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Class: ECON 20210 Assignment: 2

Todo list

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Problem 1. Explaining lower real interest rates.

(1)

Solution:

Figure 1: Fred Graph

□

(2)

Solution: $n_{90} = 0.0122699866, g_{90} = 0.00785, \delta_{90} = 0.05, s_{90} = 0.07$
$n_{10} = 0.00679, g_{10} = 0.0044, \delta_{10} = 0.04, s_{10} = 0.073$ □

(3)

Solution: We can see that $r = \frac{\partial Y}{\partial K_t} - \delta$. We can compute this as the following:

$$\frac{\partial Y}{\partial K_t} = \alpha K_t^{\alpha-1}(A_t N_t)^{1-\alpha} = \alpha k^{\alpha-1}$$

In steady state, note that $\Delta k_t = 0$. This implies that

$$\begin{aligned}\Delta k_t &= s k_t^\alpha - (n+g+\delta)k_t \\ 0 &= s k_t^\alpha - (n+g+\delta)k_t \\ \frac{n+g+\delta}{s} &= k_t^{\alpha-1} \\ \alpha\frac{n+g+\delta}{s} &= \alpha k_t^{\alpha-1}\end{aligned}$$

Thus,

$$r = \alpha\frac{n+g+\delta}{s} - \delta$$

□

(4) Let $\alpha = \frac{1}{3}$. We can see that:

$$r_{90} = \frac{0.0 - 123 + 0.0079 + 0.05}{3(0.073)} - 0.05 = 0.2689$$
$$r_{10} = \frac{0.0068 + 0.0044 + 0.04}{3(0.073)} - 0.05 = 0.183$$

we can see that the model successfully explains the decline in the real interest. However, the model does overestimate the interest rate for both time frames, as

Risk,

(5)

Solution: The Real Treasury Yield is not a good estimate for the Solov model's growth rate. Real Treausry Yield will be lower than the solov model, because treasury yield are risk free (low risk) and inside solov model, r is return on physical captial, which is inherentlt riskier than the treasury yield. □

Problem 2. Transitional Dynamics in Solow Growth Model

(1)

Solution: Given the given parameters, we can see that:

$$k_t = \frac{K_t}{A_t N_t} \quad y_t = \frac{y_t}{A_t N_t}$$

Thus

$$\begin{aligned}
K_{t+1} &= K_t(1-\delta) + I_t \\
K_{t+1} &= K_t(1-\delta) + s(A_t N_t)^{1-\alpha} K_t^{\alpha} \\
K_{t+1} - K_t &= s(A_t N_t)^{1-\alpha} K_t^{\alpha} - \delta K_t \\
\frac{\Delta K_t}{K_t} &= \frac{s(A_t N_t)^{1-\alpha} K_t^{\alpha}}{K_t} - \delta \\
\frac{\Delta K_t}{K_t} &= \frac{s K_t^{\alpha-1}}{(A_t N_t)^{\alpha-1}} - \delta \\
\frac{\Delta K_t}{K_t} &= s k_t^{\alpha-1} - \delta
\end{aligned}$$

Note that $K_t = A_t N_t k_t$ This implies:

$$\begin{aligned}
\frac{\Delta K_t}{K_t} &= s k_t^{\alpha-1} - \delta \\
\frac{\Delta k_t A_t N_t}{k_t A_t N_t} &= s k_t^{\alpha-1} - \delta \\
\frac{\Delta k_t}{k_t} + \frac{\Delta A_t}{A_t} + \frac{\Delta N_t}{N_t} &= s k_t^{\alpha-1} - \delta \\
\frac{\Delta k_t}{k_t} + g + n &= s k_t^{\alpha-1} - \delta \\
\Delta k_t &= s k_t^{\alpha} - (n+g+\delta) k_t
\end{aligned}$$

□

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(2)

Solution: At steady state, $\Delta k_t = 0$ For notational state, let $x = k_{ss}$. This implies that

$$\begin{aligned} 0 &= sx^\alpha - (n+g+\delta)k_t \\ (n+g+\delta)x &= sx^\alpha \\ \frac{n+g+\delta}{s} &= x^{\alpha-1} \\ x &= \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

□

(3)

Solution: We are interested in the following optimization problem:

$$\max \quad k_t^\alpha - (n+g+\delta)k_t$$

Taking the first order deriative with respect to k_t allows to see:

$$\alpha k_t^{\alpha-1} - (n+g+\delta) = 0 \implies k_{gr} = \left(\frac{\alpha}{n+g+\delta}\right)^{\frac{1}{\alpha-1}}$$

This implies that $s = \alpha$ □

(4)

Solution: Code for the simulation:

```
 % Setting parameters
s = 0.4;
delta = 0.06;
n = 0.02;
g = 0.02;
alpha = 1/3;
z = 100; % Number of iterations

% x axis creation
X = 0:1:z;
X = X';

K = zeros(z+1, 1);
A = zeros(z+1, 1);
N = zeros(z+1, 1);
k = zeros(z+1,1);
y = zeros(z+1,1);
Y = zeros(z+1,1);
```

```
% setting values
A(1) = 1;
K(1) = 1;
N(1) = 1;
Y(1) = K(1)^alpha * (A(1) * N(1))^(1-alpha);

% Time iteration
for i = 1:(z+1)
    A(i + 1) = A(i) * (1 + g);
    N(i + 1) = N(i) * (1 + n);
    K(i + 1) = K(i) * (1 - delta) + s * (A(i) * N(i))^(1 - alpha) *
       K(i)^alpha;
    k(i) = (K(i) / (A(i) * N(i)));
    Y(i) = K(i)^alpha * (A(i) * N(i))^(1-alpha);
    y(i) = Y(i) / (A(i) * N(i));
end

figure;

subplot(2, 2, 1);
plot(X, y);
title('Plot of y vs X');
grid on;

subplot(2, 2, 2);
plot(X, Y);
title('Plot of Y vs X');
grid on;

subplot(2, 2, 3);
plot(X, k);
title('Plot of k vs X');
grid on;

subplot(2, 2, 4);
plot(X, K(1:101));
title('Plot of K vs X');
grid on;
```

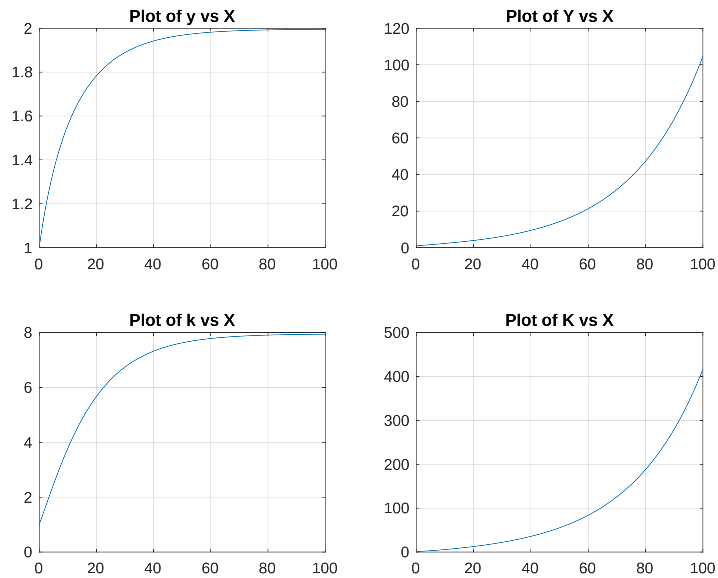

Figure 2: Figure Econ 20210 Problem 3 Question 5

Note that k and y are appraoch the steady state behaviors and Y and K approach infinity, which resemble the Inada conditions. □

(5)

Solution:

```
% Problem 3 Q5

% Setting values
k_steady_state = (s / (n+g+delta))^(1.5);
k_1 = zeros(z, 1);
```

```
k_1(1) = k_steady_state;
s = 0.35;

for i=1:z
    k_1(i+1) = s * k_1(i)^alpha - (n + g + delta) * k_1(i) + k_1(i);
end

figure;

plot(X,k_1)
```

Consumptio is higher

$\square$

(6)

Solution: Note that $c = (1-s)y_{ss} = (1-s)k_{ss}^{\alpha}$. Since consumption is higher in new path, as savings rate has decreased and k_{ss} has decreased, we know that this new path is more dynamically efficient as savings rate is closer to the golden rule savings rate. $\square$

REDO

Problem 3. Cookie Eating - Part 1

(1)

Solution: We can see the law of depreciation is:

$$W_{t+1} = W_t - c_t \quad \text{s.t} \quad W_0 > 0$$

$\square$

(2)

Solution: Note that $W_{t+1} = W_t - c_t$ and thus $W_t = W_{t-1} - c_{t-1}$. This implies that via a recursive arguement:

$$W_{t+1} = W_t - c_t \implies W_{t+1} = W_0 - \sum_{t=1}^{T} c_t$$

such that $W_{t+1} \geq 0$ $\square$

(3)

Solution: The Langrangian is as follows:

$$L = s - \lambda \left(W_{t+1} - W_0 + \sum_{0}^{t} c_t \right)$$

with the following FOCs:

$$[c_i] \quad \left(\frac{\partial u}{\partial c}\bigg|_{c_i} \right) \cdot \beta^i + \lambda \leq 0$$

$$[\lambda] \quad W_{t+1} \leq W_0 - \sum_{t=1}^{T} c_t$$

Note that W_{t+1} has to be 0, as no utility is derived from W_{t+1} period. $\square$

(4)

Solution: From a $[c_{i+1}]$ and $[c_i]$, we see that:

$$u'(c_{t+1}) = \frac{\lambda}{\beta^{t+1}} \quad u'(c_t) = \frac{\lambda}{\beta^t}$$

This implies

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{\frac{\lambda}{\beta^t}}{\frac{\lambda}{\beta^{t+1}}} = \beta \iff u'(c_t) = \beta u'(c_{t+1})$$

□

(5)

Solution: From (4), we see that

$$\beta c_t = c_{t+1} \iff \frac{c_t}{\beta} = c_{t-1}$$

This implies that using the $[\lambda]$ condition, we are innterested in solving:

$$W_0 = \sum_{i=0}^{t} \beta^{-i} c_t$$

which is equivalent to

$$c_t \sum_{i=0}^{t} = W_0 \iff c_t = \frac{W}{\sum_{i=0}^{t} \beta^{-i}}$$

□ Do final part

Problem 4. Crusoe's Intratemporal Choice

(1)

Solution:

$$\begin{aligned} &\max U(c,l) \\ &\text{s.t} \quad c = \frac{1}{1-\theta}(l-\bar{l})^{1-\theta} \\ &\text{s.t} \quad 0 < \theta < 1 \end{aligned}$$

□

(2)

Solution: The FOCs are as follows:

$$
\begin{aligned}
&[c] \quad \frac{\alpha}{c} - \lambda = 0 \\
&[l] \quad \frac{1-\alpha}{1-l} - \lambda(l-\bar{l})^{-\theta} = 0 \\
&[\lambda] \quad c = \frac{1}{1-\theta}(l-\bar{l})^{1-\theta}
\end{aligned}
$$

We see that using the [l] and [λ] constraint, we see that

$$
\begin{aligned}
-\frac{1-\alpha}{1-l} + \frac{a}{c}(l-\bar{l})^{-\theta} &= 0 \\
\frac{\alpha}{c}(l-\bar{l})^{-\theta} &= \frac{1-\alpha}{1-l} \\
\frac{\alpha}{c(l-\bar{l})^{\theta}} &= \frac{1-\alpha}{1-l}
\end{aligned}
$$

Note that $(l-\bar{l})^{\theta}$ is the weight of trade off between the consumption and labor. If θ increases, working more does become as beneifical. □

(3)

Solution: Using the derived optimality condition, we can see the following:

$$
\begin{aligned}
\frac{1-\alpha}{1-l} &= \frac{\alpha}{c}(l-\bar{l})^{-\theta} \\
c(1-\alpha) &= \alpha(1-l)(l-\bar{l})^{-\theta} \\
c &= \frac{\alpha(1-l)(l-\bar{l})^{-\theta}}{1-\alpha} \\
\frac{1}{1-\theta}(l-\bar{l})^{1-\theta} &= \frac{\alpha(1-l)(l-\bar{l})^{-\theta}}{1-\alpha} \\
\frac{l-\bar{l}}{1-\theta} &= \frac{\alpha(1-l)}{1-\alpha} \\
\frac{l}{1-\theta} + \frac{\alpha l}{1-\alpha} &= \frac{\alpha}{1-\alpha} + \frac{\bar{l}}{1-\theta} \\
l\left(\frac{1}{1-\theta} + \frac{\alpha}{1-\alpha}\right) &= \frac{\alpha(1-\theta) + \bar{l}(1-\alpha)}{(1-\alpha)(1-\theta)} \\
l\left(\frac{1-\alpha+\alpha(1-\theta)}{(1-\theta)}(1-\alpha)\right) &= \frac{\alpha(1-\theta) + \bar{l}(1-\alpha)}{(1-\alpha)(1-\theta)} \\
l &= \frac{\alpha(1-\theta) + \bar{l}(1-\alpha)}{1-\alpha+\alpha(1-\theta)}
\end{aligned}
$$

This implies that:

$$\begin{aligned}
c &= \frac{1}{1-\theta}\left(\frac{\alpha(1-\theta)+\bar{l}(1-\alpha)}{1-\alpha+\alpha(1-\theta)}-\bar{l}\right)^{1-\theta} \\
&= \frac{1}{1-\theta}\left(\frac{\alpha(1-\theta)+\bar{l}(1-\alpha)}{1-\alpha+\alpha-\alpha\theta}-\bar{l}\right)^{1-\theta} \\
&= \frac{1}{1-\theta}\left(\frac{\alpha(1-\theta)+\bar{l}(1-\alpha)}{1-\alpha\theta}-\bar{l}\right)^{1-\theta} \\
&= \frac{1}{1-\theta}\left(\frac{\alpha(1-\theta)+\bar{l}(1-\alpha)-\bar{l}(1-\alpha\theta)}{1-\alpha\theta}\right)^{1-\theta} \\
&= \frac{1}{1-\theta}\left(\frac{\alpha(1-\theta)+\bar{l}[(1-\alpha)-(1-\alpha\theta)]}{1-\alpha\theta}\right)^{1-\theta} \\
&= \frac{1}{1-\theta}\left(\frac{\alpha(1-\theta)+\bar{l}(\alpha\theta-\alpha)}{1-\alpha\theta}\right)^{1-\theta} \\
&= \frac{1}{1-\theta}\left(\frac{\alpha(1-\theta)+\alpha(\theta-1)\bar{l}}{1-\alpha\theta}\right)^{1-\theta} \\
&= \frac{1}{1-\theta}\left(\frac{\alpha(1-\theta)(1-\bar{l})}{1-\alpha\theta}\right)^{1-\theta}
\end{aligned}$$

Therefore:

$$l = \frac{\alpha(1-\theta)+\bar{l}(1-\alpha)}{1-\alpha+\alpha(1-\theta)} \quad c = \frac{1}{1-\theta}\left(\frac{\alpha(1-\theta)(1-\bar{l})}{1-\alpha\theta}\right)^{1-\theta}$$

□

(4)

Solution:

$$\frac{\partial l}{\partial \bar{l}} = \frac{1-\alpha}{1-\alpha+\alpha(1-\theta)} \quad \frac{\partial c}{\partial \bar{l}} = -\left(\frac{\alpha(1-\theta)}{1-\alpha\theta}\right)^{1-\theta}(1-\bar{l})^{-\theta}$$

□