

**Problem 2.***Solution:*

```
1 % Compute runtime and spectral norm of symmetric random matrices
2
3 % List of matrix sizes to test
4 sizes = [36 , 50, 100, 200, 400, 500, 600, 700, 800, 1000, 1100,
          1200, 1400, 1450, 1500];
5
6 numSizes = numel(sizes);
7 runtimes = zeros(1, numSizes);
8 spectralNorms = zeros(1, numSizes);
9
10 for k = 1:numSizes
11     n = sizes(k);
12
13     % Generate random symmetric matrix
14     A = randn(n);
15     A = (A + A')/2;
16
17     % Time the eigenvalue computation
18     tic;
19     ev = eig(A); % for a symmetric A, eig uses a symmetric solver
20     runtimes(k) = toc;
21
22     % Record the spectral norm (max absolute eigenvalue)
23     spectralNorms(k) = max(abs(ev));
24 end
25
26 % Plot runtime vs. matrix size
27 figure;
28 plot(sizes, runtimes, '-o', 'LineWidth', 1.5, 'MarkerSize', 8);
29 xlabel('Matrix size n');
30 ylabel('Runtime (seconds)');
31 title('Runtime of Eigenvalue Decomposition vs. Matrix Size');
32 grid on;
33
34 % Plot spectral norm vs. matrix size
35 figure;
36 plot(sizes, spectralNorms, '-o', 'LineWidth', 1.5, 'MarkerSize', 8);
37 xlabel('Matrix size n');
38 ylabel('Spectral Norm (max |\lambda|)');
39 title('Spectral Norm vs. Matrix Size');
40 grid on;
```

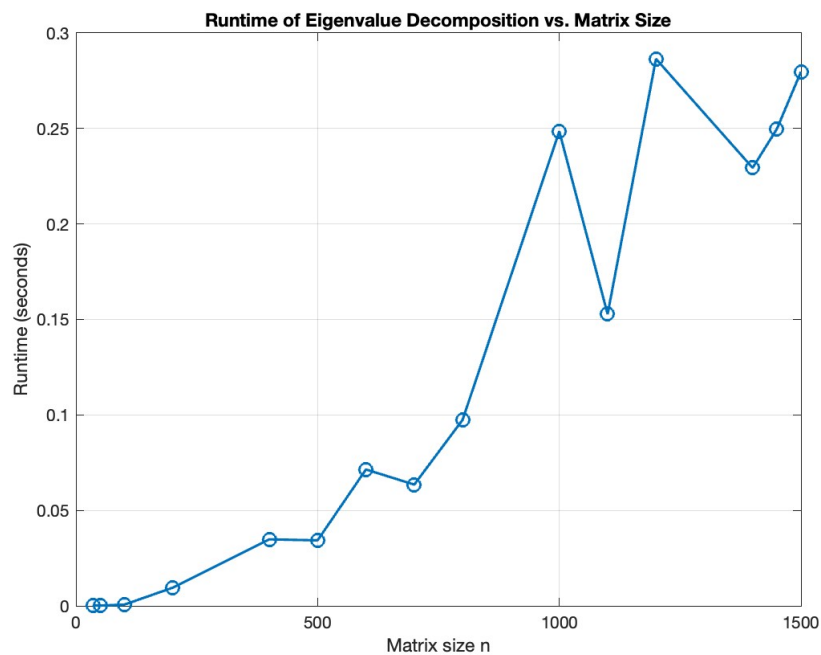


Figure 1: Runtime of Eigenvalue versus Matrix Size

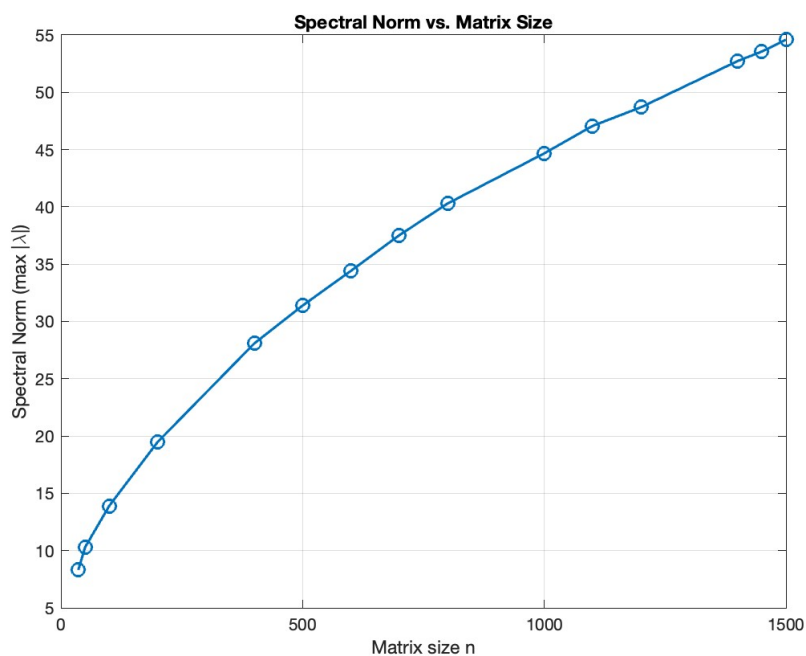


Figure 2: Spectral Norm versus Matrix Size

The run time of the Eigenvalue decomposition seems to be  $\mathcal{O}(n^3)$  and the spectral norm seems to be  $\mathcal{O}(\sqrt{n})$  □

**Problem 3.**

*Solution:* Since  $U$  and  $V$  are orthonormal, they are basis for the subspace in  $\mathbb{R}^3$ , which in this case is a 2-d plane. By definition, the angle between two subspaces are defined by the smallest angle between any two vectors, where one comes from each respective subspace. Thus, we can proceed to with the calculation of these principal angles. To calculate these angles, we must calculate the inner product of between each vector in the basis, or  $U^T V$ , where we take the SVD to reveal that:

$$U^T V = J \Sigma P^T$$

where  $J$  denotes the left singular matrix and  $P$  represents the right singular matrix. Each respective singular value of  $U^T V$  represents the cosine of each respective □