Problem 1. MATLAB Tutorial

Solution: N/A □

Problem 2. Dynamics in a Non-linear system.

(1) 123

Solution:

□

Problem 3. Level vs. Growth Rate

Solution: Write your solution here. □

Problem 4. Real GDP as a measure of welfare?

(1)

Solution: Setting up the optimization problem:

$$\begin{aligned} \max \quad & U \\ \text{s.t} \quad & \sum_{i=1}^{n} p_i x_i = M \end{aligned}$$

with the following FOCs

$$[x_i] \quad \frac{\partial U}{\partial x_i} = \lambda p_i$$

and λ is the marginal utility of income. □

(2)

Solution: Taking the total differenial of U, we can see that:

$$dU(x_1, x_2, \ldots, x_n) = \sum_{i}^{n} \frac{\partial U}{\partial x_i} dx_i$$

□

(3)

Solution: Since $\frac{\partial U}{\partial x_i} = \lambda p_i$ at the optimum, we see that:

$$dU = \sum_{i}^{n} \frac{\partial U}{\partial x_i} dx_i = \lambda \sum_{i}^{n} p_i dx_i$$

The statements holds to be true. □

Problem 5. Intertemporal Consumption Choice

(1)

Solution:

$$A_0 K_0^\alpha = c_0 + K_1$$
$$A_1 K_1^\alpha = c_1$$

as in time period 1, the individual consumes all of y □

(2)

$$y_0 = c_0 + K_1 \iff y_0 - c_0 = K_1$$

thus,

$$A_1 (y_0 - c_0)^\alpha = c_1$$

(3) Draw the graph

Draw the graph

Solution: See graph. The slope represents the rate of change between the consumption in the current time period and the next time period. □

(4)

Solution: See graph □

(5)

draw the graph.

Solution: See graph above. If A_0 increases, y_0 increases as technology in the current period would increase. This implies that budget constraint shifts outward to the right. Since the consumer now has more "budget" of corn, he now was more income to consume more. Thus,

- y_0 increases
- c_0 increases
- c_1 increases

□

(6)

Draw Graph

Solution: See graph above. If A_1 increases, this means that y_1 strictly increases. This implies that the maximum possible value of c_1 will increases, and make the graph steeper. Thus,

- c_1 increases
- y_1 increases
- c_0 does not change, as production is not impacted.

□

(7) Algebra manipulation.

Solution: We are interested in the following optimization problem:

$$\begin{aligned} \max \quad & \ln(c_0) + \beta \ln(c_1) \\ \text{s.t} \quad & A_1(y_0 - c_0)^\alpha = c_1 \end{aligned}$$

Subsituting the constraint into the objective function yields:

$$\ln(c_0) + \beta \ln(A_1(y_0 - c_0)^\alpha)$$

taking the deriative with respect to c_0 yields:

$$\frac{1}{c_0} - \frac{\alpha\beta}{c_0 - y_0} = 0$$

which, after some algebra, yields:

$$c_0^* = \frac{y_0}{1 + \beta\alpha}$$

Thus, this implies that

$$c_1^* = A_1 \left(\frac{\alpha\beta y_0}{1 + \beta\alpha} \right)^\alpha$$

Thus, we can see that since A_1 only appears in c_1, this implies that c_1 and y_1 increases. Additionally, we can see that increasing A_0 would increase y_0 which in turn increase all values. □

Problem 6. Exact Price Index from the Economic Approach

(1)

Solution:

$$\begin{aligned} \max \quad & \ln x + \ln y \\ \text{s.t} \quad & p_x x + p_y y = M \end{aligned}$$

we have the following FOCs:

$$\begin{aligned} [x] \quad & \frac{1}{x} = \lambda p_x \\ [y] \quad & \frac{1}{y} = \lambda p_y \\ [\lambda] \quad & M = p_x x + p_y y \end{aligned}$$

Note that the FOCs imply that $p_x x = p_y y$ and thus, using the budget constraint, we find that:

$$p_x x + p_y y = M \iff 2p_x x = M \iff x = \frac{M}{2p_x}$$

and by symmetry

$$y = \frac{M}{2p_y}$$

which indicates that expenditure share is one half. □

(2)

Solution: The indirect utilty function is

$$V(M,P) = \ln\left(\frac{M}{2p_x}\right) + \ln\left(\frac{M}{2p_y}\right)$$

□

(3)

Solution: We aim to use duality to prove this. Let

$$U = 2\ln(M) - \ln(4) - \ln(p_x p_y)$$

□

(4)

Solution: Using duality, we see that:

$$\begin{aligned} U &= 2\ln(M) - \ln(4) - \ln(p_x p_y) \\ \ln(M) &= 0.5\ln(4) + \ln(\sqrt{p_x p_y}) \\ M &= 2e^{\frac{U}{2}}\sqrt{p_x p_y} \end{aligned}$$

Using Shephard's Lemma, we see that:

$$x^h = \frac{\partial e(p,U)}{\partial p_x} = \sqrt{\frac{e^U p_y}{p_x}}$$

and by symmetry

$$y^h = \frac{\partial e(p,U)}{\partial p_y} = \sqrt{\frac{e^U p_x}{p_y}}$$

□

(5)

Solution:

$$M = \frac{2e^{\frac{U_t}{2}}\sqrt{p_x^t p_y^t}}{2e^{\frac{U_0}{2}}\sqrt{p_x^0 p_y^0}} = e^{\frac{U_t - U_0}{2}}\sqrt{\frac{p_x^t p_y^t}{p_x^0 p_y^0}}$$

□

Check this

(6)

Solution: This is the fisher price index. □

Double check this