ECON 20210 Notes

Anthony Yoon

April 5, 2025

Contents

Todo list

1 Lecture 1

In Macroeconomics, we want to look at the economy in the whole, hence the "macro" aspect of the economy. In this field. we try to analyze the "trend" within the economy. For example. we may be interested in how the economy reacts when we give the population a stimulus check. However, we cannot be always sure that the previous trends will be representative of that of the current trends.

However, when we think about the economy as a whole, there are a lot of data we need to work with. Hence, we can collapse the data into aggregate values, which are as follows:

- GDP (Gross Domestic Product): How well is the local economy doing?
- GNP (Gross National Product): How well are the nationals of a an economy doing?
- Unemployment: How is the labor market functioning?
- Inflation: How much money do you have to have now to buy the same basket of goods you bought in 2000.
- Stock price: How valuable are corporations.
- CPI (Consumer Price index): Measure of the general goods and prices. Done by keeping track of a certain collection of goods.

We can see that these are a lot easier to work with, rather than working with high-dimensional data.

1.1 GDP explained

In it of itself, GDP is a flow of money. Specfically, the *final* dollar amount produced per unit of time. We can measure GDP in mainly 3 ways. All goods and services purchased, produced, and all income earned. All of these have to add up to the same value, and if the product is unsold, then they are treated as self-bought goods. Usually, that value is the price. However, prices change over time, and we thus have to account of the temporal nature of prices. Thus, we have the following indicators of GDP, where we let P indicate quantity and Q denote the quantity of good, and t be time, and i be the index of the good:

- Y_t^n or the nominal GDP, values products at their *current* dollars at a time t.

$$Y_t^n = \sum_i P_{i,j} Q_{i,t}$$

- Y_t^r or the real GDP, values products at a *constant* dollars at time 0:

$$Y_t^r = \sum_i P_{i,0} Q_{i,t}$$

- P_t: GDP Deflator (price index), which serves as a baseline comparison between the year of interest and the baseline year.

$$\begin{aligned} P_t &= \frac{Y_t^n}{Y_t^r} \times 100 \\ &= \frac{\sum_i P_{i,j} Q_{i,t}}{\sum_i P_{i,0} Q_{i,t}} \approx \frac{P_t}{P_0} \end{aligned}$$

May need to but in chain weighted GDP here later.

1.2 Expenditure

We can calculate expenditure, the total amount of money spent, as the following:

$$Y = C + I + G + EX - IM$$

where

- C denotes the consumption madde by Households
- I denotes the physical investment, purchases of new capital goods by businesses.
- G Government expenditures (purchases and investments) us all expenditure for all levels of government. However, we exclude transfer payments.
- EX denotes the cost of exports
- IM denotes the cost of imports[1]

And we set this equal to the income, as noted by our assumption above, or

$$Y = C + I + G + EX - IM = wL + \pi rK + T$$

where

- wL denotes the wage and compensation to workers
- rK denotes the compensation to capital owners
- π denotes the corporate profits
- T denotes Taxess

we also have the notion of production functions, which are very similar to that but **ECON 20210**, where

$$y = f(A, K, L, X)$$

where

- A denotes technology
- K denotes capital stock
- L denotes labor
- X denotes aother factors of production

1.2.1 Criticisms of GDP as a measure of Economic Well-being. One of the good things aout GDP is that GDP per capita is often correlated with measures as infact mortality rate (-), life expectancy (+), and literacy rate (+). However, GDP does not include non-market activities such as household production and other activities, as leisure is an important to one's well being.

[1] NX denotes net exports, which is just $EX - IM$

1.3 Level versus Growth

However, note that GDP (X) is a time dependent variable. Hence, we should be interested in *growth* as well as the *level* of the good. If the we can describe the growth in a *discrete* manner, we can see that:

$$\gamma = \frac{X_{t+1} - X_t}{X_t} \quad X_{t+1} = (1+\gamma)X_t$$

if we have a continous and exponentially growing GDP, we have the following:

$$\lim_{n\to\infty} \left(1 + \frac{\gamma}{n}\right)^n$$

and since $e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$, we see that:

$$\lim_{n\to\infty} \left(1 + \frac{\gamma}{n}\right)^n = e^\gamma \implies X_t = X_0 \cdot e^{\gamma t} \iff \ln X_t = \ln X_0 + \gamma t$$

This implies that if we take an contionus GDP, take the natural log of GDP with respect time and it is linear, then the GDP rate of growth is constant.

2 Lecture 2

2.1 CPI

Inflation is usally measured by tracking the price of a basket of goods, hence we are not interested in the nominal or the real discussion of GDP. There are many ways to track inflation in the following ways:

- GDP Deflator: Basket of goods and services produced domestically
- Consumer Price Index: Basket of goods and services consumed by househoulds
- Personal Consumption Expenditure (PCE) price index: Chained the prices togther, which yields to higher coverage.
- Producer Price Index: Basket of goods purchased by producers.

To calcluate the CPI, we can do the following. We fix the basket of goods, say $Q_{i,0}$, and we track the price throughout time, like the following:

$$X_t = \sum_i P_{i,t} Q_{i,0}$$

We can calculate the price index as taking the expenditure of a year and and comparing it to the base year. Or:

$$P_t = \frac{X_t}{X_0} \times 100$$

Inflation is calcluated as the change in price index, or

$$\pi_t = \frac{P_{t+1} - P_t}{P_t}$$

Inflation can be expressed as the arithmetic mean of indvidual inflation rates weighted by the expenditure share, or rather:

$$\frac{P_{i,t}Q_{i,0}}{\text{Expenditure}} \cdot \frac{P_{i,0}}{P_{i,0}} \implies \frac{P_{i,t}}{P_{i,0}}$$

We can also calcualte the GDP deflator as:

$$\frac{\text{Nomial GDP}}{\text{Real GDP}} = \frac{\sum P_{i,t}Q_{i,t}}{\sum P_{0,t}Q_{i,t}}$$

CPI tends to be greater than of the GDP deflator. The CPI that we have been familar with until this point has been referred to as the **Lasperes** index which tends to over estimate the true cost of livin, as

$$c(u_o, P_t) \leq \sum p_{i,t}q_{i,0} \implies \frac{\sum_i p_{i,t}q_{i,0}}{\sum_i p_{i,0}, q_{i,0}} \geq \frac{C(u_0, P_t)}{C(u_0, P_0)}$$

because the hicksian demand function will always produce the amount with the lowest cost. We can also refer to the **Passche** index, which is where:

$$\frac{\sum P_{i,t}Q_{i,t}}{\sum P_{i,0}Q_{i,t}}$$

which tends to underestimate the rate of inflation for u_1 as We can also view price index as the cost of achieving the same level of utility across multiple time periods. We can see that we can use the Hicksian Demand Functions, $C(u, P)$ where u denotes utility and P denotes the price vector. Thus, we can define the true inflation rate as

Write in proof here

$$X_t = \frac{C(u_0, P_t)}{C(u_0, P_0)}$$

and also note that the changing of the price vector can be seen as a substitution efffect of the preference of the goods. We can also define the **Fisher index** as the geometric average of these two indices.

2.2 Proof that Lasperes index at base time approximates the true cost of living

Consdier the first order expansion of $C(u_0 l P_t)$ where we evaluate it where $p_T = p_0$.

$$C(u_0, P_t)\big|_{P_t = P_0} \approx C(u_0, P_0) + \sum_i \frac{\partial C(u_0, P_t)}{\partial P_{i,t}}\bigg|_{P_t = P_0} (P_{i,t} - P_{i,0})$$

Using Shephard's Lemma, we know that:

$$\frac{\partial C(u_0, P_0)}{\partial P_{i,0}} = Q_{i,0}(u_0, P_0)$$

which implies that

$$C(u_0, P_t) \approx +\sum_i Q_{i,0}(P_{i,t} - P_{i,0}) = \sum_i P_{i,t}Q_{i,0}$$

which implies that the Laspreyes index approximates the true cost of living.

2.3 Chained Index

Over a long timer period, the assumptions that $Q_t \approx Q_0$ or $P_t \approx P_0$ are no longer tenable. This can be the idea where more goods are added to the basket at a time, and we can consider the Laspreyes index, but note that:

$$\frac{P_{2024}}{P_{1970}} = \frac{P_{1971}}{P_{1970}}\frac{P_{1972}}{P_{1971}}\cdots\frac{P_{2023}}{P_{2022}}\frac{P_{2024}}{P_{2023}}$$

or similarly:

$$\frac{P_{2024}}{P_{1970}} = \frac{\sum p_{1971}q_{1970}}{\sum p_{1970}q_{1970}}\frac{\sum p_{1972}q_{1971}}{\sum p_{1971}q_{1971}}$$

2.4 Unemployment

People who are over the age of 16 are only considered. People are classified as unemployed if they do not have a job, acgtively looked for work, in the prior 4 weeks, and those currently available for work.

3 Lecture 3

One of the models we study in Macroeconomics is the **Solov growth modwl**. [2] We begin with the following observations:

1. Share of labor incomes and captial incomes remain fixed at constant ratios.
2. The value of capital per worker is slowly rising
3. The value of output per worker is slowly rising.
4. Capital and output grow at the same rate.
5. The rate of return on capital is approximatley constant.
6. Some economies grow faster than others.

The solov growth model was an attempt to explain these trends.

3.1 Solov Growth Model version 1

We begin with the following notation:

- K denotes the growth from accumulation of capital. This is how we measure "growth"
- C denotes consumption.
- I denotes investment
- Y denotes output
- A denotes the aggregate productivity factor, or a measure of technology in the economy.
- N denotes labor

Check this definition of N

[2] This model has a lot of symbols and stuff, so it does get very overwhelming.

- δ denotes the depreciation rate.
- s denotes the saving rate

We work with the following production function:

$$Y = AK^{\alpha}N^{1-\alpha}, \alpha \in [0, 1]$$

which is a Cobb Douglas Production function and CRS production technology. We introduce the following the assumptions:

1. Saving rate s is constant and exogenous.
2. We do not consider the international sector
3. No government involved
4. K follows the evolution that
$$K_{t+1} = K_t(1-\delta) + I_t$$
5. We fix A and N to be constant for all time periods.
6. K_0 is also given.

In summary, we know that $\{s, \delta, A, \alpha, N\}$ are all exogenous. We also have the following technical asssumptions on the production function, where

$$\lim_{K\to\infty} F_K(K, N) = 0 \quad \lim_{K\to\infty} F_K(K, N) = \infty$$

and $F_K(K, N) = 0$ and $F_K(K, N) = \infty$. These assumptions allow us to see that $Y = C + I$. This also implies that $C = Y(1-s)$ and $I = s(Y)$, where $Y = AK_t^{\alpha}N^{1-\alpha}$. [3]. With all this combined informaion, we get the following equation:

$$K_{t+1} = (1-\delta)K_t + sAK_t^{\alpha}N^{1-\alpha}$$

We can see that if we are given a K_0, it is very easy to calcualte the discrete sequence of K_t. Note the following, if $K_{t+1} = K_t$, we are at a **steady state** solution. We can see this graphically: Fill this in

We can see that if $K_0 = 0$ or $K_0 > 0$, the Inada conditions ensure there is a unique and stable steady state solution.

3.1.1 Implications of the Solow Growth Model Version 1.

Note that:

$$\begin{aligned}
K_{t+1} &= (1-\delta)K_t + sAK_t^{\alpha}N_t^{1-\alpha} \\
K_{t+1} - K_t &= sAK_t^{\alpha}N^{1-\alpha} - \delta K_t \\
\frac{K_{t+1} - K_t}{K_t} &= sAK_t^{\alpha-1}N^{1-\alpha} - \delta \\
\frac{\Delta K_t}{K_t} &= sAK_t^{\alpha-1}N^{1-\alpha} - \delta
\end{aligned}$$

[3] Note that time is on a discrete time

This implies that the growth rate will decrease as the size of the economy increases. For a competitve factor market, we can see that factor price must equal the marginal product [4] We can see that the version covers all the assumptions from (1,3,4). But we fail to explain why the value of capital per worker is slowly rising and why the rate of return on capital is constant and the why some economices grow faster than others.

3.2 Solov Growth Model Version 2

We now have the same assumptions as before but allow A and N to change over time. We define the following:

$$\frac{A_{t-1}}{A_t} = g \quad \frac{N_{t+1}}{N_t} = n$$

We are now left with the following production function:

$$Y_t = (A_t N_t)^{1-\alpha} K_t^\alpha$$

we also consider the following:

$$k_t = \frac{K_t}{A_t N_t} \quad y_t = \frac{y_t}{A_t N_t}$$

These are the capital and output per effective unit of labor. We do this to account for the increases in labor and production. Since we allow for time variance, we should rewrite our growth of capital in order forms:

$$\begin{aligned}
K_{t+1} &= K_t(1-\delta) + I_t \\
K_{t+1} &= K_t(1-\delta) + s(A_t N_t)^{1-\alpha} K_t^\alpha \\
K_{t+1} - K_t &= s(A_t N_t)^{1-\alpha} K_t^\alpha - \delta K_t \\
\frac{\Delta K_t}{K_t} &= \frac{s(A_t N_t)^{1-\alpha} K_t^\alpha}{K_t} - \delta \\
\frac{\Delta K_t}{K_t} &= \frac{s K_t^{\alpha-1}}{(A_t N_t)^{\alpha-1}} - \delta \\
\frac{\Delta K_t}{K_t} &= s k_t^{\alpha-1} - \delta
\end{aligned}$$

Note that $K_t = A_t N_t k_t$ This implies:

$$\begin{aligned}
\frac{\Delta K_t}{K_t} &= s k_t^{\alpha-1} - \delta \\
\frac{\Delta k_t A_t N_t}{k_t A_t N_t} &= s k_t^{\alpha-1} - \delta \\
\frac{\Delta k_t}{k_t} + \frac{\Delta A_t}{A_t} + \frac{\Delta N_t}{N_t} &= s k_t^{\alpha-1} - \delta \\
\frac{\Delta k_t}{k_t} + g + n &= s k_t^{\alpha-1} - \delta \\
\Delta k_t &= s k_t^\alpha - (n+g+\delta) k_t
\end{aligned}$$

[4] Think of marginal revenue and marginal product