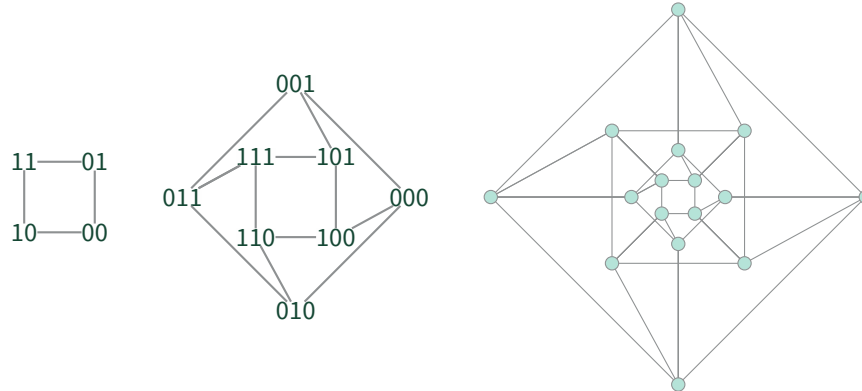# Graph Theory Set 6

27. The **cube graph** Q_n has vertices the bit strings of length n (these are sequences of 0's and 1's, such as 10110110) with an edge between two bit strings if and only if the strings differ in exactly one position. Below are Q_2 and Q_3 and an unlabeled Q_4:

a. Show that Q_n has 2^n vertices and $n2^{n-1}$ edges.

b. Show that Q_n is bipartite.

c. Use Menger's theorems to show that $\varepsilon(Q_n) = \kappa(Q_n) = n$.

d. Show that Q_n is Hamiltonian by induction on n. One Hamiltonian cycle is highlighted on an unlabeled Q_5 graph below:
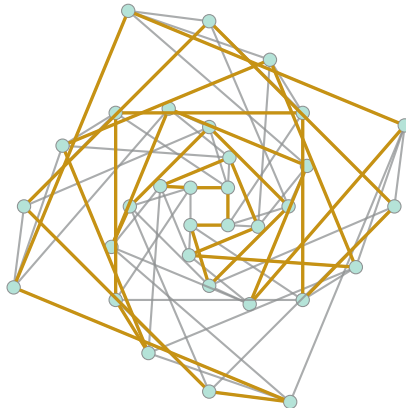
28. Suppose G is critical (see Exercise 12). Show that $\kappa(G) \neq 1$.

29. Show that deleting an edge from G reduces $\kappa(G)$ by at most 1.

30. Show that if every vertex in G has an even degree, then $\varepsilon(G)$ is even.

31. Suppose $\kappa(G) \geq 3$ and u, v, w are vertices in G. Show that G has a cycle containing u and v but not w.