Math 248 Midterm 2 Review

Midterm topics include: Proof by contrapositive and contradiction, proving biconditionals and existence, proofs involving subsets and set equality, disproving statements, conjectures, induction, relations and equivalence relations, and functions.

Sample exercises

1. Define a sequence g_n such that $g_1 = g_2 = 1$ and $g_{n+2} = g_{n+1} + 2g_n$ for $n \geq 1$. Proof or counterexample: $g_{n+5} = 4g_{n+3} + g_n$ for all $n \geq 1$.

2. Let $\mathbb{Z}/R$ be the set of equivalence classes the relation R on $\mathbb{Z}$ defined by aRb provided $a - b$ is even.

a. Define $F : \mathbb{Z}/R \to \mathbb{Z}$ by $F([a]) = a$. Does this define a function? Why or why not?

b. Define $F : \mathbb{Z}/R \to \mathbb{Z}$ by $F([a]) = \begin{cases} 1 & \text{if } a \text{ even,} \\ -1 & \text{if } a \text{ odd.} \end{cases}$ Does this define a function? Why or why not?

3. Let R be the relation on $\mathbb{Z} \times \mathbb{Z}$ such that $(a, b)R(c, d)$ provided $a - b = c - d$. Verify this is an equivalence relation and describe $[(1, 3)]$.

4. Prove that 3 divides a if and only if 3 divides a^2.

5. Prove that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$ for all $n \in \mathbb{N}$.

6. Let A, B and C be sets. Proof or counterexample: $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$.

7. Prove that there are not $a, b \in \mathbb{N}$ such that $a^2 - b^2 = 1$.