Honors Econ PSET 5

Anthony Yoon and Chimin Liu

11/12/2024

Problem 18

1

Given the utility function:

$$U(h, s) = 150h - h^2 + 300s - 2s^2$$

Note that this is a quadratic utility function. This means that there must exist a bliss point, which would be the "center" of the circle. Thus, we can do the following operations:

$$\begin{aligned}
U(h, s) &= 150h - h^2 + 300s - 2s^2 \\
U(h, s) &= 75^2 + 150 - h^2 + 2(75^2) + 300s - 2s^2 - (75^2 + 2(75^2)) \\
U(h, s) &= (75 - h)^2 + 2(75 - s)^2 - (75^2 + 2(75^2))
\end{aligned}$$

This means that the bliss point is 75 pounds of hard and soft candy.

2

To see what the total budget would be, we would need to assume that the consumer were to sell their entire endowment $(\omega_h, \omega_s) = (60, 100)$ at the current prices. That can be calculated in the following:

$$m_0 = 4\omega_h + 2\omega_s = 4(60) + 2(100) = 440$$

Hence, the budget constraint would be:

$$4h + 2s = m_0 = 440$$

Which the graph can be seen as follows:

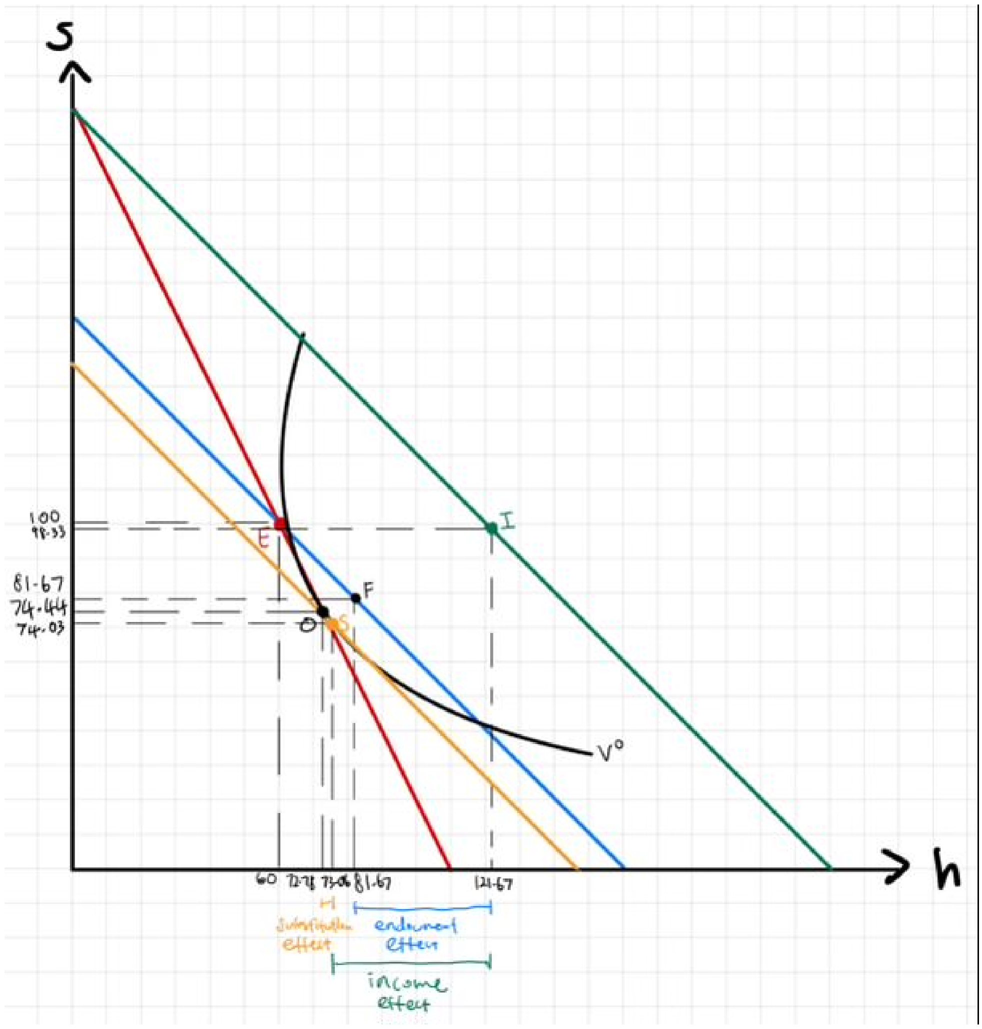

Figure 1: Graph for this PSET

3

We will first solve the [UMP] and use duality to relate gross and net demand fucntions, under the assumption that the consumer exhausts their entire budget. So given the following [UMP] problem:

$$\begin{array}{ll} \max & 150h - h^2 + 300s - 2s^2 \\ s.t & 4h + 2s = m_o \end{array}$$

The first order conditions are:

$$\begin{aligned} [h]&\ 150 - 2h = 4\lambda \\ [s]&\ 300 - 4s = 2\lambda \\ [\lambda]&\ 4h + 2s = 440 \end{aligned}$$

Which yields the following:

$$\begin{aligned} \frac{150 - 2h}{4} &= \frac{300 - 4s}{2} \\ 75 - h &= 300 - 4s \\ h - 75 &= 4s - 300 \\ h &= 4s - 225 \end{aligned}$$

Solving for h yields:

$$\begin{aligned}
440 &= 4(4s - 225) + 2s \\
440 &= 16s - 900 + 2s \\
1340 &= 18s \\
s &= \frac{670}{9} = 74\frac{4}{9} \iff h = \frac{655}{9} = 72\frac{7}{9}
\end{aligned}$$

And if we let $\overline{U} = V(h, s)$, by duality, we konw that $h_m = h_h$ and $s_m = s_h$.Thus, the consumer's net and gross demand for the goods are $\frac{670}{9}$ pounds of soft candy and $\frac{655}{9}$ pounds of hard candy.

4

Since prices changed before Patrick sold his endowment, we must calculate a new m, denoted as m_f. Since our endowment bundle has not changed, we can calculate m_o as follows:

$$m_f = 2\omega_h + 2\omega_s = 2(60) + 2(100) = 320$$

Note that our utilty function has not changed, meaning that U_s, U_h has not changed. Therefore, when we solve the [UMP] again while changing m_o to be m_f, we get the following:

$$\begin{aligned}
\frac{U_s}{p_s} &= \frac{U_h}{p_h^f} \\
\frac{300 - 4s}{2} &= \frac{150 - 2h}{2} \\
150 - 2s &= 75 - h \\
h &= 2s - 75
\end{aligned}$$

solving for the optimal values of s and p, we get that:

$$\begin{aligned}
2h + 2s &= 320 \\
2(2s - 75) + 2s &= 320 \\
4s - 150 + 2s &= 320 \\
6s &= 470 \\
s &= \frac{235}{3} = 78\frac{1}{3} \iff h = \frac{245}{3} = 81\frac{2}{3}
\end{aligned}$$

Thus, the demand for hard candy has increased by $\frac{80}{9}$ pounds and the demand for soft candy has increased by $\frac{35}{9}$. See graph above for edits to graph.

5

To calculate the substition, income, and endowment effect, we first must see the decomposition of the total price effect for any good x as follows:

$$x^f - x^o = (x^i - x^o) + (x^f - x^i)$$

$(x^f - x^i)$ represents the endowment effect, which is represented by

$$x^f - x^i = x_m(p_x^f, p_y, m_f) - x_m(p_x^f, p_y, m_o)$$

$(x^i - x^o)$ represents the Marshallian Price change, where we can further decompose that into:

$$x^i - x^o = (x^i - x^s) + (x^s - x^o)$$

where $x^i - x^s$ represents the income effect and $x^s - x^o$ represnts the substition effect. Hence, the total effect is:

$$x^f - x^o = (x^i - x^s) + (x^s - x^o) + (x^f - x^i)$$

We can represent these more accurately as follows:

$$x^i - x^s = x_m(p_x^f, p_y, m_o) - x_h(p_x^f, p_y, \overline{U} = \text{Level of Utility reached from original bundle.})$$

and

$$x^s - x^o = x_h(p_x^f, p_y, \overline{U} = \text{Level of Utility reached from original bundle.}) - x_m(p_x^o, p_y, m_o)$$

We have already calculated what $s_m(p_h^f, p_s, m^f)$ and $h_m(p_h^f, p_s, m^f)$, $s_m(p_h^o, p_s, m^o)$ and $h_m(p_h^o, p_s, m^o)$ in **4** and **3** respectively. Thus, we need to calculate h^i, s^i and h^s, s^s are.

Calculating h^i and s^i

We know that

$$h^i = h_m(p_s^f, p_y, m_o)$$

and

$$s^i = s_m(p_s^f, p_y, m_o)$$

Thus, we have the following [UMP] problem:

$$\max_{h,s} \; 150h - h^2 + 300s - 2s^2$$
$$s.t \;\; 2h + 2s = 440$$

Note that this is very similar to the first order conditions in **4**, except we are working with m_o instead of m_f. Hence, the relation $h = 2s - 75$ still holds. Now, solving for h, s with m_o yields:

$$\begin{aligned} 2h + 2s &= 440 \\ 2(2s - 75) + 2s &= 440 \\ 6s - 150 &= 440 \\ 6s &= 590 \\ s &= \frac{295}{3} = 98\frac{1}{3} \iff h = \frac{365}{3} = 121\frac{2}{3} \end{aligned}$$

So, $s^i = \frac{295}{3}$ and $h^i = \frac{365}{3}$.

Calcualting h^s and s^s

The level of utility that we reached from our original bundle was:

$$U(s = \frac{670}{9}, h = \frac{655}{9}) = 150\left(\frac{655}{9}\right) - \left(\frac{655}{9}\right)^2 + 300\left(\frac{670}{9}\right) - 2\left(\frac{670}{9}\right)^2 = 16869.4$$

Hence, we have the following [EMP] problem:

$$\min \; p_s s + p_h^f h$$
$$s.t \;\; U(h, s) = 16869.4$$

The Lagrangian is as follows:

$$L = p_s s + p_h^f h + \eta(16869.4 - U(h, s))$$

with the following first order conditions:

$$\begin{aligned} &[s] \;\; 2 = (300 - 4s)\eta \\ &[h] \;\; 2 = (150 - 2h)\eta \\ &[\eta] \;\; U(h, s) = 16869.4 \end{aligned}$$

Setting both η equal to each other yields:

$$\begin{aligned}\frac{2}{300-4s} &= \frac{2}{150-2h}\\ \frac{1}{150-2s} &= \frac{1}{75-h}\\ 150-2s &= 75-h\\ h &= 2s-75\end{aligned}$$

Nowe, we can substitute this value into the utility function to get:

$$\begin{aligned}150h - h^2 + 300s - 2s^2 &= 16869.4\\ h(150-h) + 300s - 2s^2 &= 16869.4\\ (2s-75)(225-2s) + 300s - 2s^2 &= 16869.4\\ -(2s-75)(2s-225) + 300s - 2s^2 &= 16869.4\\ -(4s^2 - 600s + 16875) + 300s - 2s^2 &= 16869.4\\ -4s^2 + 600s - 16875 + 300s - 2s^2 &= 16869.4\\ -6s^2 + 900s - 16875 - 16869.4 &= 0\\ 6s^2 - 900s + 33744.4 &= 0\end{aligned}$$

Using the quadratic formula for s yields that s can be either 75.96 or 74.03. Using the relation $h = 2s - 75$, we find that the two bundles, in the form (h, s), are $(76.92, 75.96)$ and $(73.06, 74.03)$. Since the prices are the same, we know that whatever bundle has the smallest amount will be the cheapest. Hence, to sastify the goal of expenditure minimization, we have to get that $h^s = 73.06$ and $s^s = 74.03$.

Calculating the effects

Finally, we can calculate the effects. The effects are as follows. First Hard Candy:

- **Endowment Effect for Hard Candy**: $h^f - h^i$: $81\frac{2}{3} - 121\frac{2}{3} = -40$
- **Income Effect for Hard Candy**: $h^f - h^i$: $121\frac{2}{3} - 73.07 \approx 48.6$
- **Substition Effect for Hard Candy**: $h^s - h^o$ $73.07 - 72\frac{7}{9} \approx 0.29$

Now Soft Candy:

- **Endowment Effect for Soft Candy**: $s^f - s^i$: $78\frac{1}{3} - 98\frac{1}{3} = -20$
- **Income Effect for Soft Candy**: $s^i - s^s$: $98\frac{1}{3} - 74.04 \approx 24.29$
- **Substition Effect for Soft Candy**: $s^s - s^o$: $73.07 - 72\frac{7}{9} \approx 0.29$

6

Note that since we know are dealing with a fixed budget as the prices have changed after his endowment is sold. Hence, we know that

- **Endowment Effect is 0**. There is no change in budget because of a change prices, hence no endowment effect.
- **Income and Substition Effect are the same**. Since these values do not rely on the budget changing from the endowment, we can say that these two effects are the same.

7

Timing is crucial. We can compare the utility levels for the 3 scenarios:

- **Before price changes**: $\overline{U} = 16869.4$
- **Prices change before endowment is sold**: We can calculate this as follows:

$$\begin{aligned} U\left(h = \frac{245}{3}, s = \frac{235}{3}\right) &= 150\left(\frac{245}{3}\right) - \left(\frac{245}{3}\right)^2 + 300\left(\frac{235}{3}\right) - 2\left(\frac{235}{3}\right)^2 \\ &= 13609.27 \end{aligned}$$

- **Prices change after endowment is sold**: We can calculate this as follows:

$$\begin{aligned} U\left(h = 81\frac{2}{3}, s = 78\frac{1}{3}\right) &= 150\left(81\frac{2}{3}\right) - \left(81\frac{2}{3}\right)^2 + 300\left(78\frac{1}{3}\right) - 2\left(78\frac{1}{3}\right)^2 \\ &= 16808\frac{1}{3} \end{aligned}$$

We can see that selling our endowment before the prices change is more beneificial to us, indicating that indeed timing matters.