+r}{(1+r)\omega_t+\omega_{t+1}}\right)}$$

represents the value of life. and thus, a one percent increase using the formula for $\frac{\partial \omega_t}{\partial p}$, we can find the elasicity of this, so we are interested in the quantity $\frac{\partial \omega_t}{\frac{\partial p}{p}} = \frac{\partial \omega_t}{\partial p} \cdot p$. Thus, the one percent increase is the quantity

$$\frac{\beta\left(\left(\frac{\beta p(1+r)}{\beta p+1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right) - B\right)}{(\beta p+1)\left(\frac{1+r}{(1+r)\omega_t+\omega_{t+1}}\right)} \cdot p$$

9

We shall setup a new optimization problem. We note that we get a Langrangian:

$$L = -e^{-\gamma c_t} - p\beta e^{-\gamma c_{t+1}} + \beta(1-p)B + \lambda((1+r)\omega_t + \omega_{t+1} - ((1+r)c_t + c_{t+1}))$$

with the first order conditions:

$$\begin{aligned}
[c_t] &\quad \gamma e^{-\gamma c_t} = \lambda(1+r) \\
[c_{t+1}] &\quad \gamma p\beta e^{-\gamma c_{t+1}} = \lambda \\
[\lambda] &\quad (1+r)c_t + c_{t+1} = (1+r)\omega_t + \omega_{t+1}
\end{aligned}$$

and we find that once we equate the lambdas and solve, we get that:

$$c_{t+1} - c_t = \ln(p\beta(1+r))$$

where we find that

$$c_t^* = \frac{\gamma((1+r)\omega_t + \omega_{t+1}) - \ln(p\beta(1+R))}{(2+r)\gamma}$$

and

$$c_{t+1}^* = \frac{\gamma((1+r)\omega_t + \omega_{t+1}) + \ln(p\beta(1+R))}{(2+r)\gamma}$$

Note that by the Envelope Theorem, we see that

$$\frac{\partial V^*}{\partial p} = \left.\frac{\partial L}{\partial p}\right|_* = \beta(-e^{-\gamma c_{t+1}} - B)$$

and

$$\frac{\partial V^*}{\partial \omega_t} = \frac{\partial L}{\partial \omega^*} = (1+r)\lambda^* = (1+r)\gamma p\beta e^{-\gamma c_{t+1}}$$

and thus, the statistical value is

$$\frac{\partial \omega_t}{\partial p} = \frac{\beta(-e^{-\gamma c_{t+1}} - B)}{(1+r)\gamma p\beta e^{-\gamma c_{t+1}}} = \frac{\beta\left(-e^{-\gamma\left(\frac{\gamma((1+r)\omega_t+\omega_{t+1})+\ln(p\beta(1+r))}{(2+r)\gamma}\right)} - B\right)}{(1+r)\gamma p\beta e^{-\gamma\left(\frac{\gamma((1+r)\omega_t+\omega_{t+1})+\ln(p\beta(1+r))}{(2+r)\gamma}\right)}}$$

More risk averse you are, the more you value life.

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We want $\omega_t = c_t^*$. Thus, we can see that we can the following:

$$\begin{aligned}
\omega_t &= \left(\frac{1}{\beta p + 1}\right)\left(\omega_t + \frac{\omega_{t+1}}{1+r}\right) \\
(\beta p + 1)\omega_t &= \omega_t + \frac{\omega_{t+1}}{1+r} \\
\beta p \omega_t &= \frac{\omega_{t+1}}{1+r} \\
\frac{1}{\beta p \omega_t} &= \frac{1+r}{\omega_{t+1}} \\
r_{eq} &= \frac{\omega_{t+1}}{\beta p \omega_t} - 1
\end{aligned}$$

The equilbrium rate of interest in inversely proportional to that of the probablity of survivial. This implies that the rate of interest must be directly proportional with the probablity of death. This makes sense intuitively, as if I believe that I have a high chance of dying, I don't value future money as much. Hence, I value my current money more. And if my money were to carry on into the future in the low chance I survive, then my interest rate should be high enough for me to ensure that I maximize my utility in $t+1$. Since technological advances increase, p increases so r_{eq} decreases.