

## Todo list

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**Problem 1.** Explaining lower real interest rates.

**Problem 2.** Transitional Dynamics in Solow Growth Model

(1)

*Solution:* Given the given parameters, we can see that:

$$k_t = \frac{K_t}{A_t N_t} \quad y_t = \frac{y_t}{A_t N_t}$$

Thus

$$\begin{aligned} K_{t+1} &= K_t(1 - \delta) + I_t \\ K_{t+1} &= K_t(1 - \delta) + s(A_t N_t)^{1-\alpha} K_t^\alpha \\ K_{t+1} - K_t &= s(A_t N_t)^{1-\alpha} K_t^\alpha - \delta K_t \\ \frac{\Delta K_t}{K_t} &= \frac{s(A_t N_t)^{1-\alpha} K_t^\alpha}{K_t} - \delta \\ \frac{\Delta K_t}{K_t} &= \frac{s K_t^{\alpha-1}}{(A_t N_t)^{\alpha-1}} - \delta \\ \frac{\Delta K_t}{K_t} &= s k_t^{\alpha-1} - \delta \end{aligned}$$

Note that  $K_t = A_t N_t k_t$  This implies:

$$\begin{aligned} \frac{\Delta K_t}{K_t} &= s k_t^{\alpha-1} - \delta \\ \frac{\Delta k_t A_t N_t}{k_t A_t N_t} &= s k_t^{\alpha-1} - \delta \\ \frac{\Delta k_t}{k_t} + \frac{\Delta A_t}{A_t} + \frac{\Delta N_t}{N_t} &= s k_t^{\alpha-1} - \delta \\ \frac{\Delta k_t}{k_t} + g + n &= s k_t^{\alpha-1} - \delta \\ \Delta k_t &= s k_t^\alpha - (n + g + \delta) k_t \end{aligned}$$

□

(2)

*Solution:* At steady state,  $\Delta k_t = 0$  For notational state, let  $x = k_{ss}$ . This implies that

$$\begin{aligned} 0 &= s x^\alpha - (n + g + \delta) k_t \\ (n + g + \delta) x &= s x^\alpha \\ \frac{n + g + \delta}{s} &= x^{\alpha-1} \\ x &= \left( \frac{s}{n + g + \delta} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

□

(3)

*Solution:* We are interested in the following optimization problem:

$$\max k_t^\alpha - (n + g + \delta)k_t$$

Taking the first order derivative with respect to  $k_t$  allows to see:

$$\alpha k_t^{\alpha-1} - (n + g + \delta) = 0 \implies k_{gr} = \left( \frac{\alpha}{n + g + \delta} \right)^{\frac{1}{\alpha-1}}$$

□

(4)

*Solution:* Code for the simulation:

```
% Setting parameters
s = 0.4;
delta = 0.06;
n = 0.02;
g = 0.02;
alpha = 1/3;
z = 100; % Number of iterations

% x axis creation
X = 0:1:z;
X = X';

K = zeros(z+1, 1);
A = zeros(z+1, 1);
N = zeros(z+1, 1);
k = zeros(z+1, 1);
y = zeros(z+1, 1);
Y = zeros(z+1, 1);

% setting values
A(1) = 1;
K(1) = 1;
N(1) = 1;
Y(1) = K(1)^alpha * (A(1) * N(1))^(1-alpha);

% Time iteration
for i = 1:(z+1)
    A(i + 1) = A(i) * (1 + g);
    N(i + 1) = N(i) * (1 + n);
    K(i + 1) = K(i) * (1 - delta) + s * (A(i) * N(i))^(1 - alpha) *
        K(i)^alpha;
    k(i) = (K(i) / (A(i) * N(i)));
end
```

```
    Y(i) = K(i)^alpha * (A(i) * N(i))^(1-alpha);  
    y(i) = Y(i) / (A(i) * N(i));  
end  
  
figure;  
  
subplot(2, 2, 1);  
plot(X, y);  
title('Plot of y vs X');  
grid on;  
  
subplot(2, 2, 2);  
plot(X, Y);  
title('Plot of Y vs X');  
grid on;  
  
subplot(2, 2, 3);  
plot(X, k);  
title('Plot of k vs X');  
grid on;  
  
subplot(2, 2, 4);  
plot(X, K(1:101));  
title('Plot of K vs X');  
grid on;
```

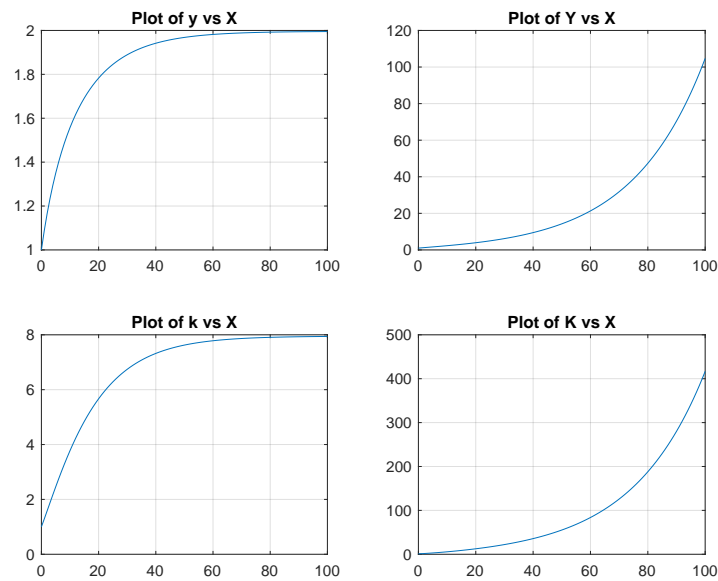


Figure 1: Figure Econ 20210 Problem 3 Question 5

Note that  $k$  and  $y$  approach the steady state behaviors and  $Y$  and  $K$  approach infinity, which resemble the Inada conditions.  $\square$

(5)

*Solution:*

```
% Problem 3 Q5

% Setting values
k_steady_state = (s / (n+g+delta))^(1.5);
k_1 = zeros(z, 1);
```

```
k_1(1) = k_steady_state;
s = 0.35;

for i=1:z
    k_1(i+1) = s * k_1(i)^alpha - (n + g + delta) * k_1(i) + k_1(i);
end

figure;

plot(X,k_1)
```

INSERT GRAPH



REDO

### Problem 3. Cookie Eating - Part 1

(1)

*Solution:* We can see the law of depreciation is:

$$W_{t+1} = W_t - c_t \quad \text{s.t.} \quad W_0 > 0$$



(2)

*Solution:* Note that  $W_{t+1} = W_t - c_t$  and thus  $W_t = W_{t-1} - c_{t-1}$ . This implies that via a recursive argument:

$$W_{t+1} = W_t - c_t \implies W_{t+1} = W_0 - \sum_{t=1}^T c_t$$

such that  $W_{t+1} \geq 0$



(3)

*Solution:* The Lagrangian is as follows:

$$L = s - \lambda \left( W_{t+1} - W_0 + \sum_0^t c_t \right)$$

with the following FOCs:

$$[c_i] \quad \left( \frac{\partial u}{\partial c} \Big|_{c_i} \right) \cdot \beta^i + \lambda \leq 0$$

$$[\lambda] \quad W_{t+1} \leq W_0 - \sum_{t=1}^T c_t$$

Note that  $W_{t+1}$  has to be 0, as no utility is derived from  $W_{t+1}$  period.



(4)

*Solution:* From a  $[c_{i+1}]$  and  $[c_i]$ , we see that:

$$u'(c_{t+1}) = \frac{\lambda}{\beta^{t+1}} \quad u'(c_t) = \frac{\lambda}{\beta^t}$$

This implies

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{\frac{\lambda}{\beta^t}}{\frac{\lambda}{\beta^{t+1}}} = \beta \iff u'(c_t) = \beta u'(c_{t+1})$$

□

(5)

*Solution:* From (4), we see that

$$\beta c_t = c_{t+1} \iff \frac{c_t}{\beta} = c_{t-1}$$

This implies that using the  $[\lambda]$  condition, we are interested in solving:

$$W_0 = \sum_{i=0}^t \beta^{-i} c_t$$

which is equivalent to

$$c_t \sum_{i=0}^t \beta^{-i} = W_0 \iff c_t = \frac{W}{\sum_{i=0}^t \beta^{-i}}$$

□

Do final part

**Problem 4.** Crusoe's Intratemporal Choice

(1)

*Solution:*

$$\begin{aligned} &\max U(c, l) \\ \text{s.t. } &c = \frac{1}{1-\theta} (l - \bar{l})^{1-\theta} \\ \text{s.t. } &0 < \theta < 1 \end{aligned}$$

□

(2)

*Solution:* The FOCs are as follows:

$$\begin{aligned} [c] \quad & \frac{\alpha}{c} - \lambda = 0 \\ [l] \quad & \frac{1-\alpha}{1-l} - \lambda(l-\bar{l})^{-\theta} = 0 \\ [\lambda] \quad & c = \frac{1}{1-\theta}(l-\bar{l})^{1-\theta} \end{aligned}$$

We see that using the  $[l]$  and  $[\lambda]$  constraint, we see that

$$\begin{aligned} -\frac{1-\alpha}{1-l} + \frac{\alpha}{c}(l-\bar{l})^{-\theta} &= 0 \\ \frac{\alpha}{c}(l-\bar{l})^{-\theta} &= \frac{1-\alpha}{1-l} \\ \frac{\alpha}{c(l-\bar{l})^\theta} &= \frac{1-\alpha}{1-l} \end{aligned}$$

Note that  $(l-\bar{l})^\theta$  is the weight of trade off between the consumption and labor. If  $\theta$  increases, working more does become as beneficial.  $\square$

**(3)**

*Solution:* Using the derived optimality condition, we can see the following:

$$\begin{aligned} \frac{1-\alpha}{1-l} &= \frac{\alpha}{c}(l-\bar{l})^{-\theta} \\ c(1-\alpha) &= \alpha(1-l)(l-\bar{l})^{-\theta} \\ c &= \frac{\alpha(1-l)(l-\bar{l})^{-\theta}}{1-\alpha} \\ \frac{1}{1-\theta}(l-\bar{l})^{1-\theta} &= \frac{\alpha(1-l)(l-\bar{l})^{-\theta}}{1-\alpha} \\ \frac{l-\bar{l}}{1-\theta} &= \frac{\alpha(1-l)}{1-\alpha} \\ \frac{l}{1-\theta} + \frac{\alpha l}{1-\alpha} &= \frac{\alpha}{1-\alpha} + \frac{\bar{l}}{1-\theta} \\ l \left( \frac{1}{1-\theta} + \frac{\alpha}{1-\alpha} \right) &= \frac{\alpha(1-\theta) + \bar{l}(1-\alpha)}{(1-\alpha)(1-\theta)} \\ l \left( \frac{1-\alpha + \alpha(1-\theta)}{(1-\theta)}(1-\alpha) \right) &= \frac{\alpha(1-\theta) + \bar{l}(1-\alpha)}{(1-\alpha)(1-\theta)} \\ l &= \frac{\alpha(1-\theta) + \bar{l}(1-\alpha)}{1-\alpha + \alpha(1-\theta)} \end{aligned}$$



This implies that:

$$\begin{aligned}
 c &= \frac{1}{1-\theta} \left( \frac{\alpha(1-\theta) + \bar{l}(1-\alpha)}{1-\alpha + \alpha(1-\theta)} - \bar{l} \right)^{1-\theta} \\
 &= \frac{1}{1-\theta} \left( \frac{\alpha(1-\theta) + \bar{l}(1-\alpha)}{1-\alpha + \alpha - \alpha\theta} - \bar{l} \right)^{1-\theta} \\
 &= \frac{1}{1-\theta} \left( \frac{\alpha(1-\theta) + \bar{l}(1-\alpha)}{1-\alpha\theta} - \bar{l} \right)^{1-\theta} \\
 &= \frac{1}{1-\theta} \left( \frac{\alpha(1-\theta) + \bar{l}(1-\alpha) - \bar{l}(1-\alpha\theta)}{1-\alpha\theta} \right)^{1-\theta} \\
 &= \frac{1}{1-\theta} \left( \frac{\alpha(1-\theta) + \bar{l}[(1-\alpha) - (1-\alpha\theta)]}{1-\alpha\theta} \right)^{1-\theta} \\
 &= \frac{1}{1-\theta} \left( \frac{\alpha(1-\theta) + \bar{l}(\alpha\theta - \alpha)}{1-\alpha\theta} \right)^{1-\theta} \\
 &= \frac{1}{1-\theta} \left( \frac{\alpha(1-\theta) + \alpha(\theta-1)\bar{l}}{1-\alpha\theta} \right)^{1-\theta} \\
 &= \frac{1}{1-\theta} \left( \frac{\alpha(1-\theta)(1-\bar{l})}{1-\alpha\theta} \right)^{1-\theta}
 \end{aligned}$$

Therefore:

$$l = \frac{\alpha(1-\theta) + \bar{l}(1-\alpha)}{1-\alpha + \alpha(1-\theta)} \quad c = \frac{1}{1-\theta} \left( \frac{\alpha(1-\theta)(1-\bar{l})}{1-\alpha\theta} \right)^{1-\theta}$$

□

(4)

*Solution:*

$$\frac{\partial l}{\partial \bar{l}} = \frac{1-\alpha}{1-\alpha + \alpha(1-\theta)} \quad \frac{\partial c}{\partial \bar{l}} = - \left( \frac{\alpha(1-\theta)}{1-\alpha\theta} \right)^{1-\theta} (1-\bar{l})^{-\theta}$$

□