LaTeX Assignment 3

Samuel Sehnert

2023/02/02

Contents

1 Instructions

There are two exercises (which were typeset using the theorem environment).

Exercise 1. *Recreate this entire document.*[1]

Exercise 2. *Create a new document containing a short description of three of your favorite books, papers, or other publications. Be sure to include a bibliography, created using BibTeX.*

An assignment which completes Exercise 2 in an interesting way or makes amusing use of mathematical typesetting will earn the coveted LaTeXer of the week distinction

1.1 When to turn it in

Please upload the `.tex` and `.bib` source files and the `.pdf` output files to your solutions to Assignment 3 on or before Sunday, January 26.

[1] How Meta

2 Euler was smart

Euler proved many statements, such as

$$\prod_{m=1}^{\infty}(1-q^m)=\sum_{n=-\infty}^{\infty}(-1)^n\,q^{\left(2n^2-n\right)/2}, \tag{1}$$

where q is an indeterminate. Equation (1) is known as Euler's pentagonal number theorem. Euler also proved Theorem 1 below.

Theorem 1 (The Basel Problem)**.** *we have* $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.

Euler's original proof of Theorem 1 makes unjustified assumptions that infinite products and sums behave like finite products and sums, but is interesting nonetheless and worth displaying.

Proof. Using the power series for $\sin x$, we have

$$\begin{aligned}\frac{\sin x}{x}&=\frac{1}{x}\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\dots\right)\\&=1-\frac{x^2}{3!}+\frac{x^4}{5!}-\dots\\&=\left(1-\frac{x}{\pi}\right)\left(1+\frac{x}{\pi}\right)\left(1-\frac{x}{2\pi}\right)\left(1+\frac{x}{2\pi}\right)\dots\end{aligned} \tag{2}$$

where te reasoning [2] behind (2) is that a polynomial can be factored if its roots are known, and the roots of $\sin x/x$ are $\pm\pi,\pm2\pi$,.... Multiplying each pair of consecutive terms in this product gives

$$\left(1-\frac{x^2}{\pi^2}\right)\left(1-\frac{x^2}{4\pi^2}\right)\left(1-\frac{x^2}{9\pi^2}\right)\dots \tag{3}$$

The coefficient of x^2 in (3) is $-\frac{1}{\pi^2}-\frac{1}{4\pi^2}-\dots=-\frac{1}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}$ and the coefficient of x^2 is $\sin x/x$ is $-1/3!$, so equating these two expressions proves the theoem. $\square$

[2]This reasoning is actually true, but needs further justification.