Random Subset Majority Voting

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1 Parable

There was once a family of five: Amy, Bob, Carol, Darren, and Emily. Every night, the family would have to decide what to order for dinner. They decided to do this using the following mechanism: they would go down the restaurant, and vote for each restaurant. They would only order from a restaurant if it got at least 3 out of 5 votes.

The family quickly realized that this mechanism split the set of restaurants into two groups: restuarants which would pass the vote, and restaurants that wouldn't. This was disappointing to many, as many members' favorite restaurants were in the latter category! Strongly liked by some, but sufficiently disliked by others that they would never pass the vote. Moreover, everyone started to get bored of the handful of restaurants that would always pass.

The family thus decided to try a new dinner-choice mechanism. Every night, three out of five people would be randomly chosen to decide dinner. The dinner restaurant would then only have to get two out of three votes.

This dramatically expanded the set of restaurants that could be ordered from, because occasionally, the 3 voting people could impose their preferences on the other 2. People sometimes had to put up with stuff they didn't like, but this was outweighted by higher variety, and being able to have the stuff they did like. The handful of restaurants that always passed were still ordered most often, but there was some chance that other stuff passed the vote also. This mechanism was also obviously still fair, and respected everyone's preferences on average. And thus, through the power of market design, everyone lived happily ever after.

2 Setup

Suppose you run a DAO, or some other form of club, with a large number of members (which we'll model as a continuum). Club members need to vote on whether to admit new applicants to the club. We'll construct a simple model of members' preferences, applicants' types, and how different voting rules affect which applicants are likely to get admitted to the club.

Each club member i has a "type" t_i, which can be thought of as a measure of "preferences" of some sort. For example, high vs low t_i's might represent:

- **Politics:** Republicans vs democrats.
- **Aesthetics:** Rock vs EDM fans.
- **Beliefs:** BTC vs ETH maxi-ness.

People with extreme positive or negative values of t_i hold strong views, people with values of t_i close to 0 are close to neutral. To simplify the math, we'll assume that t_i is distributed standard normally, with mean 0 and SD 1.

New applicants try to apply to the club, and existing members need to vote whether to admit new applicants. We assume that existing members have a simple utility function: members want to hang out with people similar to them. Thus, a member with type t_i has positive utility for admitting a new applicant with type t_j, if t_j is within Δ of i's type:

$$\left|t_j - t_i\right| \leqslant \Delta$$

For example, if $\Delta = 0.3$, then a moderate Republican with $t_i = 0.4$ would vote to admit anyone with t_j above 0.1, or below 0.7, but would try to vote down anyone more extreme in either direction.

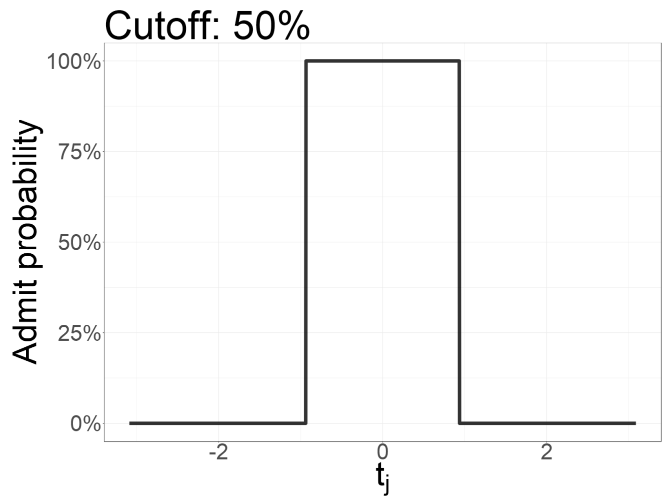
We introduce the problem as voting for "new applicants", but the model could equally well be interpreted as voting over some kind of decision. For example:

- **Politics:** j's could instead represent policy proposals. A moderate Republican voter with $t = 0.4$ would vote for any moderately republican policies, with t_j in $[0.1, 0.7]$, but not for democratic policies, or extreme right-wing policies.
- **Aesthetics:** i's might control certification for, for example, a virtual art gallery. Effectively, the members judge whether a piece of music/art/etc is "worthy" of certification by the group. A moderate EDM-oriented voter with $t = 0.4$ might vote down all rock, but be comfortable with some rock-influenced EDM ($t = 0.1$), and in the other direction would also vote down any over-technical EDM ($t > 0.7$).
- **Beliefs:** A moderate BTC maxi votes in favor of anything close to her views, as above.

The question we want to study is how different voting mechanisms affect which members get voted into the club.

3 Simple majority voting

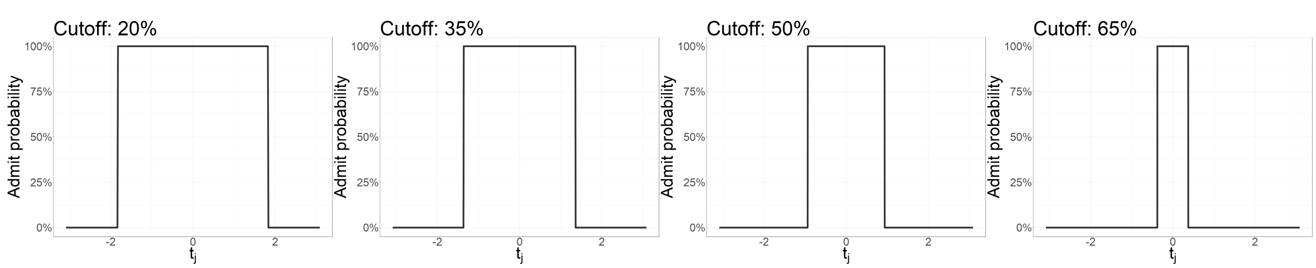
First, let's analyze the simplest possible mechanism: the whole club votes on any new applicant. The next plot shows the probability that a new member with type t_j can successfully join, as a function of t_j.



Notice that this is a rectangle! Everyone with moderate types, above around -0.93 and below around 0.93, always gets in the club. Anyone with t_j even *slightly* higher than the cutoff has *no* chance of getting in. Why is this? The reason is that, with a lot of people, simple majority voting has *no uncertainty.* Right above the cutoff, agents have just above 50% of votes, and *just* make it in. Right below the cutoff, they'll never make it in.

This seems like a very non-smooth way to make decisions about who to admit. Also, the set of members admitted has a "bias towards the middle". Since you need to be acceptable to a majority of the club to get in, extreme types will have literally no chance to ever make it in. Ironically, there are people in the club, who would never make it in, if they had to apply under majority voting.

How might we fix this? One guess might be to relax the majority voting threshold. The next plot shows how admission probabilities change when we lower the threshold: say, if we need only 30% or 10% of voters to vote in favor to join the club.



When the threshold is lower, more types join. But the admission probability plot is still a rectangle! It's still the case that you're either just above the cutoff, or just below. We've succeeded in making admissions more permissive: but the cost is that extreme applicants close to the cutoffs, who are unacceptable to many existing applicants, have just as high a chance of getting in as moderate applicants!

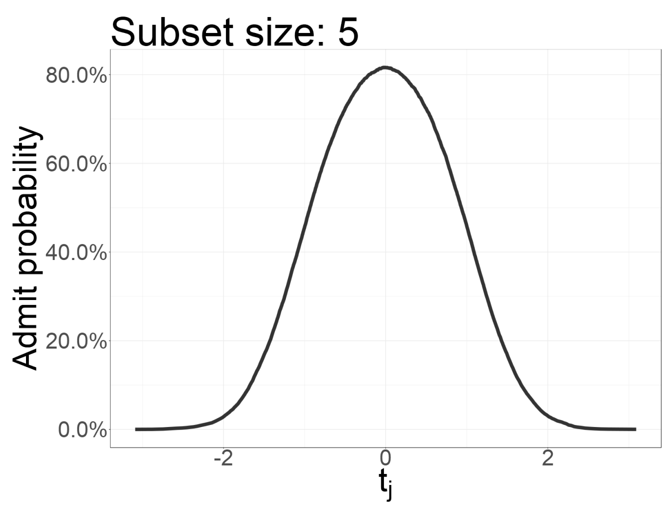
A simple question: is there any voting rule, which:

- Gives any type of applicants a chance to get in, but,
- Gives *higher chances* of getting in, to applicants who are acceptable to more of the existing club members?

We'll introduce a mechanism which accomplishes this: *random subset majority voting.*

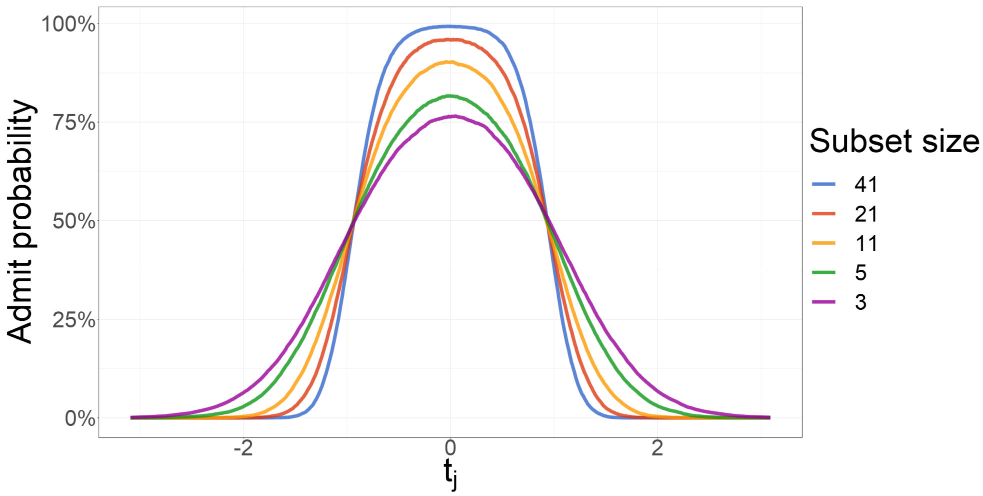
4 Random subset majority voting (RSMV)

Suppose, instead of *everyone* voting, we decide on admissions as follows. For each applicant, a random odd-numbered set of N club members is selected, and they vote on whether the applicant is admitted. The next plot shows the acceptance probability curve.



We've eliminated the rectangle! Now we have a nice, smooth curve. Intuitively, what is going here is that, under RSMV, *everyone* has a chance of getting admitted: even for applicants with very high t_j's, it's possible they'll get lucky and get a set of voters with high t_i's, and thus get in. But applicants with t_j closer to the median of club members will be *more likely* to get admitted, because they're acceptable to more club members, so it's more likely they'll get a set of voters that vote in their favor. Quantitatively, under these parameter settings, an applicant who's right in the middle, $t_j = 0$, has a very high 76.3% chance of admission, since most subsets will vote in their favor. An applicant with $t = 1$, more extreme than 84% of club members, still has a 47.2% chance of admission. An applicant with $t = 2$, more extreme than 97% of members, has a positive, but much lower – 6.5% – chance of admission.

Varying the parameters. We can also play around with the size of the subset N. The next plot below shows how admissions probabilities change as we vary N.



When N is bigger, the curve puts more weight in the middle and less at the extremes. Intuitively, the bigger N is, the more conservative decisions are: it's more likely that the subset is representative of the club as a whole, so less likely that extreme-type applicants make it into the club. As N becomes very large, everyone in the club votes, so we approach the rectangle in the first figure.

Hence, this note makes a simple point: *random subset majority voting* is a simple mechanism that, in some cases, makes "better behaved" decisions than simple majority voting. RSMV admits applicants probabilistically, depending basically on how close the applicants are to the preferences of group members. No one gets in for sure, but applicants acceptable to more group members have higher likelihood of being admitted. This is a mechanism which allows groups to do some exploration, while also ensuring that admissions decisions are "fair" and reflect equally the preferences of all existing group members on average.

5 Random subset voting in practice

- **Art contest judging.** Similarly, art, music, and other contests tend to be judged by a relatively small number of judges. This might be motivated mostly by logistics: it's infeasible to get the universe of qualified judges together for any given event. But this paper points out an interesting advantage of this mechanism: we get some randomness in who wins contests, while preserving the tastes of the field on average.

- **Academic peer review.** In academic peer review, papers are generally sent pseudo-randomly to a small number of referees, who decide whether the paper should be published or note.
- **Random examiner oral exams.** Again probably motivated mostly by logistical considerations, oral exams in various fields seem to generally have a small number of randomly selected judges.

In all of these settings, random subset majority voting is used. It's probably motivated mostly by logistical considerations: it's too costly to get everyone to vote on every possible policy/artwork/applicant/etc. But this paper suggests there's an additional benefit, that RSMV seems to make acceptance decisions in a "saner" way than simple full-sample majority.