

Exercise 1.

Every mathematical statement, such as $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$, should be part of a sentence. Even equations need punctuation!

The notation $\lim_{x \rightarrow a} f(x) = L$ means that for every $\varepsilon > 0$ there is a $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) - L| < \varepsilon$. An incorrect way to typeset this definition is

$$\forall(\varepsilon > 0) \exists(\sigma > 0) \ni (|x - a| < \varepsilon \implies |f(x) - L| < \varepsilon).$$

The symbols \forall , \exists , \ni , and \implies should be used only in the context of the mathematical subject of formal logic and should not replace the words “for all”, “there exists”, and “such that”, and “implies”.

One of your instructor’s favorite mathematical statements is

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n,$$

otherwise known as Stirling’s formula. As an example, $100!$ is approximately equal to $\sqrt{200\pi}(100/e)^{100} \approx 9.32 \times 10^{157}$.

The following is true:

$$\begin{aligned} \left| \int_1^a \frac{\sin x}{x} dx \right| &\leq \int_1^a \left| \frac{\sin x}{x} \right| dx \\ &\leq \int_1^a \frac{1}{x} dx \\ &= \ln a. \end{aligned}$$

After first simplifying using the exponential and the natural log functions, L’Hôpital’s rule can be used to evaluate $\lim_{x \rightarrow 2^-} (4 - x)^{1/(2-x)}$.

Take $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$. The inner product of \mathbf{x} and \mathbf{y} is defined by $\langle \mathbf{x}, \mathbf{x} \rangle = \mathbf{x}^T \mathbf{x}$. It follows that

$$\langle \mathbf{x}, \mathbf{x} \rangle = \mathbf{x}^T \mathbf{x} = \|\mathbf{x}\|^2$$

Which is always a non-negative real number.

Exercise 2. Identify some of the many typesetting errors that occur between the lines:

$\det(A)$ should be $\det(A)$ in two cases.

In the third line, the “-2” is not written in math mode. It should look like -2 instead.

The long right arrow should not be used in place of the word “implies”.

A “\,” should be used before the dx in all three integrals.

The left and right parentheses in the equation on the separate line should be preceded by `\left` or `\right` in each case.

Exercise 3. One of my favorite mathematical statements is the Pythagorean Identity:

$$\sin^2 \theta + \cos^2 \theta = 1$$

It states that given any angle θ , the sum of the squares of the sine and cosine of θ is 1.

Another of my favorite mathematical statements is the Binomial Theorem:

$$(x + 1)^n = \binom{n}{1} + \binom{n}{2}x + \dots + \binom{n}{n}x^n$$

In other words, $x + 1$ raised to the n th power is the polynomial in x of degree n , where the coefficient, a_m , for each term, $a_m x^m$, is the number of permutations of m objects chosen from a set of n , read “ n choose m ”.

Another mathematical statement that I really like is the formula for the n th Fibonacci Number:

$$F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right).$$

It says that the n th Fibonacci number can be expressed as the difference of the n th powers of $1 + \sqrt{5}$ and $1 - \sqrt{5}$, divided by $2^n \sqrt{5}$, which I just think is neat and unexpected.