Math 344 Midterm 2 Practice

Midterm 2 topics

Topics on Midterm 2: Frobenius differential equations, Legendre polynomials, Orthogonality, Gram-Schmidt Projection matrices, Normal equation, inner products on $PS[a,b]$.

Exercise Set 11 is due next Tuesday after the exam, but please consider completing exercises 1, 2, and 4 on Set 11 as practice.

Sample problems

1. Find the projection matrix P for the projection onto the span of $\begin{bmatrix}1\\1\\0\end{bmatrix}$ and $\begin{bmatrix}2\\1\\1\end{bmatrix}$ and use it to find the vector in the span of these two vectors closest to $\begin{bmatrix}a\\b\\c\end{bmatrix}$.

2. Which function of the form $ax + bx^2$ in $PS[0,1]$ is closest to the function 1? Of course the answer may be left as a sum or quotient of fractions.

3. Verify that $2\|f\|^2 + 2\|g\|^2 = \|f+g\|^2 + \|f-g\|^2$ for any functions f, g in $PS[a,b]$.

4. Describe how the Gram-Schmidt procedure can be used to find the dimension of span$\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$.

5. Find the first two nonzero terms in the two series solutions of $2x^2y + xy' - (1+x)y = 0$.

6. Find the line $f(x) = ax + b$ that best fits $\{(0,0),(1,1),(1,2),(2,1)\}$.

7. Let P the projection matrix onto the span of some vectors in $\mathbb{R}^n$. Show that $(I + P/2)^{-1} = I - P/3$.

8. Suppose the Dirac delta function $\delta(x)$ is written as a sum of Legendre polynomials.

$$\delta(x) = a_0 p_0(x) + a_1 p_1(x) + a_2 p_2(x) + \cdots$$

Here $p_k(x)$ is the kth Legendre polynomial. What is a_3?