# Master 1 Finance - Empirical methods
# Exam 2021/2022

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Choose one of the two following subjects. You can work as a group (up to three) and you can use any programming language for implementations. Everything can be done in Excel. Bon courage.

## 1 Log Normal asset dynamic

Let's assume log returns of a financial asset are gaussian with zero mean and 10% annual standard deviation. Simulate a daily time serie (a path) of four years (assuming 252 days a year). The steps are described below.

- Simulate $T = 252 \times 4$ standard gaussian realizations $\epsilon_t$ using:

$$\epsilon_t = \Phi^{-1}(u_t) \sim \mathcal{N}(0,1) \tag{1}$$

  where $T$ is the sample size, $\Phi^{-1}()$ is the inverse of the standard gaussian cumulative distribution function, $\epsilon_t$ is a realization of a standard gaussian, $u_t$ is a realization of a uniform on $[0,1]$. With Excel, it is done using the function ALEA(). ALEA() called $T$ times, you should have in Excel a column of $T$ rows with the $u_t$. Then, simulate one path for the log returns of the asset using:

$$r_t = \epsilon_t \times \frac{10\%}{\sqrt{252}} \tag{2}$$

- Explain why we are multiplying the standard gaussian by $\frac{10\%}{\sqrt{252}}$.
- Perform a backtesting of the one day ahead 5% Historical Simulation VaR using for each day one year of returns (you should have three years of daily VaR). How frequently do you breach the VaR ?
- Backtest as well the Expected Shortfall. Show empirically that $\frac{VaR_t}{ES_t}$ tends to one when $p$ of the $p\%$ VaR decrease.
- The asset manager has a daily VaR of $100000 EUR$, what is the maximum amount, in EUR, he can invest every day to respect the VaR ? Plot this quantity for the three years of VaR.

# 2 Subject: Market risk of a portfolio

We analyse the role of correlation on the VaR of a portfolio of correlated assets. We assume time series of four years (assuming 252 days a year, $T = 4 \times 252$).

- For the first asset $X$, simulate $T$ gaussian realizations with zero mean and a variance of one using:

$$\epsilon_t^X = \Phi^{-1}(u_t^X) \sim \mathcal{N}(0,1) \tag{3}$$

  where $T$ is the sample size, $\Phi^{-1}()$ is the inverse of the standard gaussian cumulative distribution function, $u_t^X$ is a realization of a uniform on $[0,1]$. With Excel, it is done using the function ALEA(). ALEA() called $T$ times, you should have in Excel a column of $T$ rows with the $u_t$. Then, simulate a log returns path for the asset $X$:

$$r_t^X = \epsilon_t^X \times \frac{10\%}{\sqrt{252}} \tag{4}$$

- Generate another standard gaussian vector $\epsilon_t^Y$ and a second path of log returns for asset $Y$: $r_t^Y$, $t = 1, ..., T$ using the same $\epsilon_t^X$ generated previously. Assuming a annual volatility for the asset $Y$ of 20% and a correlation $\rho$ between $r^X$ and $r^Y$ at 50%:

$$r_t^Y = \left(\rho\epsilon_t^X + \sqrt{1-\rho^2}\epsilon_t^Y\right) \times \frac{20\%}{\sqrt{252}} \tag{5}$$

  Plot the path for the two assets in level using ($S_0 = 100$ for both $X$ and $Y$):

$$S_t = S_{t-1} exp(r_t) \tag{6}$$

- Setup the portfolio $P$ as:

$$P_t = \alpha S_t^X + (1-\alpha) S_t^Y \tag{7}$$

  where $\alpha \in [0, 1]$.

- Compute the log returns of the portfolio assuming $\alpha = 0.4$, $t = 1, ..., T$. Compute its empirical variance.

- Backtest the one day 1% RiskMetrics VaR.

- Compute empirically the probability to breach this daily RiskMetrics VaR.

- Now, simulate a new time series $r_t^Y$ by changing the correlation $\rho = -0.8$. Then compute your new portfolio and its log returns ($\alpha = 0.4$). Perform the same questions as previously, ie : variance, RiskMetrics VaR backtest and maximum investment amount.

- What can you say about the correlation impact ?