Math 344 Sample Midterm 2

Midterm topics include: Series solutions to linear differential equations, Legendre polynomials, projections in $\mathbb{R}^n$, the Gram-Schmidt procedure, applications to linear regression/curve fitting/classification, projection matrices, the normal equation, inner products on $PS[a, b]$, projections for functions, orthogonal families.

The exam is closed notes/resources.

These practice problems are similar to those found on the midterm. They will not be collected.

1. Use series to solve these differential equations:

    a. $y'' - xy' - y = 0$

    b. $y'' + (1 + x)y = 0$

    c. $(1 - x^2)y'' + y = 0$.

2. Which point on the line passing through $(0, 0, 0)$ and parallel to the vector $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ is closest to $(1, 2, 3)$?

3. Let $L_k(x)$ is the polynomial solution to $\dfrac{d}{dx}\left(xe^{-x}y'\right) = -ke^{-x}y$ that satisfies $L_k(0) = 1$.

    a. Find $L_2(x)$.

    b. Show that if $k \neq m$, then $\displaystyle\int_0^\infty e^{-x} L_k(x) L_m(x)\, dx = 0$.

    c. Given $\displaystyle\int_0^\infty e^{-x} L_k^2(x)\, dx = 1$, find a formula for the constants which make the approximation

$$f(x) \approx a_0 L_0(x) + a_1 L_1(x) + \cdots + a_n L_n(x)$$

    as accurate as possible.

4. Let $p_k(x)$ be the kth Legendre polynomial and let $q(x)$ be a polynomial of degree k or less. Explain why

$$\int_{-1}^{1} q(x) p_k(x)\, dx = 0.$$

5. Find a polynomial solution to $xy'' - xy' + 2y = 0$.

6. Which function of the form $ax + bx^2$ in $PS[0, 1]$ is closest to the function 1?

7. Use the normal equation to find the line $f(x) = mx + b$ that best fits the data $\{(0, 1), (0, 0), (2, 1)\}$.

8. Find the projection matrix for the projection onto the span of your favorite three vectors in $\mathbb{R}^4$.