

Math 143 Midterm 2 Sample Questions

These identities will be given on the midterm: $\cos^2 t = (1 + \cos 2t)/2$, $\sin^2 t = (1 - \cos 2t)/2$.

1. Find the arclength and the area enclosed by the polar curve $r = \theta^2$ for $\theta \in [0, 2\pi]$.
2. Find the arclength of the curve described by the parametric equations $\begin{cases} x = 3 + e^{-2t} \\ y = 2 - e^{-2t} \end{cases}$ for $t \in [0, 1]$.
3. Graph the parametric equations $\begin{cases} x = 2 + 3 \sin t \\ y = 1 + 2 \cos t \end{cases}$ for $t \in [0, 3\pi/2]$.
4. Find two vectors of length 2 which are orthogonal to $\langle 2, 2, 3 \rangle$ and $\langle -1, 0, 2 \rangle$.
5. Find the equation of the plane which passes through the origin and is perpendicular to both $x + y + z = 3$ and $x + 2y + 3z = 3$.
6. Find the angle between the planes $x + y + z = 1$ and $x + 2y - z = 2$.
7. Consider the curve in the plane $\begin{cases} x = \cos t + t \sin t \\ y = \sin t - t \cos t \end{cases}$ where $t \in [0, 2\pi]$.
 - a. Find the (x, y) coordinates of all vertical and horizontal tangents.
 - b. Find the values of t for which this curve is concave down.
 - c. Find the arclength of the curve.
8. Consider the curve given parametrically by $\begin{cases} x = 2e^t - t \\ y = e^t - 3 \end{cases}$ for $t \in \mathbb{R}$. Find the parametric equations for the line tangent to the curve at $t = 1$.
9. Fix a vector $\mathbf{v} \in \mathbb{R}^3$. Which unit vector \mathbf{w} maximizes the dot product $\mathbf{w} \cdot \mathbf{v}$?