Math 344 Sample Final

The final exam is cumulative. The exam is closed notes/resources, but the table of Laplace transforms and Fourier transforms on the next pages will be given.

These practice problems are similar to those found on the final. They will not be collected.

1. Find the Fourier series for the function $f(x) = x + 3\cos(2\pi x) - 4\sin(3\pi x)$ on $[-1, 1]$.

2. Give a physical interpretation and solve the partial differential equation $\begin{cases} u_{xx} + u_{yy} = 0, \\ u_x(0, y) = 0, u(1, y) = 0, \\ u_x(x, 0) = 0, u(x, 1) = 1. \end{cases}$

3. Find the Fourier transform of the function that is equal to e^{-t} on $0 \le t \le \pi$ and 0 otherwise.

4. If $f(t)$ is the function equal to $\sin t$ if $0 \le t \le \pi$ and 0 otherwise, then the Fourier transform of $f(t)$ is

$$F(\omega) = \frac{1 + e^{-i\pi\omega}}{2\pi(1 - \omega^2)}$$

Use $F(\omega)$ to find the coefficient of e^{it} in the complex Fourier series for $f(t)$ on $[-\pi, \pi]$. (Hint: This is usually $F(1)$, but since $F(1)$ is undefined, instead take $\lim_{\omega \to 1} F(\omega)$ and use L'Hôpital.)

5. On $PS[0, 1]$, which function of the form $a\sqrt{x} + b$ is closest to the function x?

6. Solve these differential equations:

a. $y'' + xy' + 2y = 0$.

b. $(1 - x^2)y'' - 2y = 0$.

c. $y'' + y' + 4y = 1 + \delta(x - 1)$ with $y(0) = 1, y'(0) = 0$.

d. $x^2y'' - 3xy' + 4y = 0$.

7. Let p_k be the k^{th} Legendre polynomial. Find the projection of $|x|$ onto the span of p_0, p_1, p_2 and p_3.

8. Use the projection matrix to find the vector in the span of $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ closest to $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

9. Use the normal equation to find the line that best fits the data $(-1, 0), (-1, 1), (0, 1), (0, 2)$.

10. Set 1 data is $\{(0, 0), (1, 1), (1, 2)\}$. Set 2 data is $\{(1, 0), (1, 2), (1, 1), (2, 2)\}$. Using a linear model, does the data point $(0, -1)$ more likely to belong to Set 1 or Set 2?

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)]$	
$f(t)$	$\int_0^\infty f(t)e^{-st}\,dt$	Definition of Laplace transform
t^n	$\frac{n!}{s^{n+1}}$	Valid for $n = 0, 1, 2, \ldots$
t^r	$\frac{r}{s}\mathcal{L}[t^{r-1}]$	Valid for $r > 0$
$t^{-1/2}$	$\sqrt{\frac{\pi}{s}}$	
e^{at}	$\frac{1}{s-a}$	
$\cos at$	$\frac{s}{s^2+a^2}$	
$\sin at$	$\frac{a}{s^2+a^2}$	
$\frac{\sin at}{t}$	$\arctan\left(\frac{a}{s}\right)$	
$\frac{e^{at}-1}{t}$	$\ln\left(\frac{s}{s-a}\right)$	
$f'(t)$	$s\mathcal{L}[f(t)] - f(0)$	First derivative in t
$f''(t)$	$s^2\mathcal{L}[f(t)] - sf(0) - f'(0)$	Second derivative in t
$e^{at}f(t)$	$F(s-a)$ where $F(s) = \mathcal{L}[f(t)]$	Shifting Theorem 1
$u_a(t)f(t-a)$	$e^{-as}\mathcal{L}[f(t)]$	Shifting Theorem 2
$\delta(t-a)$	e^{-as}	Dirac delta function
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}\mathcal{L}[f(t)]$	Derivatives in s
$f(t) * g(t)$	$\mathcal{L}[f(t)]\mathcal{L}[g(t)]$	The Convolution Theorem

Table of Fourier Transforms

$f(t)$	$F(\omega) = F[f(t)]$	
$f(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt$	Definition of Fourier transform
$\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}\,d\omega$	$F(\omega)$	The Fourier relations
$u_0(t)f(t)$	$\frac{1}{2\pi}\mathcal{L}[f(t)]$ with $s = i\omega$	Relation to Laplace transform
$cf(t) + g(t)$	$cF[f(t)] + F[g(t)]$	Linearity
$f(at)$	$\frac{1}{a}F\left(\frac{\omega}{a}\right)$	Scaling
$f(t-a)$	$e^{-ia\omega}F(\omega)$	Shifting 1
$e^{iat}f(t)$	$F[f(t-a)]$	Shifting 2
$f'(t)$	$i\omega F(\omega)$	First derivative in t
$f''(t)$	$-\omega^2 F(\omega)$	Second derivative in t
$itf(t)$	$F'(\omega)$	First derivative in ω