Honors Economics PSET 2

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Problem 5

1

Given that

$$\begin{array}{cc} \max_{x,y} & x^{\alpha}y^{1-\alpha} \\ s.t & p_x x + p_y y = m \end{array}$$

We can note derived from the Lagrangian

$$L = U(x,y) - \lambda(m - p_x x - p_y y)$$

and the following first order conditions:

$$\begin{array}{ll} [x] & \alpha x^{\alpha-1}y^{1-\alpha} = \lambda p_x \\ [y] & x^{\alpha}\left(1-\alpha\right)y^{-\alpha} = \lambda p_y \\ [\lambda] & p_x x + p_y y = m \end{array}$$

Note that we equate the lambdas to each other, where we can see that

$$\lambda = \alpha x^{\alpha-1}y^{1-\alpha}\left(\tfrac{1}{p_x}\right) \quad \lambda = x^{\alpha}\left(1-\alpha\right)y^{-\alpha}\left(\tfrac{1}{p_y}\right)$$

Equating these lambdas yields:

$$\begin{aligned} \alpha x^{\alpha-1}y^{1-\alpha}\left(\frac{1}{p_x}\right) &= x^{\alpha}\left(1-\alpha\right)y^{-\alpha}\left(\frac{1}{p_y}\right) \\ \alpha x^{-1}y\frac{1}{p_x} &= \left(1-\alpha\right)\frac{1}{p_y} \\ \alpha\left(\frac{y}{x}\right)\frac{p_y}{p_x} &= \left(1-\alpha\right) \\ \left(\frac{y}{x}\right)\frac{p_y}{p_x} &= \frac{1-\alpha}{\alpha} \\ \frac{y}{x} &= \left(\frac{p_x}{p_y}\right)\left(\frac{1-\alpha}{\alpha}\right) \end{aligned}$$

Now that we have a ratio between y and x, we solve for y and x and substitute this into the budget constraint:

$$\begin{aligned} p_x x + p_y y &= m \\ p_x x + p_y x\left(\frac{p_x}{p_y}\right)\left(\frac{1-\alpha}{\alpha}\right) &= m \\ p_x x + p_x x\left(\frac{1-\alpha}{\alpha}\right) &= m \\ p_x x\left(1 + \frac{1-\alpha}{\alpha}\right) &= m \\ p_x x\left(\frac{1}{\alpha}\right) &= m \\ x = x^* &= \frac{\alpha m}{p_x} \end{aligned}$$

Since we know what x is, we can calculate y using the derived ratio above:

$$y = x\left(\frac{p_x}{p_y}\right)\left(\frac{1-\alpha}{\alpha}\right)$$
$$y = \left(\frac{\alpha m}{p_x}\right)\left(\frac{p_x}{p_y}\right)\left(\frac{1-\alpha}{\alpha}\right)$$
$$y = y^* = \frac{(1-\alpha)\,m}{p_y}$$

Plugging back into the utility function, we see that

$$v\left(p_x, p_y, m\right) = \left(\frac{\alpha m}{p_x}\right)^{\alpha}\left(\frac{(1-\alpha)\,m}{p_y}\right)^{1-\alpha}$$

2

Note that we can rewrite the indirect utility function as:

$$v = \left(\frac{\alpha^{\alpha}m^{\alpha}}{p_x^{\alpha}}\right)\left(\frac{(1-\alpha)^{1-\alpha}m^{1-\alpha}}{p_y^{1-\alpha}}\right)$$
$$v = \frac{\alpha^{\alpha}m(1-\alpha)^{1-\alpha}}{p_x^{\alpha}p_y^{1-\alpha}}$$
$$\frac{\partial v}{\partial m} = \frac{(\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{p_x^{\alpha}p_y^{1-\alpha}}$$

Since we know that $\alpha \in (0,1)$ and prices are always positive, this implies the above quantity is always positive

3

We would interpret that as when the prices are held constant, change in income will always lead to a change in the maximum possible utility at the optimum bundle.

4

From part 1, we can see that:

$$\lambda = \alpha x^{\alpha-1}y^{1-\alpha}\left(\tfrac{1}{p_x}\right) \quad \lambda = x^{\alpha}\left(1-\alpha\right)y^{-\alpha}\left(\tfrac{1}{p_y}\right)$$

All we need to do is to insert the optimal values at their respective places. We will use the equation containing p_y:

$$\lambda^* = \frac{(1-\alpha)\left(\frac{(1-\alpha)m}{p_y}\right)^{-\alpha}\left(\frac{\alpha m}{p_x}\right)^{\alpha}}{p_y}$$
$$\lambda^* = \frac{(1-\alpha)(1-\alpha)^{-\alpha}m^{-\alpha}\alpha^{\alpha}m^{\alpha}}{p_y^{1-\alpha}p_x^{\alpha}}$$
$$\lambda^* = \frac{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}{p_y^{1-\alpha}p_x^{\alpha}}$$

Note that prices are always positive and $\alpha \in (0,1)$ means that both α and $1-\alpha$ are always positive.

5

Note that we can rewrite the above quantity in the following form:

$$v = (\alpha m)^{\alpha}px^{-}\alpha(1-\alpha)^{1-\alpha}m^{1-\alpha}p_y^{a-1}$$

Thus we can compute the following

$$\frac{\partial v}{\partial p_x} = -\alpha p_x^{-\alpha-1}(\alpha m)^{\alpha}(1-\alpha)^{1-\alpha}m^{1-\alpha}$$

Since $\alpha \in (0,1)$, we can see because of the negative sign, the above quantity will always be negative. Now to compute the other partial:

$$\frac{\partial v}{\partial p_y} = (\alpha - 1)(p_y)^{\alpha-2}(p_x)^{-\alpha}(1-\alpha)^{1-\alpha}m^{1-\alpha}$$

since $(\alpha - 1)$ will always be negative as $\alpha \in (0,1)$ and every other quantity in this equation will always be positive, we can see that both quantities are negative.

6

This is Roy's identity for x^*:

$$x^*_m(p_x, p_y, m) = -\frac{\frac{\partial v(p_x,p_y,m)}{\partial p_x}}{\frac{\partial v(p_x,p_y,m)}{\partial m}}$$

We can see that

$$\begin{aligned} x^*_m(p_x, p_y, m) &= -\frac{-\alpha p_x^{-\alpha-1}(\alpha m)^{\alpha}(1-\alpha)^{1-\alpha}m^{1-\alpha}}{\frac{(\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{p_x^{\alpha}p_y^{1-\alpha}}} \\ &= \frac{\alpha m}{p_x} \end{aligned}$$

And similarly:

$$\begin{aligned} y^*_m(p_x, p_y, m) &= -\frac{(\alpha-1)(p_y)^{\alpha-2}(p_x)^{-\alpha}(1-\alpha)^{1-\alpha}m^{1-\alpha}}{\frac{(\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{p_x^{\alpha}p_y^{1-\alpha}}} \\ &= \frac{(1-\alpha)\, m}{p_y} \end{aligned}$$

Problem 6

1

We can write the cost minimization as:

$$\begin{array}{cc} \min_{x,y} & \frac{1}{2}Wi + F(\frac{Y}{W}) \\ s.t & U(x,y) = U \end{array}$$

Let

$$c(w) = \frac{1}{2}Wi + F(\frac{Y}{W})$$

be the cost function. Note:

$$\begin{aligned} \frac{dc}{dW} &= \frac{1}{2}i - F(\frac{Y}{W^2}) \\ 0 &= \frac{1}{2}i - F(\frac{Y}{W^2}) \\ \frac{FY}{W^2} &= \frac{1}{2} \\ W^2 &= \frac{2FY}{i} \\ W &= \sqrt{\frac{2FY}{i}} \end{aligned}$$

Note that

$$\frac{1}{2}Wi = \frac{1}{2}(\sqrt{\frac{2FY}{i}})i = \sqrt{\frac{iFY}{2}}$$

and

$$\frac{FY}{W} = FY\sqrt{\frac{i}{2FY}} = \sqrt{\frac{iFY}{2}}$$

We can then substitute the values into the equation:

$$c(w) = \sqrt{\frac{iFY}{2}} + \sqrt{\frac{iFY}{2}} = \sqrt{2FYi}$$

The value of W we found is the optimal value of money to withdraw to minimize cost as we set the derivative equal to 0.

2

We computed the direct costs because the number of times that an individual goes to a bank is determined by the his balance divided by his constant withdrawal amount. which can be said as $\frac{Y}{W}$. Then, the direct cost would be calculated by multiplying the withdrawal amount to get $F\frac{Y}{W}$The indirect cost was calculated because the amount in the individual's account $\frac{W}{2}$ gaining i interest, thus multiplying the two to get $\frac{Wi}{2}$ to get the indirect cost. The tradeoff here is contained within the W term. When we increase W, the number of times we go to the banks decrease, thus decreasing costs from withdrawing money. However, in doing so, the individual leaves more of his own money on hand without depositing to the bank, which increases indirect cost.

3

As given in the problem,W^* is the money withdrawn from the bank. The Money Demand function is $M^* = \frac{1}{2}W^*$. We found the optimal value $W* = \sqrt{\frac{2FY}{i}}$ and thus, we can see that

$$W^* = \sqrt{\frac{2FY}{i}} \quad M^* = \sqrt{\frac{FY}{2i}}$$

4

We can compute the elasticity of money demand with respect to the (nominal) interest rate as follows. To do so note

$$\frac{\partial M}{\partial i} = -\frac{1}{2}M^*i^{-1}$$

And we compute elasticity, which is really the percent change of one thing over the other, as follows:

$$\begin{aligned}\epsilon &= \frac{\partial M}{\partial i} \times \frac{i}{M^*} \\ &= -\frac{1}{2}M^*i^-1 \times \frac{i}{M} \\ &= -\frac{1}{2}\end{aligned}$$

The negative sign means that there is a tradeoff between the money demand and the nominal interest rate. Money demand is inversely proportional to the nominal interest rate. This means that if there is a two percent increase in nominal interest rate, there is a one percent decrease in money demand. This intuitively makes sense, as we can see that increasing the nominal interest rate, there would be less of a desire to withdraw money as the money in the bank would gain more interest. The indirect cost of withdrawing increases in terms of interest forgone, thus minimizing withdrawals and maximizing the money in the bank.

5

Note that

$$\frac{\partial M}{\partial Y} = \frac{1}{2\sqrt{Y}}\left(\sqrt{\frac{F}{2i}}\right)$$

And using the formula for elasticity, which is as follows:

$$\frac{\partial M}{\partial Y} = \frac{\partial M}{\partial i} \times \frac{Y}{m^*}$$

we can compute elasticity as:

$$\begin{aligned}\epsilon &= \frac{\partial M}{\partial i} \times \frac{Y}{M} \\ &= \left(\frac{1}{2\sqrt{Y}}\right)\left(\sqrt{\frac{F}{2i}}\right)\left(\frac{Y}{M}\right) \\ &= \frac{1}{2}\left(\sqrt{\frac{FY}{2i}}\right)\left(\frac{1}{M}\right) \\ &= \frac{1}{2}\end{aligned}$$

The positive sign means that there is no tradeoff between money demand and a person's income. In fact, they are directly proportional. According to elasticity, we can see that a 2 percent increase in income will yield a 1 percent increase in money demand. This intuitively makes sense. If we have more of an income, we would want more money to spend on expenditures, and also can afford higher direct and indirect costs, thus increasing money demand.

6

Note that:

$$\frac{\partial M}{\partial F} = \frac{1}{2\sqrt{F}}\left(\sqrt{\frac{Y}{2i}}\right)$$

And using the formula for elasticity:

$$\begin{aligned}\epsilon &= \frac{\partial M}{\partial F} \times \frac{F}{M} \\ &= \left(\frac{1}{2\sqrt{F}}\right)\left(\sqrt{\frac{Y}{2i}}\right)\left(\frac{F}{M}\right) \\ &= \frac{1}{2}\left(\sqrt{\frac{FY}{2i}}\right)\left(\frac{1}{M}\right) \\ &= \frac{1}{2}\end{aligned}$$

The positive sign means that there is no tradeoff between the money demand and the brokerage fee; they are directly proportional. According to the definition of elasticity, we can see that a 2 percent increase in brokerage fee will yield a 1 percent increase in the money demand. This intuitively makes sense as if the brokerage fee increases, the individual is incentivized to withdraw more money at once as fewer trips to the ATM reduce cost. This is because the marginal cost of increasing the withdrawal amount decreases more than proportionately to the marginal increase in interest forgone. This increase in withdrawing more money is indicative of how the money demand increases. Mathematically, the money demand is proportional to the amount withdrawn, which means that increasing amount withdrawn should mean that money demand increases.

7

Let us define money illusion as the phenomenon that an individual approaches their money at face value (or nominal value), rather than adjust it for inflation and other economic factors (real value).

In this example, this individual is actually not going to fall for the money illusion. We know this because when we take a look at the individual's demand function, we see that it is written in the form:

$$M^* = \frac{1}{2}W^*$$

but when we derived W^*, we can see that W^* is dependent on the factors F, Y, i. Thus, this means that M^* is also dependent on the factors F, Y, i. And since the individual adjusts their money demand, how much money that they want, and the amount withdrawn, based on the factors such as brokerage fee, Income, and Nominal Interest rate, this individual sees their money based on the economic factors given the problem, and not just the amount that is in front of him.

$$W^* = \sqrt{\frac{FY}{2i}}$$

We can see that if we were to multiply the factors F and Y by some scalar T that denotes an increase in interest rate or any other conversion rate, note the following:

$$\begin{aligned}W^*(i, tF, tY) &= \sqrt{\frac{t^2 FY}{2i}} \\ &= t\sqrt{\frac{FY}{2i}} \\ &= tW^*(i, F, Y)\end{aligned}$$

We also see that this function is homogeneous in degree 1, which also indicates that the individual takes in changes in interest/exchange rate into consideration, as he adjusts his money demand proportionately to maintain the same real purchasing power.

8

For this question, we must note that Y is a very large value, and we define harm as the magnitude of the individual's cost as defined above. According to the envelope theorem, we can note that

$$\frac{dc^*}{dF} = \frac{\partial c}{\partial i}\Bigg|_*$$

and

$$\frac{dc^*}{di} = \frac{\partial c}{\partial i}\Bigg|_*$$

we can see that the following holds:

$$\frac{dc^*}{dF} = \frac{Y}{W^*}$$

and

$$\frac{dc^*}{di} = \frac{1}{2}W^*$$

When we evaluate these values at the optimal value, W^*, we can see that

$$\begin{aligned}\frac{dc^*}{dF} &= \frac{Y}{\sqrt{\frac{2FY}{i}}}\\ &= \sqrt{\frac{iY}{2F}}\end{aligned}$$

and

$$\begin{aligned}\frac{dc^*}{di} &= \frac{1}{2}\sqrt{\frac{2FY}{i}}\\ &= \sqrt{\frac{FY}{2i}}\end{aligned}$$

Note when we tax the brokerage fee, this means that F increases, and taxing the interest rate means that i decreases. Thus, when F increases, the rate in which costs increase for individuals actually decreases. But when i decreases, this increases the magnitude of $\frac{dc}{di}$, which means that any individual is hurt more when the interest rate is taxed. Note that income does not matter in both cases, as the Y is not being affected in any matter.

Problem 7

1

We cannot know that. If the consumer spends all his money on goods x and y, that could be any point on the budget constraint. Since the budget constraint has an infinite amount of points, as we assume goods are divisible, it is impossible to know which one he chooses.

2

The market demand would be determined by the aggregate behavior of every consumer. And if we assume that we have a large enough N, we can see that there will be a pattern that emerges from the middle of the budget constraint, sort of like a probability distribution. This is because since each impulsive consumer allocates their income randomly between the 2 goods, the proportion of income spent on good x is uniformly distributed between 0 and 1, and thus the expected proportion will be $\frac{1}{2}$ (likewise for good y). So, we denote the budget constraint as

$$p_x x + p_y y = m$$

And note that the x and y intercepts can be written as:

$$\text{x intercept } = (\frac{m}{p_x}, 0) \qquad \text{y intercept } = (0, \frac{m}{p_y})$$

And taking the midpoint of these two lines, we can see that it is:

$$(\frac{m}{2p_x}, \frac{m}{2p_y})$$

Therefore, we can see that the market demand here is going to be each component is

$$(\frac{Nm}{2p_x}, \frac{Nm}{2p_y})$$

as we need to multiply each x and y by a value of N. Thus, we can see that indeed the Law of Demand does hold. Since we know that the law of demand states that a consumer is less willing to purchase a quantity of goods given a higher price, we can see that increasing either p_x or p_y will increase the denominator which will then decrease the magnitude. This means that as price increases, less of that good is acquired.

3

No, we still cannot determine whether a consumer's bundle. Now, we are dealing with the area bounded by the budget constraint and the axes, compared to a line. We can see that in this area, there are infinitely many points for us to choose, and it is impossible for us to know for certain where this individual is without knowing some sort of utility function. But in this case, since there is nothing of that nature, we do not know for sure.

4

With a similar logic to question 2, we are interested in finding the middle point of this area, as with a large enough N, we can see that in the long run, most consumers will gravitate towards a middle point. In this case, we would be concerned about finding the centroid of the triangle created by the area of the budget constraint.

We denote the formula of the centroid in a coordinate system as follows:

$$(O(x), O(y)) = (\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3})$$

where $x_1, x_2, x_3, y_1, y_2, y_3$ denote the coordinates of the corner of the triangle respectively. Solving this equation yields:

$$\begin{aligned} O(x) &= \frac{0 + 0 + \frac{m}{p_x}}{3} \\ &= \frac{m}{3p_x} \end{aligned}$$

and

$$\begin{aligned} O(y) &= \frac{0 + \frac{m}{p_y} + 0}{3} \\ &= \frac{m}{3p_y} \end{aligned}$$

Therefore the centroid times the number of consumers, and in this case the market demand, is

$$(O(x), O(y)) = (N\frac{m}{3p_x}, N\frac{m}{3p_y})$$

Note that when we increase price of a good, the quantity of that respective good the market desires will decrease. Thus, this behavior follows the law of demand.

5

Within the scope of this problem, we can note that order referencing and purposive behavior does not really matter when it comes to determining the market demand. (1) We can see that without order preferencing, we still can determine market demand. (2) Thus, we can choose the approach we want depending on the scenario of the question. (3) If it is easier to do the problem in accordance without a utility function, we can do so and vice versa. (4) We should note that for an individual, the introduction of assumptions such as order preferencing is important because that we need to introduce a framework for to help narrow down the choices of that individual such that we can find the bundle that they would consume. (5)

6

One we can determine this is to assume that a consumer is influenced by their neighboring. We determine this as

$$x_i = x_j$$

where i and j represent different consumers. We can note that if a consumer is affected by their neighbor, we can eventually reach the market demand as eventually there will be a common consensus reached from the interactions from all consumers. This holds because if the consumer is not following their own unique preference system, but rather their neighbor, which sastifies the question. Thus, we can still calculate market demand, which would be similar to the equations we derived in part 2 and 4, depending on our assumptions on how a consumer spend their money. The bundle that this individual chooses would be reliant on the bundle that the majority of people chooses, which would be determined by their budget divided by the price for each respective good. Note that from here, we can note that law of demand still holds, as increasing the price will decrease the quantity of the respective good in the market bundle.

Problem 8

We know that the perfect complement utility function is:

$$U(x, y) = \min\left\{\frac{x}{a}, \frac{y}{b}\right\}$$

and we want to find the minimized expenditure function, which is

$$\begin{array}{cc} \min_{x,y} & p_x x + p_y y \\ s.t & \min\left\{\frac{x}{a}, \frac{y}{b}\right\} = \bar{U} \end{array}$$

Note that graphically speaking, these graphs are right angles. And for the budget constraint and these graphs to be tangent to each other, the budget constraint and the indifference must be intersecting at the right angle point of the indifference curve. And mathematically speaking, this tangent intersection is the minimum, and is the point that we should only be worried in. We can see this graphically:

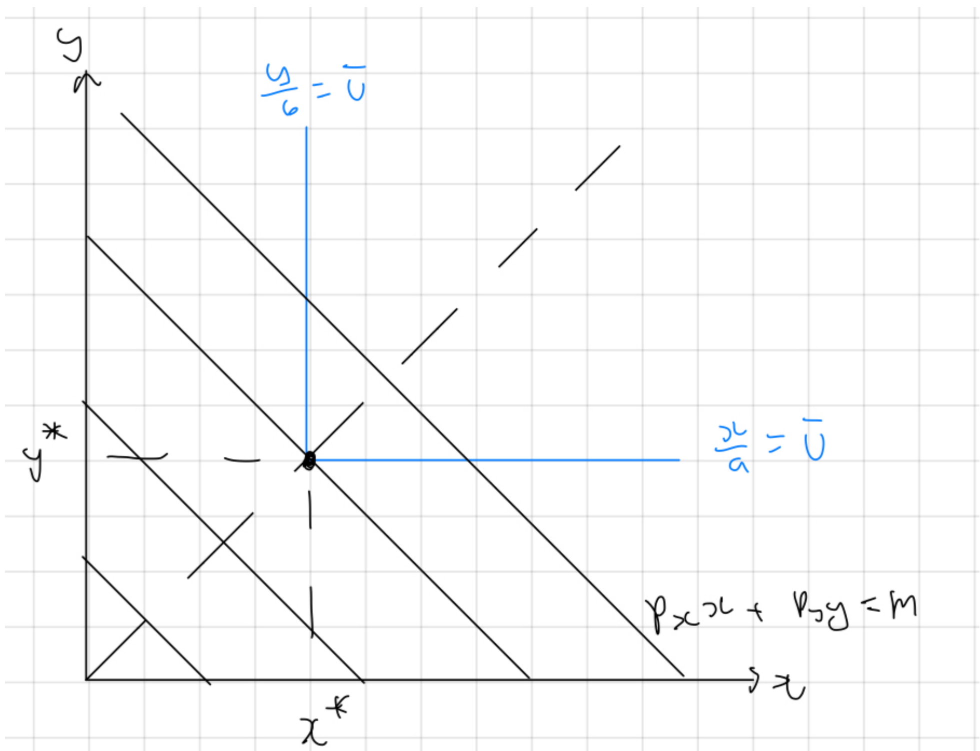

Figure 1: Graph for Problem 8

The formulation of this utility function implies that for the right angle point to be the minimum, the following must hold:

$$\frac{x}{a} = \frac{y}{b} = \bar{U}$$

where $\bar{U}$ is the desired utility level. Thus, when rearrange these terms, we get

$$x^* = a\bar{U} \quad y^* = b\bar{U}$$

and when put into the expenditure function, we get

$$e = \bar{U}(ap_x + bp_y)$$

This does make sense. The reason why this makes sense is that perfect complement goods want to buy those 2 goods together.Thus, we can note that within the perfect complement function, the smallest amount of goods that we can get is the fixed ratio of goods denoted by $\frac{ax}{by}$ that still hits the level $\bar{U}$ required. Then, we have to account for the prices associated with each good. This is represented by the equation inside the parenthesis, and that also indicates that in this case, the total expenditure is directly proportional to the prices of the goods, as these goods must be purchased together.

Problem 9

We know that the perfect substitute utility function is written as follows:

$$U(x, y) = ax + yb$$

So if we were to calculate the expenditure function, we are interested in minimizing the expenditure equation given some utility. It could be written as follows, with the assumption prices are held constant:

$$\begin{array}{cc} \min_{x,y} & p_x x + p_y y \\ s.t & ax + yb = \bar{U} \end{array}$$

Note that in this problem, we cannot apply the lagrangian method to this equation. However, we can note something unique about this situation. Since the utility function is a linear line, it has x and y intercepts on the graph. and to minimize expenditure while maintaining tangency of the budget constraint and the utility function, the tangency point of the 2 functions must lie on either the x or y intercept. But to determine which intercept it is, we must consider the slopes of the budget constraint and the utility function.

When we calculate the slope of each graph, we get $-\frac{a}{b}$ and $-\frac{p_x}{p_y}$. Note that if $\frac{b}{a} = \frac{p_x}{p_y}$, this implies that the lines are exactly the same, as if they weren't the two lines would be parallel. This means that any bundle on the lines are valid. If $\frac{a}{b} > \frac{p_x}{p_y}$, this means the consumer will only buy goods y, which is y^*. Similarly, if $\frac{a}{b} < \frac{p_x}{p_y}$, the consumer will only buy goods x, which would be x^*. This is demonstrated graphically as shown

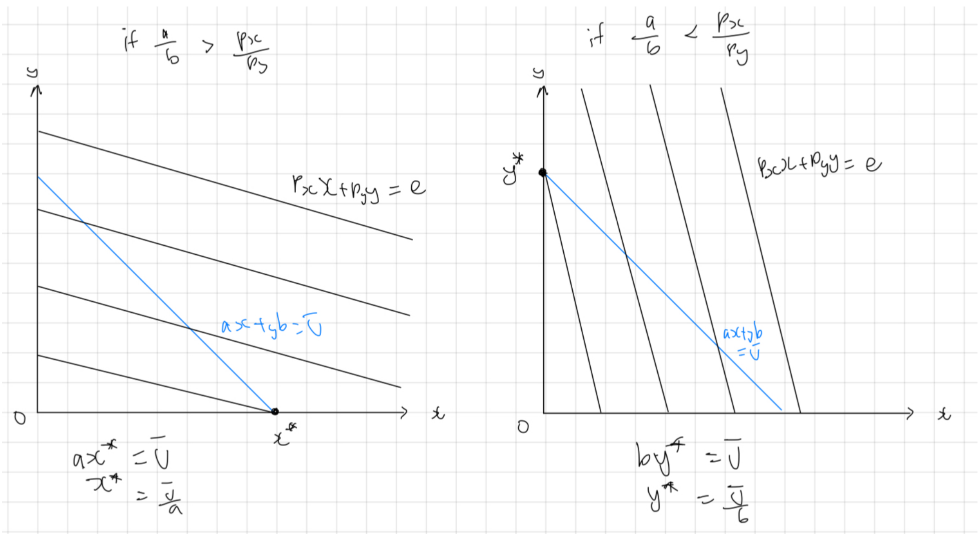

Figure 2: Figure for Problem 9

We can then calculate from here. Note that if we buy all goods x, we can see:

$$ax^* = \bar{U} \iff x^* = \frac{\bar{U}}{a}$$

and for the other case

$$by^* = \bar{U} \iff y^* = \frac{\bar{U}}{b}$$

But, since we are trying to minimize the expenditure here, all we have to do is choose the value with the lower total cost. Hence the expenditure function is

$$e(p_x, p_y, \bar{U}) = \min\left\{\frac{\bar{U}p_x}{a}, \frac{\bar{U}p_y}{b}\right\}$$

These results do make sense, because we can see that if the goods are equally exchangeable, the consumer is not only trying to minimize their expenditure, but they are also trying to maximize their utility. Hence, individual's are trying to find the highest utility possible (or in this case, hit the utility level) while keeping things as cheap as possible.