Math 143 Midterm 1 Review

Topics on Midterm 1

1. Taylor polynomials of degree n for $f(x)$ centered at $x = a$, namely $y = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$.
2. The error when $f(a)$ is approximated using the degree n Taylor polynomial for $f(x)$ centered at $x = 0$ is less than $M/(n+1)!|a|^{n+1}$ where $|f^{(n+1)}(x)| \le M$ for $-a \le x \le a$.
3. Taylor series for $f(x)$ centered at $x = a$, namely $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$.
4. The top 3 Taylor series:
   (a) The geometric series: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $-1 < x < 1$
   (b) The exponential function: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all x.
   (c) The sine function: $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ for all x.
5. Ways to test for the convergence of infinite series:
   (a) Recognize a known series (a geometric series, a p-series)
   (b) Compare the series to a larger convergent series
   (c) Compare the series to a smaller divergent series
   (d) Compare the series to an improper integral of the form $\int_1^{\infty} f(x)\,dx$
   (e) Use the limit comparison test
   (f) Use the Ratio test
   (g) Use the Alternating series test (and error bounds for alternating series!)
6. The interval and radius of convergence for functions of the form $\sum_{n=0}^{\infty} a_n x^n$.

Sample questions

1. Do these series converge? If so, why?

a. $\sum_{n=2}^{\infty} \frac{4n^2-2}{3n^2+2}$

b. $\sum_{n=2}^{\infty} \frac{n}{n^2+1}$

c. $\sum_{n=2}^{\infty}(-1)^n\frac{n}{n^2+1}$

d. $\sum_{n=2}^{\infty}\frac{\ln n}{2^{n^2}}$

e. $\sum_{n=2}^{\infty}\frac{(1+n)^3}{(1+\sqrt{n})^4\ln n}$

f. $\sum_{n=0}^{\infty}\frac{n^2+2^n}{n^4+2^n}$

g. $\sum_{n=2}^{\infty}\frac{n}{n^2\ln n+1}$

2. Find the interval and radius of convergence for these series

a. $\frac{1}{1}+\frac{1}{1\cdot 6}x+\frac{1}{1\cdot 6\cdot 11}x^2+\frac{1}{1\cdot 6\cdot 11\cdot 16}x^3+\cdots$

b. $\sum_{n=1}^{\infty}\frac{\ln(n+2)}{(2n)^2}x^n$

c. $\sum_{n=0}^{\infty}(x+4)^n/n^4$

d. $\sum_{n=0}^{\infty}(x-1)^{2n}$

3. Find the Taylor series for $1/(1-2x)$ centered at $x=0$.

4. Find the degree 5 Taylor polynomial for $x+\sqrt{x}$ centered at $x=1$.

5. Approximate the value of $\sum_{n=1}^{\infty}\frac{(-1)^n\ln n}{n}$ to within $1/1000$ of the true value. (The answer may be left as a finite sum of fractions with $\cdots$ in the middle).

6. Find the exact values of $\sum_{n=2}^{\infty}(-1)^n\frac{2^n}{3^{n-1}}$ and $\sum_{n=2}^{\infty}(-1)^n\frac{2^n}{(n-1)!}$.

7. Let $f(x)=\frac{4}{5}(1+x)^{5/2}$.

a. Find the degree 2 Taylor polynomial for $f(x)$ at $x=0$.

b. Find a bound on the error when approximating $f(-1/2)$ by taking $x=-1/2$ in part a.