

# Math 244 Sample Midterm 2 Questions

1. Solve  $y'' - 3y' + y = x + \sin x$ .

2. Solve the initial value problem  $\begin{cases} x^2 y'' + xy' - 4y = x^3 \\ y(1) = 1/5, \\ y'(1) = 1. \end{cases}$ .

3. Solve  $\begin{cases} (1 + x^2)y' + y = 1, \\ y(0) = 0. \end{cases}$

4. Let  $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ . Find a matrix  $S$  and a diagonal matrix  $D$  such that  $A = SDS^{-1}$  or state why this is impossible.

5. Give an example of a matrix  $A$  with eigenvalue/eigenvector pairs  $2, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $-3, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ .

6. Give an example of a square matrix  $A$  which is not diagonalizable.

7. Find a differential equation with solutions  $x$  and  $\sin x$  (among other possible solutions).

8. Show that the solutions to a linear homogeneous differential equation form a subspace. (Note: We use this fact whenever we write all solutions as  $C_1 y_1 + C_2 y_2$  where  $y_1, y_2$  are linearly independent solutions to the differential equation.)

9. Solve  $xy'' + (2x + 2)y' + (x + 2)y = 2e^{-x}$  if one solution to  $xy'' + (2x + 2)y' + (x + 2)y = 0$  is  $y = e^{-x}$ .

10. Solve  $y''' + y' = 1$ .

11. Give an example of a  $3 \times 3$  matrix  $A$  which has eigenvalue 2 with corresponding eigenvector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  and

has eigenvalue  $1/3$  with corresponding eigenvectors  $s \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ .