Discrete Midterm 2

Name: ______________________________

1. Prove the q-binomial theorem: $(1+xq^0)\cdots(1+xq^{n-1}) = \sum_{k=0}^{n} q^{\binom{k}{2}} \begin{bmatrix} n \\ k \end{bmatrix}_q x^k.$

2. Prove that $\lim_{n\to\infty} \begin{bmatrix} n \\ k \end{bmatrix}_q = \dfrac{1}{(1-q)\cdots(1-q^k)}$.

3. Prove that (the number of $\lambda \vdash n$ with distinct parts) is odd if and only if $n = \dfrac{k(3k-1)}{2}$ for some $k \in \mathbb{Z}$.

4. Prove that (the number of $\lambda \vdash n$ without any part a perfect square) is equal to (the number of $\lambda \vdash n$ such that a part of length i appears fewer than i times).

5. Let $e_n(x_1, x_2, x_3)$ be the elementary symmetric function in the variables x_1, x_2, x_3. Express $e_3e_2e_1$ as a sum of monomial symmetric functions of the form $m_\lambda(x_1, x_2, x_3)$ for $\lambda \vdash 6$.