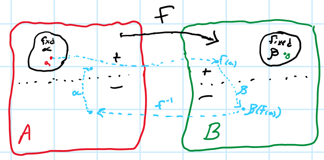
Let A, B be finite sets w/ $|A|=|B|$
Then sort elements of A & B into "+", "−" elements
Let $f: A \to B$ be a bijection that preserves sign
Let $\alpha: A \to A$ be a sign reversing involution w/ only "+" fixed points
Let $\beta: B \to B$ be a sign reversing involution w/ only "+" fixed points



Thm There is a bijection g: fixed $\alpha \to$ fixed β

$g(a) = (\alpha \circ f^{-1} \circ \beta \circ f)^k(a)$, k times till you land in fxd β

Thm: ($\#\lambda \vdash n$ w/ no even parts) = ($\#\lambda \vdash n$ w/ no repeated parts)

Proof: Let $A_1 = \{2\}$, $A_2 = \{4\}$, $A_3 = \{6\}$, $A_4 = \{8\}$, ...
& $B_1 = \{1,1\}$, $B_2 = \{2,2\}$, $B_3 = \{3,3\}$, $B_4 = \{4,4\}$, ...

Define $A = \{(\lambda, S) : S$ is a subset of the indicies i w/ $\lambda \vdash n$ having disease $A_i\}$
$B = \{(\lambda, S) :$ " " " $B_i\}$

Ex: $((\underbrace{4,4,3,2,1,1}_{\lambda}), \underbrace{\{2\}}_{S}) \in A \xrightarrow{\alpha} (\lambda, \{1,2\})$
$((3,3,1,1), \{\emptyset\}) \in A$

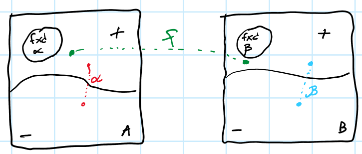
Define the sign of $(\lambda, S) = (-1)^{|S|}$

$\alpha: A \to A$ by $\alpha((\lambda,S)) = \begin{cases} (\lambda, S \setminus \{m\}) & \text{if } m \in S \\ (\lambda, S \cup \{m\}) & \text{if } m \notin S \\ (\lambda, \emptyset) & \text{if otherwise} \end{cases}$ where m is the minimum disease index that affects λ

$\beta: B \to B$ defined similarly w/ B_i's instead of A_i's
$f: A \to B$ by replacing A diseases in (λ, S) w/ B diseases

Ex (f): $((6,5,5,2,1), \{1,3\})$
$\downarrow f$
$((5,5,3,3,1,1,1), \{1,3\})$

11/7 The involution principal: $\exists$ a bijection g: Fxd $\alpha \to$ Fxd β



Thm: Remmel's Bijection Machine

If $\{A_1, A_2, \dots\}$ & $\{B_1, B_2, \dots\}$ are list of pairwise disjoint multisets
s.t $|A_i| = |B_i|$ $\forall i$, then ($\#\lambda \vdash n$ w/ no A_i in parts) = ($\#\lambda \vdash n$ w/ no B_i in parts)
sum of elements in the multiset.

Ex: ($\#\lambda \vdash n$ w/ parts $\equiv \pm 1 \mod 6$) = ($\#\lambda \vdash n$ w/ distinct parts $\equiv \pm 1 \mod 3$)
$A_1 = \{2\}$, $A_2 = \{3\}$, $A_3 = \{4\}$, $A_4 = \{6\}$, $A_5 = \{8\}$ ....
$B_1 = \{1,1\}$, $B_2 = \{3\}$, $B_3 = \{2,2\}$, $B_4 = \{6\}$, $B_5 = \{4,4\}$ ...

Recall: $A = \{(\lambda, S) :$ w/ S a subset of A_i diseases in $\lambda\}$, $B = \{(\lambda, S) : S$ is a subset of B_i diseases in $\lambda\}$

The sign $(\lambda, S) = (-1)^{|S|}$, & $\alpha((\lambda,S)) = (\lambda, \hat{S})$ w/ $\hat{S}$ has the smallest index added/removed from S

$\beta \sim$ similar to α, & $f(\lambda, S) = (\hat{\lambda}, S)$, w/ diseases from A replaced w/ disease from B.

Ex: $\circlearrowleft A \xrightarrow{\alpha} A \xrightarrow{f} B \xrightarrow{\beta} B \circlearrowleft f^{-1}$

$(1^3 5 7, \emptyset) \in A \xrightarrow{f} (1^3 5 7, \emptyset) \in B \xrightarrow{\beta} (1^3 5 7, \{1\}) \xrightarrow{f^{-1}} (1 2 5 7, \{1\}) \to$
$\xrightarrow{\alpha} (1 2 5 7, \emptyset) \xrightarrow{f} (1 2 5 7, \emptyset)$ the bijection sends $1^3 5 7$ to $1 2 5 7$

Ex 2: $A_1 = \{3\}$, $A_2 = \{6\}$, $A_3 = \{9\}$, ... $B_1 = \{1,1,1\}$, $B_2 = \{2,2,2\}$ $B_3 = \{3,3,3\}$ ...

Consider $(1^6 2 8^4, \emptyset) \xrightarrow{f} (1^6 2 8^4, \emptyset) \xrightarrow{\beta} (1^6 2 8^4, \{1\}) \xrightarrow{f^{-1}} (2 3^2 8^4, \{1\}) \xrightarrow{\alpha} (2 3^2 8^4, \emptyset)$
$\xrightarrow{f} (2 3^2 8^4, \emptyset) \xrightarrow{\beta} (2 3^2 8^4, \{8\}) \xrightarrow{f^{-1}} (2 3^2 8 24, \{8\}) \xrightarrow{\alpha} (2 3^2 8 24, \{1,8\}) \xrightarrow{f} (1^6 2 8^4, \{1,8\})$ ....