

Linear Analysis II Exercise Set 2

1. Use the known Laplace transforms of t^n , e^{at} , $\cos(at)$, and $\sin(at)$ given in the table of Laplace transforms on our web site to find

$$\mathcal{L}^{-1} \left[\frac{1}{s^5} + \frac{2s+1}{s^2+16} + \frac{6+4s^2}{s(s^2+2)} + \frac{a}{s^2-a^2} \right].$$

Hint: Use partial fractions on the last two terms.

2. Let y and x be functions of t . Use Laplace transforms to solve the following:

- a. $y' - y = e^{2t}$ with $y(0) = 3$.
- b. $y'' - 4y = 0$ with $y(0) = 1, y'(0) = 2$.
- c. $y'' - y' = -t$ with $y(0) = 0, y'(0) = 1$.
- d. $\begin{cases} x' = 9y \\ y' = -x \end{cases}$ with $x(0) = 0, y(0) = 1$.
- e. $\begin{cases} x' = -4x - 2y \\ y' = x - y, \end{cases}$ with $x(0) = 0, y(0) = 1$.
- f. $\begin{cases} x' = x + y + 1 \\ y' = x + y - 1, \end{cases}$ with $x(0) = 1, y(0) = 0$.

3. Verify the following identities involving the Laplace transform:

- a. $\mathcal{L} \left[\int_0^t f(x) dx \right] = \frac{1}{s} \mathcal{L}[f(t)]$.
- b. $\frac{d}{ds} \mathcal{L}[f(t)] = -\mathcal{L}[tf(t)]$.