Linear Analysis II Set 9

1. Use orthogonal projections to find the distance from the point $(2, 3, 4)$ to the plane $2x + y + z = 0$.

2. Show that if $\mathbf{u}_1, \ldots, \mathbf{u}_k$ are pairwise orthogonal vectors such that $\|\mathbf{u}_i\| = 1$ for all i, then

$$\|c_1\mathbf{u}_1 + \cdots + c_k\mathbf{u}_k\|^2 = c_1^2 + \cdots + c_k^2.$$

3. a. Let $\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$. Find these four matrix products: $\mathbf{x}^\top\mathbf{y}$, $\mathbf{y}\mathbf{x}^\top$, $\mathbf{y}^\top\mathbf{x}\mathbf{y}$ and $\mathbf{y}\mathbf{y}^\top\mathbf{x}$.

b. Show that $\mathbf{x} \cdot \mathbf{y} = \mathbf{x}^\top\mathbf{y}$ holds for any vectors $\mathbf{x}, \mathbf{y}$ in $\mathbb{R}^n$.

c. Let $\mathbf{v} \in \mathbb{R}^n$ and let $P = \dfrac{1}{\mathbf{v}^\top\mathbf{v}}\mathbf{v}\mathbf{v}^\top$. Show the $\mathbf{w} \in \text{span}\{\mathbf{v}\}$ that minimizes $\|\mathbf{x} - \mathbf{w}\|^2$ is $\mathbf{w} = P\mathbf{x}$.

4. Use the Gram-Schmidt procedure to orthogonalize the span of the vectors $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

5. Find the vector in the span of $\begin{bmatrix} 4 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \end{bmatrix}$ closest to $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.

6. Find the line $f(x) = mx + b$ that best to fits the data below. (For these type of exercises, use a calculator or computer to help with the calculations.) Using this model, what is an estimate for the percentage of children in the US who were obese in 2010?

Percentage of children in the US classified as obese								
year	1962	1974	1980	1994	1999	2002	2006	2008
percent	4	4	7	11	15	16	17	20

7. Find a function of the form $f(x) = ax^2 + bx + c$ to fit this data:

Oil production in the US										
year	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
(barrels/day)/1000	1210	2460	4107	5407	7035	9637	8597	7355	5822	5500