PSET 5 and 6

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1

Set of players: $\{\text{Frank Underwood}, \text{Raymond Tusk}\} = \{p1, p2\}$.
Set of actions: $\{\text{Compromise}, \text{Not Compromise}\} = \{C, NC\}$
Action profile:$A = \{(\text{C}, \text{C}), (\text{C}, \text{NC}), (\text{NC}, \text{C}), (\text{NC}, \text{NC})\}$ We can define:

$$u_1(C,C) = 3 \quad u_2(C,C) = 3$$

$$u_1(C,NC) = 1 \quad u_2(C,NC) = 2$$

$$u_1(NC,C) = 2 \quad u_2(NC,C) = 1$$

$$u_2(NC,NC) = 0 \quad u_2(NC,NC) = 0$$

The table is as follows:

	C	NC
C	$(3,3)$	$(1,2)$
NC	$(2,1)$	$(0,0)$

We can see that the Nash Equilbrium is $(3,3)$

2

We first define the set of players as $\{1, 2\}$. We can also define the set of actions as $\{\text{Sit}, \text{Stand}\} = \{I, T\}$ and $A = \{(I, I), (I, T), (T, I), (T, T)\}$, we the note that:

$$u_i(I, D) \succ u_i(I, I)$$

with respective I, D for each person.

a

We define the payoffs as follows:

$$u_1(I,T) = 5 \quad u_2(I,T) = 0$$
$$u_1(I,I) = 3 \quad u_2(I,I) = 3$$
$$u_1(T,I) = 0 \quad u_2(T,I) = 5$$
$$u_1(T,T) = 0 \quad u_2(T,T) = 0$$

with the following table:

	I	T
I	$(3, 3)$	$(5, 0)$
T	$(0, 5)$	$(0, 0)$

We can see that the nash equilbrim is (I, I)

b

We can define the payoffs as the following:

$$u_1(I,I) = 1 \quad u_2(I,I) = 1$$
$$u_1(I,T) = 2 \quad u_2(I,T) = 3$$
$$u_1(T,I) = 3 \quad u_2(T,I) = 2$$
$$u_1(T,T) = 0 \quad u_2(T,T) = 0$$

We see that the payoff table is as follows.

	I	T
I	$(1, 1)$	$(2, 3)$
T	$(3, 2)$	$(0, 0)$

We can see that there exists no nash equilbrim.

c

Refer to above

3

We can see that:

$$B_1(L) = \{M\} \quad B_2(T) = \{L, C\}$$
$$B_1(C) = \{T\} \quad B_2(M) = \{L\}$$
$$B_1(R) = \{T\} \quad B_1(B) = \{L\}$$

Thus, we can see that that $\{M, L\}$ is the Nash equilbrim

4

a

First, define the players of the game as $\{1, 2, \ldots n\}$, where $n = 10$ and the actions that the can take as $\{Hare, Stag\}$. We now consider the following cases:

- Everyone hunts the stag
- Everyone hunts a Hare
- Without a loss of generality, assume that one person hunts a hare and everyone else hunts the stag
- Without a loss of generality, assume that one person hunts a stag and everyone hunts a hare.

We can see that if everyone hunts the stag, then we are in a Nash Equilbrium, as if one person goes to hunt a Hare, they are strictly worse off. A similar logic applies to that of everyone hunting a hare, as if one individual were to hunt the stag, then they would be strictly worse off.

Thus, we can see that if we consider the two other cases, we can see that these are not Nash Equilbrium. We can see that in the third case that if the one person hunting a hare goes to go hunt the stag, we will be better off and by symmetry a similar argueement holds for final case. Thus, the Nash Equilbrium is everyone hunting the hare of the stag.

b

We now modify the arguement here. Let k equal to number of people hunting the stag. We now consider the following cases:

- Everyone hunts the Stag
- Everyone hunts a Hare
- We have $k \geq 6$
- We have $k < 6$

By a similar arguement to that above, we know that Everyone hunting the Stag and everyone hunting the Hare is a Nash equilbrim. Now we analyze the case where $k < 6$. If $k < 6$, then k people hunting the stag can move to hunting a Hare and be strictly better off, which means that this is not a Nash Equilbrium. If $k \geq 6$, we see that each indvidual recieves $\frac{100}{k}$ dollars. Assume that $k = 10$, we see that $\frac{100}{10} = 10$, which implies for all $k \geq 6$, we see that the hunters will be strictly better off hunting the stag, and anyone currently hunting a hare can unilateraly deviate to hunting the stag and be strictly better off, so $k \geq 6$ is not a nash equilbrim. By symmetry, we see that if $k < 6$, that a similar logic is repeated but the stag hunting people will want to hunt a hare, thus making no nash equilbrium. Thus, the same nash equilbrium derived from before holds.

c

We have the following cases:

- Everyone hunts the Stag
- Everyone hunts a Hare
- We have $k = 6$
- We have $k < 6$
- WE have $k > 6$

Note that $60/6 = 10$. Thus, by the same logic as above, everyone hunting the hare and the stag is a Nash Equilbrium. Consider the case $k = 6$. If $k = 6$, we can see that each individual who hunts the stag recieves 10 dollars. If a person hunting a hare unilateraly deviates to a stag, now everyone who hunts the stag recieves $10/7 \approx 8.57 < 9$, which makes that person worse off. Simialrly, if a person hunting a stag goes to hunt a hare, we see that $10 > 9$, which makes the worse off. Thus, $k = 6$ is a nash equilbrim. By a simillar arguement to above, $k < 6$ and $k > 6$ are not nash equilbriums.

5

a

The monopolist solves:

$$\max_q P(q)q - cq$$

Note: If $a \leq p$, this implies that $q^d = 0 = p$. Thus, we see that we have to let $a > p$, which means that $q = a - p$ Thus, using this assumption, we can solve the following:

$$\max q(q-a) - qc$$

The FOCS indicate that:

$$c = 2q_m + a \iff q_m = \frac{a+c}{2}$$

Thus, we can see that $q_m = \frac{a-c}{2}$ which implies that $p_m = \frac{a+c}{2}$

b

In PE, we are concerned with the Profit Maximzation Problem where:

$$\max py - cy = \max y(p-c)$$

Using results from derived from previous homeworks, we know that $p = c$, which means that equilbrim price is c and the quantity demanded is $a - c = q^d$ Thus, we can see that:

$$\frac{a-c}{2} > c \quad \frac{a-c}{2} < a - c$$

Which meanns that the monopoly has a higher price and supplies less than that of partial equilbrim.

c

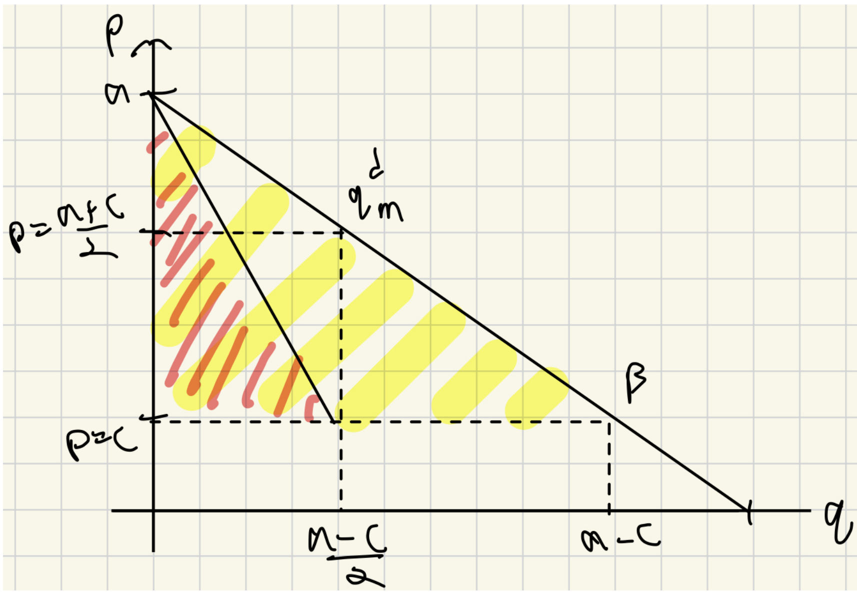

Figure 1: CS for 5

we can see that the red highlighted part corresponds to CS for the monopolist and the yellow part is the CS assocaited withthe PE case. Thus, we can see through the calculation of area that:

$$CS_{PE} = \frac{(a-c)^2}{2} \quad CS_M = \frac{(a-c)^2}{4}$$

d

We see that $q^d = p^{-2}$ Thus, we see that:

$$\frac{\partial Q}{\partial P}\frac{P}{Q} = -2p^{-3}\left(\frac{p}{q}\right) = -2$$

e

With the new demand function, we see that:

$$\max qp(q) - cq \iff q^{\frac{1}{2}} - cq$$

FOCs for optimization inidcate that

$$0.5q^{-\frac{1}{2}} = c \iff q = \frac{1}{4c^2}$$

Thus, we see that $p = 2c$. Using results from the lecture slides, we see that:

$$p^*(q_m) = \frac{c}{1 - \frac{1}{1-|\mathcal{E}|^{-1}}} = \frac{c}{\frac{1}{2}} = 2c$$

Verifies results on lecture slides.

6

a

Let q_1 and q_2 denote the output for Nobil and Chel respectively, where $q_1 + q_2 = Q$ Thus, we have the following profit functions:

$$\pi_1(q_1, q_2) = q_1(P(q_1, p_2) - 3) \quad \pi_2(q_1, q_2)q_2(P(q_1, q_2) - 2)$$

We begin with an analysis of π_1. Note that if $q_2 \geq 0.0001(5-3) = 20,000$, then $q_1^* = 0$ as q_1 produced would shut the market down. Thus, assuming that $q_2 < 20,000$, we find that:

$$\max q_1(5 - 0.00001(q_1 + q_2) - 3)$$

Take deriative with respect to q_1 and we get the following FOC:

$$5 - 0.0001(q_1 + q_2) - 3 + q_1(-0.0001) = 0$$

$$q_1^* = 10000 - 0.5q_2 \quad q_2 < 20000$$

or more specifcaly:

$$B_1(q_2) = \begin{cases} 10000 - 0.5q_2 & q_2 < 20000 \\ 0 & \text{otherwise} \end{cases}$$

and by symmetry we find that

$$B_2(q_1) = \begin{cases} 15000 - 0.5q_1 & q_1 < 30000 \\ 0 & \text{otherwise} \end{cases}$$

Thus, let $q_1^* = B(q_2)$ and see that:

$$10000 - 0.5(15000 - 0.5q_1) = q_1 \iff q_1 = \frac{10000}{3}, q_2 = \frac{400000}{3}$$

Thus, this implies that Chel owns more of the market share and the above is the equilbrim.

b

$$p = 5 - (0.0001)\left(\frac{50000}{3}\right) = \frac{10}{3}$$

thus we see that $\pi_1 = 1111\frac{1}{9}$ and $\pi_2 17777\frac{7}{9}$, which means that $\pi_2 > \pi_1$

c

Refering to section **a**, we see that Chell must produce at least 20000 units to make the other firm not produce. Thus, we can refer to a general best response function for Chell to see that:

$$\max q_1(5 - 0.0001(q_1 + q_2) - \tilde{c})$$

and assuming $q_2 < x$ where x is some constant, then:

$$q_1^* = \frac{10000(5 - \tilde{c}) - q_2}{2}$$

Thus, setting $q_1^* = 0$, we see that $\tilde{c} = 1$

7

We modify the above question as follows, as cost is variate. Note that the only possibilty where each firm will actually produce anything is when $q_i > 0$ and $Q \leq 50000$. With these assumptions, we begin with the problem:

$$\max_{q_1} q_1(5 - 00001(q_1 + q_2) - (5000 - 2q_1))$$

taking the deriative with respect to q_1 we find that:

$$3 - 0.0002q_1 - 0.0001q_2 = 0 \iff q_1 = 15000 - 0.5q_2$$

By symmetry, we find that:

$$q_2 = 15000 - 0.5q_1$$

Thus, if we let:

$$q_1 = 15000 - 0.5(15000 - 0.5q_1)$$

and solve this system of equations, we get that:

$$q_1 = q_2 = 10000$$

which is the Nash Equilbrium. note that changing 5000 to 15000 does not change anything as marginal cost stays the same.

8

a

Let $\beta \in (0,1)$ denote the fraction obtained by Firm 1. This implies that:

$$\pi_1(p_1,p_2) = \begin{cases} (p_1 - c_1)(\alpha - p_1) & p_1 < p_2 \\ \beta(p_1 - c_1)(\alpha - \beta) & p_1 = p_2 < c_2 \\ (p - 1 - c_1)(\alpha - p_1) & p_1 = p_2 \geq c_2 \\ 0 & p_1 > p_2 \end{cases}$$

$$\pi_2(p_1,p_2) = \begin{cases} 0 & p_1 < p_2 \\ (1-\beta)(p_2 - c_2)(\alpha - p_2) & p_1 = p_2 < c_2 \\ 0 & p_1 = p_1 \geq c_2 \\ (p_2 - c_2)(\alpha - p_2) & p_1 > p_2 \end{cases}$$

We wish to prove that (c_2, c_2) is the only nash equilbrim. Consider the following cases, where we analyze the best response of firm 1, and let $c_2 = p_1$, and we see that $\pi_1(c_2, c_2) = (c_2 - c_1)(\alpha - c_2)$:

- $p_1 = c_2$, then $\pi_1(p_1, c_2) = 0$
- $p_1 < c_2$ then $\pi_1(p_1, c_2) = (p_1 - c_1)(\alpha - p_1)$. Note that this quantity is increasing.

Thus, this implies that $p_1^* = c_2$. Consider firm 2's cases. Assume that $p_1 = c_2$ We see the following cases:

- if $p_2 > c_2$, $\pi_2 = 0$ as firm 2 does not have any demand.
- if $p_2 < c_2$, $\pi_2 \leq 0$, which forces a shutdown

Thus, we see that (c_2, c_2) is the only Nash Equilrbium. To prove uniqueness that this is the only nash equilbrim, consider the following cases:

1. If $p_1 > p_2 > c_2$, the firm wants to decrease their price such that $p_1 = p_2$ and thus, this takes the market.
2. If $p_1 = p_2 > c_2$, the firm wants to decrease their price such that firm 2 always wants to have a price that is less than than of p_1, which will let firm 2 take the market.
3. If $p_2 > p_1 = c_2$, this is a similar to case 1
4. If $p_2 > p_1 > c_2$ Same logic to 2
5. If $p_1 > p_2 = c_2$ firm 2's price will deviate in between p_1, p_2

6. If $p_1 \geq c_2 > p_2$, we see that firm 2's profit will be less than 0, which implies that $p_2 = 0$ as the firm chooses not to produce.
7. If $p_2 \geq c_2 > p_1$, we see that $p_1 = c_2$. thus π_1 increases
8. If $p_2 < p_1 = c_2$, thus in equilbrium, $p_2 < c_2$ so in inequilbirum $\pi_2 = 0$

Thus, we see that (c_2, c_2) is the only Nash Equilrbium.

b

We have already done most of the mundane casework. Therefore, consider the following cases:

- If $p_1 = p_2 = c_2$, we see that firm 1 will always want to decrease the price relative to c_2, which increases their profits. Thus, the given allocation is noot a Nash Equilrbium.