Honors Econ PSET 4

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Problem 14

1

We begin by setting up the [UMP] problem as follows:

$$\begin{array}{ll} \max_{b,c} & bc \\ s.t & 2b + 5b = 100 \end{array}$$

2

Note that we can set up the Langrangian in the following manner:

$$L = bc + \lambda(100 - 2b - 10c)$$

which gives us the following first order conditions:

$$\begin{array}{ll} [b] & c = 2\lambda \\ [c] & b = 10\lambda \\ [\lambda] & 100 = 2b + 10c \end{array}$$

And using these first order conditions, we can derive the fact by noting that:

$$\frac{c}{2} = \lambda$$
$$\frac{b}{10} = \lambda$$

And from here, we can derive the relation

$$5c = b$$

From here, we can plug this back into our budget constraint to get that the optimal bunlde in this case is 5 cigerattes and 25 beers.

3

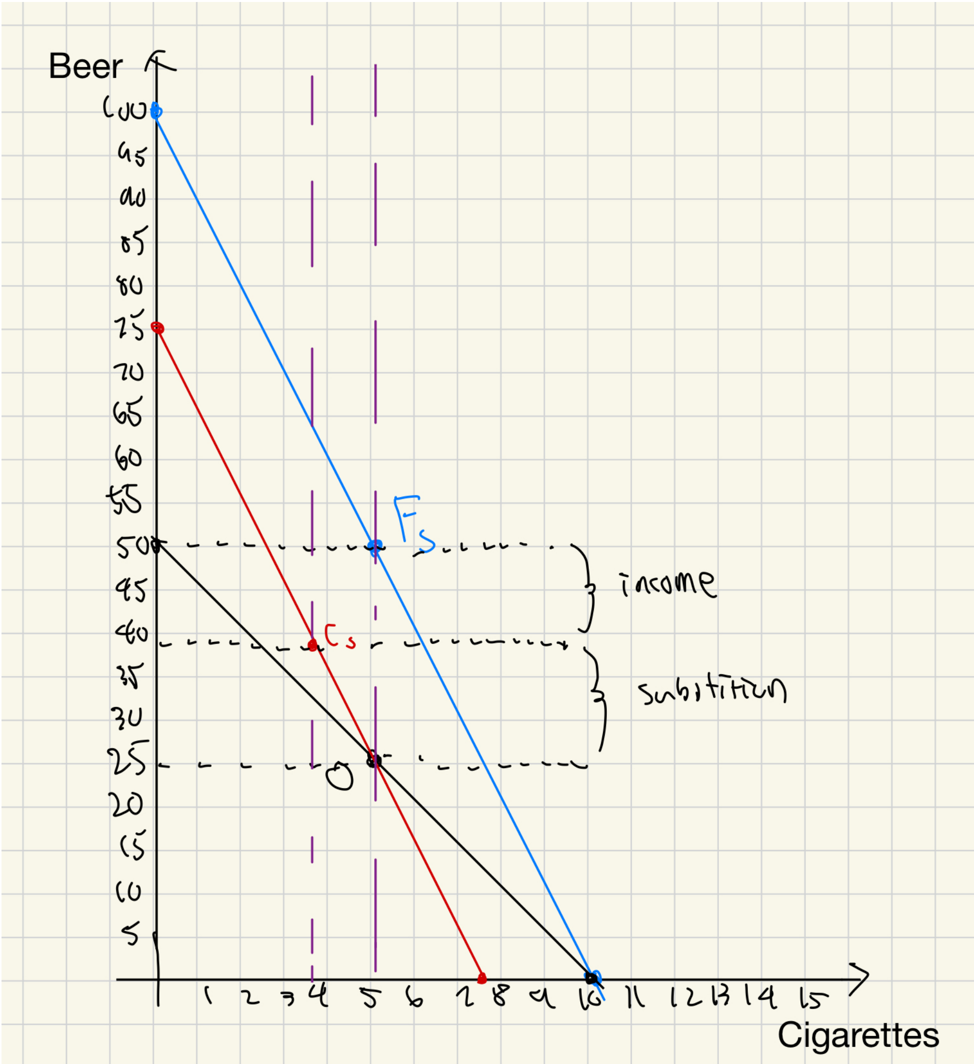
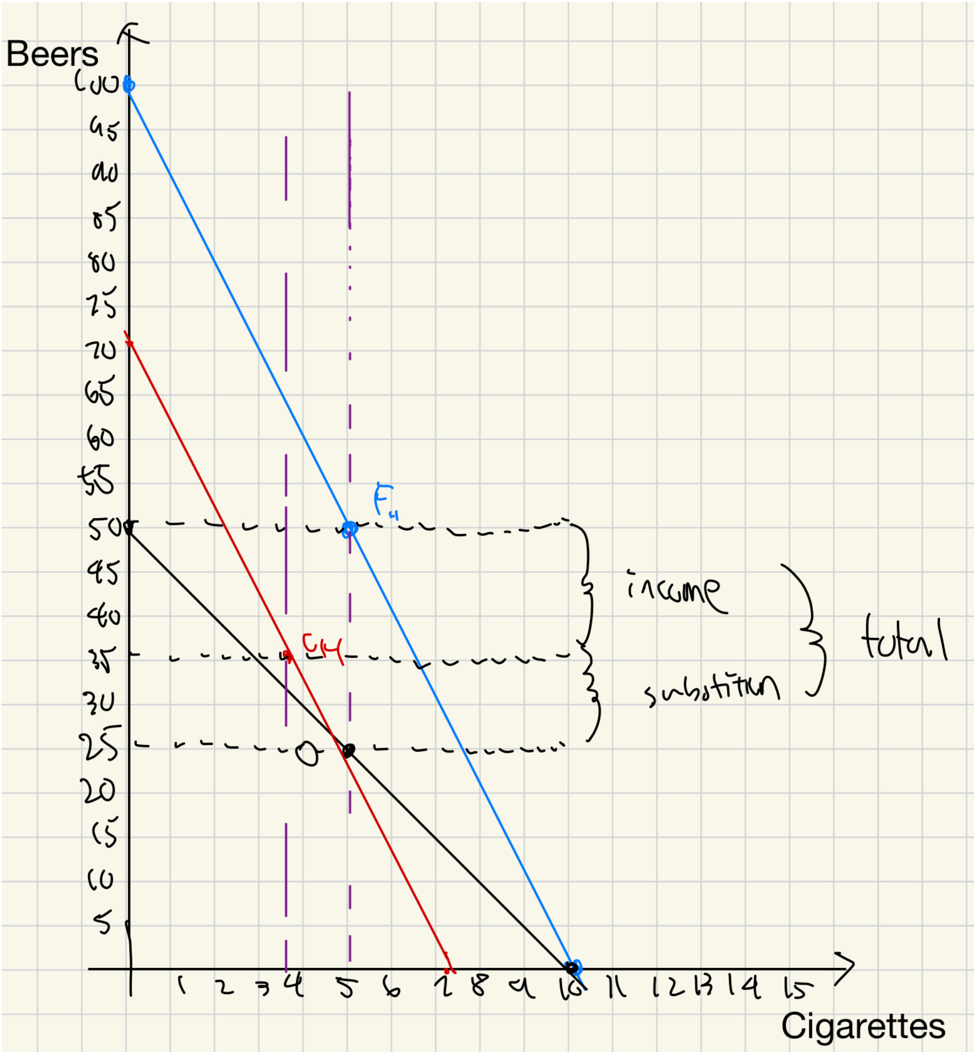


Figure 1: Graph for Slutsky Compensation



4

Our new budget constraint can be written as the following:

$$b + 10c = m_f$$

where we note that if we were to compensate the consumer such that the consumer can barely afford the new bundle, we only have to plug the bundle into the new equation to get m_f. Thus, we get the following,

$$\begin{aligned} b + 10c &= m_f \\ 25 + 10(5) &= m_f \\ m_f &= 75 \end{aligned}$$

We can note that this is defined as the Slutsky compensation.

5

Refer to graphs above

6

When we change the price of beer here, we can see that the first order conditions that we derived earlier would change such that we can derive the following ratio:

$$b = 10c$$

and given this new fact, we can now use this ratio to find the new optimal bundle, which is done as follows:

$$\begin{aligned} b + 10c &= 75 \\ 20c &= 75 \\ c &= 3.75 \\ b &= 37.5 \end{aligned}$$

Thus, the consumer would be buying 37.5 beers and 3.75 packs of cigerattes.

7

We can note that this substitution effect has caused the individual to buy less cigerattes but less beers. We can see that in this case, consumption of beer has increased by 12.5 beers and consumption of cigerattes has decreased by 1.25 packs.

8

We note that the first order condtions for this condition are similar to that of **4** and **5**, with he only exception that $m_f = 100$. Thus, we can use this idea to show that

$$\begin{aligned} b + 10c &= 100 \\ 20c &= 100 \\ c &= 5 \\ b &= 50 \end{aligned}$$

Thus, the consumer at the new income level will buy 5 packs of cigerattes and 50 beers.

9

See above graph

10

we can see that utility can be derived in the following:

$$U(25, 5) = 25 \cdot 5 = 125$$

And note that we can express the new bundle of interest (b', c') by noting that:

$$b'c' = 125 \iff b' = \frac{125}{c'}$$

Thus, when we substitute this into the budget constraint where our new budget can be expressed as m':

$$\begin{aligned} 10c' + b' &= m' \\ 10c' + \frac{125}{c'} &= m' \end{aligned}$$

Differentiating with respect to c' yields and optimizing m yields:

$$\begin{aligned} \frac{dm}{dc} &= 10 - \frac{125}{c'^2} \\ 0 &= 10 - \frac{125}{c'^2} \\ \frac{125}{c'^2} &= 10 \\ 10c'^2 &= 125 \\ c' &= \frac{5}{\sqrt{2}} \approx 3.53 \end{aligned}$$

We can find b' to be $\frac{50}{\sqrt{2}} \approx 35.35$. Thus, we can calculate m' in the following fashion:

$$m' = 10\left(\frac{5}{\sqrt{2}}\right) + \frac{50}{\sqrt{2}} = \frac{100}{\sqrt{2}} \approx 70.7$$

Therefore, the new budget is 70.7 dollars. This is also Hicks Copmensation.

11

See above graph

12

In this scenario, the consumer buys approximately 3.53 packs of cigerattes and 35.35 beers.

13

The substitution effect here makes the consumer buy less packs of cigerattes and less beer. The difference here being that the consumer will by approximately 2.5 less packs of cigerattes and buys about 10 more beers.

14

The answer here is the same as **8**. 50 beers and 5 packs of cigerattes.

15

See above graph

16

The substitution effect here is the following:

- **Hicks**: Approximately 10.36 bottles of beer
- **Slutsky**: 12.5 bottles

The income effect here is the following:

- **Hicks**: Approximately 14.64 bottles of beer
- **Slutsky**: 12.5 bottles of beer

17

The Slutsky Compensation tends to overcompenate more. The reason why we tend to overcompenate is because we are only giving the consumer enough money for him to afford the same bundle as before. However, we are not controlling for the fact that we may have a different level of utility and thus be not on the same indifference curve. Therefore, the consumer has a higher relative purchasing power with Slutsky compared to Hicks, which allows the consumer to reach a higher utility after the "overcompensation"

18

The income effect within the Hicks Compensation is higher than that of the Slutsky compensation. They are not the same because Hicks Compensation precisely adjusts income to keep the consumer at the same utility level, which accurately reflects the change in purchasing power needed to maintain the same utility. But Slutsky compensation overcompenates the consumer, leading to a diferent indifference curve. Thus, this inadverntently includes part of the income effect in its overestimation of the substitution effect, making the income effect smaller than it actually is.

19

The economic importance of breaking down the price effect down into the substitution and income effect is that we can see what influenced the change in prices more: The change in relative prices (substitution effect) or the change in the consumer's purchasing power (income effect).

Problem 15

Given

$$x_m(p_x, p_y, m) = x_h(p_x, p_y, \bar{U} = v(p_x, p_y, m))$$

Differentiating both sides with respect to p_x, we get the following:

$$\frac{\partial x_m(p_x, p_y, m)}{\partial p_x} = \frac{\partial x_H}{\partial p_x} + \frac{\partial x_H}{\partial v} \cdot \frac{\partial v}{\partial p_x}$$

By Roy's Identity, we can see that

$$\frac{\partial v(p_x, p_y, m)}{\partial p_x} = -x_m^* \cdot \frac{\partial v(p_x, p_y, m)}{\partial m}$$

Thus,

$$\frac{\partial x_m(p_x, p_y, m)}{\partial p_x} = \left(\frac{\partial x_h(p_x.p_y, v)}{\partial v}\right) \cdot \left(\frac{\partial v}{\partial m}\right)(-x_M^*) + \frac{\partial x^H}{\partial p_x}$$

And using the given statement in the problem, we can rewrite the above in the following form:

$$\frac{\partial x_m(p_x, p_y, m)}{\partial p_x} = \left(\frac{\partial x_m(p_x, p_y, m)}{\partial m}\right) \cdot (-x_M^*) + \frac{\partial x^H}{\partial p_x}$$

Problem 16

1

We can set up the problem as

$$\begin{aligned} \max_{c,r} \quad & 3\log(c) + r \\ s.t \quad & 5c + 4r = 60 \end{aligned}$$

2

The Langrangian is set as

$$L = 3\log(c) + r + \lambda(60 - 5c - 4r)$$

and the first order condtions are set as:

$$\begin{aligned} [c] \quad & \frac{3}{c} = 5\lambda \\ [r] \quad & 1 = 4\lambda \\ [\lambda] \quad & 5c + 4r = 60 \end{aligned}$$

Note that

$$\lambda = \frac{1}{4}$$

From this, we can derive the fact that

$$\frac{3}{c} = 5\lambda \iff \frac{3}{c} = \frac{5}{4} \iff c = \frac{12}{5}$$

Thus, this implies that $r = 12$ Thus, Peter consumes 12 pounds of raisins and 2.4 pounds of cereal.

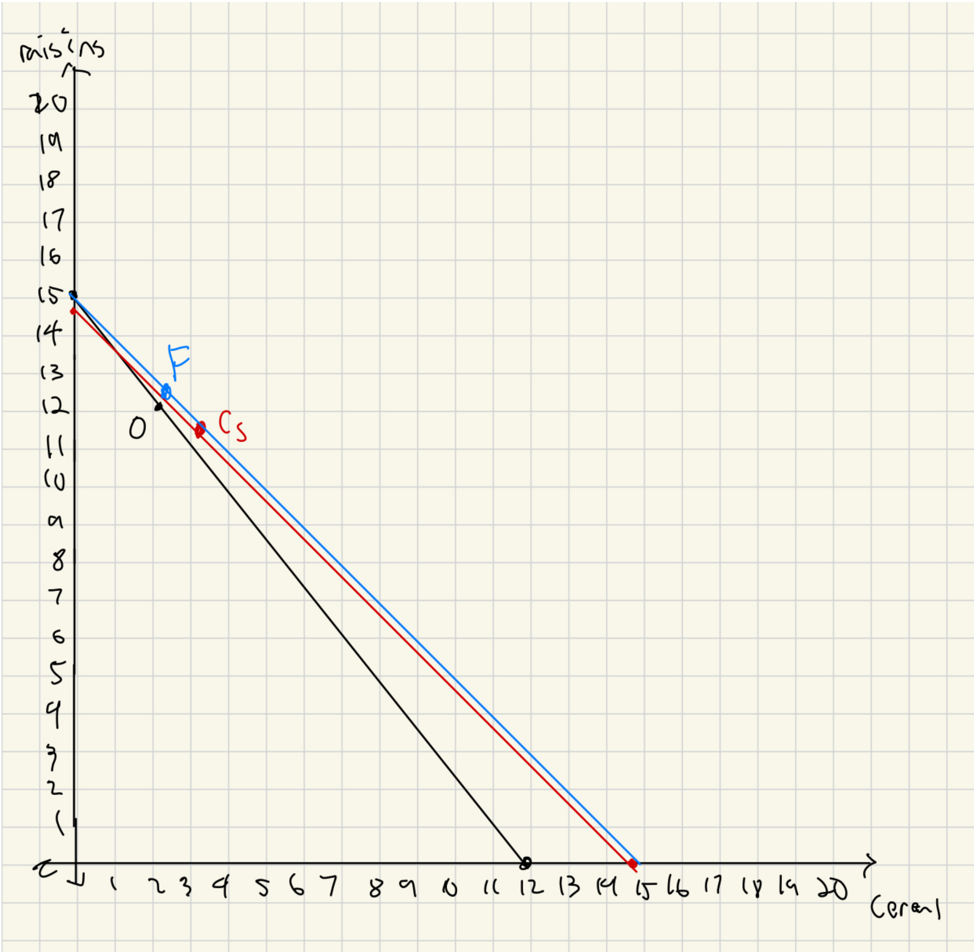


Figure 2: Graph for **Question 4**

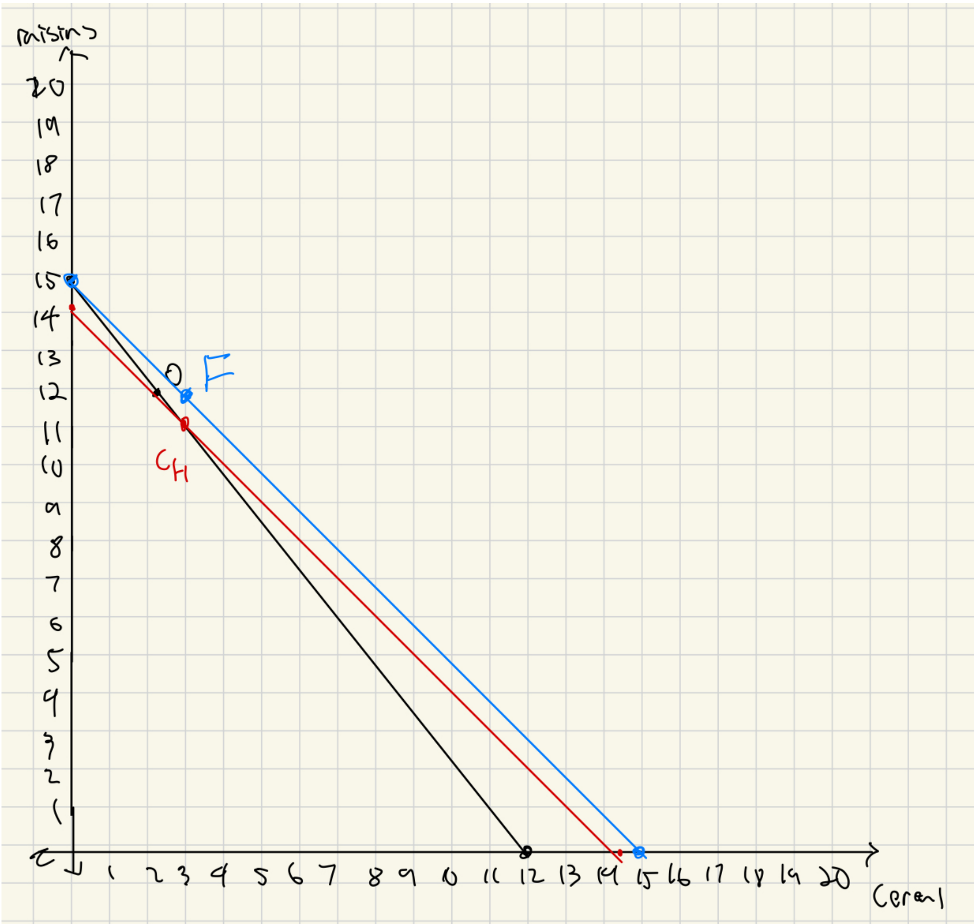


Figure 3: Graph for **Question 5**

4

When we see that tne price of cereal drops to 4 dollars a pound, we can use the bundle derived earlier to see that our new budget can be comprised as

$$4(2.4) + 12(4) = 57.6$$

Thus, our new budget would be 57.6 dollars. We can also see that since the price of cereal has dropped to 4 dollars, our first order conditions must change. This is mainly reflected in the following:

$$\begin{aligned} [c] \quad & \frac{3}{c} = 4\lambda \\ [\lambda] \quad & 4c + 4r = 57.6 \end{aligned}$$

Note that $\lambda = \frac{1}{4}$ must hold. Thus, we can see that

$$\frac{3}{c} = 4\lambda \iff c = 3$$

Putting this back into the budget constraint, we can see that

$$\begin{aligned} 4c + 4r &= 57.6 \\ 4(3) + 4r &= 57.6 \\ 4r &= 45.6 \\ r &= 11.4 \end{aligned}$$

Thus, the new optimal bundle is 3 pounds of cereal and 11.4 pounds of raisins.

5

Since we are dealing with the same consumer Peter, we note that we can view this problem either from an [EMP] or [UMP] persepctive and still get the same result. Since we already have an target utility level that we want to hit, we can view this as an [EMP] problem.Given our previous optimal bundle of 2.4 pounds of cereal and 12 pounds of raisins, we can find that our utlity at this bundle was:

$$U(2.4, 12) = 3\log(2.4) + 12$$

We can then set up the Langrangian as follows:

$$L = 4c + 4r + \eta(3\log(2.4) + 12 - 3\log(r) - c)$$

and derive the following first order conditions:

$$\begin{aligned} [r] &\quad 4 = \lambda \\ [c] &\quad 4 = \frac{3\lambda}{c} \\ [\lambda] &\quad 3\log(c) + r = 3\log(2.4) + 12 \end{aligned}$$

We can see that $[c]$ and $[r]$ imply that $c = 3$. Thus, we can use $[\lambda]$ to get:

$$\begin{aligned} 3\log(c) + r &= 3\log(2.4) + 12 \\ 3\log(3) + r &= 3\log(2.4) + 12 \\ r &= 3(\log(2.4) - \log(3) + 12) \approx 11.33 \end{aligned}$$

Thus, new optimal bundle here would be 3 pounds of cereal and 11.33 pounds of raisins. Thus, when we plug this bundle back into the budget constraint, we find that our new budget that we are concerned with is $3(4) + 4(11.33) = \$57.32$.

6

If Peter still had original income of 60 dollars, we can note that the first order conditions from **4** still holds, with one change:

$$\begin{aligned} [c] &\quad \frac{3}{c} = 4\lambda \\ [r] &\quad 1 = 4\lambda \\ [\lambda] &\quad 4c + 4r = 60 \end{aligned}$$

Thus, we can find that the fact that $c = 3$ holds. Thus, we can use this fact to do the following:

$$\begin{aligned} 4c + 4r &= 60 \\ 4(3) + 4r &= 60 \\ 4r &= 48 \\ r &= 12 \end{aligned}$$

Thus, the bundle in this case would be 3 pounds of cereal and 12 pounds of raisins. When compared to **4**, we can see that Peter consumes the same amount of cereal but less raisins. The same can said about **5**. Peter consumes the same amount of cereal but less raisins. We can summarize the changes numerically as follows:

- **Substitution Effect Hicks**: Loss of about 0.67 pounds of raisins
- **Income Effect Hicks**: Gain of about 0.67 pounds of raisins.
- **Substitution Effect Slutsky**: Loss of about 0.6 pounds of raisins
- **Income Effect Slutsky**: Gain of about 0.6 pounds of raisins.

7

See graphs above

8

Note that as cereal dropped in price, increasing the relative price of raisin, Peter will substitute away from raisin to consume more cereal (0.6 pounds more). So substitution effect applies to both cereal and raisin. But due to logarithmic utility of cereals, cereal has diminishing marginal utility, so past the optimal quanity of cereal, the additional income cannot increase utility derived from cereal anymore, and hence income effect does not affect cereal. Instead, Peter would allocate his additional income toward raisins which is linear in the uitlity function. So to increase total utility to hit $\bar{U}$ (original utility), he can only increase the utility derived from raisins as it doesn't have a utility ceiling like cereal. Thus, income effect increases raisins consumed, which coincides in cancelling out the quantity of raisins substituted away from.

Problem 17

1

We set up the problem in the following manner:

$$\begin{aligned} \min \quad & p_x^o x + p_y y \\ s.t \quad & \bar{U} = \log(x) + y \end{aligned}$$

Which gives us the following first order conditions:

$$\begin{aligned} &[x] \quad p_x^o = \eta \left(\frac{1}{x}\right) \\ &[y] \quad p_y = \eta \\ &[\eta] \log(x) + y = \bar{U} \end{aligned}$$

We can then note that in this case, we can use these first order conditions:

$$x_h^* = \frac{p_y}{p_x^o}$$

And thus, we can use $[\eta]$ to get

$$y_h^* = \bar{U} - \log\left(\frac{p_y}{p_x^o}\right)$$

From here, we can see that the expenditure function can be written in the following:

$$\begin{aligned} e(p_x, p_y, \bar{U}) &= \left(\frac{p_y}{p_x^o}\right) p_x^o + p_y \left(\bar{U} - \log\left(\frac{p_y}{p_x^o}\right)\right) \\ &= p_y \left(1 + \bar{U} - \log\left(\frac{p_y}{p_x^o}\right)\right) \end{aligned}$$

2

Upon invoking duality, we can see that $x_h^* = x_m^*$ and $y_h^* = y_m^*$ if and only if we set $m = e(p_x, p_y, \bar{U})$. Thus, note that

$$\begin{aligned} v(p_x, p_y, m) &= U(x^*, y^*) \\ &= \log\left(\frac{p_y}{p_x^0}\right) + \bar{U} \log\left(\frac{p_y}{p_x^o}\right) \\ &= \bar{U} \end{aligned}$$

Thus the level of utility here is $\bar{U}$ Doing some algebraic manipulations yields:

$$
\begin{aligned}
m &= p_y\left(1+\bar{u}-\log\left(\frac{p_y}{p_x^o}\right)\right)\\
\frac{m}{p_y} &= 1+\bar{U}-\log\left(\frac{p_y}{p_x^o}\right)\\
\bar{U}^o &= \frac{m}{p_y}-1+\log\left(\frac{p_y}{p_x^o}\right)
\end{aligned}
$$

3

The consumer's original consumption bundle is $\left(\frac{p_y}{p_x^o}, \bar{U}-\log\left(\frac{p_y}{p_x^o}\right)\right)$ and the level utility derived from here is $\bar{U}$.

4

Note that if we were to increase the price, only the values of p_x^o would be changed to p_x^f, and in regards to calculations, we can see that we will still stay at the same level of utility, which is because we have already set this value in the [EMP] problem. We can see that the bundle them optimizes itself to be $\left(\frac{p_y}{p_x^f}, \bar{U}-\log\left(\frac{p_y}{p_x^f}\right)\right)$. Doing some algebraic manipulations yields:

$$
\begin{aligned}
m &= p_y\left(1+\bar{u}-\log\left(\frac{p_y}{p_x^f}\right)\right)\\
\frac{m}{p_y} &= 1+\bar{U}-\log\left(\frac{p_y}{p_x^f}\right)\\
\bar{U}^f &= \frac{m}{p_y}-1+\log\left(\frac{p_y}{p_x^f}\right)
\end{aligned}
$$

5

The Slutsky Equation is as follows:

$$\frac{\partial x_m}{\partial p_x} = \frac{\partial x_h}{\partial p_x} - \frac{\partial x_m}{\partial m}x_m^*$$

However, note that

$$\frac{\partial x_m}{\partial m} = 0$$

as there is no m in x_m under duality assumptions. This implies that

$$\frac{\partial x_m}{\partial p_x} = \frac{\partial x_h}{\partial p_x} = -\frac{p_y}{p_x^2}$$

This means that the total price effect id determined only by the chaneg in relative prices. The consumer only changes their demand of their good based on the fact that the relative prices of the goods are changing, rather than the fact that a consumer could have lost purchasing power from the increase in price.

6

Note by the setup of the Quasilinear Utility Function (which is the name of the utility function here), we are incentivized to pruchase goods of x until a certain limit, then start increasing the amount of good Y we purchase as the MRS of this can be given by

$$MRS = \frac{U_x}{U_y} = \frac{\frac{1}{x}}{1}$$

as $\frac{1}{x}$ will approach 0 as x increases. Thus, an example of a good that would represent this behavior is table salt. Each consumer here is has a finite limit of the amount of salt they want before they start allocating resources to other goods. Thus, there is no income effect when the price of salt decreases. There is only purely the substitution effect. Additionally, if the price of salt decreases, then the consumer may replace more expensive spices with salt. If salt price increases, then the consumer may replace salt with cheaper spices.

7

CV is calculated as

$$CV = e(p_x^f, p_y, v^f) - e(p_x^f, p_y.v^o)$$

Thus, we can calculate things in the following

$$\begin{aligned}
CV &= e(p_x^f, p_y, v^f) - e(p_x^f, p_y.v^o) \\
&= p_y(1 + \bar{U}^f - \log\left(\frac{p_y}{p_x^f}\right)) - p_y(1 + \bar{U}^0 - \log\left(\frac{p_y}{p_x^f}\right)) \\
&= p_y(\bar{U}^f - \bar{U}^o) \\
&= p_y(\Delta \bar{U})
\end{aligned}$$

The CV determined how much income that will have to be compensated if the consumer willing accepts a change in prices.

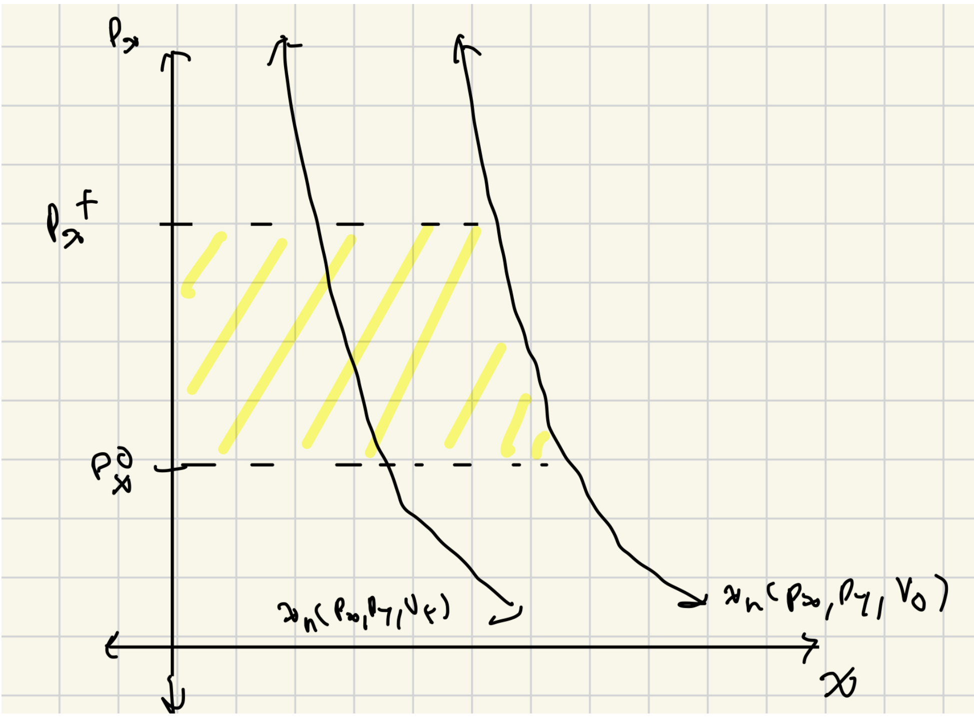


Figure 4: Graph for **CV**

8

EV is calculated as

$$EV = e(p_x^o, p_y, v_f) - e(p_x^f, p_y, v_f)$$

Thus we can calculate things in the following manner:

$$
\begin{aligned}
EV &= e(p_x^o, p_y, v_f) - e(p_x^o, p_y, v_0) \\
&= p_y(1 + \bar{U}^f - \log\left(\frac{p_y}{p_x^o}\right)) - p_y(1 + \bar{U}^0 - \log\left(\frac{p_y}{p_x^o}\right)) \\
&= p_y(\bar{U}^f - \bar{U}^o) \\
&= p_y(\Delta \bar{U})
\end{aligned}
$$

This means how much a consumer is willing to forgo to keep prices constant.

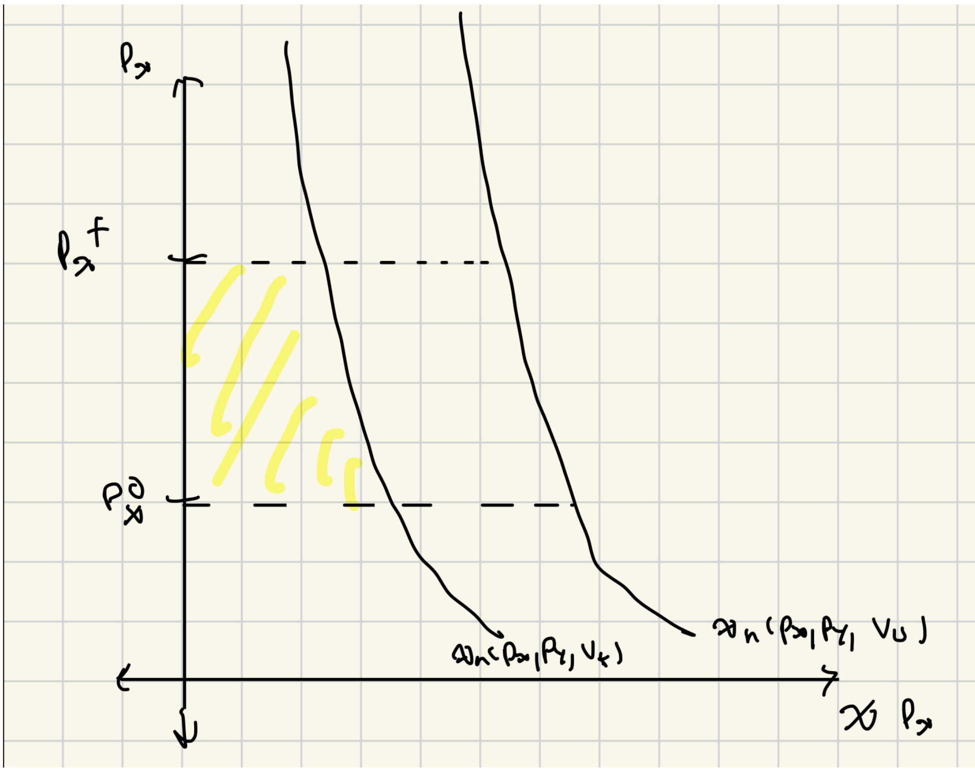


Figure 5: Graph for **EV**

9

CS is calculated as:

$$CS = -\int_{p_x^o}^{p_x^f} x_m(p_x, p_y, m) dp_x$$

We have already derived the value of x_m already, and thus, we can see that we can do the following:

$$
\begin{aligned}
CS &= -\int_{p_x^o}^{p_x^f} x_m \, dp_x \\
&= \int_{p_x^f}^{p_x^o} x_m \, dp_x \\
&= \int_{p_x^f}^{p_x^o} \frac{p_y}{p_x} \, dp_x \\
&= p_y \ln(p_x) \Big|_{p_x^f}^{p_x^o} \\
&= p_y \left(\ln(p_x^o) - \ln(p_x^f)\right) \\
&= p_y \ln\left(\frac{p_x^o}{p_x^f}\right).
\end{aligned}
$$

Note that we can see that if we use the functions we derived in **2** and **4**, we get:

$$CS = p_y(\bar{U}^f - \bar{U}^o) = p_y(\Delta\bar{U})$$

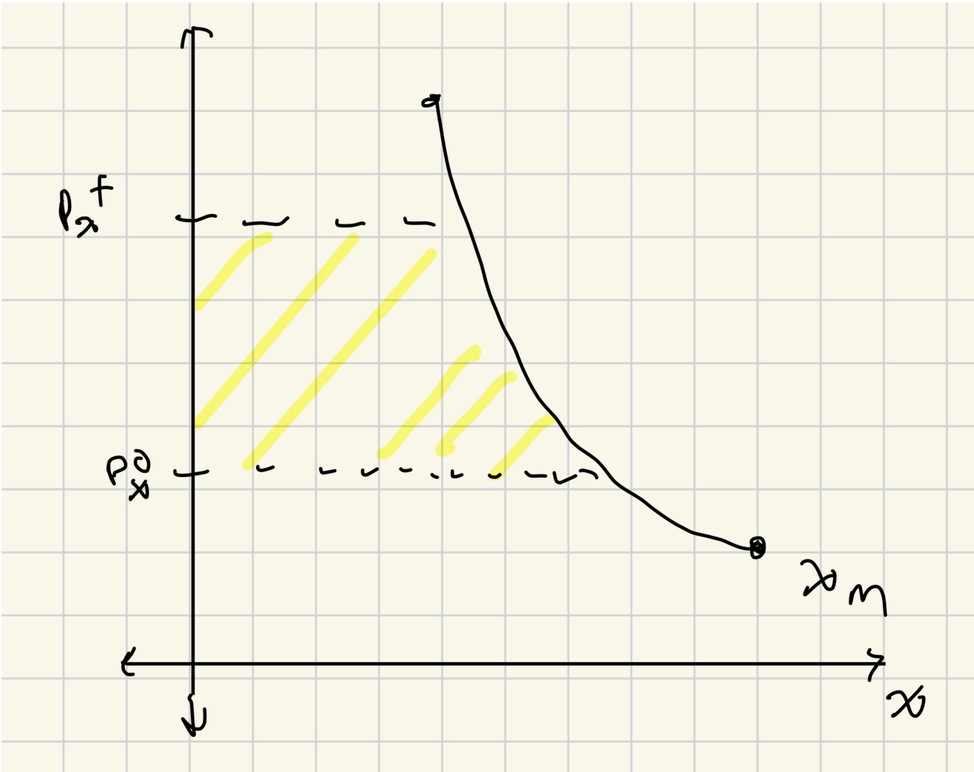


Figure 6: Graph for **CS**

10

We should note that CS, CV, EV are all equal. We can see that this is true because the utility function is quasilinear, thus price changes only lead to the substitution effect and not the income effect, which means that according to the Slutsky equation:

$$\frac{\partial x_m}{\partial p_x} = \frac{\partial x_h}{\partial p_x}$$

which implies that the integrals of CV, EV, and CS will be the same. This means that regardless of the change in welfare we want to measure, they will all be constant.