Math 344 Sample Midterm 2

These are questions that may be similar to the ones on the first midterm exam. The actual midterm has only 5 questions.

1. Verify that $2\|f\|^2 + 2\|g\|^2 = \|f+g\|^2 + \|f-g\|^2$ for any functions f, g in $PS[a,b]$.

2. Which function of the form $ax + bx^2$ in $PS[0,1]$ is closest to the function 1? Of course the answer may be left as a sum or quotient of fractions.

3. Find the projection matrix P for the projection onto the span of $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and use it to find the vector in the span of these two vectors closest to $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

4. Let $f_1, \ldots, f_n$ be functions in $PS[a,b]$. Describe how the Gram-Schmidt procedure can be used to find the dimension of $\operatorname{span}\{f_1, \ldots, f_n\}$.

5. Find the first two nonzero terms in the two series solutions of $2x^2y + xy' - (1+x)y = 0$.

6. Use the normal equation to find the line $f(x) = ax + b$ that fits $\{(0,0), (1,1), (1,2), (2,1)\}$.

7. Suppose

$$x = a_0 + \sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right)$$

on $PS[-L,L]$. What is the constant a_0? What is the constant b_3? What are the constants a_n?

8. Let P the projection matrix onto the span of some vectors in $\mathbb{R}^n$. Show that $(I + P/2)^{-1} = I - P/3$.

9. Suppose the Dirac delta function $\delta(x)$ is written as a sum of Legendre polynomials.

$$\delta(x) = a_0 p_0(x) + a_1 p_1(x) + a_2 p_2(x) + \cdots$$

Here $p_k(x)$ is the kth Legendre polynomial. What is a_3?