PSET5

Anthony Yoon

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a

We can first begin calulating MRS for each individual. Without a loss of generality, we can begin to note the following:

$$MRS_i = \frac{\alpha(x_1^i)^{\alpha-1}(x_2^i)^{\beta}}{\beta(x_2^i)^{\beta-1}(x_1^i)^{\alpha}}$$

This above quantity implies that $\frac{x_2^1}{x_1^1} = \frac{x_2^2}{x_1^2}$. Let $e_1 = x_1^1 + x_1^2$ and $e_2 = x_2^1 + x_2^2$. Using this equalities as well as the implication derived from the MRSes, we can find that we get:

$$x_2^1 = \frac{e_2}{e_1}x_1^1 \quad x_2^2 = \frac{e_2}{e_1}x_1^2$$

Since both the above quantities are linear in nature with no intercept, this implies that indeed the contact curve is that of connecting endpoints.

b

Since we we are working with different utility functions, we can find that after similar caluclations to above that:

$$MRS_1 = \frac{\alpha(x_1^1)^{\alpha-1}(x_2^1)^{1-\alpha}}{(1-\alpha)(x_1^1)^{\alpha}(x_2^1)^{-\alpha}} = \frac{\alpha x_2^1}{(1-\alpha)x_1^1}$$

and using similar calculations, we find that:

$$MRS_2 = \frac{\beta x_2^2}{(1-\beta)x_1^2}$$

Since we know that $1 > \alpha > \beta > 0$, we find that:

$$\frac{\alpha}{1-\alpha} > \frac{\beta}{1-\beta}$$

Thus, we can see that for MRS to equal to each other, we know that:

$$\frac{x_2^1}{x_1^1} < \frac{x_2^2}{x_1^2}$$

Using the equations derived above, we can find that:

$$x_2^1 < \frac{e_2}{e_1} x_1^1$$

this implies that the graph still intersects the origins, but now $x_2^1 < x_1^1$, where we have all a curve that will be strictly below that of the original line derived in **a**

c

For the contract curve to exist, we want $MRS_1 = MRS_2$. Let $e_1 = x_1^1 + x_1^2$. We can see that

$$MRS_1 = MRS_2 \implies \alpha(x_1^1)^{\alpha-1} = \beta(x_1^2)^{\beta-1}$$

Thus, substuting the endowment, we find that:

$$(x_1^1)^{\alpha-1} = \beta(e_1 - x_1^1)^{\beta-1}$$

So we see that as $x_1^1 \to e_1^1$, we find that the consumers will not consume any

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