Math 241 Midterm 2 Review

Topics

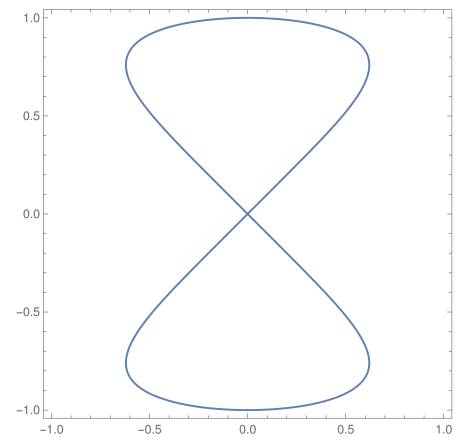
1. Double and triple integrals to find areas, volumes, averages, and center of mass
2. Rectangular, cylindrical and spherical coordinate systems
3. Change of variables and the Jacobian
4. Line integrals and calculating the work done by a vector field along a curve
5. Properties of conservative vector fields and the curl.
6. Greens theorem: $\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \operatorname{curl} \mathbf{F} \cdot \mathbf{k}\, ds$
7. The surface integral of $f(x, y, z)$ over surface parameterized by $\mathbf{r}(u, v)$ is

$$\iint_S f(x,y,z)\, d\sigma = \iint_R f(x,y,z) |\mathbf{r}_u \times \mathbf{r}_v|\, dA.$$

Sample questions

1. Find the work done by $\mathbf{F} = \langle -y, x \rangle$ in moving along $\begin{cases} x = t\cos t, \\ y = \sin t, \end{cases}$ for $t \in [0, \pi/2]$.

2. Using $\begin{cases} x = u\sqrt{v}, \\ y = \sqrt{v}, \end{cases}$ find the area enclosed by $x^2 + y^6 = y^2$:



3. Find the center of mass for the solid in the first octant bounded by the plane $z = 2 - 3x - y$.

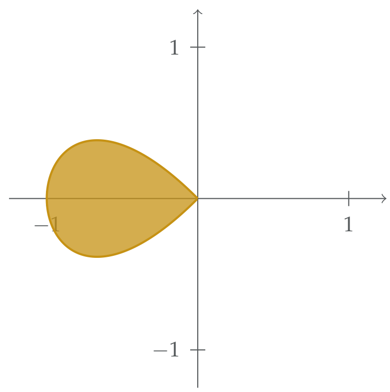
4. Find the volume of the portion of the cylinder $x^2 + y^2 = 4$ which lies above the x, y and below the plane $x + y + z = 4$.

5. Find the volume of the solid below the graph of $z = 3 - 8x^2 - y^2$ and above the graph of $z = x^2 + 8y^2$.

6. How much work is done by the vector field $\mathbf{F} = \langle xy^2 + y, x^2 y \rangle$ when a particle travels along the graph of $y = \cos x$ for $x \in [0, \pi/2]$, then travels straight down to the point $(0, -\pi/2)$, and then straight to $(0, 1)$?

7. Evaluate $\iiint_E xy \, dV$ where E is the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

8. Use Green's theorem to find the center of mass for a flat, uniformly dense object in the shape enclosed by the curve described by the parametric equations $\begin{cases} x = t^2 - 1, \\ y = t^3 - t, \end{cases}$ for $-1 \le t \le 1$.



9. Evaluate $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} z \, dz \, dy \, dx$ by converting to cylindrical coordinates.

10. Rewrite $\displaystyle\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} f(x, y, z) \, dz \, dy \, dx$ using the order $dx \, dy \, dz$.

11. Find the center of mass for the solid inside the cylinder $x^2 + y^2 = 1$, inside the sphere $x^2 + y^2 + z^2 = 4$, and above the x, y plane.

12. Convert into spherical coordinates (but do not evaluate):

$$\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} \, dz \, dx \, dy.$$

13. Find the center of mass for the region of the cylinder $x^2 + y^2 = 1$ which lies between the planes $x + y + z = 2$ and $x + 2y + z = -2$.

14. Find the center of mass for the solid described in spherical coordinates by $\theta \in [0, 2\pi]$, $\varphi \in [\frac{\pi}{4}, \frac{\pi}{2}]$, and $\rho \in [2, 3]$.

15. Write the integral $\displaystyle\int_0^1 \int_x^{2x} \int_0^1 1 \, dz \, dy \, dx$ the other 5 ways.