Math 142 Midterm 1 Review

Topics

1. The "four step process" can be used to explain why integrals appear in a variety of contexts. We used this process to find:
   - The area under a curve is $\int f(x)\,dx$ and the area between two curves is $\int f(x) - g(x)\,dx$.
   - The volume of a solid with cross sectional area $A(x)$ is $\int A(x)\,dx$.
   - As a special case of the last fact, the volume created when the graph of $f(x)$ is rotated around the x-axis is $\int \pi f(x)^2\,dx$.
   - The volume created when rotating the graph of $f(x)$ around the y-axis is $2\pi \int xf(x)\,dx$.
   - The average of the function $f(x)$ on $[a,b]$ is $\frac{1}{b-a}\int f(x)\,dx$.
   - The work done by a variable force $f(x)$ is $\int f(x)\,dx$.
2. The integration technique of u-substitution.
3. The definition of a function and its inverse. If $f'(x)$ exists, then $\frac{d}{dx}(f^{-1}(x)) = 1/f'(f^{-1}(x))$.
4. The definition and properties (the derivative, integral, limits, graph, etc.) of $\ln x$, e^x, a^b, and $\log_a x$.

Sample Questions

The blue book exercises are a good measure of difficulty for exam questions. A good way to prepare for the exam is to redo these exercises with small changes (change the constants, change the functions, etc.).

The following questions have appeared on previous Math 142 exams.

1. Evaluate $\dfrac{d}{dx}\left(x^{-x+2}\right)$.

2. Find the equation for the line tangent to $x^2 \ln(\sqrt{x})$ at $x = 2$.

3. Let $a > 1$. Why does $\log_a x$ have an inverse on $(0,\infty)$? If $g(x)$ is the inverse, find $g'(x)$ in terms of $g(x)$.

4. Evaluate $\displaystyle\int \cot(2t)\,dt$.

5. Evaluate $\displaystyle\int_1^4 \frac{2x}{x^2+4}\,dx$.

6. What is the volume of the solid created by rotating $\dfrac{1}{1+2x^2}$ on the interval $[0,1]$ around the y axis?

7. A demolition crane has a 200 kg ball suspended from a 20 m cable that weighs 1 kg/m. How much work is needed to wind up the ball the entire 20 m?

8. Let $f(x)$ be any function you can integrate. Find $\int_0^{\pi} f(\sin x)\ \cos x\, dx$.

9. Find the equation for the line tangent to $x^2 2^{-x}$ at $x = -2$.

10. Simplify $\int_0^t \frac{e^x}{(1+e^x)^3}\, dx$. Use your result to evaluate $\lim_{t\to\infty} \left(\int_0^t \frac{e^x}{(1+e^x)^3}\, dx \right)$.

11. Why does $f(x) = x^3 + 2x + \sin x$ have an inverse? What is the slope of the line tangent to this inverse at $x = 0$?