Math 143 Set 7

1. Find the radius of convergence R for each of these power series:

a. $\sum_{n=0}^{\infty} (-1)^n (\ln n)(x-2)^n$

b. $\sum_{n=0}^{\infty} \frac{x^{2n}}{n^4}$

c. $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{(3n+1)!}$

d. $\sum_{n=0}^{\infty} 3^{\sqrt{n}}(x+1)^n$

e. $\sum_{n=0}^{\infty} n!(x-9)^{2n}$.

2. These are the top 3 Taylor series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad (R=\infty)$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad (R=1)$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \qquad (R=\infty)$$

By differentiating, integrating, or otherwise manipulating one of the above series, find the Taylor series for each of the functions below. Include the radius of convergence R for each power series.

a. $\frac{\sin(x^2)}{x}$

b. $\frac{1}{1-x^2}$

c. $\frac{e^{-x^2}-1}{x^2}$

d. $x^3 \cos(x^2)$

e. $\int \frac{\sin x}{x}\, dx$

f. $\int e^{-x^3}\, dx$

g. $\frac{d}{dx}\left(\frac{1-\cos\left(\sqrt{x}\right)}{x}\right)$

3. By multiplying known series, find the first 4 terms in the Taylor series at $x = 0$ for:

a. $e^{2x}\sin(x/2)$

b. $e^{-x^2}/(1-x)$

4. Approximate $\int_0^1 \sin(x^2)\, dx$ to within $1/100$ of the true value.

5. Approximate $\int_0^1 \frac{\sin x}{x}\, dx$ to within $1/100$ of the true value.