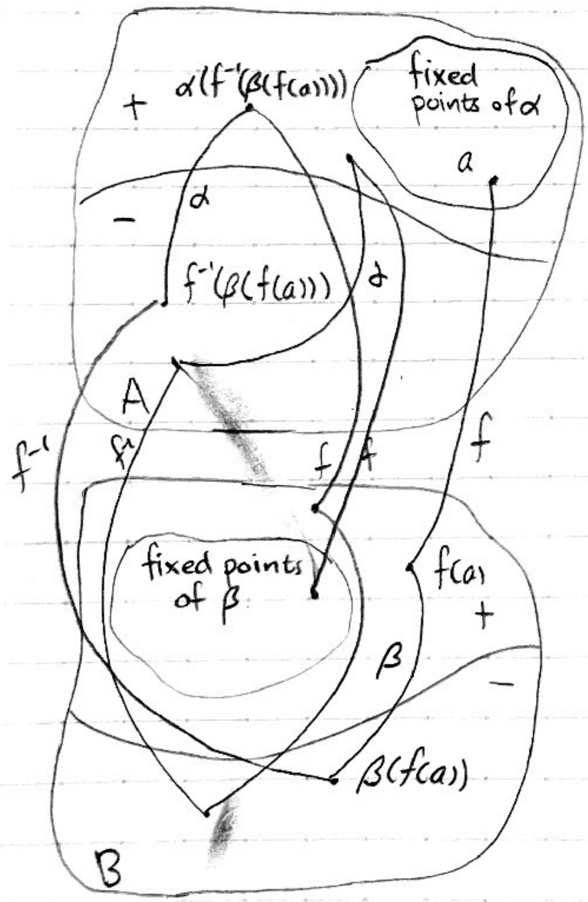
Let A and B be finite sets with $|A| = |B|$. Sort the elements in A and B into "+" elements and "−" elements.



Let $f: A \to B$ be a bijection that preserves sign.
Let $\alpha: A \to A$ be a sign reversing involution with only positive fixed points. An involution is a function that is equal to it's own inverse.
Let $\beta: B \to B$ be a sign reversing involution with only positive fixed points.

Theorem) There is a bijection g: fixed points of $\alpha \to$ fixed points of β.
Proof) Since our functions are all bijections, we never encounter the case of returning to the same point when iterating the composition of f, α, β, and f^{-1}.
This process terminates since the sets are finite. Once g reaches the fixed points of β, it remains in the fixed points of β by definition.

$g(a) = f \circ (\alpha \circ f^{-1} \circ \beta \circ f)^k (a)$
where k is the necessary amount of iterations.

Theorem) $(\#\lambda \vdash n$ with no even parts$) = (\#\lambda \vdash n$ with no repeated parts$)$

Proof) Let $A_1 = \{2\}$, $A_2 = \{4\}$, $A_3 = \{6\}$, $A_4 = \{8\}$, $A_5 = \{10\}$

Example) $\lambda = (5,2,2,1)$ contains A_1 twice

$\lambda = (10,4,3,3)$ contains A_5 and A_2.

Let $B_1 = \{1,1\}$, $B_2 = \{2,2\}$, $B_3 = \{3,3\}$, $B_4 = \{4,4\}$, $B_5 = \{5,5\}, \dots$

Example) $\lambda = (5,2,2,1)$ contains B_2

$\lambda = (10,4,3,3)$ contains B_3

Define $A = \{(\lambda, S) \mid S$ is a subset of the indices i with $\lambda \vdash n$ containing $A_i\}$

$B = \{(\lambda, S) \mid S$ is a subset of the indices i with $\lambda \vdash n$ containing $B_i\}$

Example) $((4,4,3,2,1,1), \{2\}) \in A \quad ((3,3,1,1), \phi) \notin A$

Define the sign of $(\lambda, S) = (-1)^{|S|}$.

Now define $\alpha : A \to A$ by $\alpha((\lambda,S)) = \begin{cases} (\lambda, S \setminus \{m\} & \text{if } m \in S \\ (\lambda, S \cup \{m\} & \text{if } m \notin S \\ (\lambda, \phi) & \text{otherwise} \end{cases}$

where m is the minimum disease index that afflicts λ, with A diseases

Here, α maps $((4,4,3,2,1,1), \{2\}) \longmapsto ((4,4,3,2,1,1), \{1,2\})$

The fixed points of α have no diseases of A_i

Now define $\beta : B \to B$ by $\beta((\lambda, S)) = \begin{cases} (\lambda, S \setminus \{m\} & \text{if } m \in S \\ (\lambda, S \cup \{m\} & \text{if } m \notin S \\ (\lambda, \phi) & \text{otherwise} \end{cases}$

where m is the minimum disease index that afflicts λ, with B diseases

Define $f : A \to B$ by replacing any A diseases in (λ, S) with B diseases

Here, f maps $((6,5,5,\boxed{2},1), \{1,3\}) \longmapsto ((5,5,3,3,\boxed{1,1},1), \{1,3\})$

This is our bijection and the result now follows by the previous theorem.