STAT 24310 Notes

Anthony Yoon

April 23, 2025

Contents

1 Lecture 1: Complexity/Accuracy/Stability/Condition Number

Abstract

Notes from STAT24310. Taught by Professor Yuehaw Khoo. I try to include most things in here, including some related proofs and ideas. I'm also presuming some basic linear algebra knowledge.

 $\mathbf{2}$

1 Lecture 1: Complexity/Accuracy/Stability/Condition Number

This class is heavily based on Trefethen and Bau's Textbook. Take a look at it if you have the time.

When we run an algorithm, we often are interested in how long it takes to run the algorithm. For example, if we have a matrix $A \in \mathbb{R}^{n \times n}$ an $x, b \in \mathbb{R}^n$, and we are interested in solving

$$Ax = b$$

and n is very large, say n = 100,000, a concern is that the algorithm takes forever to run. In this case, we may be worried about the worst case time complexity, denoted as big O notation. In this case, solving $x = A^{-1}b$ is $\mathcal{O}(n^3)$. But what is this notation:

Definition 1.1. If there exists a $C \in \mathbb{R}$ such that for fg, where $g \geq 0$, where for all sufficiently t large enough, such that $|f(t)| \leq C \cdot g(t)$, then $f(t)\mathcal{O}(g(t))$

or equivantly:

Definition 1.2. If there exists a constant $C \in \mathbb{R}$ and a real number t_0 such that for all $t \geq t_0$, $|f(t)| \leq C \cdot g(t)$, where $g(t) \geq 0$, then we write $f(t) = \mathcal{O}(g(t))$ as $t \to \infty$.