# Linear Algebra Midterm 1 Review Questions

**1.** If possible, find the inverse to the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and use it to solve the linear system $A\mathbf{x} = \mathbf{b}$ for a fixed vector $\mathbf{b} \in \mathbb{R}^3$.

**2.** Write the linear transformation defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ x \\ y \end{bmatrix}$ as $T(\mathbf{x}) = A\mathbf{x}$ for some $A$. Can you describe what this linear transformation does to vectors? Does this linear transformation have an inverse?

**3.** Is $\begin{bmatrix} 1 \\ -1 \\ 3 \\ 2 \end{bmatrix}$ a linear combination of the vectors $\begin{bmatrix} 1 \\ 4 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix}$?

**4.** Are the vectors $\begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ linearly independent? Do these vectors span $\mathbb{R}^3$? Why or why not?

**5.** Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$. Describe what the function $f(\mathbf{x}) = A\mathbf{x}$ does to vectors $\mathbf{x} \in \mathbb{R}^2$, drawing pictures when possible.

**6.** Give an example of a matrix $A$ for which $A^{-1}$ does not exist but $A\mathbf{x} = \mathbf{0}$ has a unique solution.

**7.** Find all solutions to the system $\begin{cases} x + y - 2z = 1, \\ x + y - z = 0, \\ y - 2z = 3. \end{cases}$

**8. T**rue or **F**alse:

______ **a.** Let $A$ be an $m \times n$ matrix. If $A\mathbf{x} = \mathbf{0}$ has a unique solution, then $A^{-1}$ exists.

______ **b.** A set of linearly dependent vectors can span $\mathbb{R}^n$.

**9.** Give an example of a linear system of equations that have a solution set that can be written as the span of two linearly independent vectors.

**10.** Find a formula for $\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}^{-1}$ provided $a, d,$ and $f$ are nonzero.

**11.** Let $A$ be a matrix which is not square. Explain why $A^{-1}$ does not exist.

**12.** Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & -1 & -1 \\ 0 & 0 & 3 & 3 \end{bmatrix}$. Write the solutions to $A\mathbf{x} = \mathbf{0}$ as a span of vectors in $\mathbb{R}^4$.

**13.** Find all solutions to the system $\begin{cases} x + y - 2z = 1, \\ x - 2z = -1, \\ 3y - 2z = 1. \end{cases}$