AAE 2200: Introduction to Aerospace Engineering

Take-Home Portion of Final Exam

FLIGHT PERFORMANCE ANALYSIS
OF THE GRUMMAN AA-5B TIGER

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Lab Section: Monday 11:30

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Purpose

Aircraft performance analysis allows for both parties to understand and articulate how a certain aircraft will perform under certain conditions, and what it can and cannot physical do. This will allow for necessary design changes to an aircraft in order for it to meet certain requirements.

Background

The Grumman AA-5B has a maximum takeoff weight of 2400 pounds. It has a wing area of 140.4 square feet, a wing span of 31.5 feet, and a maximum lift coefficient of 1.61. The power plant is a Lycoming O-360-A4K air-cooled, 4-cyl, horizontally-opposed. It's Cruise SFC is .49 (lb/hr)/shp, with a usable fuel volume of 51 gallons, and it uses 100LL fuel which has a density of 6 lbs/gal.

Task I: Drag Polar

Theory:

Below is Equation 1, which is the equation used to find the total drag coefficient of the aircraft.

$$C_D = C_{D_0} + C_{D_i} \tag{1}$$

In this equation, C_D is the drag coefficient of the aircraft, C_{D_0} is the parasitic drag coefficient. which was estimated to be 0.0236 in class. Lastly, C_{D_i} is the induced drag coefficient. The induced drag coefficient can be found using Equation 2 below.

$$C_{D_i} = \frac{{C_L}^2}{\pi e AR} \tag{2}$$

Where C_L is the lift coefficient which was given by the instructor as a range values of 0 to 1.61, e is the Oswald efficiency factor, and AR is the aspect ratio of the wing. The aspect ratio of the wing can be calculate using Equation 3 below.

$$AR = \frac{b^2}{S} \tag{3}$$

Where b is the span of the wing (wingspan) and S is the surface area of the wing. The next equation is Equation 4, which shows the relationship between the lift-to-drag ratio and the ratio of the lift coefficient to the drag coefficient.

$$\frac{L}{D} = \frac{C_L}{C_D} \tag{4}$$

In this equation, $\frac{L}{D}$ represents the lift-to-drag ratio, and C_L and C_D have the same meanings that they do in Equation 1.

Results:

To find the parasitic drag of the aircraft, two methods were used in junction; Aerodynamic Cleanliness and Component Build-Up. First, the method of Aerodynamic Cleanliness was used. This method can be broken down to three main steps. Step one is to estimate the coefficient of skin friction for the aircraft using historical data. Using Table 4.2 on the Historical Drag Data pdf provided by the instructor, a skin friction coefficient of 0.0095 was selected. This value comes from the data for a PA-28 which is a plane with a very similar structural style as the Grumman AA-5B Tiger; due to it also being a single prop, low wing, fixed gear aircraft. Step two is to calculate the surface area of the wing, fuselage, and the horizontal and vertical tails from that, then take the total value. Afterwards, calculate the wetted area of each surface which in general is the previously calculated surface area multiplied by two, to account for both sides of each surface. At this point, C_{D_0} can be found by multiplying the skin friction coefficient with the total wetted area and then dividing by the total surface area. From this, I calculated a value of approximately 0.0266, however in class the value was 0.0268. Now, the Component Build-Up method is used. Using the “Drag of Airplane Components” table on page 3 of the handout provided in class, C_{D_π} values are collected for each surface of the aircraft. These values are the multiplied by the corresponding wetted surface area values, in order to get f_π values. The f_π values are then summed up. Next, 25% of f_π is added on to the total to account for landing gear. Next 10% of f_π is added to the total value to account for interference drag. Now C_{D_0} can be found again by dividing f_π by the total surface area of the plane. For this value of C_{D_0} I calculated approximately 0.0203. Finally, take the average value of the two C_{D_0} values. I calculated an average value of 0.0235, however, the value used for the class is 0.0336. Using Equation 1, a drag coefficient was calculated for each lift coefficient value, since the drag coefficient varies with the lift coefficient. After the calculations, a drag polar relating C_D vs C_L was created, and is shown below as Figure 1.

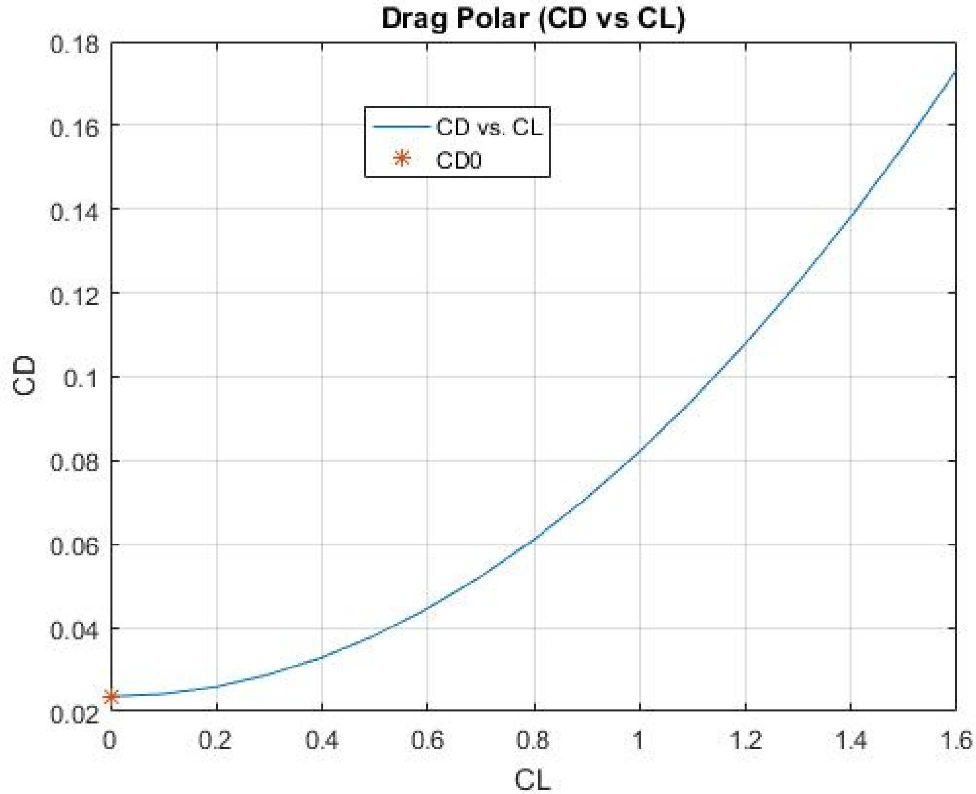

Figure 1: Drag Polar ($C_D$vs C_L)

By observing Figure 1, it can be seen that as the value of the lift coefficient increases, so does the value of the drag coefficient; however, the lift coefficient is always greater than the drag coefficient except for the condition where C_L is equal to 0. At this point, the drag coefficient is equal to only the value of the parasitic drag coefficient (marked as the red asterisk on the figure); because the induced drag coefficient depends on C_L which is equal to 0. Using the values of C_L and C_D, the lift-to-drag ratio for each value of the lift coefficient was calculated using Equation 4. After the values of $\frac{L}{D}$ were calculated, a plot of the lift-to-drag ratio vs lift coefficient was generated and can is shown as Figure 2 below.

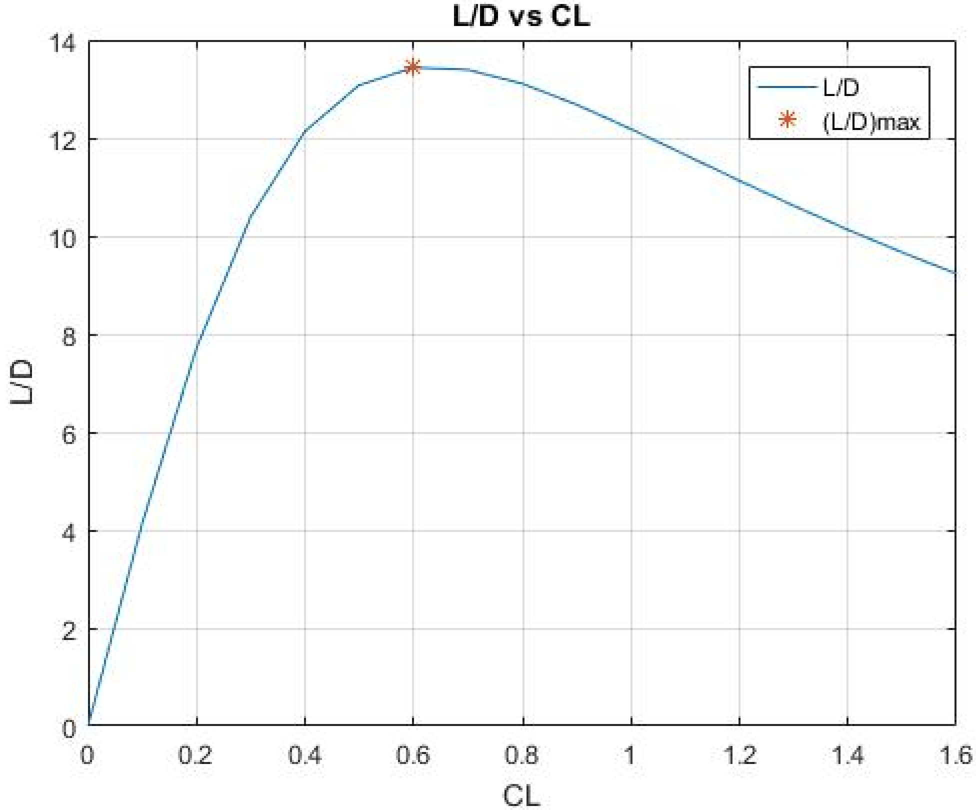

Figure 2: lift-to-drag ratio vs lift coefficient

Figure 2 shows that as the lift coefficient increases, the lift-to-drag ratio also increases, until the lift coefficient reaches a value of exactly 0.6. At this value of C_L, the lift to drag ratio is at its maximum value of approximately 13.4. Immediately after the lift-to-drag ratio reaches its maximum value $(\frac{L}{D})_{max}$, the lift-to-drag ratio begins to decrease.

Discussion:

Relating Equation 1 and Figure 1, it is proven that the drag coefficient is directly affected by the lift coefficient. As the lift coefficient increases, the induced drag increases; which increases the total drag of the aircraft. When there is no induced drag (i.e. C_L = 0) the total drag is equal to the parasitic drag of the aircraft. Figure 2 shows that the maximum lift-to-drag ratio is 13.4, and occurs at a C_L of 0.6. This means that when flying with a lift coefficient of 0.6, the aircraft will be flying most efficiently. It will get the most lift for every pound of drag force produced. This condition requires the least amount of thrust to be produced in order for the plane to fly.

Task II: Power Required

Theory:

Equation 6 below, is the equation that was used to calculate the power required as a function of velocity of the aircraft.

$$P_R = q * S * \left(C_{D_0} + \frac{{C_L}^2}{\pi eAR}\right) * V_\infty \qquad (6)$$

In this equation, S is the surface area of the wing, V_∞ is the freestream velocity, All other parameter retain their previous meanings; however q is the dynamic pressure which is represented in Equation 7 below.

$$q = \frac{1}{2} * \rho * {V_\infty}^2 \tag{7}$$

Where ρ is the freestream air density which can be found in the standard atmosphere tables appendix D of the "Introduction to Flight" text book by John D Anderson Jr; which gives conditions of the atmosphere at different altitudes, assuming standard conditions. V_∞ is the freestream velocity, and represents the same thing in Equation 6. The freestream velocity ranged from speeds of 40 to 160 knots. In Equation 6, C_L changes with altitude and airspeed and was calculated using Equation 8 below.

$$C_L = \frac{2*(\frac{w}{s})}{\rho*{V_\infty}^2} \tag{8}$$

Note that a term for the value $\frac{w}{s}$ is wing loading. Equation 8 shows that for low relatively low airspeeds, the C_L will be high due to the low air density at higher altitudes. This also means that for relatively low airspeeds, the aircraft will require more power to fly at higher altitudes.

Results:

Using Equation 6 and Equation 7, the power required as a function of velocity was calculated for the Grumman AA-5B at sea level, an altitude of 5,000 feet, and an altitude of 10,000 feet. Power required vs velocity was plotted and is shown below in Figure 3.

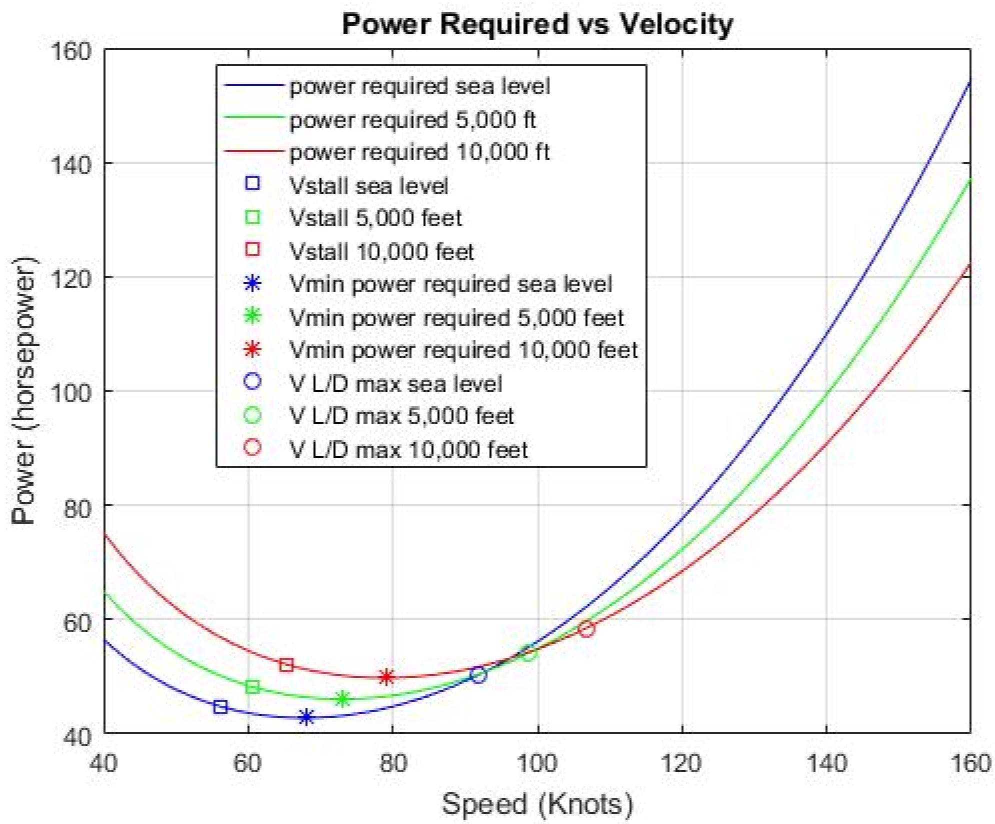

Figure 3: Power Required vs. Velocity

Figure 3 shows the power required vs. airspeed in knots for each of the fore mentioned altitudes. Figure 3 also marks the airspeed at which the aircraft will stall (denoted Vstall on the figure), the airspeed that yields the minimum power required, and the airspeed at which the lift-to-drag ratio is at its maximum value (denoted V L/D max on the figure).

Discussion:

Figure 3 shows values of different important airspeeds in terms of analyzing the flight performance of the Grumman AA-5B. V_{stall} is the airspeed at which the aircraft is flying at an angle of attack that yields $C_{L_{max}}$, and will stall out. V_{stall} is a limiting factor on how slow the aircraft can fly in given flight conditions. If V_{stall} occurs at a higher airspeed than the airspeed for minimum power required, then V_{stall} is the lowest airspeed that the plane will physically be able to fly. This same airspeed will also later be used again when analyzing takeoff and landing performance. $V_{(L/_D)max}$ is the velocity at which the maximum lift-to-drag ratio occurs and therefore is the most efficient airspeed to fly at. However, this airspeed is not the same as $V_{P_{R_{min}}}$ which is the velocity that requires the least amount of power. $V_{(L/_D)max}$ and $V_{P_{R_{min}}}$ are two different values and represent two different conditions. Figure 3 also shows that towards the back side of the power curve, it requires more power to fly at higher altitudes. Equation 6, shows that for relatively low airspeeds, it will require more power to fly at higher altitudes due to the C_L value being higher with these same conditions; due to the lower air density at the higher altitudes.

Task III: Power Available

Theory:

Equation 9, shown below, is the equation that gives the power available.

$$P_A = \eta P \tag{9}$$

Where η is the propeller efficiency and P is the maximum shaft horsepower, which had values provided for the same three altitudes stated in the previous section. The propeller efficiency is defined by Equation 10, below.

$$\eta = 0.90(1 - (\frac{35}{V_\infty})^2) \tag{10}$$

In this equation, V_∞ is the freestream velocity in knots.

Results:

P_A was plotted versus airspeed in knots to create Figure 4, which can be found in the appendix. From Figure 4, Table 1 was generated.

Table 1: Speeds from Power Curve

	Minimum Speed			Speed for L/D_{max}			Maximum Speed		
Altitude	V_∞ (kts)	P_R (hp)	P_A (hp)	V_∞ (kts)	P_R (hp)	P_A (hp)	V_∞ (kts)	P_R (hp)	P_A (hp)
Sea Level	56.2	44.8	98.7	91.7	50.3	138.6	160	154.6	154.2
5,000 ft	60.5	48.3	86.1	98.8	54.2	114.2	154	124.8	123.8
10,000 ft	65.4	52.1	71.2	106.7	58.5	93.3	142	93.6	93.0

Discussion:

The power available comes directly from the power that the power plant of the aircraft is able to provide at a given altitude. If the difference is taken between power available and power required at the same velocity and for the same altitude, then that result will be what is called excess power. Excess power is what allows the aircraft to ascend or descend using power from the power plant. Table 1 and Figure 4, show that the most excess power exists at point where the airspeed is equal to $V_{L/D_{max}}$. At the points where there is no excess power, the aircraft cannot perform a powered climb or descent. At the back end of the power curve where excess power is equal to zero, is where the minimum speed for the aircraft would be; however, since the aircraft would stall before reaching that point, then V_{stall} is the minimum speed.

Task IV: Climb Performance

Theory:

Equation 11 shows the equation used to calculate the rate of climb for the aircraft.

$$R/C = \frac{P_A - P_R}{W} \tag{11}$$

In this equation, R/C is the syntax for rate of climb, and is not a fraction. P_A is the power available, P_R is the power required, and W is the maximum takeoff weight of the aircraft, which was given as 2400 pounds. The equation used to calculate the time it takes for the aircraft to climb is represented below, as Equation 12.

$$t = trapz(\frac{1}{R/C}) \tag{12}$$

Where t represents the time to climb, $trapz()$ is a MatLab function that performs a trapezoidal numerical integration. The time to clime is equal to the integral of R/C^{-1}, and $trapz()$ returns the approximate integral of R/C^{-1} since that is the value inside the parenthesis. The next equation relates the velocity of the plane in the vertical direction with the freestream velocity. This is Equation 13, and is as follows.

$$V_v = V_\infty \sin(\theta) \tag{13}$$

Where V_v is the velocity of the plane in the vertical direction (vertical velocity), V_∞ is the freestream velocity, and θ is the climb angle. The next equation relates the velocity of the plane in the horizontal direction with the freestream velocity. This is Equation 14, and is as follows.

$$V_h = V_\infty \cos(\theta) \tag{14}$$

Where V_h is the velocity of the plane in the horizontal direction (horizontal velocity), and the other parameters are the same as in Equation 13.

Results:

Figure 5 located in the appendix, shows the Grumman AA-5B's rate of climb versus airspeed for flight conditions at sea level, 5,000 feet, and 10,000 feet. It also depicts the maximum rate of climb and the airspeeds associated with them, for each of the previously stated altitudes. The next figure, Figure 6, shows the values of the max rate of climb at their corresponding altitudes; and gives a general trend-line that was later used to calculate the time it takes to climb to 10,000 feet in this aircraft. Figure 6 is provided below.

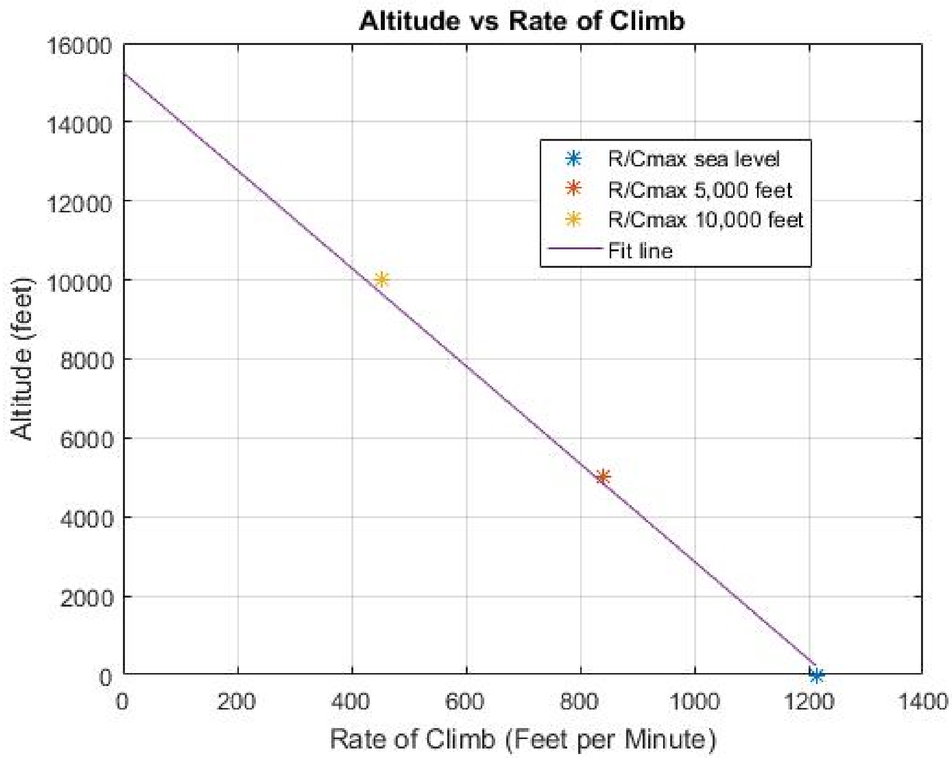

Figure 6: Altitude vs Rate of Climb

Figure 6 shows that the maximum rate of climb for the Grumman AA-5B Tiger is approximately 1,2133 feet per minute at sea level (altitude of 0 feet), 840.2 feet per minute at 5,000 feet, and 451.9 feet per minute at 10,000 feet. The data shows that as altitude is increases, the rate of climb decreases. From this figure, the absolute and service ceilings can also be observed. The absolute ceiling is the altitude at which the rate of climb is 0. From observation of the trend line, that altitude is approximately 15,500 feet. The service ceiling is the altitude that corresponds to a rate of climb of 100 feet per minute. From observation of the trend line, that altitude is approximateley13900 feet. By using Equation 12, the time to climb to 10,000 feet is calculated to be approximately 8 minutes and 36 seconds. Figure 7, is a climb hodograph for sea level climb at maximum takeoff weight.

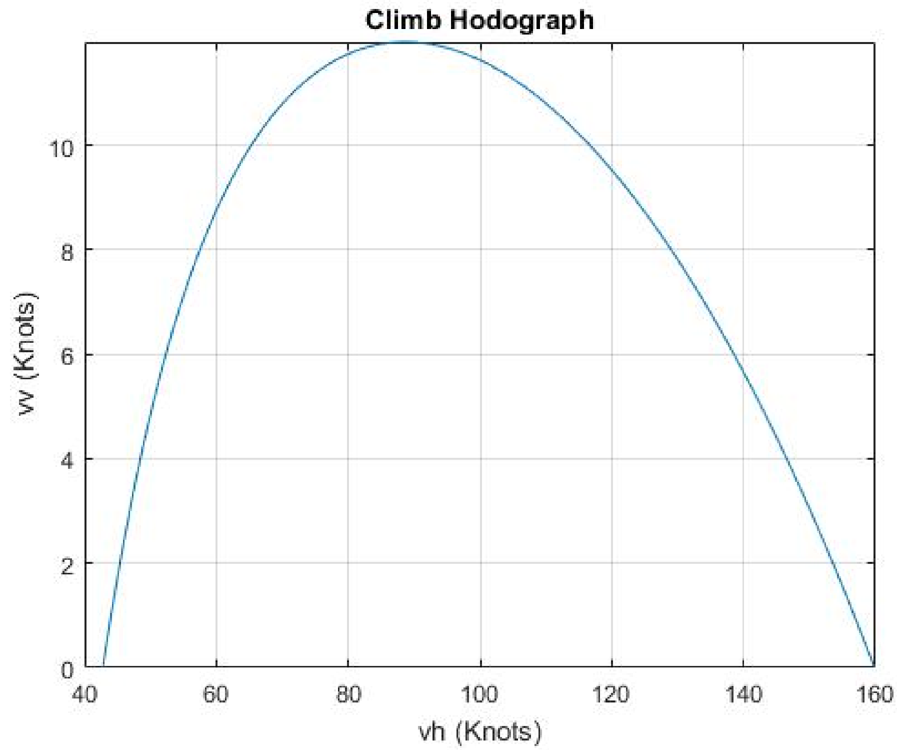

Figure 7: Climb Hodograph

The climb hodograph shows the relationship between rate of climb and horizontal velocity. From Figure 7, the best rate of climb and also the best climb angle can be determined. Data from Figure 7 was used to generate Table 2, provided below.

Table 2: Best Climb Conditions

	Rate of Climb (ft/min)	Climb Angle (degrees)	Velocity (knots)
Best Rate of Climb Condition	1,2133	7.7366	89.0
Best Climb Angle Condition	1,0811	8.7723	70.0

Table 2 shows that the best rate of climb for this aircraft is 1,2133 feet per minute, and that will occur at a climb angle of approximately 7.7 degrees. A freestream velocity of 89.0 knots at sea level yields these conditions, which are the conditions for the aircraft to produce the best rate of climb. Table 2 also shows that the best climb angle for this aircraft at sea level is approximately 8.8 degrees. This climb angle will yield a rate of climb of 1,0811 feet per minute and is produced at a freestream velocity of 70.0 knots at sea level. These conditions are needed for the aircraft to climb at its best climb angle.

Discussion:

Figure 5 and Figure 6 both show that the aircraft will have its best climb performance at lower altitudes, where the air is denser. Equation 11 and Figure 6 aligns with Figure 3 in showing that it takes more power to fly at higher altitudes at relatively low speeds, due to the low air density at those higher altitudes. When climbing, the aircraft will be travelling at lower airspeeds that its cruise velocity, so the low density of air is a direct cause of slower climb rates at higher altitudes for this aircraft. V_x and V_y are two important airspeeds for pilots when considering takeoff performance. V_x yields the highest climb angle which is useful for clearing obstacles at the end of a runway, and V_y yields the fastest rate of climb which is what a pilot would want to fly at in order to quickly get to a cruising altitude. V_x and V_y are shown in Table 2. V_x is the freestream velocity in the "Best Climb Angle Condition" row of Table 2. V_x is 70.0 knots for this airplane. V_y is the freestream velocity in the "Best Rate of Climb Condition" row of Table 2. V_y is 89.0 knots for this airplane.

Task V: Range and Endurance

Theory:

The Berguet Range equation is defined by Equation 15 below.

$$R = \frac{\eta}{c} * \left(\frac{C_L}{C_D}\right)_{max} * \log(\frac{W_0}{W_1}) \tag{15}$$

Where η is still the propeller efficiency, c is the specific fuel consumption in consistent, standard units, $\left(\frac{C_L}{C_D}\right)_{max}$ represents the maximum lift-to-drag ratio since this equation is trying to find the maximum range of the aircraft, W_0 is the weight of the aircraft, and W_1 is the weight of the fuel being used. W_1 can be found using Equation 16 below.

$$W_1 = W_0 - W_F \tag{16}$$

W_F in the equation represents the weight of the fuel. The weight of the fuel was calculated using the following equation, Equation 17

$$W_F = \rho_{fuel} * 0.9 * V \tag{17}$$

Where ρ_{fuel} is the density of the fuel in pounds (mass) per gallon, V is the usable fuel volume in gallons, and these parameters are being multiplied by 0.9 because the Range calculation in this section of the performance analysis is based around using only 90% of the fuel on board. This gives an answer in pounds (mass) however, an assumption is being made that 1 pound (mass) is equal to 1 pound (force). Next is the Endurance relation, modeled in Equation 18 below.

$$E = \frac{\eta}{c} * \frac{{C_L}^{3/2}}{C_D} * (2\rho_\infty s)^{1/2}({W_1}^{-1/2} - {W_0}^{-1/2}) \tag{18}$$

In this equation, ρ_∞ is the freestream density of the air, s is the surface area, and the other variables still hold their same meanings as in Equation 15.

Results:

After performing Equation 15, it was concluded that this aircraft has a maximum range of approximately 980.2 nautical miles. After performing Equation 18 with ρ_∞ being set equal to the freestream density of air at 10,000 feet, it was concluded that this aircraft has a maximum endurance of approximately 3 hours and 18 minutes.

Discussion:

These results determine how far and for how long the aircraft can theoretically travel in one flight, and still land safely. These parameters are paramount to the safety of the users of this aircraft. These parameters can be translated into how much fuel someone would need to complete a certain flight, and also give a n idea of when the pilot should consider landing the plane to make a stop, should their journey exceeds these parameters.

Task VI: Gliding Flight

Theory:

The forces of gliding flight in the direction parallel with the glide path are represented below in Equation 19.

$$\Sigma F = W sin(\theta) - D \tag{19}$$

Where W is the aircraft weight, θ is the glide path angle, and D is the drag force. The forces of gliding flight in the direction perpendicular with the glide path are represented below in Equation 20.

$$\Sigma F = L - W cos(\theta) \tag{20}$$

Where L is the lift force, and the other parameters remain the same as in the previous equation. These forces are both set equal to 0 because the glide is assumed to not be accelerated. When the ratio of Equation 19 and Equation 20 is taken, an expression to find the glide angle can be derived. The equation is Equation 21, shown below.

$$\theta = tan^{-1}(\frac{1}{L/D}) \tag{21}$$

Where all the variables still represent the same parameters, as in the previous equations. The freestream velocity during a glide is represented as Equation 22, below.

$$V_\infty = \sqrt{\frac{2*W*coS(\theta)}{\rho_\infty*S*C_L}} \tag{22}$$

Where V_∞ is the freestream velocity of the glide path angle, S is the surface area, ρ_∞ is the freestream sir density and C_L is the lift coefficient. Using Equation 13 and Equation 14, V_v and V_h can be calculated, respectively.

Results:

Figure 8, provided below, is the glide hodograph that was constructed using the above equations, assuming that the aircraft is at maximum takeoff weight and at an altitude of 5,000 feet.

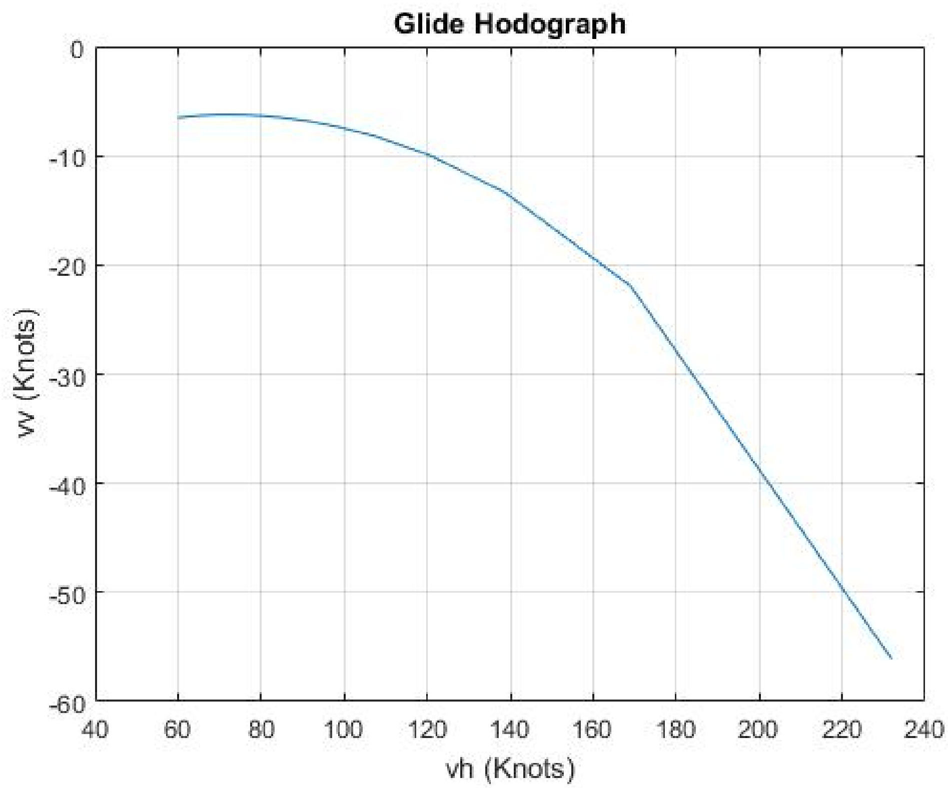

Figure 8: Glide Hodograph

V_v in this case is also known as the sink rate. As shown by Figure 8, the sink rate becomes more negative as the horizontal velocity increases.

Discussion:

Glide performance dictates how the aircraft would perform if the engines were not active. This could be due to an engine failure perhaps. For example, if the Grumman AA-5b were to experience complete engine failure at a pressure altitude of 10,000 feet, it would be able to glide for approximately 22 nautical miles until reaching sea level. This means that if it were to experience this engine failure at 10,000 feet, the pilot would be able to land it on a runway that is at sea level at a range of 22 nautical miles away. From the start of the glide. To maximize the glide distance, the pilot should fly at an indicated airspeed of 98 knots.

Task VII: Turning Flight

Theory:

Equation 23 below, represent the equation for load factor on a wing.

$$n = \frac{\frac{1}{2} * \rho_\infty * V_\infty^2 * C_{L_{max}} * S}{W} \quad (23)$$

Where n is the load factor, $\rho_\infty$is the freestream air density, $V_\infty$is the freestream velocity, $C_{L_{max}}$ is the maximum lift coefficient, S is the surface area of the wing, and W is the weight of the aircraft. The load factor was provided as being a maximum positive value of 3.8 and a maximum negative value of -1.52; and V_∞ is defined as a range of 0 to V_{ne} (never exceed speed) with V_{ne} being equal to 178 knots.

Results:

After calculating the range of load factors as a function of airspeed, Figure 9 was generated by plotting load factor versus airspeed. Figure 9 is provided below.

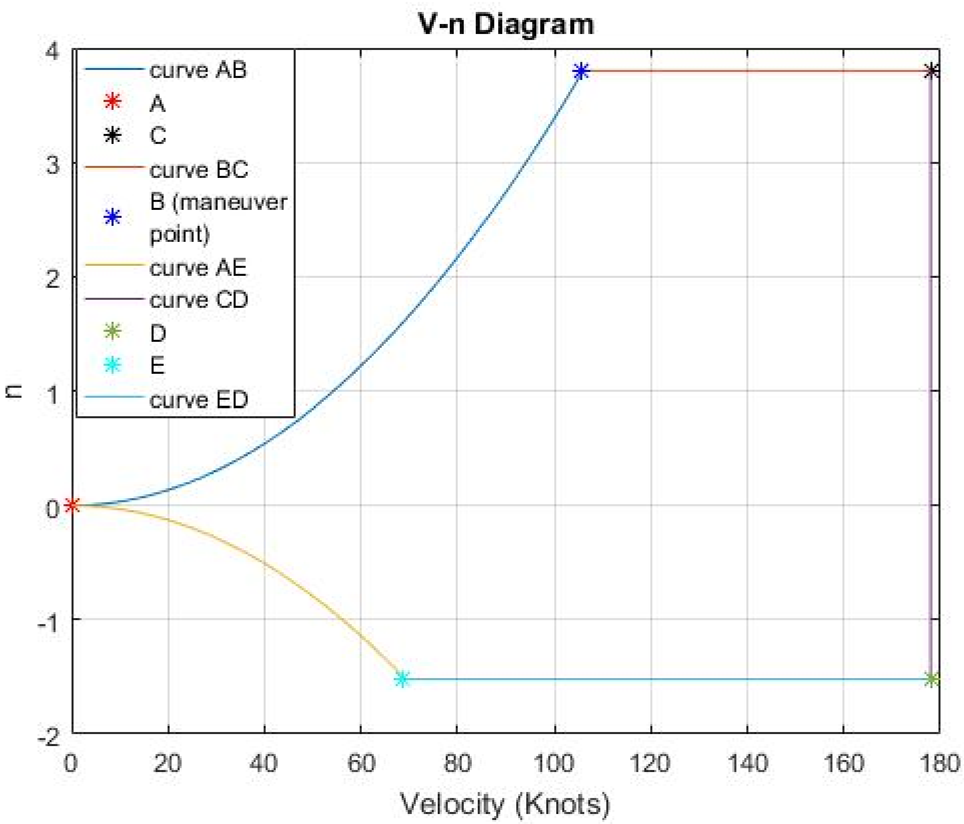

Figure 9: V-n Diagram

The V-n diagram creates something called a flight envelope that contains safe flight conditions for the Grumman AA-5B. If they aircraft is operating within and up to (inclusive) the boundary curves, it will not stall or incur structural damage.

Discussion:

Figure 9, creates what is called a flight envelope. The Grumman AA-5B should only operate within and up to the area bound by each of the curves in the figure. The maneuver point indicates the condition at which the aircraft will be able to make a turn with the smallest turn radius and at the maximum rate. The airspeed associated with this point is denoted as V^*. V^* is known as the maneuver speed. For load factors on curve AB, ranging from an airspeed of 0 to the maneuver speed, if the aircraft is moving too slowly, it could stall. If it flies faster than V_{ne} then it will begin to have structural damage. This will also be true if it is flying at an airspeed between the maneuver speed and V_{ne} , and manages to increase its load factor. For load factors on curve AE, ranging from an airspeed of 0 to point D on the figure, if the aircraft is moving too slowly, it could stall. It will begin to have structural damage if it flies faster than V_{ne} . It will also encounter structural damage if it is flying at an airspeed between point D and V_{ne} , and manages to increase its load factor.

Task VIII: Takeoff and Landing Performance

Theory:

Equation 24, shown below, is the equation used to calculate the takeoff ground roll distance.

$$S_{TO} = \left(\frac{W}{2*g_0*F_{AVG}}\right) * {V_{TO}}^2 \tag{24}$$

Where S_{TO} represents the takeoff ground roll distance, g_0 is the acceleration due to gravity and was assumed to be a constant value of 32.17 $^{ft}/_{s^2}$. V_{TO} is the takeoff speed, which is defined by Equation 25 below.

$$V_{TO} = 1.2 * V_{stall} \tag{25}$$

Equation 25 shows that the takeoff speed is 20% greater than the stall speed of the aircraft. Referring back to Equation 24, F_{AVG} is the forces that act on the airplane during takeoff evaluated at an average velocity which is equal to 0.707 multiplied by the takeoff speed. F_{AVG} is represented below in Equation 26.

$$F_{AVG} = T - D_{AVG} - R_{AVG} \tag{26}$$

In this equation, T represents the thrust being produced, D_{AVG} is the average drag force, and R_{AVG} is the average force of rolling resistance. The equation for thrust is shown below as Equation 27.

$$T = \frac{P_A}{V_{TO}} \tag{27}$$

In this equation, all the parameters retain their previous meanings. D_{AVG} is represented by the following equation, Equation 28.

$$D_{AVG} = \frac{1}{2} * \rho * (0.707 * V_{TO})^2 * S * (C_{D_0} + \phi * (\frac{{C_{L_{OPT}}}^2}{\pi eAR})) \tag{28}$$

In Equation 28, S is the surface area of the wing as opposed to distance. ϕ is a variable that accounts for ground effect which occurs during takeoff, and $C_{L_{opt}}$ is the lift coefficient that will yield the best performance during takeoff. ϕ is modeled below in Equation 29, and $C_{L_{opt}}$ is modeled in Equation 30.

$$\phi = \frac{16*(\frac{h}{b})^2}{1+(16*(\frac{h}{b})^2)} \tag{29}$$

Where h is the height of the wing above the ground, and b is the wingspan. The height of the wing above the ground was measured to be 3.52 feet. $C_{L_{opt}}$ is modeled below in Equation 30.

$$C_{L_{OPT}} = \frac{\mu\pi eAR}{2\phi} \tag{30}$$

In Equation 30, μ is the takeoff coefficient of friction, which was 0.02 due to the assumption that the takeoff will occur on a paved runway. All other parameters retain their previous meanings. Referring to Equation 26, R_{AVG}, the rolling resistance force, is shown below with Equation 31.

$$R_{AVG} = \mu * (W - (\frac{1}{2} * \rho * (0.707 * V_{TO})^2 * S * C_{L_{OPT}}) \tag{31}$$

The equation that represents landing ground roll distance is very similar to Equation 24. One important difference to note is that during landing, μ is now assumed to be a value of 0.4, to account for the aircraft applying brakes while landing on the paved runway. This will directly impact $C_{L_{OPT}}$. $C_{L_{OPT}}$ will become relevant again in later equations for calculating the landing ground roll distance, due to the physics determining the distance being very similar to that of the physics that determines takeoff ground roll distance. Refer to Equation 30 to clarify the impact that μ will have on $C_{L_{OPT}}$ if necessary. The equation that represents landing ground roll distance is shown below as equation 32.

$$S_L = \left(\frac{W}{2 * g_0 * F_{AVG}}\right) * {V_L}^2 \tag{32}$$

In this equation, S_L represents the landing ground roll distance. V_L is the landing speed, which is defined by Equation 33 below.

$$V_L = 1.3 * V_{stall} \tag{33}$$

Equation 33 shows that the takeoff speed is 30% greater than the stall speed of the aircraft. In Equation 32, F_{AVG} has a different definition than in Equation 24. It is shown below, in Equation 34.

$$F_{AVG} = -D_{AVG} - R_{AVG} \tag{34}$$

D_{AVG} and R_{AVG} now have slight changes in their definitions. They will now use the landing speed instead of the takeoff speed. These definitions are provided below. First, is D_{AVG} represented as Equation 35.

$$D_{AVG} = \frac{1}{2} * \rho * (0.707 * V_L)^2 * S * (C_{D_0} + \phi * (\frac{{C_{L_{OPT}}}^2}{\pi eAR})) \tag{35}$$

This equation shows that the landing speed is used in place of the takeoff speed. The same thing can be observed in Equation 36 below, which defines R_{AVG} for this aircraft during a landing.

$$R_{AVG} = \mu * (W - (\frac{1}{2} * \rho * (0.707 * V_L)^2 * S * C_{L_{OPT}}) \tag{36}$$

Note that all other variables in Equation 32 represent the same parameters as they do in Equation 24.

Results:

Using Equation 24, takeoff ground roll distance at maximum takeoff weight at standard sea level conditions was calculated to be approximately 963.6 feet. Using the same equation, the takeoff ground roll distance at maximum takeoff weight at 5,000 feet was calculated to be 1,479.9 feet. Using Equation 32, landing ground roll distance at maximum weight and standard sea level conditions was calculated to be approximately 644.5 feet. Using Equation 33 again, landing ground roll distance at maximum weight at 5,000 feet was calculated to be approximately 748.0 feet.

Discussion:

Comparing Equation 25 and Equation 33, it is observed that the takeoff speed is 10% less than that of the landing speed. These parameters were established by the FAA and are not physical properties of the aircraft. They are used to ensure safe takeoff and landing procedures. If an aircraft is operating close to its stall speed, it is in danger of not being able to be controlled, due to the flow separation occurring over the control surfaces. Altitude has an impact on the takeoff and landing performance of aircraft. Higher altitudes yield longer distances for both takeoff and landing compared to a lower altitude. The altitude effects could be simulated by the ambient air temperature wherever the plane is taking off or landing; and should be considered very carefully. It is important that the runway being used is long enough for the aircraft to safely takeoff and land. The length needed to ensure this can be calculated prior to takeoff and landing, given certain conditions.

Conclusions

Flight performance analysis directly impacts the users of aircraft and also those who design aircraft. Aircraft performance analysis also allows for suggestions to be made in terms of how to operate the aircraft safely or how to maximize its performance in various situations. Aircraft performance analysis gives an understanding of the practical uses of the aircraft being analyzed. From doing this project I learned that $V_{L/D_{max}}$occurs at the point of maximum excess power on the power curve; and also how important the lift-to-drag ratio is for an aircraft. I also learned what a V-n diagram and a flight envelope is, and how they articulate performance restraints of an aircraft. One more thing I learned is how theoretical equations and analysis turn into practical information to be distributed.

References

Anderson, John D., Jr. Introduction to Flight, 8thedition, McGraw-Hill, Boston, 2016.

Appendix A: Figures

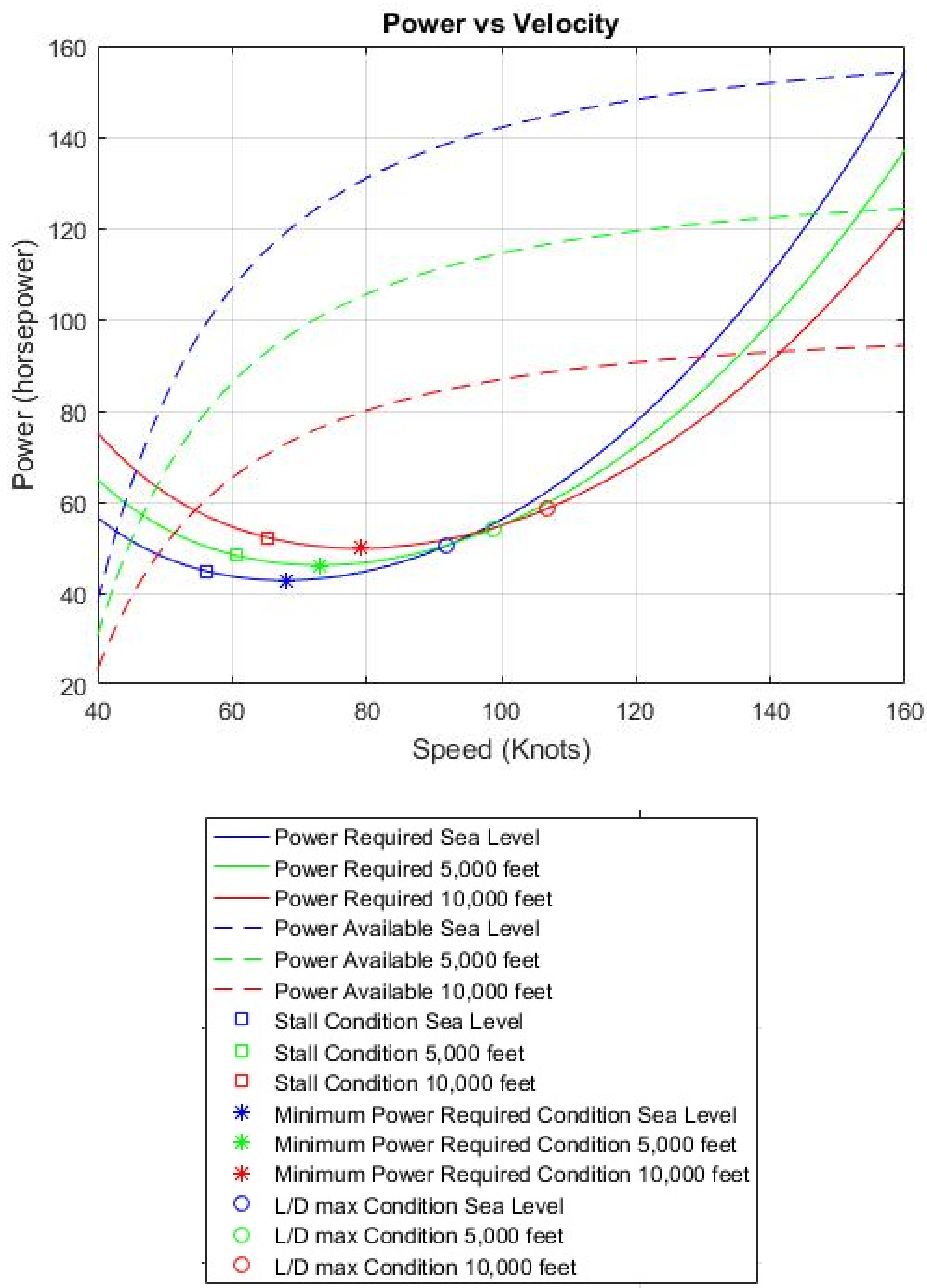

Figure 4: Power Curves

Figure 4 shows the power available vs power required for the Grumman AA-5B at sea level, 5,000 feet, and 10,000 feet. The difference between two points at a specific airspeed for each corresponding curve is the excess power for that airspeed at that altitude. The key for this figure is shown separately in order to make both the figure, and the key readable.

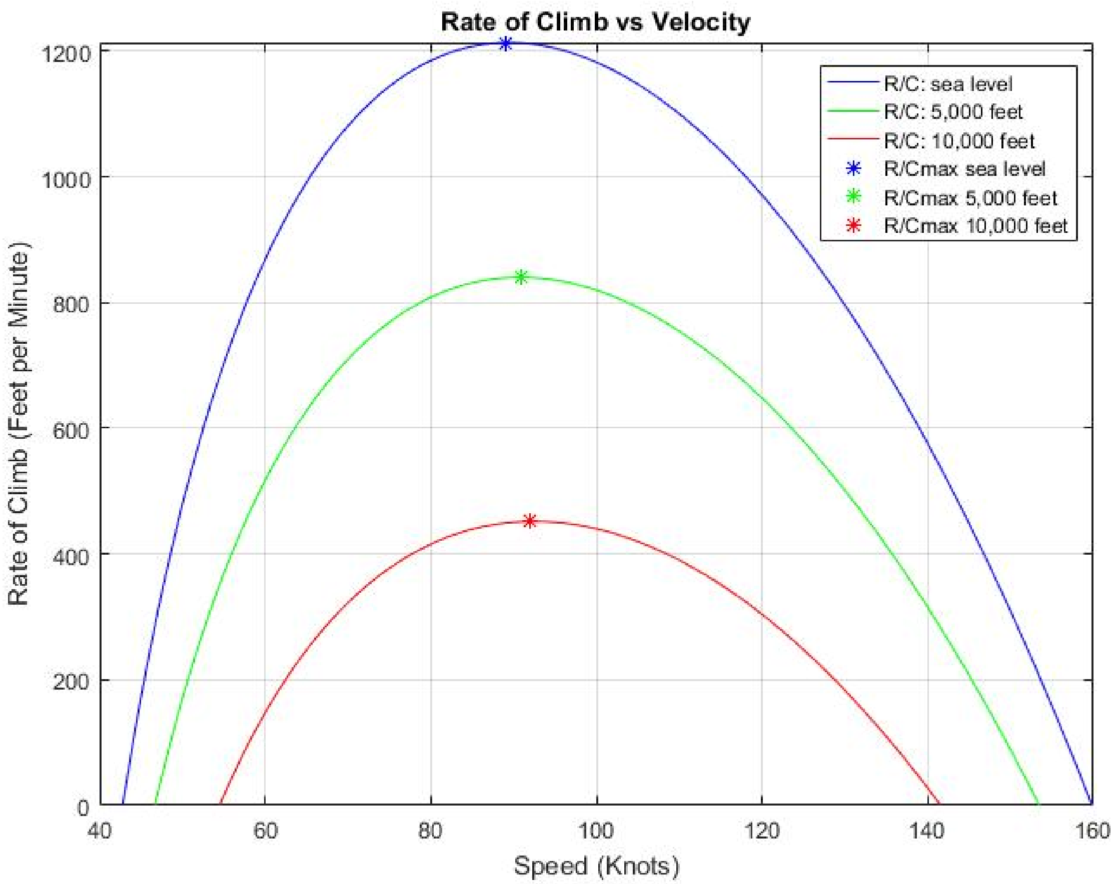

Figure 5: Rate of Climb vs. Velocity

Figure 5 shows The rate of climb for the Grumman AA-5B at sea level, 5,000 feet, and 10,000 feet. This figure shows that higher altitudes have an adverse impact on the rate of climb.

Appendix B: MatLab Code

```
%Anthony Jones
%7/12/2017

close all;
clear;
clc;

%variables that won't change:
S = 140.4; %square feet
b = 31.5; %feet
CD0 = 0.0236; %unitless
eff = 0.77; %unitless
AR = b^2 / S; %unitless
weight = 2400; %pounds
wl = weight / S; %wing loading
%CL_MAX = 1.6

%%
%task 1: plot drag polar & L/D vs CL

%CD = CD0 + (CL^2 / (pi * eff * AR))
CL = [0:.1:1.61];

%get drag coefficients
for i = 1:1:size(CL,2)

    CD(i) = CD0 + ((CL(i))^2 / (pi * eff * AR));

end

%plot drag polar
figure(1);
plot(CL,CD);
hold on;
grid on;
plot(CL(1), CD(1), '*');
hold off;
title('Drag Polar (CD vs CL)');
xlabel('CL');
ylabel('CD');

%get L/D
for i = 1:1:size(CL,2)
```

```
    LD(i) = CL(i) / CD(i);

end

[N,I] = max(LD); %I will give me the index of LD max so I can plot it
%value of LD max is N

%plot L/D vs CL
figure(2);
plot(CL,LD);
hold on;
grid on;
plot(CL(I), LD(I), '*');
hold off;
title('L/D vs CL');
xlabel('CL');
ylabel('L/D');

%%
%{
task 2:
Graph power required vs Velocity.
Locate the stall speed, speed for minimum PR, and
speed for maximum lift-to-drag ratio on this graph
for each altitude.
%}

%{
power required equation:

PR = q * S * (CD0 + (CL^2 / (pi * eff * AR))) * v
%}
%q = (1/2) * rho * v^2
%rho =  P / R * T;

R_sl = 1716; % ft-lb / slug deg.R
T = [518.69, 500.86, 483.04];
Press = [2116.2, 1760.9, 1455.6];

%values for rho:
for i  = 1:1:3

    rho(i) = Press(i) / (R_sl * T(i));

end
```

```
v_kts = [40:160];

%v in feet per second conversion
for i = 1:1:size(v_kts,2)

    v_fps(i) = (v_kts(i) * 6076) / 3600;

end

for i = 1:1:size(v_fps,2)

    %dynamic pressure for each altitude depending on speed
    q_sl(i) = (1/2) * rho(1) * (v_fps(i))^2;
    q5k(i) = (1/2) * rho(2) * (v_fps(i))^2;
    q10k(i) = (1/2) * rho(3) * (v_fps(i))^2;

    %lift coefficient for each altitude depending on speed
    CL_sl(i) = (2 * wl) / (rho(1) * (v_fps(i))^2);
    CL5k(i) = (2 * wl) / (rho(2) * (v_fps(i))^2);
    CL10k(i) = (2 * wl) / (rho(3) * (v_fps(i))^2);

end

%Power Required for each altitude depending on speed
for i = 1:1:size(v_fps,2)

PR_sl(i) = (q_sl(i) * S * (CD0 + (((CL_sl(i))^2) / (pi * eff * AR))) * v_fps(i))/550;
PR5k(i) = (q5k(i) * S * (CD0 + (((CL5k(i))^2) / (pi * eff * AR))) * v_fps(i))/550;
PR10k(i) = (q10k(i) * S * (CD0 + (((CL10k(i))^2) / (pi * eff * AR))) * v_fps(i))/550;

end

%{
Locate the stall speed, speed for minimum PR, and
speed for maximum lift-to-drag ratio on this graph for each altitude.
%}

%stall speed
v_stall_sl = sqrt((2*wl)/(rho(1)*1.6)); %feet per second
v_stall5k = sqrt((2*wl)/(rho(2)*1.6));
v_stall10k = sqrt((2*wl)/(rho(3)*1.6));

%speed for minimum PR
[minPR_sl,index1] = min(PR_sl);
[minPR5k,index2] = min(PR5k);
[minPR10k,index3] = min(PR10k);
```

```
%the velocities at these indicies will be the speed for min PR at altitude

%speed for maximum lift-to-drag ratio
v_LDmax_sl = sqrt((2*wl)/(rho(1)*CL(I))); %feet per second
v_LDmax5k = sqrt((2*wl)/(rho(2)*CL(I)));
v_LDmax10k = sqrt((2*wl)/(rho(3)*CL(I)));
%**CL(I) = coefficient of lift where L/D is at its max value**

%convert to knots for plot:
kts_vStall_sl = (v_stall_sl * 3600) / 6076;
kts_vStall5k = (v_stall5k  * 3600) / 6076;
kts_vStall10k = (v_stall10k  * 3600) / 6076;

kts_vLDMax_sl = (v_LDmax_sl * 3600) / 6076;
kts_vLDMax5k = (v_LDmax5k * 3600) / 6076;
kts_vLDMax10k = (v_LDmax10k * 3600) / 6076;

%find corresponding PR
qStall_sl = (1/2) * rho(1) * v_stall_sl^2;
PRstall_sl = (qStall_sl * S * (CD0 + ((1.6^2) / (pi * eff * AR))) * v_stall_sl)/550;

qStall5k = (1/2) * rho(2) * v_stall5k^2;
PRstall5k = (qStall5k * S * (CD0 + ((1.6^2) / (pi * eff * AR))) * v_stall5k)/550;

qStall10k = (1/2) * rho(3) * v_stall10k^2;
PRstall10k = (qStall10k * S * (CD0 + ((1.6^2) / (pi * eff * AR))) * v_stall10k)/550;

qLDmax_sl = (1/2) * rho(1) * v_LDmax_sl^2;
CL_LDmax_sl = (2 * wl) / (rho(1) * (v_LDmax_sl^2));
PR_LDmax_sl = (qLDmax_sl * S * (CD0 + (((CL_LDmax_sl)^2) / (pi * eff * AR))) *
v_LDmax_sl)/550;

qLDmax5k = (1/2) * rho(2) * v_LDmax5k^2;
CL_LDmax5k = (2 * wl) / (rho(2) * (v_LDmax5k^2));
PR_LDmax5k = (qLDmax5k * S * (CD0 + (((CL_LDmax5k)^2) / (pi * eff * AR))) *
v_LDmax5k)/550;

qLDmax10k= (1/2) * rho(3) * v_LDmax10k^2;
CL_LDmax10k = (2 * wl) / (rho(3) * (v_LDmax10k^2));
PR_LDmax10k = (qLDmax10k * S * (CD0 + (((CL_LDmax10k)^2) / (pi * eff * AR))) *
v_LDmax10k)/550;
%%
%task 3: determine the power available at the three altitudes.

%PA = nP <-P is different for each altitude
%n = 0.90 * (1-((35/v_kts)^2))
```

```
%propeller efficiency (n) values:
for i = 1:1:size(v_kts,2)

    n(i) = 0.90 * (1-(35/v_kts(i))^2);

end

P = [180, 145, 110];

for i = 1:1:size(v_kts,2)

    PA_sl(i) = n(i) * P(1); %horsepower
    PA5k(i) = n(i) * P(2);
    PA10k(i) = n(i) * P(3);

end


figure(3);
%power required
plot(v_kts, PR_sl,'b');
hold on;
grid on;
plot(v_kts, PR5k,'g');
plot(v_kts, PR10k,'r');
%power available
plot(v_kts, PA_sl,'--b');
plot(v_kts, PA5k,'--g');
plot(v_kts, PA10k,'--r');
%v_stall
plot(kts_vStall_sl,PRstall_sl,'bs');
plot(kts_vStall5k,PRstall5k,'gs');
plot(kts_vStall10k,PRstall10k,'rs');
%v_PRmin
plot(v_kts(index1),PR_sl(index1),'b*');
plot(v_kts(index2),PR5k(index2),'g*');
plot(v_kts(index3),PR10k(index3),'r*');
%v_LDmax
plot(kts_vLDMax_sl,PR_LDmax_sl,'bo');
plot(kts_vLDMax5k,PR_LDmax5k,'go');
plot(kts_vLDMax10k,PR_LDmax10k,'ro');
hold off;
title('Power vs Velocity');
xlabel('Speed (Knots)');
ylabel('Power (horsepower)');
```

```
%will help with finding max conditions
for i = 1:1:size(PR_sl,2)

   excess_sl(i) = PA_sl(i) - PR_sl(i);
   excess5k(i) = PA5k(i) - PR5k(i);
   excess10k(i) = PA10k(i) - PR10k(i);

end

%%
%task 4:

%Rate of climb:
%RC = (PA - PR) / weight [feet / second]
%RC_fpm = RC * 60 [feet / minute]

for i = 1:1:size(PA_sl,2)

    RC_sl(i) = ((PA_sl(i) - PR_sl(i)) / weight) * 550; %feet per second
    RC5k(i) = ((PA5k(i) - PR5k(i)) / weight) * 550;
    RC10k(i) = ((PA10k(i) - PR10k(i)) / weight) * 550;

end

for i = 1:1:size(RC_sl,2)

    RC_fpm_sl(i) = RC_sl(i) * 60; %feet per minute
    RC_fpm5k(i) = RC5k(i) * 60;
    RC_fpm10k(i) = RC10k(i) * 60;

end

[RCmax_sl, ind] = max(RC_fpm_sl);
[RCmax5k, ind1] = max(RC_fpm5k);
[RCmax10k, ind2] = max(RC_fpm10k);

%part a
figure(4);
plot(v_kts,RC_fpm_sl,'b');
hold on;
grid on;
plot(v_kts,RC_fpm5k,'g');
plot(v_kts,RC_fpm10k,'r');
plot(v_kts(ind),RCmax_sl,'b*');
plot(v_kts(ind1),RCmax5k,'g*');
```

```
plot(v_kts(ind2),RCmax10k,'r*');
ylim([0 RCmax_sl]);
xlabel('Speed (Knots)');
ylabel('Rate of Climb (Feet per Minute)');
title('Rate of Climb vs Velocity');


RC_max_vals = [RCmax_sl, RCmax5k, RCmax10k, 0];
alt = [0, 5000, 10000, 15000];
f = polyfit(RC_max_vals, alt,1);
x1 = RC_max_vals;
y1 = polyval(f, x1);

%part b
figure(5);
plot(RCmax_sl,0,'*');
hold on;
grid on;
plot(RCmax5k,5000,'*');
plot(RCmax10k,10000,'*');
plot(x1,y1);
xlabel('Rate of Climb (Feet per Minute)');
ylabel('Altitude (feet)');
title('Altitude vs Rate of Climb');

%part c
time_to_climb = trapz(1./RC10k)

%part d
for i = 1:1:size(v_kts,2)

    theta(i) = asind(RC_sl(i)/v_fps(i));

end

for i = 1:1:size(v_kts,2)

    vh(i) = v_fps(i) * cosd(theta(i));

end

for i = 1:1:size(v_kts,2)

    vh_kts(i) = (vh(i) * 3600) / 6076;
    vv(i) = (RC_fpm_sl(i) * 60) / 6076;
```

```
end

figure(6);
plot(vh_kts,vv);
ylim([0 max(vv)]);
grid on;
xlabel('vh (Knots)');
ylabel('vv (Knots)');
title('Climb Hodograph');
%best rate of climb: vv(max)
%best climb angle: theta(max)

%%
%{
task 5: Use the Breguet Range and Endurance relations
to determine the maximum range and endurance while cruising
at 10,000 ft and using 90% of the fuel on board.
%}
%{
Max Range:
R = (eta / c) * (CL/CD)[max] * ln(weight/W1)

Max Endurance:
E = (eta / c) * ((CL^3/2)/CD) * (2 * rho * S)^1/2 * (W1^(-1/2) - W0^(-1/2))
%}

eta = .9;
SFC = .49;
c = SFC / (550 * 3600);
WF = 6 * (.9 * 51);
W1 = weight - WF;

%Max Range @ 10,000 feet using 90% fuel on board:
Range = (eta / c) * LD(I) * log(weight/W1);
Range_nm = Range / 6076;
%Max Endurance @ 10,000 feet using 90% fuel on board::
E = (eta / c) * (((CL(I)^(3/2)))/CD(I)) * (2 * rho(3) * S)^1/2 * (W1^(-1/2) - weight^(-1/2));
E_hours = E / 3600;


%%
%task 6:Construct a glide hodograph

%theta_glide = atand(1/LD);
%v_glide = sqrt((2*wl*cosd(theta_glide)) / (rho*CL));
%vv_glide = v_glide * sind(theta_glide);
```

```
%vh_glide = v_glide * cosd(theta_glide);

for i = 1:1:size(LD,2)

    theta_glide(i) = atand(1/LD(i));

end

for i = 1:1:size(theta_glide,2)

   v_glide(i) = sqrt((2*wl*cosd(theta_glide(i))) / (rho(2)*CL(i)));

end

for i = 1:1:size(v_glide,2)

    vv_glide(i) = v_glide(i) * sind(theta_glide(i)); %fps
    vh_glide(i) = v_glide(i) * cosd(theta_glide(i));

end

for i = 1:1:size(v_glide,2)

    kts_vv_glide(i) = (-vv_glide(i) * 3600) / 6076;
    kts_vh_glide(i) = (vh_glide(i) * 3600) / 6076;

end

%part a:
figure(7);
plot(kts_vh_glide(2:17),kts_vv_glide(2:17));
grid on;
xlabel('vh (Knots)');
ylabel('vv (Knots)');
title('Glide Hodograph');

%part b:
max_glide_range = 10000 * LD(I) %feet
m_nm = max_glide_range / 6076
%v for max range is same v for theta min.
%the index for theta min is same index as LD max:
v_mgr = (v_glide(I) * 3600) / 6076 %knots

%%
%task 7: Create a V-n diagram for standard sea level conditions
```

```
Vne = 178; %kts
Vne_fps = (Vne * 6076) / 3600;

v1 = [0:178];
%load_factor = -1.52:3.8;
maneuver_speed = sqrt( (2 * 3.8 * wl) / (rho(1) * CL(17)) );
v2 = 0:maneuver_speed;

for i = 1:1:length(v1)

    vfps(i) = (v1(i) * 6076) / 3600;
    %lift_coefficient(i) = (2*wl) / (rho(1)*(vfps(i))^2);
    load_factor(i) = ((1/2)*rho(1)*(vfps(i))^2*CL(I))/wl;
    v2_kts(i) = v2(i) * 3600 / 6076;

end

maneuver_speed_kts = maneuver_speed * 3600 / 6076;%higher than Vne
R_min = (2 * weight) / (rho(1) * 1.6 * 32.17 * S);
man_speed = sqrt( (2 * -1.52 * wl) / (rho(1) * -CL(17)) );
v3 = 0:man_speed;
for i = 1:1:length(v3)

    neg_load_factor(i) = ((1/2)*rho(1)*(v3(i))^2*CL(17))/wl;
    v3_kts(i) = v3(i) * 3600 / 6076;

end

figure(8);
plot(v2_kts, load_factor);
hold on;
grid on;
plot(0,0,'r*');
plot(Vne,3.8,'k*');
plot([max(v2_kts),Vne], [3.8,3.8]);
plot(max(v2_kts), 3.8, 'b*'); %gives good looking plot
%negative load factor?
plot(v3_kts, -neg_load_factor);
plot([Vne,Vne], [3.8,-1.52]);
plot(Vne,-1.52,'*');
plot(max(v3_kts), -1.52, 'c*');
plot([max(v3_kts),Vne], [-1.52,-1.52]);
hold off;
title('V-n Diagram');
xlabel('Velocity (Knots)');
ylabel('n');
```

```
%%
%task 8: Takeoff and Landing Performance

%part a: takeoff ground roll distance at sea level:
%{
Sto = (weight / 2 * g0 * Favg) * Vto^2 feet
Vto = 1.2 * Vstall (sea level)
g0 = 32.2 ft/s^2

Favg:
Favg = T - Davg - R

Tavg = PA / Vto

Davg = (1/2) * rho(1) * (.707*Vto)^2 * S * (CD0 + (phi*(CLopt^2/(pi*eff*AR))))
phi = ((16*(h/b))^2) / (1 + ((16*(h/b))^2))
CLopt = (mu * pi * eff * AR) / (2 * phi)

R = mu * (weight - ((1/2) * rho(1) * (.707*Vto)^2 * S * CLopt))
%}

g0 = 32.17; %ft/s^2
Vto = [1.2*v_stall_sl, 1.2*v_stall5k]; %fps
Vto_kts = [1.2*kts_vStall_sl, 1.2*kts_vStall5k]; %kts
mu = .02; %paved runway
h = 3.52; %feet
phi = ((16*(h/b))^2) / (1 + ((16*(h/b))^2));
CLopt = (mu * pi * eff * AR) / (2 * phi);

%takeoff ground roll at seal level (part a)
T_sl = (((.9*(1-(35/Vto_kts(1))^2))*P(1)) * 550) / Vto(1); %lbs
Davg_sl = (1/2) * rho(1) * (.707*Vto(1))^2 * S * (CD0 + (phi*(CLopt^2/(pi*eff*AR))));
R_sl = mu * (weight - ((1/2) * rho(1) * (.707*Vto(1))^2 * S * CLopt));

Favg_sl = T_sl - Davg_sl - R_sl; %lbs

Sto_sl = (weight / (2 * g0 * Favg_sl)) * Vto(1)^2; %feet

%takeoff ground roll at 5000 feet (part b)
T5k = (((.9*(1-(35/Vto_kts(2))^2))*P(2)) * 550) / Vto(2); %lbs
Davg5k = (1/2) * rho(2) * (.707*Vto(2))^2 * S * (CD0 + (phi*(CLopt^2/(pi*eff*AR))));
R5k = mu * (weight - ((1/2) * rho(2) * (.707*Vto(2))^2 * S * CLopt));

Favg5k = T5k - Davg5k - R5k;

Sto5k = (weight / (2 * g0 * Favg5k)) * Vto(2)^2; %feet
```

```
%part a: takeoff ground roll distance at sea level:
%{
Sland = (-weight / 2 * g0 * Favg) * (1.69 * v_stall_sl^2) feet
Vl = 1.3 * Vstall (sea level)
g0 = 32.2 ft/s^2

Favg:
Favg = -Davg - R

Davg = (1/2) * rho(1) * (.707*Vl)^2 * S * (CD0 + (phi*(CLopt^2/(pi*eff*AR))))
phi = ((16*(h/b))^2) / (1 + ((16*(h/b))^2))
CLopt = (mu * pi * eff * AR) / (2 * phi)

R = mu2 * (weight - ((1/2) * rho(1) * (.707*Vl)^2 * S * CLopt))
%}

%Landing ground roll distance at sea level (part c)
mu2 = .4; %brakes applied
CLoptl = (mu2 * pi * eff * AR) / (2 * phi)
Vland = [1.3*v_stall_sl, 1.3*v_stall5k]; %fps

Davg_land = (1/2) * rho(1) * (.707*Vland(1))^2 * S * (CD0 + (phi*(CLoptl^2/(pi*eff*AR))));
Rland_sl = mu2 * (weight - ((1/2) * rho(1) * (.707*Vland(1))^2 * S * CLoptl));

Favg_land = -Davg_land - Rland_sl;

Sland = (-weight / (2 * g0 * Favg_land)) * Vland(1)^2; %feet

Davg_land5k = (1/2) * rho(2) * (.707*Vland(2))^2 * S * (CD0 + (phi*(CLoptl^2/(pi*eff*AR))));
Rland5k = mu2 * (weight - ((1/2) * rho(2) * (.707*Vland(2))^2 * S * CLoptl));

Favg_land5k = -Davg_land5k - Rland5k;

Sland5k = (-weight / (2 * g0 * Favg_land5k)) * Vland(2)^2; %feet
```