Math 142 Sample Midterm 2

Topics

1. Know everything (graphs, limits, derivatives, integrals, ...) about
   - $\sinh x = \dfrac{e^x - e^{-x}}{2}$ and $\cosh x = \dfrac{e^x + e^{-x}}{2}$
   - $\arcsin x$ and $\arctan x$

2. Integration techniques.
   - Just know $\int 1/x \, dx$, $\int 1/(1+x^2) \, dx$, $\int 1/\sqrt{1-x^2} \, dx$, $\int e^x \, dx$, $\int x^k \, dx$, $\int \sin x \, dx$, $\int \cos x \, dx$.
   - u-substitution
   - Integration by parts
   - Integration of powers of $\sin x$ and $\cos x$
   - Integration of rational functions
   - Integration involving $\sqrt{a^2 \pm x^2}$

3. Applications involving exponential growth and solutions to the differential equation $y'(x) = ay(x)$

4. Numerical integration using Simpson's rule

5. L'Hôpital's rule

6. Improper integrals

Sample Questions

The blue book exercises are a good measure of difficulty for exam questions. A good way to prepare for the exam is to redo these exercises with small changes (change the constants, change the functions, etc.). The exam material covers blue book exercises starting with exercise 18.

The following questions have appeared on previous Math 142 exams.

1. Integrate $\displaystyle\int \sin^{\text{red}-1} x \cos^{\text{blue}-1} x \, dx$ where red and blue are randomly chosen from $\{1, \ldots, 6\}$.

2. Integrate $\displaystyle\int x \sinh(x^2) \, dx$.

3. Integrate $\int_0^\infty x^2 e^{-x} \, dx$.

4. Integrate $\displaystyle\int_0^1 \frac{x^3}{\sqrt{2-x^2}} \, dx$.

5. Integrate $\displaystyle\int \frac{x}{16 + x^4} \, dx$.

6. Evaluate $\displaystyle\int_0^1 \frac{3x}{(x-2)(x+1)}\,dx$.

7. Integrate $\displaystyle\int \frac{x^2+2x+1}{x(x^2+1)}\,dx$.

8. Integrate $\displaystyle\int_0^\infty (x+2)^2 e^{-2x}\,dx$.

9. Integrate $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx$.

10. Integrate $\displaystyle\int \frac{1}{a^2-x^2}\,dx$ where a is a positive number.

11. Integrate $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx$ where a is a positive number.

12. Integrate $\displaystyle\int \frac{1}{x^2+a^2}\,dx$ where a is a positive number.

13. Integrate $\displaystyle\int_0^{\ln(3)/2} \frac{e^x}{e^{2x}+1}\,dx$.

14. Integrate $\displaystyle\int_1^{\sqrt{3}} \frac{2x^2+1}{x^2(x^2+1)}\,dx$.