Linear Analysis II Exercise Set 12

1. Find the constants a_0, a_n and b_n for the Fourier series for the unit step function

$$u_0(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right)$$

on $PS[-L, L]$.

2. Find the Fourier series for the Dirac delta function $\delta(x)$ on $[-L, L]$.

3. a. Find the Fourier series for the function x^2 on $[-L, L]$.

b. After taking $x = \pi$ and $L = \pi$ in the Fourier series in the previous part of this exercise, we have

$$\pi^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{4}{n^2} \cos(n\pi).$$

Use this expression to find a formula for $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

4. Give a physical interpretation and solve these partial differential equations:

a. $\begin{cases} u_t = k u_{xx} \\ u_x(0,t) = 0, u(L,t) = 0, \\ u(x,0) = f(x), \end{cases}$

b. $\begin{cases} 0 = u_{xx} + u_{yy} \\ u_x(0,y) = 0, u_x(L,y) = 0, \\ u(x,0) = 0, u(x,H) = f(x). \end{cases}$