Math 241 Quiz 5

Names: _____

1. Use Stokes' theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle z^2, x^2, y^2 \rangle$ and C is the curve of intersection of $z = 4 - x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

2. Let $\hat{\mathbf{n}}$ be the (outward) unit normal vector to a closed surface S with surface area A that encloses the region D in $\mathbb{R}^3$. Use the divergence theorem to find $\iiint_D \nabla \cdot \hat{\mathbf{n}} \, dV$.

3. Let $f(x, y, z)$ be a function from $\mathbb{R}^3$ to $\mathbb{R}$. Show that the vector field $f\nabla f$ is conservative and then write div $(f\nabla f)$ in terms of $f_x, f_y, f_z, f_{xx}, f_{yy}$, and f_{zz}.

4. Show that if $f(x, y, z)$ is a function such that $\nabla \cdot \nabla f = 0$ and S is a closed surface that encloses a domain D, then the flux of $f\nabla f$ through S is equal to $\iiint_D |\nabla f|^2 \, dV$.