2. Numerical value function iteration

$$\max_{(c_t, k_{t+1})_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

s.t. $A_t k_t^{\alpha} = c_t + k_{t+1} - (1-\delta) k_t$

given: k_0

1. $c_t = A_t k_t^{\alpha} - k_{t+1} + (1-\delta) k_t$

Bellman equation $V(k) = \max_{k'} \frac{[A_t k_t^{\alpha} - k_{t+1} + (1-\delta) k_t]^{1-\sigma}}{1-\sigma} + \beta V(k')$

☆ I derived euler through the lagrange method, but it was the same as my group members using FOCs here

2: $\mathcal{L}: \sum \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} + \lambda (A_t k_t^{\alpha} - c_t - k_{t+1} + (1-\delta) k_t)$

$[c_t]\ \frac{\beta^t}{c_t^{\sigma}} = \lambda_t \qquad [c_{t+1}]\ \frac{\beta^{t+1}}{c_{t+1}^{\sigma}} = \lambda_{t+1}$

$[k_{t+1}]\ \lambda_t = \lambda_{t+1} (\alpha A_t k_t^{\alpha-1} + 1 - \delta)$

mixing all 3 $\Rightarrow$ $c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} (\alpha A_t k_t^{\alpha-1} + 1 - \delta)$

also $c_{t+1}^{\sigma} = \beta c_t (1 + MPK - \delta)$

which is $u'(c_t) = \beta u'(c_{t+1}) (1 + MPK - \delta)$

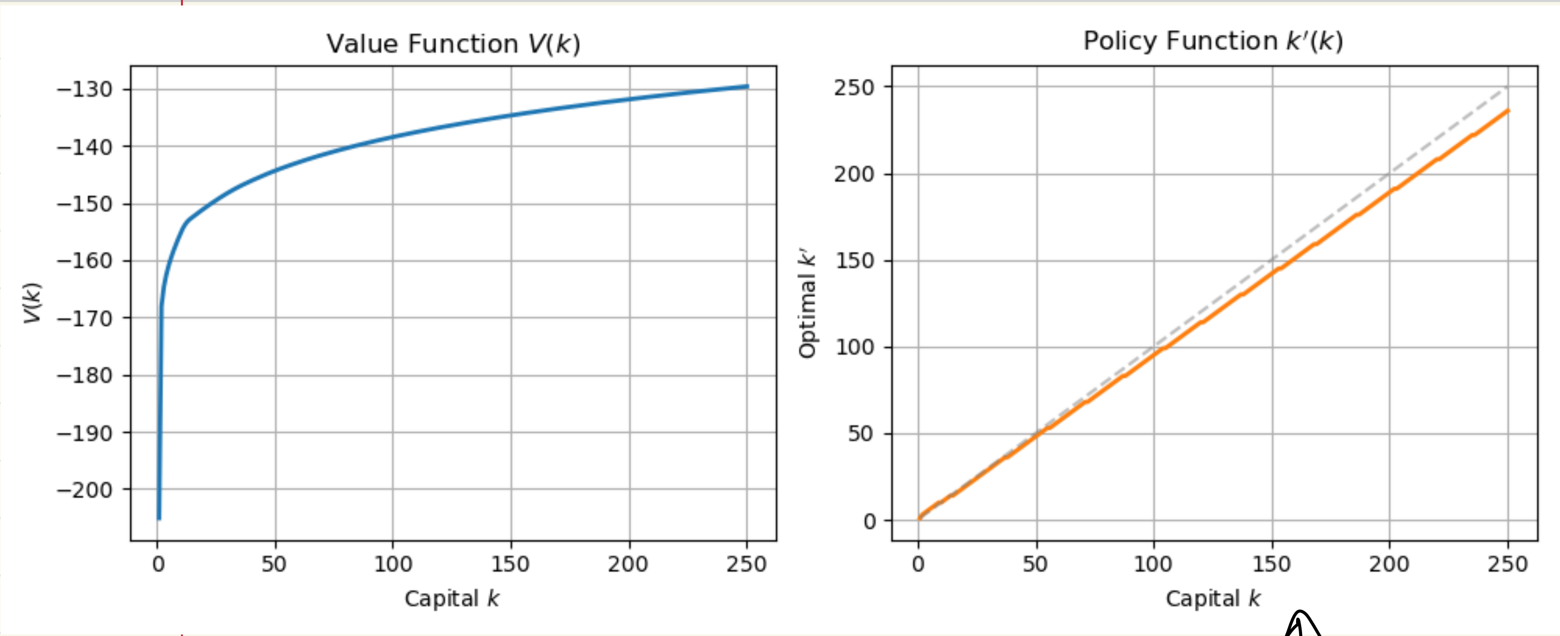
Meaning of σ:
for higher values of σ, the difference between c_t and c_{t+1} reduces for the same $(1 + MPK - \delta)$ (refer to blue equation)

This means that consumption is smoother at the optimum when σ is higher

2. I wrote a python code for this using numpy and plotted the graphs using matplot.

Key details: points go from 1 to 250 with step size = 1
→ Tolerance = 10^{-3} (1e-3)
→ max iterations = 1000 but it stopped at 766

<u>2&3</u> Graphs for <u>2 & 3</u>.

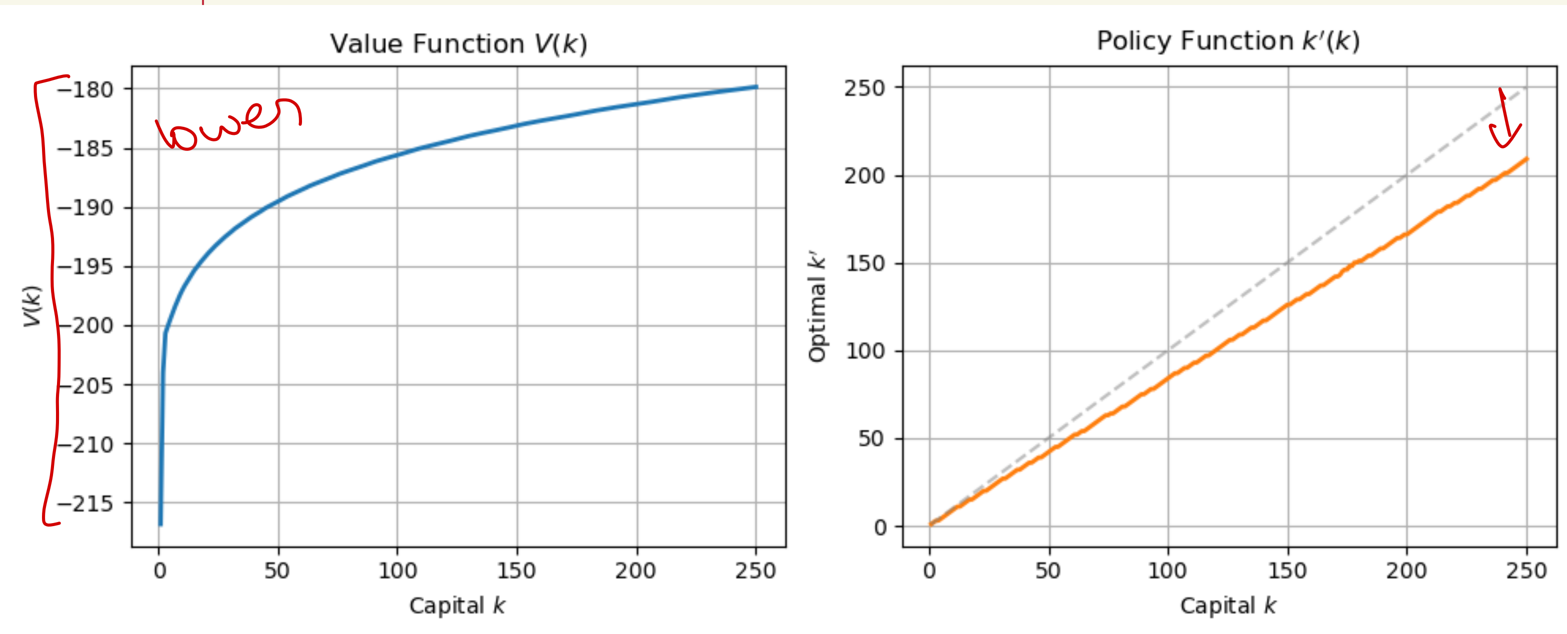


took 766 iterations

$k' = 0.94k$
as per the line

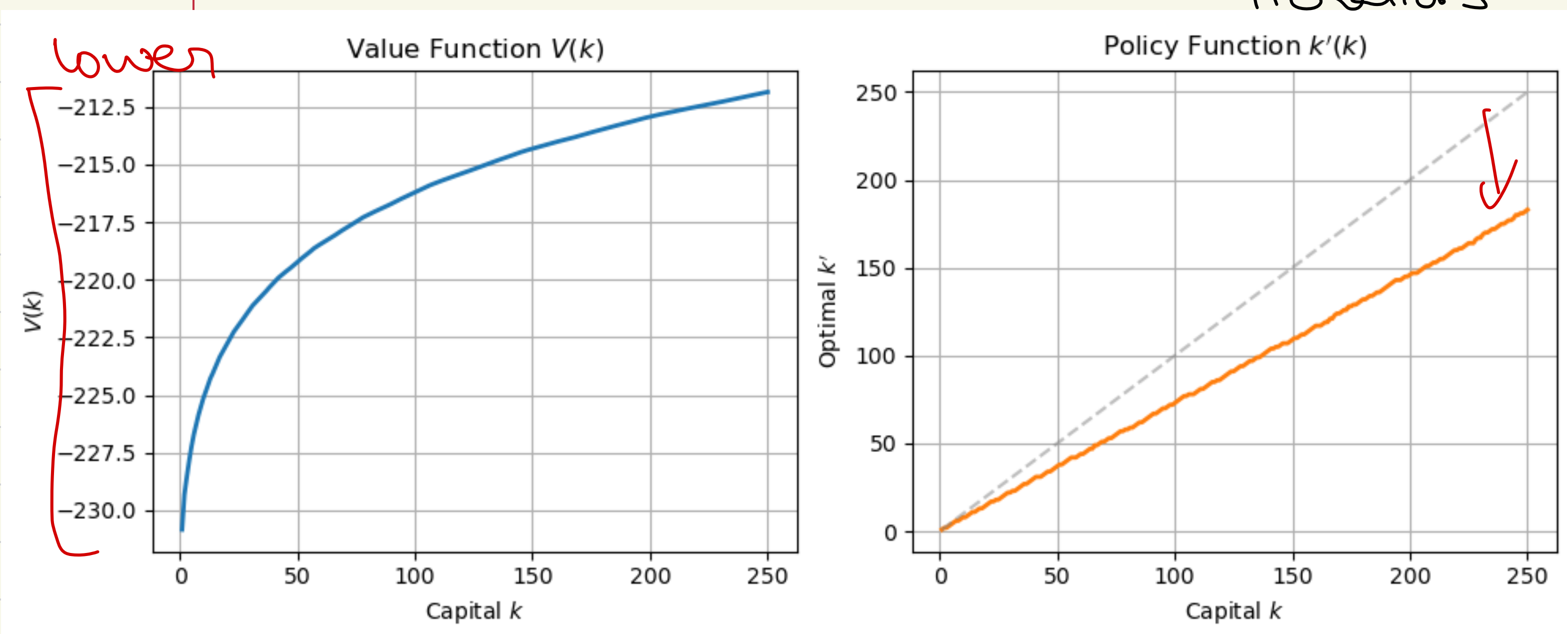
so $g(k) = 0.94k$

4. $\sigma = 0.15$ (up from 0.05) 766 iterations



lower

$\sigma = 0.25$ 772 iterations



lower

When I increase depreciation:
- → value function shifts down and gets more curved (increases less sharply)
- → the policy function assigns less k" to the next time period
- → the effects are proportionate to the increase in σ.

Anthony Yoon
Min Seo Kim
Pratuysh Sharma

Problem 3a.

Solution: If income is IID, let π_g denote the probablity of getting income y_g and $1 - \pi_g$ denote the probability of getting income y_b. Thus, we set up the following Bellman equation:

$$V(b_t, y_t) = \max_{b_{t+1}} \ln(y_t + (1+r)b_t - b_{t+1}) + \beta \mathbb{E}[V(b_{t+1}, y_{t+1})]$$

$$= \max_{b_{t+1}} \ln(y_t + (1+r)b_t - b_{t+1}) + \beta[\pi_g V(b_{t+1}, y_{t+1})] + (1-\pi_g)V(b_{t+1}, y_{t+1})$$

□

Problem 3b.

Solution: Consider the following:

$$[b_{t+1}] \quad \frac{1}{y_t + (1+r)b_t - b_{t+1}} = \beta[\pi_g V(b_{t+1}, y_{t+1})] + (1-\pi_g)V(b_{t+1}, y_{t+1})$$

$$[EC] \quad V_1(b_t, y_t) = \frac{1+r}{y_t + (1+r)b_t - b_{t+1}}$$

The Euler Equation can be derived from the above two equations:

$$\frac{1}{y_t + (1+r)b_t - b_{t+1}} = \beta[\pi_g \frac{1+r}{y_g + (1+r)b_t - b_{t+1}} + (1-\pi_g)\frac{1+r}{y_b + (1+r)b_{t+1} - b_{t+2}}]$$

$$\frac{1}{c_t} = \beta\left(\pi_g \frac{1+r}{c^g_{t+1}} + (1-\pi_g)\frac{1+r}{c^b_{t+1}}\right)$$

where c^g_t is the house holds consumption if they have income y_g and c^b_t is the HH's consumption if they have income y_b □

Problem 3c.

Solution: Consider the case where we have a first order Markov Process, where

$$\Pi = \begin{bmatrix} \pi_{gg} & \pi_{gb} \\ \pi_{bg} & \pi_{bb} \end{bmatrix}$$

and thus, the bellman equations are as follows:

$$V_{b_t, y_t} = \max_{b_{t+1}} \ln(y_i + (1+r)b_t - b_{t+1}) + \beta[\pi_{ig} V_1(b_{t+1}, y_{t+1}) + \pi_{ib} V_1(b_{t+1}, y_{t+1})] \quad i \in \{g, b\}$$

□

Problem 3d.

Solution: We can solve the following FOCs, where:

$$[b_{t+1}] \quad \frac{1}{c_t} = \beta[\pi_{ig} V_1(b_{t+1}, y_{t+1}) + \pi_{ib} V_1(b_{t+1}, y_{t+1})]$$
$$[EC] \quad V_1(b_t, y_t) = \frac{1+r}{y_i + (1+r)b_t - b_{t+1}}$$

After some algebra, we can find that the Euler Equation is"

$$\frac{1}{c_t} = \beta[\pi_{ig} \frac{1+r}{c^g_{t+1}} + \pi_{ib} \frac{1+r}{c^b_{t+1}}]$$

□

Problem 5.1.

Solution: Using equations derived from class,

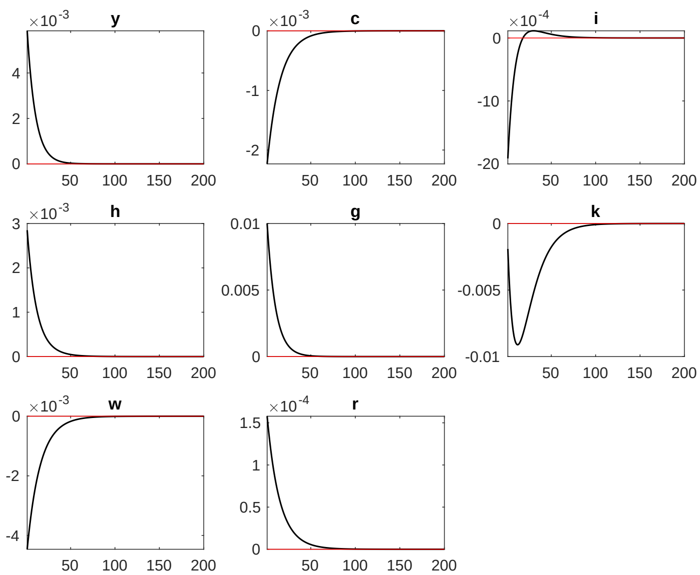


Figure 1: Dynare Graphs

□

Problem 5.2.

Solution: Note that the increase in government expenditure is a negative income effect. Thus, to make up for this loss of purchasing power, the household should supply more labor. However, since every consumer is less rich, consumption and investment must drop as well.

□