Problem 2. Dynamics in a Non-linear system.

(1)

Solution: For the graph to have a stable solution, the graph must asymptoically approach a value, to encourage convergance. □

(2)

Solution: We can consider 3 cases: $|a| < 1, |a| = 1, |a| > 1$. Note that if $|a| = 1$, we have the linear difference equation $x_{t_1} = x_t + b$, which implies that there is not stable solution, and thus approachs infinity. If $|a| < 1$ or $|a| > 1$, this means that the exponetial term grows slower and faster than the linear term respectively. However, as noted in **1**, we can see that if the slope of graph $|ax_t^{a-1}| < 1$, we will asymptocally approach a solution. Else, we may diverge and not approach a steady state solution. □

Suppose conditions under local stability, what conditions connverge above or below . show 100

(3)

Solution:

$$\begin{aligned}\overline{x} &= \overline{x}_t^{0.5} + 2\\ \overline{x} - \overline{x}_t^{0.5} &= 2\\ (\overline{x}^{0.5} - 2)(\overline{x}^{0.5} + 1) &= 0\end{aligned}$$

The only possible solution to the above is $\overline{x} = 4$ □

(4)

Do the graphically

Solution:

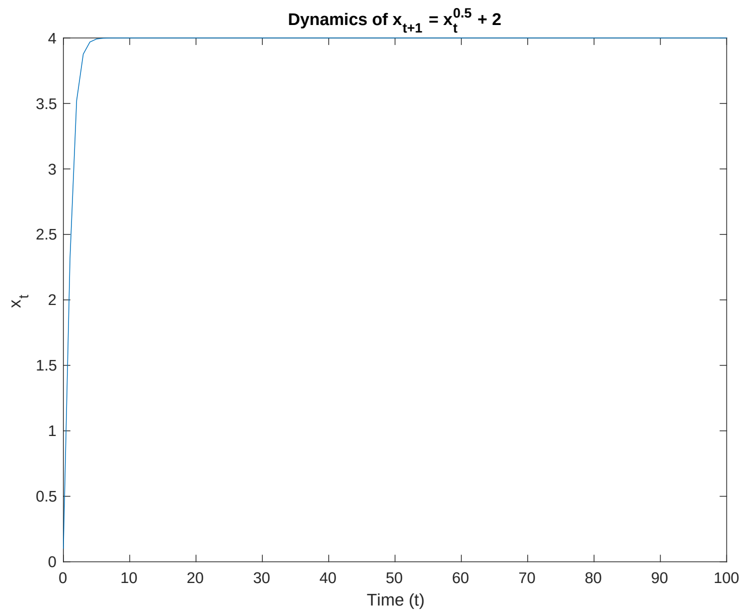

Figure 1: Intial value of 0.1

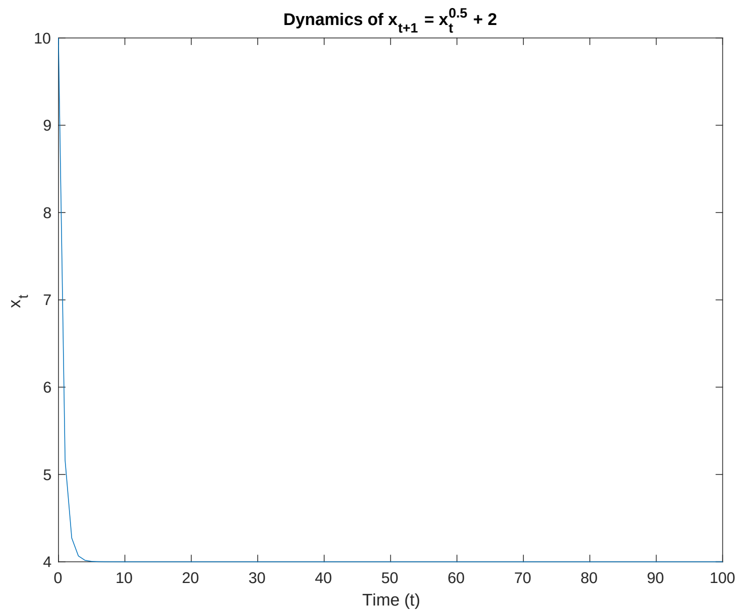

Figure 2: Intial Value of 10

$\square$

(5)

Solution:

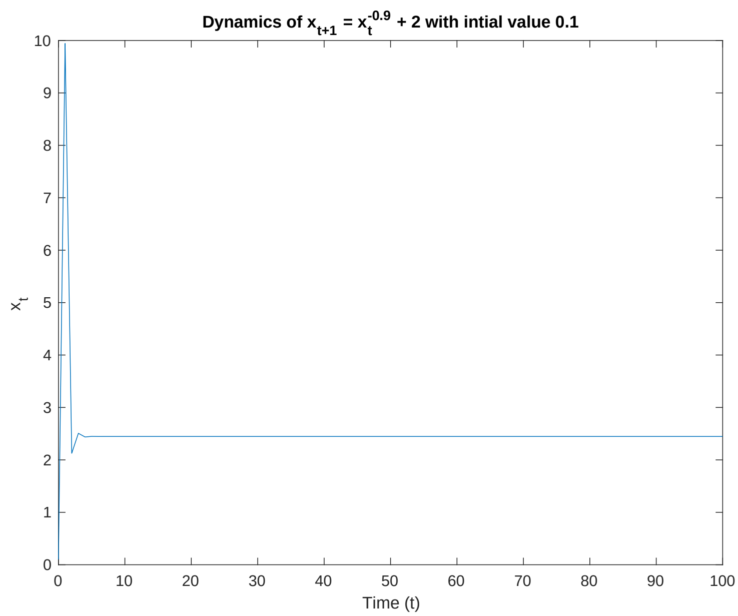

Figure 3: Intial value of 0.1

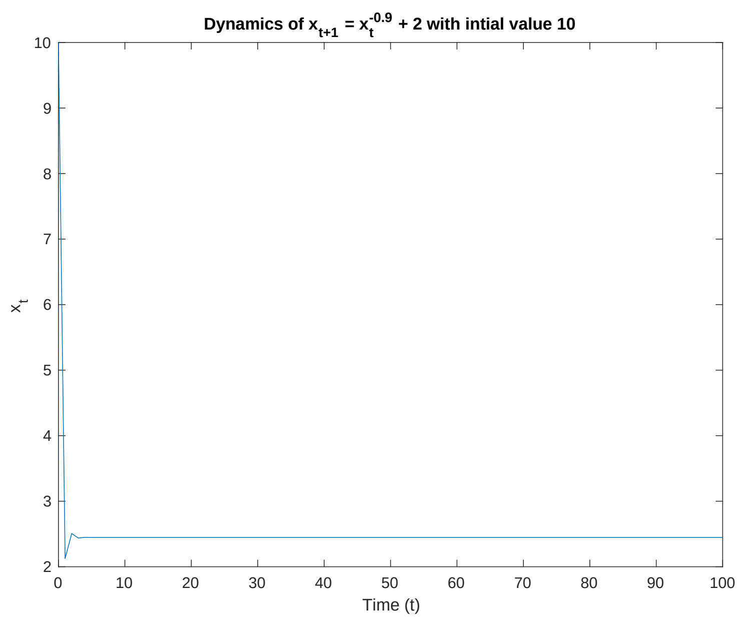

Figure 4: Intial Value of 10

The steady state changes to approximately 2.45. Both trajectories seem to either steeply rise and fall or fall and rise for intial values of 0.1 and 10 resepctively. This is constrast to the other set of parameters as in the original case, there are no kinks in the graph. □

Check, different trajectories. Monotone convergence, osciallation

(6)

Solution: We are given $x_{t+1} = x_t^a + b$ where $f(x) = x^a + b$. Let $\overline{x}$ such that $f(\overline{x}) = \overline{x}$ and $\overline{x} \in (x_t - \epsilon, x_t + \epsilon), \forall \epsilon > 0$. Consider the first degree Taylor expansion around $\overline{x}$. We can see that:

$$x_{t+1} \approx f(\overline{x}) + f'(x)(x_t - \overline{x})$$
$$x_{t+1} - \overline{x} \approx f'(x)(x_t - \overline{x})$$

Let $d_t = x_t - \overline{x}$ and similarly for d_{t+1}. Thus,

$$x_{t+1} - \overline{x} \approx f'(x)(x_t - \overline{x}) \implies d_{t+1} = f'(x)d_t$$

which implies that the general solution:

$$d_t = (f'(\overline{x}))^t d_0$$

Using this equation, we see that:

$$\begin{aligned} d_t &= (f'(\overline{x}))^t d_0 \\ (x_t - \overline{x}) &= (f'(\overline{x}))^t (x_0 - \overline{x}) \\ x_t &= (f'(\overline{x}))^t x_0 + \overline{x}(1 - f'(x)^t) \end{aligned}$$

Note that $f'(x) = -0.5x_t^{-0.5}$ and for any $x \in [0, \infty]$ that $|f'(x)| < 1$. Thus, for any t sufficently large enough, we see that $x_t \to \overline{x}$, which indicates that this equation approximates it very well. □

Problem 4. Real GDP as a measure of welfare?

(1)

Solution: Setting up the optimization problem:

$$\begin{aligned} \max \quad & U \\ \text{s.t} \quad & \sum_{i=1}^{n} p_i x_i = M \end{aligned}$$

with the following FOCs

$$[x_i] \quad \frac{\partial U}{\partial x_i} = \lambda p_i$$

and λ is the marginal utility of income. □

(2)

Solution: Taking the total differenial of U, we can see that:

$$dU(x_1, x_2, \ldots, x_n) = \sum_{i}^{n} \frac{\partial U}{\partial x_i} dx_i$$

□

(3)

Solution: Since $\frac{\partial U}{\partial x_i} = \lambda p_i$ at the optimum, we see that:

$$dU = \sum_{i}^{n} \frac{\partial U}{\partial x_i} dx_i = \lambda \sum_{i}^{n} p_i dx_i$$

The statements holds to be true. □

Problem 5. Intertemporal Consumption Choice

(1)

Solution:

$$A_0 K_0^\alpha = c_0 + K_1$$
$$A_1 K_1^\alpha = c_1$$

as in time period 1, the individual consumes all of y □

(2)

$$y_0 = c_0 + K_1 \iff y_0 - c_0 = K_1$$

thus,

$$A_1(y_0 - c_0)^\alpha = c_1$$

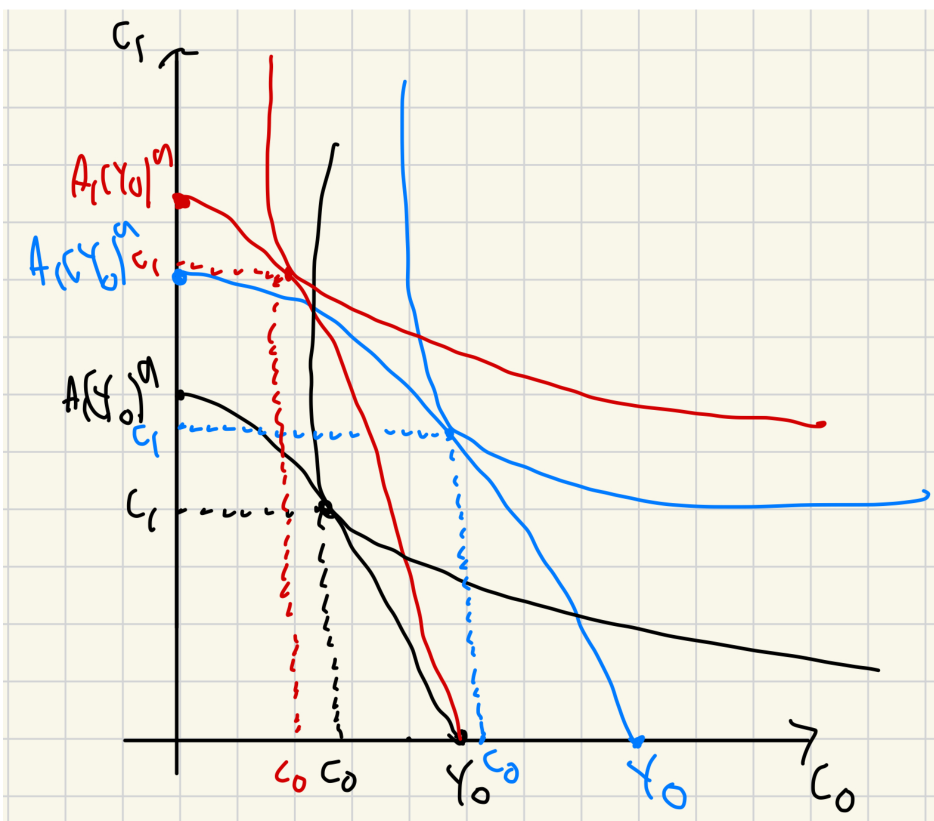

Figure 5: Graph for 3,4,5,6

(3)

Solution: See graph above in black ink. The slope represents the rate of change between the consumption in the current time period and the next time period. □

(4)

Solution: See graph above in black ink □

(5)

Solution: See graph above, blue line. If A_0 increases, y_0 increases as technology in the current period would increase. This implies that budget constraint shifts outward to the right. Since the consumer now has more "budget" of corn, he now was more income to consume more. Thus,

- y_0 increases
- c_0 increases
- c_1 increases

□

(6)

Solution: See graph above, red line. If A_1 increases, this means that y_1 strictly increases. This implies that the maximum possible value of c_1 will increases, and make the graph steeper. Thus,

- c_1 increases
- y_1 increases
- c_0 decreases, as consumer will substitute away from the good.

□

(7) Algebra manipulation.

Solution: We are interested in the following optimization problem:

$$\begin{aligned} \max \quad & \ln(c_0) + \beta \ln(c_1) \\ \text{s.t} \quad & A_1(y_0 - c_0)^\alpha = c_1 \end{aligned}$$

Subsituting the constraint into the objective function yields:

$$\ln(c_0) + \beta \ln(A_1(y_0 - c_0)^\alpha)$$

taking the deriative with respect to c_0 yields:

$$\frac{1}{c_0} - \frac{\alpha\beta}{c_0 - y_0} = 0$$

which, after some algebra, yields:

$$c_0^* = \frac{y_0}{1 + \beta\alpha}$$

Thus, this implies that

$$c_1^* = A_1 \left(\frac{\alpha \beta y_0}{1 + \beta \alpha} \right)^{\alpha}$$

Thus, we can see that since A_1 only appears in c_1, this implies that c_1 and y_1 increases. Additionally, we can see that increasing A_0 would increase y_0 which in turn increase all values. □

Problem 6. Exact Price Index from the Economic Approach

(1)

Solution:

$$\begin{aligned} \max \quad & \ln x + \ln y \\ \text{s.t} \quad & p_x x + p_y y = M \end{aligned}$$

we have the following FOCs:

$$\begin{aligned} [x] \quad & \frac{1}{x} = \lambda p_x \\ [y] \quad & \frac{1}{y} = \lambda p_y \\ [\lambda] \quad & M = p_x x + p_y y \end{aligned}$$

Note that the FOCs imply that $p_x x = p_y y$ and thus, using the budget constraint, we find that:

$$p_x x + p_y y = M \iff 2p_x x = M \iff p_x x = \frac{M}{2}$$

and by symmetry

$$p_y y = \frac{M}{2}$$

which indicates that expenditure share is one half. □

(2)

Solution: The indirect utilty function is

$$V(M, P) = \ln \left(\frac{M}{2p_x} \right) + \ln \left(\frac{M}{2p_y} \right)$$

□

(3)

Solution: We aim to use duality to prove this. Let

$$U = 2\ln(M) - \ln(4) - \ln(p_x p_y)$$

□

(4)

Solution: Using duality, we see that:

$$\begin{aligned}
U &= 2\ln(M) - \ln(4) - \ln(p_x p_y) \\
\ln(M) &= 0.5\ln(4) + \ln(\sqrt{p_x p_y}) \\
M &= 2e^{\frac{U}{2}}\sqrt{p_x p_y}
\end{aligned}$$

Using Shephard's Lemma, we see that:

$$x^h = \frac{\partial e(p,U)}{\partial p_x} = \sqrt{\frac{e^U p_y}{p_x}}$$

and by symmetry

$$y^h = \frac{\partial e(p,U)}{\partial p_y} = \sqrt{\frac{e^U p_x}{p_y}}$$

□

(5)

Solution:

$$M = \frac{2e^{\frac{U_t}{2}}\sqrt{p_x^t p_y^t}}{2e^{\frac{U_0}{2}}\sqrt{p_x^0 p_y^0}} = e^{\frac{U_t - U_0}{2}}\sqrt{\frac{p_x^t p_y^t}{p_x^0 p_y^0}}$$

Since the level of utility remains fixed, we can see that we are left with the following expression.

$$\sqrt{\frac{p_x^t p_y^t}{p_x^0 p_y^0}}$$

□

(6)

Solution: This is the fisher price index. Geometric mean of passches index and . Compare geometric with arithmetic mean, compare AM - GM inqualitu Hold basket fixed, not taking into account sub effect □

Double check this

Same basket