

Econ Research Project

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1 Introduction

With over 7,000 students, the University of Chicago's Undergraduate College provides education to numerous students. Due to the on-campus housing requirement, the College also provides ample room and board opportunities for students to take advantage of so that students can socialize with one another during their underclassmen years. This on-campus requirement has also helped foster UChicago's coveted "House culture," where students go on field trips and participate in bonding activities together in the dorms, providing them with a reliable social network from the day they set foot on campus.

However, once students are released from the on-campus housing requirement, many (54% in 2014) choose to opt out of the dorms for off-campus apartments around Hyde Park. UChicago could benefit from having students pay for on-campus housing to develop community building between lowerclassmen and upperclassmen, to optimize the success of students who have easier access to classroom building and study spaces, and to have a steady revenue stream to fund housing operations, dining services, campus maintenance, on top of higher occupancy rates in the rented dorm building, Woodlawn Residential Commons.

Inspired by the many variables that motivate students to leave the dorms as well as UChicago's preference to have students stay, we chose to examine incentives that would lead UChicago students to stay in the dorms for our research project

In this paper, we treat UChicago as the firm and undergraduate students as the consumer. Specifically, UChicago wants to optimize for the number of students staying on campus. The students (the consumers) are choosing to maximize their utility by consuming one of either good, on or off campus housing. In this paper, we ask the question: **How can we model the incentives UChicago wishes to provide to motivate students to stay in on-campus housing using an anonymous game with externalities?**

2 Associated Costs of Living

We begin with a perspective of the various costs of living on and off campus. According to the University of Chicago Financial Aid Website and the US News report in 2014, the costs and the quantity of people are as follows:

	On campus	Off Campus
Room and Board	\$20109	\$17502
Num. of People	4133	3520

Under the assumption that there are approximately 1913 students per grade level and all underclassmen must stay on campus, we see that there are approximately 307 upperclassmen that choose to stay on campus. This indicates that the majority of upperclassmen choose to

live off campus. We also assume that there are about 100 occupancies per house, and with 48 houses within the dorms, that there are approximately 4800 spots for students to live on campus. Assuming that these numbers have remained constant throughout time, the majority of upperclassmen still choose to live off campus today. This means that currently the university has about 670 open spots throughout a given school year, implying a wastage of capital, and thus a potential loss of profit. Thus, the university wants to introduce incentives to promote higher occupancy rates among upperclassmen.

3 Assumptions of the model

Before we introduce the model, we would like to introduce the core assumptions of this model.

- The number of occupancies in the dormitories are bounded by some constant $M \in \mathbb{R}^+$ where given our estimates for the new academic year, $M = 4800$
- When the demand for rooms is greater than M , the university will increase each rooms' occupancy by adding beds, making all students living on campus worse off.
- The incentives we propose in our model is symmetric such that the improvements and detriments to the quality of life that a student receives is constant for all students.
- We assume that the university is strictly better off the more rooms that they fill. Hence, all occupancies being filled would be the optimal outcome.
- All assumptions presented in class will hold in this analysis

4 Model: Housing Game as an Anonymous Game with Externalities

4.1 Modeling student's choice to move on or off campus

Given that we know the current number of upperclassmen staying on campus is suboptimal, the university decides to introduce a housing game before the next academic year in the hopes of increasing the upperclassmen's on-campus occupancy to the optimal level. Let the status quo number of upperclassmen living on campus be the bound ϕ and the optimal number of upperclassmen ξ . Utilizing the estimates from earlier, ϕ and ξ must be in $[0, M]$ where $\xi = M$. Now consider the following one-shot game. We define the players of the game as the upperclassmen at the University.

$$Players = \{1, 2, \dots, N\}$$

and with the actions to live off or on campus, or denoted as follows:

$$Actions = \{F, O\}$$

with the base payoffs in utility for each individual as follows:

$$U_i(F) = \Lambda \quad U_i(O) = \gamma$$

for $i \in \text{players}$. Note that $\Lambda > \gamma$ as the price for living off campus is cheaper than that of living on campus. We can see if that there was no incentive provided by the university, than everyone would move off campus, making that the Nash Equilibrium.

The university sends out the 2025-2026 housing application to upperclassmen with complete information of standard of living will change contingent on number of upperclassmen who live on campus. Let K equal the improvement or detrition to the standard of living.

Based on this information, the upperclassmen choose whether to live on or off campus. However, depending on the number of upperclassmen staying on campus, the University has actions that it can take. Using the bounds that were derived before, the following cases emerge:

1. If less than or equal ϕ student stay on campus, the University is worse off compared to last year. Thus, to compensate for this, the quality of life is reduced on campus. Thus, the payoffs can be defined as

- $U_i(O): \gamma - K$
- $U_i(F): \Lambda - K$

2. If the number of classmen $\in (\phi, \xi)$ students stay on campus, the school has not hit their target number of upperclassmen, and to incentive more upperclassmen to stay off campus, they increase the quality of life for only those living on campus. Thus, the payoffs can be defined as

- $U_i(O): \gamma + K$
- $U_i(F): \Lambda$

3. If there are exactly ξ upperclassmen stay on campus, the University's target has been hit, and thus, they have enough budget and the ability to raise the quality of life across the board. Thus, the payoffs can be defined as

- $U_i(O): \gamma + K$
- $U_i(F): \Lambda + K$

4. If there are more than ξ upperclassmen, the university crams students into dorms by adding more beds and the quality of living lowers. Thus, the payoffs can be defined as

- $U_i(O): \gamma - K$
- $U_i(F): \Lambda$

4.2 Solving for optimal incentives

The university aims to have ξ number of upperclassmen living on campus. To simplify notation, let x be the number of upperclassmen who choose to live on campus. Thus, the incentive, or K , that the university provides must have only one feasible nash equilibria, such that ξ number of upperclassmen choose to live on campus. Thus, we must derive some relationship between Λ, ϕ, K to enforce this notion. To do so, consider the following. If $x = \xi$, and denoting unilateral deviation as the following:

$$\begin{aligned} U_i(O) = \gamma + K &\rightarrow U_i(F) = \Lambda \\ U_i(F) = \Lambda + K &\rightarrow U_i(O) = \gamma - K \end{aligned}$$

Note that if we want no profitable deviation, we want the following conditions to hold:

$$\begin{aligned} n \leq \gamma + k &\iff k \geq \Lambda - \gamma > 0 \\ \gamma - K \leq \Lambda + K &\iff 2k \geq \gamma - \Lambda \end{aligned}$$

Note that the second constraint is strictly negative, which implies that if $K \geq \Lambda - \gamma$, then the scenario when $\xi = x$ will be a nash equilibrium. To increase the strength of this relation, consider strict inequality. Thus, we claim that $K > \Lambda - \gamma$ is the relationship between K, Λ, γ , and we check for any other Nash Equilibria.

4.2.1 Casework

1. $x = 0$

Since everyone is living off campus, moving to on campus would make them strictly worse off, thus, this makes this case a Nash Equilibria.

2. $0 < x < \phi$

Consider the possible deviations:

$$\begin{aligned} u_i(O) = \gamma - K &\rightarrow u_i(F) = \Lambda - K \\ u_i(F) = \Lambda - K &\rightarrow u_i(O) = \gamma - K \end{aligned}$$

We see that $\Lambda - k > \gamma - k$ or $\gamma - k > \Lambda - K$ must hold to force a profitable deviation. Thus, we see that the first condition is always true, which implies that in this scenario, there is no Nash Equilibrium, regardless of K

3. $n = \xi - 1$

To force a profitable deviation, consider all the possible deviations:

$$\begin{aligned} u_i(O) = \gamma + K &\rightarrow u_i(F) = \Lambda \\ u_i(F) = \Lambda &\rightarrow u_i(O) = \gamma + K \end{aligned}$$

This implies that either $\Lambda > \gamma + k$ or $\gamma + k > \Lambda$ forces a profitable deviation, which the second condition matches our proposed condition.

4. $x = \xi$

This is the solution derived above. Therefore, this is a nash equilibrium.

5. $x = \xi + 1$

Consider the possible deviations:

$$\begin{aligned} u_i(O) = \gamma - K &\rightarrow u_i(F) = \Lambda + K \\ u_i(F) = \Lambda &\rightarrow u_i(O) = \gamma - K \end{aligned}$$

We see that for a profitable deviation to exist either $2k > \gamma - \Lambda$ or $k < \gamma - \Lambda$, which the first inequality is true from our derived condition. Thus, this is not a Nash equilibrium.

6. $x = \phi$

Consider the possible deviations:

$$\begin{aligned} u_i(O) = \gamma - K &\rightarrow u_i(F) = \Lambda - K \\ u_i(F) = \Lambda - K &\rightarrow u_i(O) = \gamma + K \end{aligned}$$

To force a profitable deviation, we need that $\Lambda - K > \gamma - K$ or $2k > \Lambda - \gamma$, which the first condition is always true from how Λ and γ are defined.

7. $x = \phi + 1$

Consider the possible deviations:

$$\begin{aligned} u_i(O) = \gamma + K &\rightarrow u_i(F) = \Lambda - K \\ u_i(F) = \Lambda &\rightarrow u_i(O) = \gamma + K \end{aligned}$$

We see that this is very similar to $2k < \Lambda - \gamma$ and $k > \Lambda - \gamma$, of which the latter term matches our proposed condition.

8. $\phi + 2 < x < \xi - 1$

Consider the possible deviations:

$$\begin{aligned} u_i(O) = \gamma + K &\rightarrow u_i(F) = \Lambda \\ u_i(F) = \Lambda &\rightarrow u_i(O) = \gamma + K \end{aligned}$$

Note that $k > \Lambda - \gamma$ forces a profitable deviation.

9. $x > \xi + 1$

Consider the possible deviations:

$$\begin{aligned} u_i(O) = \gamma - K &\rightarrow u_i(F) = \Lambda \\ u_i(F) = \Lambda &\rightarrow u_i(O) = \gamma - K \end{aligned}$$

Note that our proposed condition causes this to not be a nash equilibrium.

10. $n = N$

Consider the following deviation:

$$U_i(O) = \gamma - K \rightarrow U_i(F) = \Lambda$$

Note that our proposed condition causes this to not be a nash equilibrium.

4.3 Analysis of casework

We can see that there are 2 Nash equilibria, $x = 0$ and $x = \xi$. However, some attention must be brought to the case where $x = 0$, as this is a case that must be avoided. This is because this is the worst possible case for the University, but we can also show that this is the worst possible case for all the upperclassmen as well. Consider the aggregated utility of the case where $x = 0$, we see that

$$\text{total utility in 0 case} = N(\Lambda - K)$$

and similarly when $x = \xi$

$$\text{total utility in } \xi \text{ case} = \xi(\gamma + K) + (N - \xi)(\lambda + K)$$

we see that the total utility in the $x = \xi$ is clearly higher than that in the $x = 0$ case. Thus, it is also in the students' interest to move away from the $x = 0$. Thus, it is in both the students' and the university's interest to encourage movement away from the $x = 0$ case.

5 Discussion

5.1 The implications of the incentive

Our model shows that for on-campus housing to be a Nash equilibrium, the university's incentive for students to stay on-campus, K , must be greater than the gap in utility represented by $\Lambda - \gamma$. $\Lambda - \gamma$ is the gap in utility for students between living off-campus Λ and on-campus γ . Thus, the university must provide enough benefits to encourage staying on campus by having on-campus living conditions confer as much utility to students as moving off-campus. This incentive structure is crucial because:

- The school can reliably predict dorm occupancy rates from year to year, allowing them to have a reliable budget (including the operational costs of housing and amenities).
- On-campus housing is more attractive than off-campus housing, so students feel that on-campus housing is desirable, despite the higher price.
- It ensures that once ξ students choose dorms, the remaining students experience no incentive to deviate from their decision to live on-campus, as going below or above ξ would decrease the utility of the students as a collective.

Our model finds two Nash equilibria in the housing decision game, which makes the on-campus vs. off-campus housing decision a collective coordination game. We know that students are collectively better off if exactly ξ upperclassmen stay on campus. However, without proper coordination, there is a risk of reaching the worst-case scenario for both students and the university, where no upperclassmen choose on-campus housing, as it is also an equilibrium. For the university, this scenario leads to revenue losses (since on-campus housing provides a stable revenue stream) and unused resources for the university. For students, this scenario presents the possibility of an oversaturated off-campus housing market, which could lead to higher rent costs, as well as reduced on-campus academic and social engagement from the lessened presence of upperclassmen in the dorms. To prevent this worst-case scenario, the university could introduce methods of communication that facilitate the coordination of students' choices with one another.

5.2 Extension: Cheap Talk Round Before Housing Game

One potential solution to the problem presented by the collective coordination is a sequential live poll, which provides a costless method of communication before students participate in formal on-campus housing selection. This poll would allow students to see their peers' decision between on-campus and off-campus housing before making their own choices, which reduces uncertainty and enables coordination, allowing students to progress toward the optimal equilibrium. The poll would work in this manner:

- **Random Order Voting:** The poll assigns each student to a random position in a queue to vote anonymously, choosing either “On” (O) for on-campus housing or “Off” (F) for off-campus housing. Students who are indifferent between staying on and off campus could skip the voting and they will be shifted to the end of the queue, allowing people who confer more utility from staying on-campus to indicate that they would like to take on-campus spots first.
- **Live Distribution of Votes** After each vote, the running total of “O” vs. “F” is displayed publicly, allowing students to see the distribution between the two options in real time.

In the Random Order Voting implementation, the poll itself is an example of cheap talk, which is “cheap” in two senses: it has no direct payoff consequences, and there is no enforceable requirement for it to be truthful. Players may attempt to communicate their private information, or their intended actions in the game to come. Thus, to successfully implement this model, we have to impose the following assumptions:

- Students can alter their responses in the actual housing selection process after viewing the answers of their peers in the live poll.
- There is no financial or academic penalty for changing one’s vote in the formal process in response to peers.

It is important to note that since poll action is costless and non-binding, they only serve to change players’ beliefs about what others will do and not to change a player’s individual payoffs.

We thus expect that rational students who prefer the ξ equilibrium (to maximize their utility) will vote so that the distribution between on-campus and off-campus housing approaches this equilibrium, increasing the likelihood of reaching the optimal outcome. As the poll progresses, rational students will react to the information given in the poll to alter their decision in the housing selection. If the number of individuals who choose to vote to live off campus approaches ξ , those who were undecided may “jump on the bandwagon,” knowing that reaching this threshold maximizes their payoff. Conversely, students who strongly prefer off-campus housing may still opt for F , but their decision does not disrupt the equilibrium as long as enough students opt for on-campus housing to reach ξ .

Ideally, if every student is rational, utility-maximizing and believes in each other, once the number of O votes reaches ξ , or the number of votes for living off campus approaches $N - \xi$, the remaining votes become unnecessary in collectively maximizing utility for the students. In the first case, any voter after seeing that the number of on campus votes equals ξ will know that on the real housing form, they should pick F . Similarly, in the second case, any voter after seeing $F = N - \xi$ will know that on the real housing form, they should pick O . If the poll convincingly signals that ξ threshold is feasible and stable, a student has no reason to unilaterally deviate on the actual sign-up.

5.3 Challenges and Limitations

Although the live polling method could provide many benefits in the coordination game, this method also presents a number of limitations:

- **Credibility Concerns:** While the live tally reduces feelings of uncertainty students have about other students' choices, some students might attempt to mislead others by voting one way in the poll but making a different choice in the official housing form. While our current cheap talk stage allows the housing game to conclude with only the optimal $x = \xi$ equilibrium, we had assumed that all students had reason to be rational, honest and believe each other. However, this is because we did not consider different action profiles that are possible if we formally define that students can be split into 2 types: (1) Those who value-communal living and (2) Those who value independence and personal space (or even a 3rd indifferent type).

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If there are such defined types, we would have a separating equilibrium or a pooling equilibrium depending on whether different types send different messages (i.e. Type 1 votes "O" and Type 2 votes "F" — all students are honest/rational) or all types send the same messages (i.e. all students vote "O" regardless of their type — not all students are honest/rational). Our current game implicitly assumes the former. But in the latter case, also known as a babbling equilibrium, no real information is conveyed in the cheap talk stage (i.e. all votes are effectively meaningless "babbling"), and hence students act in the housing game disregarding the poll entirely, leading to $x = 0$ and $x = \xi$ both being possible equilibria. In this paper, we have thus not presented a complete model of the cheap talk stage, but only a simplified version of it with assumptions

5.4 Future Research Directions

Future research could explore communication methods that help solve these limitations. Potential extensions of the discussion include:

- Designing different incentives to strengthen students' desire to commit to the $x = \xi$ equilibrium.
- Conducting empirical studies or experimental trials to discover whether students actually follow the live poll predictions in real-world housing decisions.

- Investigating whether the category of incentive (i.e., financial vs. non-financial perks) can affect student preferences differently.
- Properly modeling the cheap talk game

5.5 Conclusion

Our project aims to design the housing game for the university and derive the lower bound for K that optimizes their occupancy rate. Given that students make decisions on housing based on the difference in utility of partaking in on-campus and off-campus living, we found that students will want to participate in off-campus housing unless the university introduces a minimum incentive K that can offset the gap in utility between off-campus and on-campus housing.

In order to offset the higher cost of living on campus, the inequality shows that the university needs to offer a level of benefit K that at least equals this disadvantage relative to that of off-campus alternatives. So, we found that the condition $K > (\Lambda - \gamma)$ is the necessary lower bound for incentives. After meeting this specific threshold, we can achieve a Nash equilibrium where ξ students remain on campus.

If K is not sufficiently large, the game will only have one equilibrium at $x = 0$. This is the worst-case scenario where no upperclassmen stay in dorms, leading to losses in revenue for the university, as well as lower total student utility. The housing game thus requires the correct incentive structure to have students' optimal choices lead to the fulfillment of the university's optimal occupancy goals.

By understanding the mechanics of this game and the constraints on K , the university can develop targeted policies to balance student preferences, financial feasibility, and dormitory utilization, ensuring that the equilibrium remains at ξ rather than collapsing to 0.