

## Math 344 Quiz 3

Names: \_\_\_\_\_

1.  $L_k$  is the polynomial solution to  $\frac{d}{dx} [xe^{-x}y'] = -ke^{-x}y$  that satisfies  $L_k(0) = 1$ . Find  $L_2$ .

2. Show that if  $k \neq m$ , then  $\int_0^\infty e^{-x} L_k L_m dx = 0$ .

**3.** Given  $\int_0^\infty e^{-x} L_k^2 dx = 1$ , find a formula for the constant  $a_i$  which makes the approximation

$$f(x) \approx a_0 L_0 + \cdots + a_i L_i + \cdots + a_n L_n$$

as accurate as possible.

**4 (Bonus! Only try when done with other problems).** Let  $p_k(x)$  be the  $k^{th}$  Legendre polynomial and let  $q(x)$  be a polynomial of degree less than  $k$ . Explain why  $\int_{-1}^1 q(x) p_k(x) dx = 0$ .