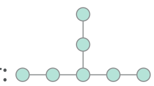
Graph Theory Set 5

21. Show that the complement of this tree on 7 vertices is planar: 

22. Find the least n such that if T is a tree with n vertices, then T^c is not planar.

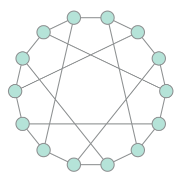
23. The **crossing number** of a connected graph G is the minimum number of edge crossings needed to draw G in the plane. Suppose G has V vertices, E edges, and smallest cycle length C.

a. Show that if G is planar, then $(C-2)E - C(V-2) \leq 0$.

b. Suppose G has crossing number K. Show that $(C-2)(E-K) - C(V-2) \leq 0$. Rewritten, this is $E - C(V-2)/(C-2) \leq K$, which gives a lower bound on K.

c. Find the crossing number for the Petersen graph.

d. Find the crossing number for the **Heawood graph**, a graph with $E = 21$, $V = 14$ and $C = 6$ that is shown below:



24. Let G be planar with V vertices, E edges, and F faces and let G^* be a dual for G.

a. How many vertices, edges, and faces does G^* have?

b. Let T be a spanning tree for G. Let T^* be the edges in G^* which do not cross edges in T when G and G^* are superimposed. Show that T^* is a spanning tree for G^*.

c. Use the above two parts of this exercise to give a new proof that $V - E + F = 2$.

25. Suppose G is connected, not a tree, does not contain $K_{1,3}$ as a subgraph, and does not contain $K_{1,3} + e$ as subgraph. Show that G is Hamiltonian.
Hint: Show that the longest cycle in G must contain every vertex.

26. Show the following graph is not Hamiltonian: