Math 244 Sample Midterm 1 Questions

1. Let $A = \begin{bmatrix} \lambda & 1 \\ 1 & \lambda \end{bmatrix}$. For which values of λ does $A\mathbf{x} = \mathbf{0}$ have an infinite number of solutions?

2. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find a basis and the dimension for the set of solutions to $A\mathbf{x} = \mathbf{0}$.

3. Are the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ linearly independent?

4. Are $1 - x, 1 + x, 2x^2, x^2 + x - 4$ linearly independent?

5. Let A be a square matrix and let $S = \{\mathbf{x} \text{ in } \mathbb{R}^n : A\mathbf{x} = A^\top \mathbf{x}\}$. Either show S is a subspace or show it is not a subspace.

6. Let $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ in } \mathbb{R}^3 : z = xy \right\}$. Either show S is a subspace or show it is not a subspace.

7. Let $S = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}$. Find a basis for S and the dimension of S.

8. Give an example of a matrix A such that the rank of A is 2 and the dimension of the subspace of solutions to $A\mathbf{x} = \mathbf{0}$ is also 2.

9. Find all solutions, by any means, to the system $\begin{cases} x + y + z - w = 1, \\ 2x + 2y - z + w = 2 \end{cases}$

10. Write the system $\begin{cases} x - y + z = 4 \\ y - z = 2 \\ -y + 2z = 0 \end{cases}$ as a matrix multiplication of the form $A\mathbf{x} = \mathbf{b}$. Solve using A^{-1}.

11. Find all eigenvalues and eigenvectors for $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

12. Let $S = \{n \times n \text{ matrices } A : A^2 = A\}$. Either show S is a subspace or show it is not a subspace.

13. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$ and let $S = \{\mathbf{x} \text{ in } \mathbb{R}^3 : A\mathbf{x} = \mathbf{0}\}$. Find a basis for S and the dimension of S.

14. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ and let S be the set of vectors $\mathbf{x}$ in $\mathbb{R}^3$ which satisfy $A\mathbf{x} = \mathbf{x}$. Is S a subspace? If yes, find a basis and the dimension of S.