Math 143 Set 3

1. Let $f(x) = \dfrac{1}{1-x}$.

a. Find the degree 3 Taylor polynomial $y(x)$ for $f(x)$ at $x = 0$.

b. Use Taylor's inequality to find a bound on the error when approximating $f(1/2)$ with $y(1/2)$.

2. Let $f(x) = \ln x$.

a. Find the degree n Taylor polynomial $y(x)$ for $f(x)$ at $x = 1$.

b. Use Taylor's inequality to find a bound on the error when approximating $f(1/2)$ with $y(1/2)$. The answer should involve n.

3. Let $f(x) = \cos 3x$.

a. Find the degree $2n$ Taylor polynomial $y(x)$ for $f(x)$ at $x = 0$.

b. Use Taylor's inequality to find a bound on the error when approximating $f(1)$ with $y(1)$. The answer should involve n.

c. Use your answer in part b. to explain why the error in this approximation has limit 0 as $n \to \infty$.

4. The approximation

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

is the best degree 8 polynomial approximation (degree 8 because the coefficient of x^8 is 0) for $\sin x$ at $x = 0$. Show that the error in using this approximation is less than 0.1 when $-\pi \leq x \leq \pi$.