Econ Research Project

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1 Introduction

With over 7,000 students, the University of Chicago's Undergraduate College provides education to numerous students. Due to the on-campus housing requirement, the College also provides ample room and board opportunities for students to take advantage of so that students can socialize with one another during their underclassmen years. This on-campus requirement has also helped foster UChicago's coveted "House culture," where students go on field trips and participate in bonding activities together in the dorms, providing them with a reliable social network from the day they set foot on campus.

However, once students are released from the on-campus housing requirement, many (54% in 2014) choose to opt out of the dorms for off-campus apartments around Hyde Park. UChicago could benefit from having students pay for on-campus housing to develop community building between lowerclassmen and upperclassmen, to optimize the success of students who have easier access to classroom building and study spaces, and to have a steady revenue stream to fund housing operations, dining services, campus maintenance, on top of higher occupancy rates in the rented dorm building, Woodlawn Residential Commons.

Inspired by the many variables that motivate students to leave the dorms as well as UChicago's preference to have students stay, we chose to examine incentives that would lead UChicago students to stay in the dorms for our research project

In this paper, we treat UChicago as the firm and undergraduate students as the consumer. Specifically, UChicago wants to optimize for the number of students staying on campus. The students (the consumers) are choosing to maximize their utility by consuming one of either good, on or off campus housing. In this paper, we ask the question: **How can we model the incentives UChicago wishes to provide to motivate students to stay in on-campus housing using an annoymous game with externalities?**

2 Associated Costs of Living

We begin with a perspective of the various costs of living on and off campus. According to the University of Chicago Finacial Aid Website and the US News report in 2014, the costs and the quantity of people are as follows:

	On campus	Off Campus
Room and Board	$20109	$17502
Num. of People	4133	3520

Under the assumption that there are approximately 1913 students per grade level and all underclassmen must stay on campus, we see that there are approximately 307 upperclassmen that choose to stay on campus. This indicates that the majority of upperclassmen choose to live off campus. We also assume that there are about 100 occupancies per house, and with 48 houses within the dorms, that there are approximately 4800 spots for students to live on campus. Assuming that these numbers have remained constant throughout time, the majority of upperclassmen still choose to live off campus today. This means that currently the university has about 670 open spots throughout a given school year, implying a wastage of capital, and thus a potental loss of profit. Thus, the university wants to introduce incentives to promote higher occupnacy rates among upperclassmen.

3 Assumptions of the model

Before we introduce the model, we would like to introduce the core assumptions of this model.

- The number of occupancies in the dormitories are bounded by some constant $M \in \mathbb{R}^+$ where given our estimates for the new academic year, $M = 4800$
- When the demand for rooms is greater than M, the university will increase each rooms' occupancy by adding beds, making all students living on campus worse off.
- The incentives we propose in our model is symmetric such that the improvements and detriments to the quality of life that a student recieves is constant for all students.
- We assume that the university is strictly better off the more rooms that they fill. Hence, all occupancies being filled would be the optimal outcome.
- All assumptions presented in class will hold in this analysis

4 Model

4.1 Modeling student's choice to move on or off campus

Given that we know the current number of upperclassmen staying on campus is suboptimal, the university decides to introduce a housing game before the next academic year in the hopes of increasing the upperclassmen's on-campus occupancy to the optimal level. Let the status quo number of upperclasmen living on campus be the bound ϕ and the optimal number of upperclassmen ξ. Utilizing the estimates from earlier, ϕ and ξ must be in $[0, M]$ where $\xi = M$. Now consider the following one-shot game. We define the players of the game as the upperclassmen at the University.

$$Players = \{1, 2, \ldots, N\}$$

and with the actions to live off or on campus, or denoted as follows:

$$Actions = \{F, O\}$$

with the base payoffs in utility for each individual as follows:

$$U_i(F) = \Lambda \quad U_i(O) = \gamma$$

for $i \in players$. Note that $\Lambda > \gamma$ as the price for living off campus is cheaper than that of living on campus. We can see if that there was no incentive provided by the university, than everyone would move off campus, making that the Nash Equirlbirum.

The university sends out the 2025-2026 housing application to upperclassmen with complete information of standard of living will change contingent on number of upperclassmen who live on campus. Let K equal the improvement or detrition to the standard of living.

Based on this information, the upperclassmen choose whether to live on or off campus. However, depending on the number of upperclassmen staying on campus, the University has actions that it can take. Using the bounds that were derived before, the following cases emerge:

1. If less than or equal ϕ studenst stay on campus, the University is worse off compared to last year. Thus, to compensate for this, the quality of life is reduced on campus. Thus, the payoffs can be defined as
   - $U_i(O)$: $\gamma - K$
   - $U_i(F)$: $\Lambda - K$
2. If the number of classmen $\in (\phi, \xi)$ students stay on campus, the school has not hit their target number of upperclassmen, and to incentive more upperclassmen to stay off campus, they increase the quality of life for only those living on campus. Thus, the payoffs can be defined as
   - $U_i(O)$: $\gamma + K$
   - $U_i(F)$: Λ
3. If there are exactly ξ upperclassmen stay on campus, the University's target has been hit, and thus, they have enough budget and he ability to raise the quality of life across the board. Thus, the payoffs can be defined as
   - $U_i(O)$: $\gamma + K$
   - $U_i(F)$: $\Lambda + K$
4. If there are more than ξ upperclassmen, the university crams students into dorms by adding more beds and the quality of living lowers. Thus, the payoffs can be defined as
   - $U_i(O)$: $\gamma - K$
   - $U_i(F)$: Λ

4.2 Solving for optimal incentives

The university aims to have ξ number of upperclassmen living on campus. To simplify notation, let x be the number of upperclassmen who choose the live on campus. Thus, the incentive, or K, that the university provides must have only one feasible nash equilbria, such that ξ number of upperclassmen choose to live on campus. Thus, we must derive some relationship between Λ, ϕ, K to enforce this notion. To do so, consider the following. If $x = \xi$, and denoting unilateral deviation as the following:

$$\begin{aligned} U_i(O) = \gamma + K &\quad \rightarrow \quad U_i(F) = \Lambda \\ U_i(F) = \Lambda + K &\quad \rightarrow \quad U_i(O) = \gamma - K \end{aligned}$$

Note that if we want no profitable deviation, we want the following conditions to hold:

$$\begin{aligned} n \leq \gamma + k &\iff k \geq \Lambda - \gamma > 0 \\ \gamma - K \leq \Lambda + K &\iff 2k \geq \gamma - \Lambda \end{aligned}$$

Note that the second constraint is strictly negative, which implies that if $K \geq \Lambda - \gamma$, then the scenario when $\xi = x$ will be a nash equilbrium. To increase the strength of this relation, consider strict inquality. Thus, we claim that $K > \Lambda - \gamma$ is the relationship between K, Λ, γ, and we check for any other Nash Equilbria.

4.2.1 Casework

1. $\mathbf{x = 0}$

Since everyone is living off campus, moving to on campus would make them strictly worse off, thus, this makes this case a Nash Equilbria.

2. $\mathbf{0 < x < \phi}$

Consider the possible deviations:

$$\begin{aligned} u_i(O) = \gamma - K &\quad \rightarrow \quad u_i(F) = \Lambda - K \\ u_i(F) = \Lambda - K &\quad \rightarrow \quad u_i(O) = \gamma - K \end{aligned}$$

We see that $\Lambda - k > \gamma - k$ or $\gamma - k > \Lambda - K$ must hold to force a profitable deviation. Thus, we see that the first condition is always true, which implies that in this scenario, there is no Nash Equilbrium, regardless of K

3. $\mathbf{n = \xi - 1}$

To force a profitable deviation, consider all the possible deviations:

$$
\begin{aligned}
u_i(O) = \gamma + K &\quad\rightarrow\quad u_i(F) = \Lambda \\
u_i(F) = \Lambda &\quad\rightarrow\quad u_i(O) = \gamma + K
\end{aligned}
$$

This implies that either $\Lambda > \gamma + k$ or $\gamma + k > \Lambda$ forces a profitable deviation, which the second condition matches our proposed condition.

4. $\mathbf{x = \xi}$

This is the solution derived above. Therefore, this is a nash equilbirum.

5. $\mathbf{x = \xi + 1}$

Consider the possible deviations:

$$
\begin{aligned}
u_i(O) = \gamma - K &\quad\rightarrow\quad u_i(F) = \Lambda + K \\
u_i(F) = \Lambda &\quad\rightarrow\quad u_i(O) = \gamma - K
\end{aligned}
$$

We see that for a profitable deviation to exist either $2k > \gamma - \Lambda$ or $k < \gamma - \Lambda$, which the first inequality is true from our derived condition. Thus, this is not a Nash equilbrium.

6. $\mathbf{x = \phi}$

Consider the possible deviations:

$$
\begin{aligned}
u_i(O) = \gamma - K &\quad\rightarrow\quad u_i(F) = \Lambda - K \\
u_i(F) = \Lambda - K &\quad\rightarrow\quad u_i(O) = \gamma + K
\end{aligned}
$$

To force a profitable deviation, we need that $\Lambda - K > \gamma - K$ or $2k > \Lambda - \gamma$, which the first condition is always true from how Λ and γ are defined.

7. $\mathbf{x = \phi + 1}$

Consider the possible deviations:

$$
\begin{aligned}
u_i(O) = \gamma + K &\quad\rightarrow\quad u_i(F) = \Lambda - K \\
u_i(F) = \Lambda &\quad\rightarrow\quad u_i(O) = \gamma + K
\end{aligned}
$$

We see that this is very similar to $2k < \Lambda - \gamma$ and $k > \Lambda - \gamma$, of which the latter term matches our proposed condition.

8. $\boldsymbol{\phi + 2 < x < \xi - 1}$

Consider the possible deviations:

$$\begin{aligned} u_i(O) = \gamma + K \quad &\rightarrow \quad u_i(F) = \Lambda \\ u_i(F) = \Lambda \quad &\rightarrow \quad u_i(O) = \gamma + K \end{aligned}$$

Note that $k > \Lambda - \gamma$ forces a profitable deviation.

9. $x > \xi + 1$

Consider the possible deviations:

$$\begin{aligned} u_i(O) = \gamma - K \quad &\rightarrow \quad u_i(F) = \Lambda \\ u_i(F) = \Lambda \quad &\rightarrow \quad u_i(O) = \gamma - K \end{aligned}$$

Note that our proposed condition causes this to not be a nash equilbirum.

10. $\mathbf{n = N}$

Consider the following deviation:

$$U_i(O) = \gamma - K \quad \rightarrow \quad U_i(F) = \Lambda$$

Note that our proposed condition causes this to not be a nash equilbirum.

4.3 Analysis of casework

We can see that there are 2 Nash equilbria, $x = 0$ and $x = \xi$. However, some attention must be brought to the case where $x = 0$, as this is a case that must be avoided. This is becasue this is the worst possible case for the University, but we can also show that this is the worst possible case for all the upperclassmen as well. Consider the aggregated utility of the case where $x = 0$, we see that

$$\text{total utility in 0 case} = N(\Lambda - K)$$

and similarly when $x = \xi$

$$\text{total utilty in } \xi \text{ case} = \xi(\gamma + K) + (N - \xi)(\lambda + K)$$

we see that the total utility in the $x = \xi$ is clearly higher than that in the $x = 0$ case. Thus, it is also in the students' interest to move away from the $x = 0$. Thus, if there is some collective action among the upperclassmen or some additional university policy that encourages movement away from the $x = 0$ case.

5 Discussion

5.1 The implciations of the incentive

Our model shows that for on-campus housing to be a Nash equilibrium, the university's incentive for students to stay on-campus, K, must be greater than the gap in utility represented by $\Lambda - \gamma$. $\Lambda - \gamma$ is the gap in utility for students between living off-campus Λ and on-campus γ. Thus, the university must provide enough benefits to encourage staying on campus by having on-campus living conditions confer as much utility to students as moving off-campus. This incentive structure is crucial because:

- The school can reliably predict dorm occupancy rates from year to year, allowing them to have a reliable budget (including the operational costs of housing and amenities).
- On-campus housing is more attractive than off-campus housing, so students feel that on-campus housing is desirable, despite the higher price.
- It ensures that once ξ students choose dorms, the remaining students experience no incentive to deviate from their decision to live on-campus, as going below or above ξ would decrease the utility of the students as a collective.

As noted before, our model finds two Nash equilibria in the housing decision game, which makes the on-campus vs. off-campus housing decision a collective coordination game. A game of collective coordination means that students are collectively better off exactly ξ upperclassmen stay on campus is achieved, as this condition would allow for an equilibrium. An equilibrium maximizes total student utility. However, without proper coordination, there is a risk of reaching the worst-case scenario for both students and the university, where no upperclassmen choose on-campus housing. For the university, this scenario leads to revenue losses (since on-campus housing provides a stable revenue stream) and unused resources for the university. For students, this scenario presents the possibility of an oversaturated off-campus housing market, which could lead to higher rent costs, as well as reduced on-campus academic and social engagement from the lessened presence of upperclassmen in the dorms. To prevent this worst-case scenario, the university could introduce methods of communication that facilitate the coordination of students' choices with one another.

5.2 Policy Implementations

One potential solution to the problem presented by the collective coordination is a sequential live poll, which provides a costless method of communication before students participate in formal on-campus housing selection. This poll would allow students to see their peers' decision between on-campus and off-campus housing before making their own choices, which reduces uncertainty and enables coordination, allowing students to progress toward the optimal equilibrium. The poll would work in this manner:

- **Random Order Voting**: The poll assigns each student to a random position in a queue to vote anonymously, choosing either "On" (O) for on-campus housing or "Off" (F) for off-campus housing. Students who are indifferent between staying on and off campus could skip the voting and they will be shifted to the end of the queue, allowing people who confer more utility from staying on-campus to indicate that they would like to take on-campus spots first.
- **Live Distribution of Votes** After each vote, the running total of "O" vs. "F" is displayed publicly, allowing students to see the distribution between the two options in real time.

In the Random Order Voting implentation, the poll itself is an example of cheap talk, which is "cheap" in two senses: it has no direct payoff consequences, and there is no enforceable requirement for it to be truthful. Players may attempt to communicate their private information, or their intended actions in the game to come. Thus, to successfully implement this model, we have to impose the following assumptions:

- Students can alter their responses in the actual housing selection process after viewing the answers of their peers in the live poll.
- There is no financial or academic penalty for changing one's vote in the formal process in response to peers.

It is important to note that since poll action is costless and non-binding, they only serve to change players' beliefs about what others will do and not to change a player's individual payoffs.

We thus expect that rational students who prefer the ξ equilibrium (to maximize their utility) will vote so that the distribution between on-campus and off-campus housing approaches this equilibrium, increasing the likelihood of reaching the optimal outcome. As the poll progresses, rational students will react to the information given in the poll to alter their decision in the housing selection. If the number of individuals who choose to vote to live off campus approaches ξ, those who were undecided may "jump on the bandwagon," knowing that reaching this threshold maximizes their payoff. Conversely, students who strongly prefer off-campus housing may still opt for F, but their decision does not disrupt the equilibrium as long as enough students opt for on-campus housing to reach ξ.

Ideally, if every student is rational, utility-maximizing and belives in each other, once the number of O votes reaches ξ, or the number of votes for living off campus appraoches $N-\xi$, the remaining votes become unnecessary in collectively maximizing utility for the students. In the first case, any voter after seeing that the number of on campus votes equals ξ will know that on the real housing form, they should pick F. Similarly, in the second case, any voter after seeing $F = N - \xi$ will know that on the real housing form, they should pick O. If the poll convincingly signals that ξ threshold is feasible and stable, a student has no reason to unilaterally deviate on the actual sign-up.

5.3 Challenges and Limitations

Although the live polling method could provide many benefits in the coordination game, this method also presents a number of limitations: