Math 143 Sample Midterm 2

Midterm 2 topics include: Euler's formula and multiplication, parametric equations (derivatives, second derivatives, arclength), polar (derivatives, areas, arclength), $\mathbb{R}^3$, vectors, the dot product and cross products, lines and planes.

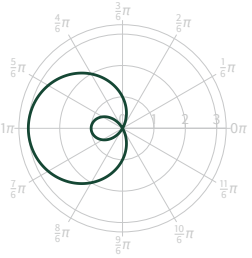
The exam is closed notes/resources but these identities will be given:

$$\cos^2 t = \frac{1+\cos 2t}{2} \qquad \sin^2 t = \frac{1-\cos 2t}{2} \qquad \sin^2 t + \cos^2 t = 1.$$

These practice problems are similar to those found on the midterm. They will not be collected.

1. Find the arclength of the parametric equations $\begin{cases} x = \cos(2t), \\ y = 2t - \sin(2t) \end{cases}$ for $t \in [0, \pi]$.

2. Find the area in the little loop of the graph $r(\theta) = 1 - 2\cos\theta$:



3. Find the arclength and the area enclosed by the polar curve $r = \theta^2$ for $\theta \in [0, 2\pi]$.

4. Find the arclength of the curve described by the parametric equations $\begin{cases} x = 3 + e^{-2t} \\ y = 2 - e^{-2t} \end{cases}$ for $t \in [0, 1]$.

5. Graph the parametric equations $\begin{cases} x = 2 + 3\sin t \\ y = 1 + 2\cos t \end{cases}$ for $t \in [0, 3\pi/2]$.

6. Find two vectors of length 2 which are orthogonal to $\langle 2, 2, 3\rangle$ and $\langle -1, 0, 2\rangle$.

7. Find the values of t for which the parametric curve $\begin{cases} x = 2t^2, \\ y = -t^3 + t^2/2 \end{cases}$ for $t \in \mathbb{R}$ is concave down.

8. Sketch the points in $\mathbb{R}^3$ that satisfy $4x^2 + z^2 = 1$ and $x^2 + 2y^2 + z^2 = 1$.

9. Find the equation of the plane which passes through the origin and is perpendicular to both $x + y + z = 3$ and $x + 2y + 3z = 3$.

10. Fix a vector $\mathbf{v} \in \mathbb{R}^3$. Which unit vector $\mathbf{w}$ maximizes the dot product $\mathbf{w} \cdot \mathbf{v}$?