Math 344 Sample Midterm 1

Midterm topics include: Laplace transforms, inverse Laplace transforms, using Laplace transforms to solve differential equations (especially those involving the Dirac delta function, the unit step function, and piecewise defined functions), the shifting theorems, convolution, Cauchy Euler differential equations and reduction of order.

The exam is closed notes/resources, but the table of Laplace transforms on the next page will be given.

These practice problems are similar to those found on the midterm. They will not be collected.

1. Find $\mathcal{L}^{-1}\left[1+\frac{2s}{s^2+2s+1}+\frac{s}{(s-a)^2+b^2}+e^{-2s}\arctan\left(\frac{\pi}{s}\right)+\frac{1}{3s-1}+\frac{2}{s(s^2+1)}\right]$.

2. Find $\mathcal{L}\left[\frac{\cos t-1}{t}\right]$ using the series for $\cos t$ and referencing the series in Exercise Set 1 Execise 3.

3. Give a physical interpretation and solve the differential equation

$$y''+2y'+5y=\delta(t-1)+\delta(t-2)$$

with $y(0)=0$ and $y'(0)=1$.

4. Simplify $\sqrt{t}*\sqrt{t}$.

5. Solve $tf(t)=\int_0^t f(x)f(t-x)\,dx$. Hint: $\mathcal{L}[tf(t)]=-\frac{d}{ds}\mathcal{L}[f(t)]$.

6. Solve $2x^2y''+3xy'-y=x$.

7. One solution to $xy''+(2x+2)y'+(x+2)y=0$ is $y=e^{-x}$. Find the general solution.

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)]$	
$f(t)$	$\int_0^\infty f(t)e^{-st}\,dt$	Definition of Laplace transform
t^n	$\frac{n!}{s^{n+1}}$	Valid for $n = 0, 1, 2, \ldots$
t^r	$\frac{r}{s}\mathcal{L}[t^{r-1}]$	Valid for $r > 0$
$t^{-1/2}$	$\sqrt{\frac{\pi}{s}}$	
e^{at}	$\frac{1}{s-a}$	
$\cos at$	$\frac{s}{s^2+a^2}$	
$\sin at$	$\frac{a}{s^2+a^2}$	
$\frac{\sin at}{t}$	$\arctan\left(\frac{a}{s}\right)$	
$\frac{e^{at}-1}{t}$	$\ln\left(\frac{s}{s-a}\right)$	
$f'(t)$	$s\mathcal{L}[f(t)] - f(0)$	First derivative in t
$f''(t)$	$s^2\mathcal{L}[f(t)] - sf(0) - f'(0)$	Second derivative in t
$e^{at}f(t)$	$F(s-a)$ where $F(s) = \mathcal{L}[f(t)]$	Shifting Theorem 1
$u_a(t)f(t-a)$	$e^{-as}\mathcal{L}[f(t)]$	Shifting Theorem 2
$\delta(t-a)$	e^{-as}	Dirac delta function
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}\mathcal{L}[f(t)]$	Derivatives in s
$f(t) * g(t)$	$\mathcal{L}[f(t)]\mathcal{L}[g(t)]$	The Convolution Theorem