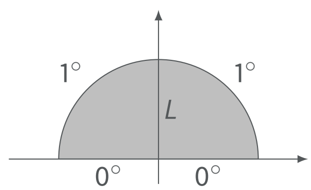
Linear Analysis II Set 13

1. Give a physical interpretation and solve these partial differential equations:

a. $\begin{cases} u_t = ku_{xx} \\ u_x(t,0) = 0, u(t,L) = 0, \\ u(0,x) = f(x), \end{cases}$

b. $\begin{cases} 0 = u_{xx} + u_{yy} \\ u_x(0,y) = 0, u_x(L,y) = 0, \\ u(x,0) = 0, u(x,H) = f(x). \end{cases}$

2. The base of a semicircular plate of radius L is held at 0° while the top arc is held at 1°:



a. Using the multivariate chain rule for partial derivatives, the steady state heat equation $u_{xx} + u_{yy} = 0$ can be transformed from rectangular coordinates into polar coordinates. When this is done, we find the steady state heat equation in polar is

$$r^2 u_{rr} + r u_r + u_{\theta\theta} = 0$$

where $u(r,\theta)$ is the steady state temperature in the plate at the polar coordinates r, θ. Assuming $u(r,\theta) = R(r)\Theta(\theta)$, turn this partial differential equation into two differential equations, one with boundary conditions.

b. Solve for Θ.

c. Solve for R. (Since $\lim_{r \to 0} R(r) < \infty$, one of the two answers for R can be disregarded.)

d. What is $u(r,\theta)$? (Be sure to give a formula for the constants in the solution.)