Math 143 Midterm 1 Sample Questions

1. Do these series converge? If so, why?

a. $\sum_{n=2}^{\infty} \frac{4n^2-2}{3n^2+2}$

b. $\sum_{n=2}^{\infty} \frac{n}{n^2+1}$

c. $\sum_{n=2}^{\infty} (-1)^n \frac{n}{n^2+1}$

d. $\sum_{n=2}^{\infty} \frac{(\ln n)^n}{2^{n^2}}$

e. $\sum_{n=2}^{\infty} \frac{(1+n)^3}{(1+\sqrt{n})^4 \ln n}$

2. Find the interval and radius of convergence for these series

a. $\sum_{n=0}^{\infty} n^{n/2} x^n$

b. $\sum_{n=0}^{\infty} (x+4)^n / n^4$

3. Let $f(x) = \sqrt{5+2x}$.

a. Find the degree 2 Taylor polynomial for $f(x)$.

b. Find a bound on the error when approximating $f(1)$ by taking $x = 1$ in part a.

4. Approximate the value of $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$ to within $1/1000$ of the true value. (The answer may be left as a finite sum of fractions with $\cdots$ in the middle).

5. Using the well known series for e^x, $\sin x$, and $1/(1-x)$, find the power series for these functions:

a. $\frac{\sin x - x}{4x^2}$

b. $\frac{e^{2x} - e^{-2x}}{x}$

6. Find the exact values of $\sum_{n=2}^{\infty} (-1)^n \frac{2^n}{3^{n-1}}$.