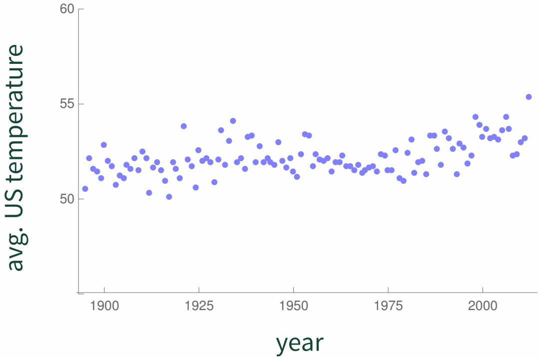
Regression Examples



Which function $f(x) = mx + b$ best fits the data?

How to find $f(x)$

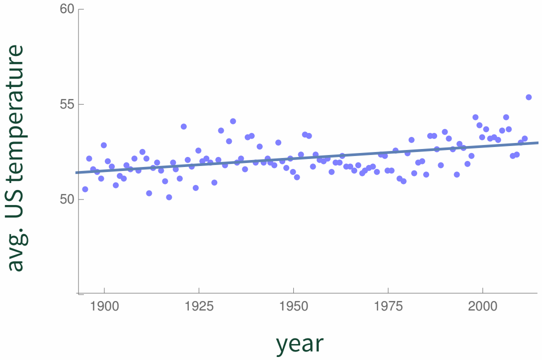
If our data is $\{(x_1, y_1), \ldots, (x_n, y_n)\}$, then

$$\begin{bmatrix} f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix} = m \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + b \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \mathbf{w}$$

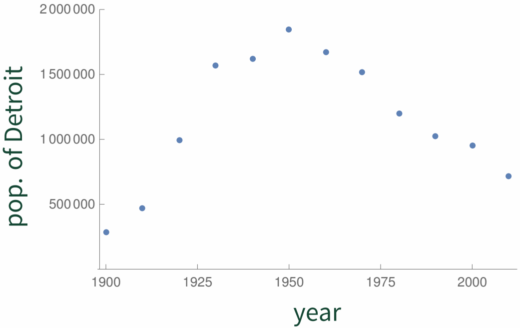
We find $\mathbf{w}$ in the span of $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ and $\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ that is closest to $\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$.

Then we use $\mathbf{w}$ to find m, b.

The function is $f(x) = 0.0128349x + 27.1186$



A quadratic fit example



Which function $f(x) = ax^2 + bx + c$ best fits the data?

The process is the same every time

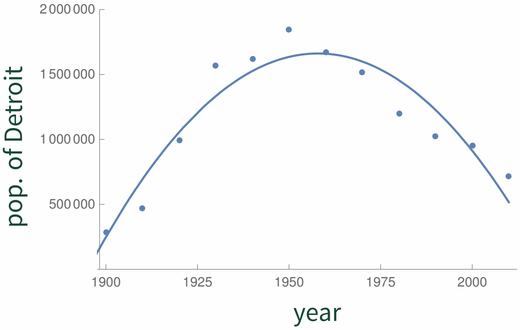
If our data is $\{(x_1, y_1), \ldots, (x_n, y_n)\}$, then

$$\begin{bmatrix} f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix} = a \begin{bmatrix} x_1^2 \\ \vdots \\ x_n^2 \end{bmatrix} + b \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + c \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \mathbf{w},$$

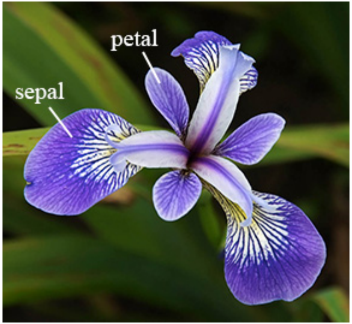
We find $\mathbf{w}$ in the span of $\begin{bmatrix} x_1^2 \\ \vdots \\ x_n^2 \end{bmatrix}$, $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ and $\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ that is closest to $\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$.

Then we use $\mathbf{w}$ to find a, b, c.

$$f(x) = -421.785x^2 + 1651621x - 1615188632$$



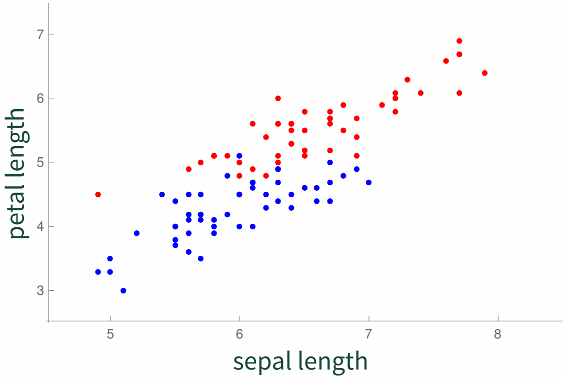
Iris classification



Irises can be Virginica or Versicolor.

Sepal and petal lengths were measured on 50 Irises of each type.

Virginica and Versicolor measurements



How do we predict the flower type given sepal/petal length?

Setting up the problem

The data: $\{(x_1, y_1), \ldots, (x_n, y_n)\}$. We want $f(x, y)$ such that

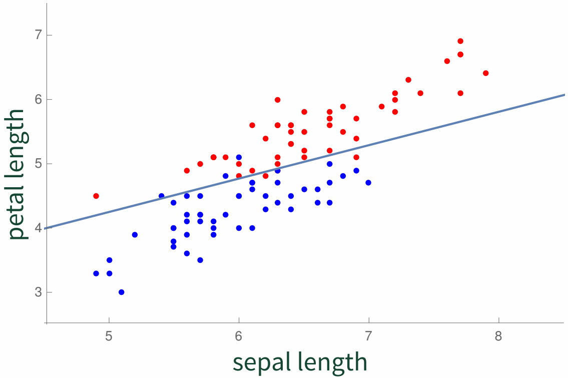
$$f(x,y) \approx \begin{cases} 1 & \text{if } (x,y) \text{ is Virginica,} \\ -1 & \text{if } (x,y) \text{ is Versicolor.} \end{cases}$$

Assuming $f(x, y) = ax + by + c$,

$$\begin{bmatrix} f(x_1, y_1) \\ \vdots \\ f(x_n, y_n) \end{bmatrix} = a \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + b \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} + c \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \mathbf{w}$$

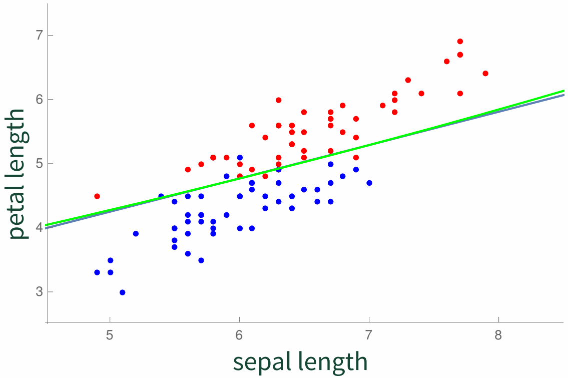
We find the $\mathbf{w}$ in the span closest to $\begin{bmatrix} \pm 1 \\ \vdots \\ \pm 1 \end{bmatrix}$ and then use $\mathbf{w}$ to find a, b, c.

$$f(x, y) = -0.760161x + 1.463y - 2.41736$$



The blue line is where $f(x, y) = 0$.

A quadratic fit



If $f(x, y) = a_1x^2 + a_2xy + a_3y^2 + a_4x + a_5y + a_6$, the green line is $f(x, y) = 0$.