Honors Economics PSET 1

Anthony Yoon and Chimin Liu

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Exericse 1

1. Given the information of this problem, we can see that the following is the budget constraint:

$$p_s s + p_b b = m$$

where p_s and p_b denotes the price of steak and bread. s and b also denote the amount of bread we buy in pounds. We also have to denote that these goods have to be positive, so we also denote that

$$s \geq 0, b \geq 0$$

Since the budget of the individual is \$60 and the price of bread and steak is 6 and 5 dollars respectively, Therefore, the budget set of the customer is

$$6s + 5b \leq 60$$

$$s \geq 0, b \geq 0$$

2. If the individual only buys steak, the proportion of carbs and proteins are 2 grams of protein to 1 gram of carbs. This can be seen here

$$\frac{y_s}{x_s} = \frac{12 \text{ grams of protein}}{6 \text{ grams of carbs}} = 2$$

Thus, an equation can be written such that the relation between the protein and carbs is:

$$y = 2x$$

The nutritional content of the steak constrains the individual's consumption of carbs and proteins because compared between the 2 goods, steak has the higher protein amount and lower amount of carbs. The upper bound of the protein consumed by the individual is only achieved when the consumer buys all steak.

3. Note

$$\frac{m}{p_s} = \frac{\$60}{6 \text{ dollars per pound of steak}} = 10 \text{ pounds of steak}$$

which means that we can

$$10 \text{ pounds of steak} \times \frac{12 \text{ grams of proteins}}{1 \text{ pounds of steak}} = 120 \text{ grams of protein}$$

$$10 \text{ pounds of steak} \times \frac{6 \text{ grams of carbs}}{1 \text{ pounds of steak}} = 60 \text{ grams of carbs}$$

Thus, the individual will consume 120 grams of protein and 60 grams of carbs.

4. With a similar logic to number 2, the individual will consume a proportion of 1 gram of protein to 3 grams of carbs. This can be seen here

$$\frac{y_b}{x_b} = \frac{5 \text{ grams of protein}}{15 \text{ grams of carbs}} = \frac{1}{3x}$$

which yields the following relation if the individual only consumes bread.:

$$y = \frac{1}{3}x$$

And thus, the nutritional content of the bread constrains the individual's consumption of carbs and proteins because bread has the lower protein content compared to steak but higher carb content compared to steak. This means that upper bound of carb consumption is only possible when we buy all bread.

5. Note:

$$\frac{m}{p_b} = \frac{\$60}{\text{5 dollars per pound of bread}} = \text{12 pounds of bread}$$

and

$$\text{12 pounds of bread} \times \frac{\text{5 grams of proteins}}{\text{1 pounds of bread}} = \text{60 grams of protein}$$

$$\text{12 pounds of bread} \times \frac{\text{15 grams of carbs}}{\text{1 pounds of bread}} = \text{180 grams of carbs}$$

Thus the individual will consume 60 grams of protein and 180 grams of carbs.

6. Note that each pound of bread has 5 grams of protein and 15 grams of carbs, and each pound of bread costs 6 dollars. Let p_x and p_y denote the price of a gram of protein and a gram of carbs respectively. Thus, we can see that if we were to construct the budget constraint for the bread, we can see that

$$xp_x + yp_y = p_b$$

Substituting the value of each variable, we see that

$$15p_x + 5p_y = 5$$

This holds because for each piece of bread, we have to spend 5 dollars to get a piece of bread, and with each loaf of bread, there are 5 grams of protein and 15 grams of carbs. We cannot change these values at all as these are immutable properties of the object (the nutritional value), and thus the relation holds. This relation also holds because the sum of the components of bread (i.e. the grams of carbs and protein in bread) multiplied by their corresponding prices p_x and p_y must equal the cost of bread.

7. A similar logic holds from above. If we were to construct the budget constraint of the steak we get:

$$xp_x + yp_y = p_s$$

Thus, when we substitute the known values, we get

$$6p_x + 12p_y = 6$$

This relation holds because to get a pound of steak, we must spend 6 dollars exactly. And within each pound of steak, there are 12 grams of protein and 6 grams of carbs. We cannot change these values at all as these are immutable properties of the object (the nutritional value), and thus the relation holds. This relation also holds because the sum of the components of steak (i.e. the grams of carbs and protein in steak) multiplied by their corresponding prices p_x and p_y must equal the cost of steak.

8. Solving the system of equations. Note:

$$15p_x + 5p_y = 5 \iff 3p_x + p_y = 1$$

and

$$6p_x + 12p_y = 6 \iff p_x + 2p_y = 1$$

Thus, by solving the system of equations

$$\begin{aligned} p_x + 2p_y &= 1 \\ 3p_x + p_y &= 1 \end{aligned}$$

multiplying the bottom equation by 2 and rearranging:

$$\begin{aligned} 6p_x + 2p_y &= 2 \\ p_x + 2p_y &= 1 \end{aligned}$$

and subtracting the equations yields:

$$5p_x = 1$$

$$p_x = 0.20$$

Substituting this value into the original equation will yield:

$$\begin{aligned} 0.20 + 2p_y &= 1 \\ 2p_y &= 0.8 \\ p_y &= 0.40 \end{aligned}$$

Thus the price of protein is 40 cents per gram and the price of carbs is 20 cents per gram.

9. Setting up the budget constraint as
$$m = p_x x + p_y y$$
and taking the total differential yields
$$dm = \frac{\partial m}{\partial p_x}dp_x + \frac{\partial m}{\partial x}dx + \frac{\partial m}{\partial p_y}dp_y + \frac{\partial m}{\partial y}dy$$
$$dm = xdp_x + p_x dx + ydp_y + p_y dy$$
now we are assuming that there is perfect competition, prices are not changing, which implies that
$$dp_y = dp_x = 0$$
so we are left with:
$$p_x dx + p_y dy = dm$$
However, we assume that the consumer always spends all of his money and there is no change in income. Therefore, the following is resultant:
$$p_x dx + p_y dy = 0$$
and if we rearrange the terms we get:
$$\begin{aligned} dx &= -\frac{p_y}{p_x}dy \\ &= -\frac{0.40}{0.20}dy \\ &= -2dy \end{aligned}$$
Remark: if we were to rearrange terms slightly more, we get
$$\frac{dy}{dx} = -\frac{p_x}{p_y} = -\frac{1}{2}$$
Therefore, to gain one additional gram of protein, we must give up 2 grams of carbs, to keep expenditure constant at income level. This value is determined by the ratio of the price of carbs to the price of protein $\left(\frac{p_y}{p_x}\right)$.

10. The constraint is
$$0.2x + 0.4y = 60$$

11. The budget set is
$$\begin{aligned} 0.2x + 0.4y &\leq 60 \\ y &\leq 2x \\ y &\geq \frac{1}{3}x \end{aligned}$$

12. Graph shown here

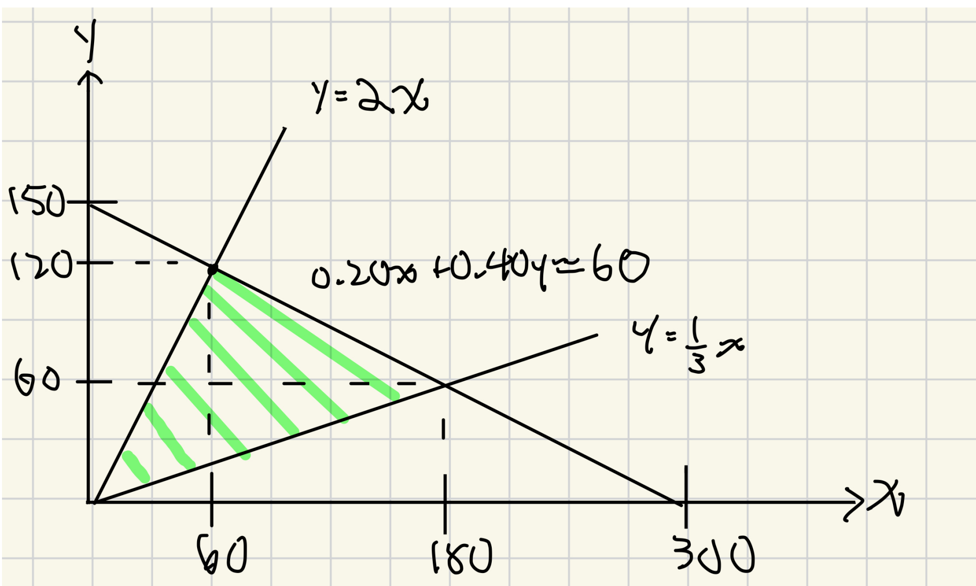

Figure 1: Budget set for Exercise 1

Explanation: The budget constraint is downward sloping because there are tradeoffs: If we want to have more of one good, we have to give up some of the other good. Because of the assumption we made for perfect competition and no changing prices, the line is linear. And the $\leq$ sign is derived from the fact that the consumer does not have to spend all of his money. The slope of the budget constraint is -0.5, as

$$-\frac{p_x}{p_y} = -\frac{0.2}{0.4} = -0.5$$

The constraint $y \leq 2x$ and $y \geq \frac{1}{3}$is derived from the ratio of nutrients in steak and bread respectively. As if we were to only buy steaks or bread, we would only have to follow on their respective line. But physically speaking, we cannot find a way to only buy protein or only buy carbs, so for example the bundles (0, 150) and (300, 0) are impossible. Therefore, mathematically the inequalities work out to their respective counterparts in steak and bread so that they bound the only possible bundles of proteins and carbs.

Exercise 2

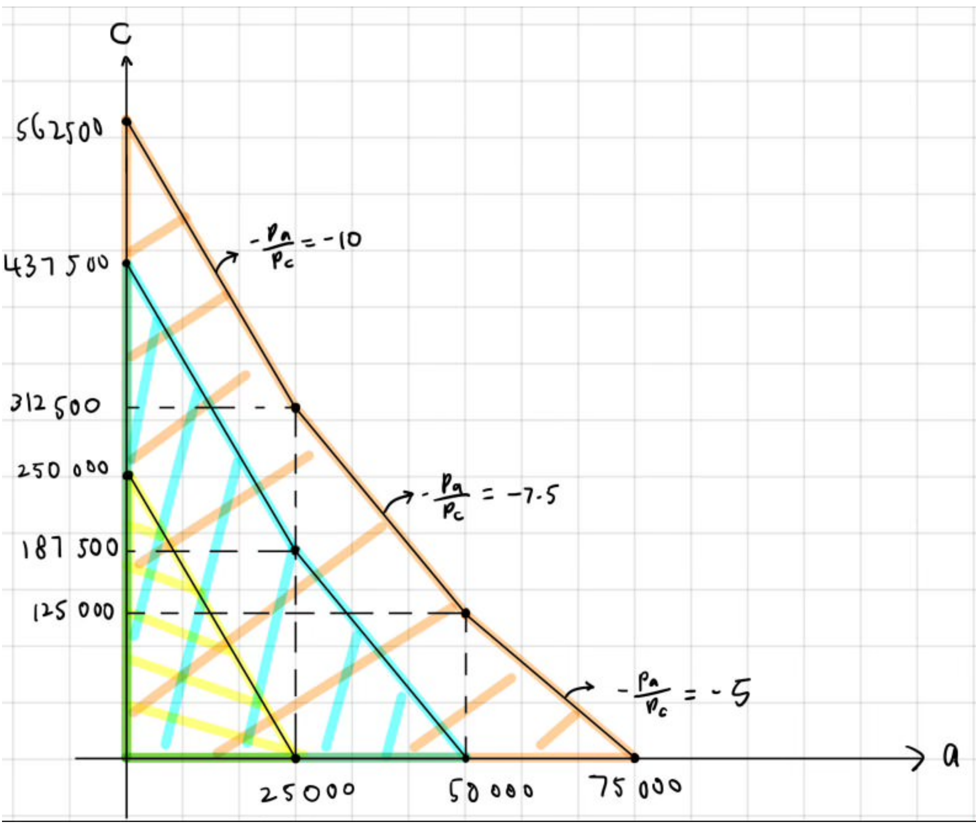

Figure 2: Graph for Exercise 2, Questions 1,2,3

1. Refer to graph above
2. Refer to graph above
3. Refer to graph above. Assume income is 562500 dollars.
4. Her budget set will always change as her income increases in a couple of ways.

   As her incomes increase, her budget constraint shifts outwards. Additionally, note that as the number of miles traveled increase, then the gradient of the constraint also decreases. The budget cionstraint also becomes a line of conjoint line segments, each with a flatter and flatter slope as the number of miles increases.

5. The budget constraint can be set as

$$m = p_a a + p_c c$$

Taking the total differential of the equation yields:

$$dm = adp_a + p_a da + cdp_c + p_c dc$$

Note that in this specific scenario, prices are not changing as well as her budget is changing, which means

$$dm = dp_a = dp_c = 0$$

thus yielding

$$0 = p_a da + p_c dc$$
$$-p_a da = p_c dc$$
$$\frac{dc}{da} = -\frac{p_a}{p_c}$$
$$\frac{dc}{da} = -\frac{10}{1} = -10$$

This means that the rate is Anne gives up 1 mile to gain 10 units of other goods.

6. Using the derived equation from above, we can see that

$$\frac{dc}{da} = -\frac{p_a}{p_c}$$

substituting values now that air fare has been decreased by 25 percent yields

$$\frac{dc}{da} = -\frac{7.5}{1} = -7.5$$

Thus, the rate is that Anne gives up 1 mile to gain 7.5 units of other goods.

7. Using the same equation

$$\frac{dc}{da} = -\frac{p_a}{p_c}$$

substituting values now that air fare has been decreased by 50 percent yields:

$$\frac{dc}{da} = -\frac{5}{1} = -5$$

Thus, the rate is that Anne gives up 1 mile to gain 5 units of other goods.

8. The graph can be seen here:

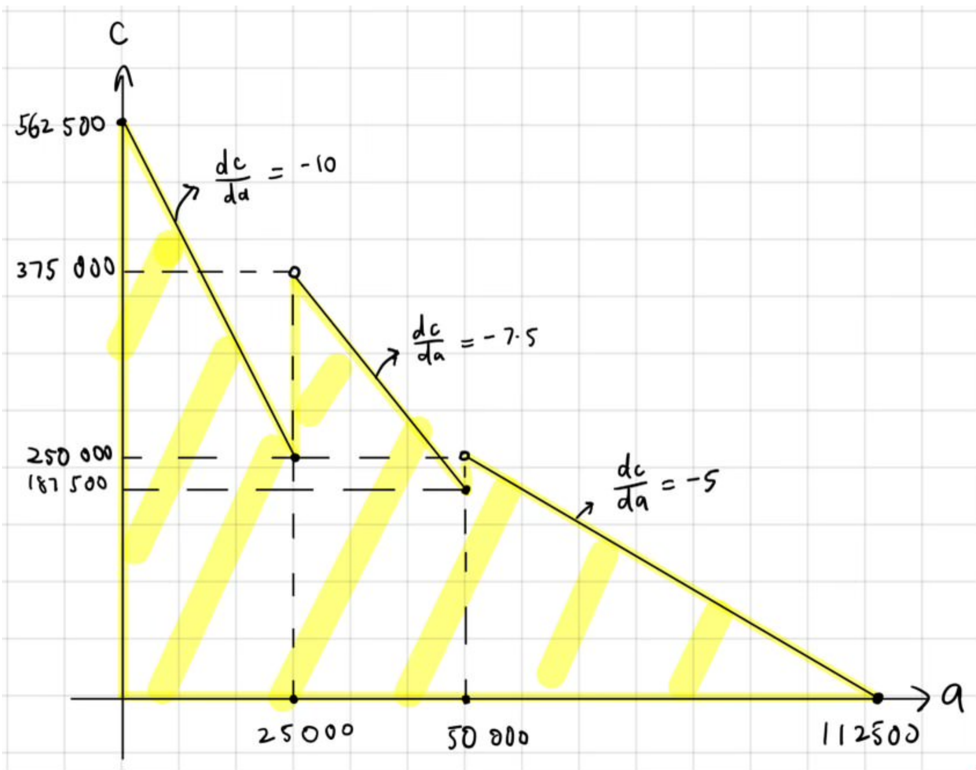

Figure 3: Graph for Exercise 2 Number 8

The reason why the graph is discontinuous is because when Annie hits 25,000 miles, the price changes to 25% off, and to 50% respectively for 50,000 miles. Thus, whenever we hit a discount barrier, we must account for this cheaper price for the miles already purchased, which means that the amount of money left to purchase the other goods increases and hence the consumption of other goods increases.

Exercise 3

1. We can see

$$U_x = \alpha x^{\alpha-1} y^{1-\alpha}$$

$$U_y = x^{\alpha}(1-\alpha) y^{-\alpha}$$

simplifying the given equation, we get the final equation:

$$\frac{dy}{dx} = -\frac{U_x}{U_y} = -\frac{\alpha x^{\alpha-1} y^{1-\alpha}}{x^{\alpha}(1-\alpha)(y^{-\alpha})} = -\frac{\alpha}{1-\alpha}\frac{y}{x}$$

Alpha is the only value that dictates the magnitude. We can see that if $\alpha < 0.5$, then the magnitude will decrease compared to if $\alpha = 0.5$ because $\frac{\alpha}{1-\alpha}$ will be less than 1, thus decreasing magnitude However, if $\alpha > 0.5$, the magnitude will increase compared to 0.5 because $\frac{\alpha}{1-\alpha}$ will be greater than 1, thus increasing magnitude. The consumption bundle (x,y) also determines the magnitude of $\frac{dy}{dx}$, as seen in the formula. A bundle with more units of good y than units of good x will increase the magnitude of $\frac{dy}{dx}$ whearas in which units of good y are less than units of good x will decrease the magnitude of $\frac{dy}{dx}$

2. Shown here

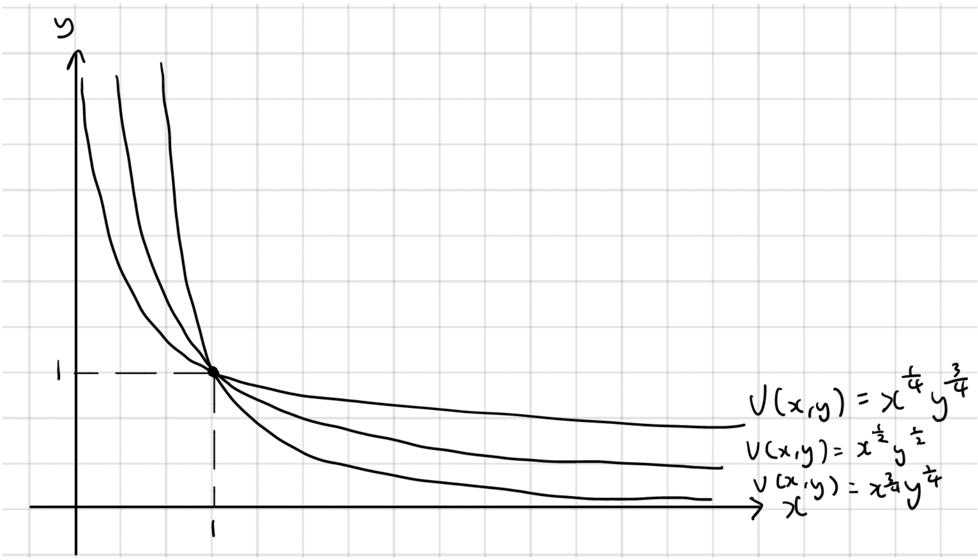

Figure 4: The different indifference curves.

3. α changes the the indifference curves by either making them steeper or less steep. We can see this mathematically by seeing that $\frac{y}{x}$ will be multiplied by $\frac{\alpha}{1-\alpha}$, which will then either increase or decrease $\frac{dy}{dx}$ depending on the α chosen. So in this case, we can see that $\alpha = 0.25$ will have smallest $\frac{dy}{dx}$ and $\alpha = 0.75$ will have the largest $\frac{dy}{dx}$, given that (x,y) are constant.

4. Since all indifference curves go through $(1, 1)$, we can assume to fix $(x, y) = (1, 1)$ When $\alpha = 0.25$

$$\frac{dy}{dx} = -\frac{y}{3x} = -\frac{1}{3}$$

when $\alpha = 0.5$:

$$\frac{dy}{dx} = -\frac{y}{x} = -1$$

when $\alpha = 0.75$:

$$\frac{dy}{dx} = -\frac{3y}{x} = -3$$

We can see that the individual with $\alpha = 0.75$ is more willing to give up 3 units of y for a 1 unit increase of x, but the individual with $\alpha = 0.25$ is the least willing to give up good y for x, as they are only willing to give up $\frac{1}{3}$ units of y for the same 1 unit of x.

5. Note that we are still assuming (x, y) are still being held constant at $(1, 1)$, which means that we can refer to the above derived quantities, which indicate that slope will be steepest indifference curves with $\alpha = 0.75$, and with $\alpha = 0.25$, the slope will nbe the least steep.

6. This is a similar question to the 4 and 5, with the same assumptions. There is the additional assumption that income is the same, which means that the point of intersection will be all the same. The individual with $\alpha = 0.75$ is more willing to give up a lot of Y (3 units exactly) for 1 unit of X, and the individual with $\alpha = 0.25$ is willing to give 3 units of X for 1 unit of Y. Thus, the individual with $\alpha = 0.75$ is most likely to buy the most units of X, and the individual with $\alpha = 0.25$ is most likely to buy the most of Y.

7. *Ceteris Paribus*, which means "all other things held equal", is important within in this question, as this allows us to compare $\frac{dy}{dx}$ with differing values of α, while ensuring we are analyzing at the same point $(1, 1)$, same utility level, and same budget constraint. Thus, this allows to effectively measure and compare marginal rate of substitution.

Exercise 4

According to the problem, A student is first and foremost interested in beer and would be willing to forgo any quantity of milk for the smallest additional quantity of beer. We can denote this using an order preference like this, where we assume the bundle of interest is (x, y) and x is milk and y is beer:

$$(x_1, y + \Delta y) \succ (x_2, y)$$

where $x_1 < x_2$ for any amount of milk x_2 and $\Delta y > 0$. This means that for an individual to stay indifferent about a good, *beer must stay the same between the 2 bundles in question* If there are any increases in the amount of beer, there will be preference towards that good. However, the second part of the statement(if beer consumption is given, the student prefers to have more milk rather than less) implies the following ordering preference

$$(x_2, y) \succ (x_1, y)$$

with the same ordering of x_1 and x_2 as above. This means that for an individual to stay indifferent about a good, *milk must stay the same between 2 bundles in question.* This is because if beer is present, then a higher quantity of milk will be always desired, thus meaning that for an individual to be indifferent, they must have two bundles with the same amount of milk. The only logical indifference curve that makes in this sense is a singular point, where an individual will stay indifferent about their preference of bundles (or bundle for this manner) at this point. Thus any point in the beer milk space will be an indifference curve, as there will be no other order preference along this curve. This also means that the individual s not indifferent between 2 bundles, and will always have a preference, as explained earlier in the question. Combining the logic shows us that for any 2 bundles to be indifferent,

$$(x_1, y_1) \sim (x_2, y_2)$$

and

$$x_1 = x_2, y_1 = y_2$$

Thus, there are no 2 bundles for which the students are indifferent and therefore all bundles are indifference curves with different utility values themselves. The curve is as follows for the bundle (x_1, y_1):

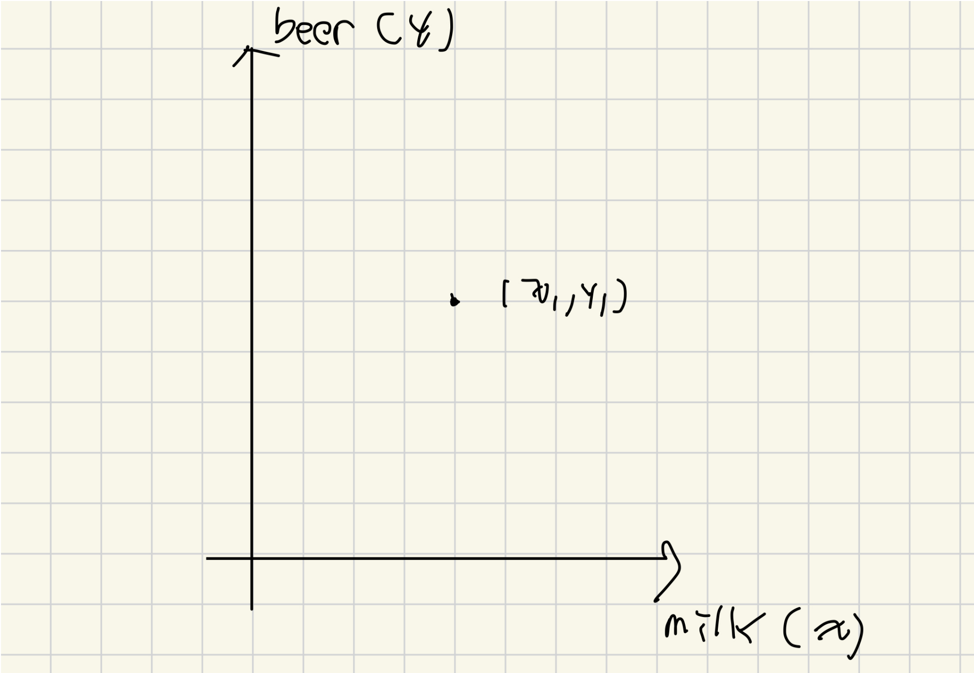

Figure 5: Indifference curve (or dot)