Math 118 Sample Midterm 2

Midterm 2 topics are the topics found in exercise Sets 6 through 10. The following practice problems are similar to those found on the midterm. They are just for practice and will not be collected.

1. Perform long division on these rational functions:

a.
$$\frac{x^5 + 3x^4 - 3x + 1}{x^2 + 1}$$

b.
$$\frac{x^3 - 1}{x^3 + 1}$$

c.
$$\frac{2x^4 + x^2 + 1}{2x^2 - 2x + 1}$$

2. State the Fundamental Theorem of Algebra.

3. Give an example of:

a. a polynomial of minimum possible degree with real coefficients and zeros equal to 0, 2 + 6i and 1 - 4i,

b. a polynomial that has 1 + i as a root of multiplicity 3 and 2 as a root of multiplicity 2.

4. Write the following polynomials in the form $a(x-c_1)\cdots(x-c_n)$ for a real number a and complex numbers c_1,\ldots,c_n :

a.
$$4x^3 + 3x^2 - x$$

b.
$$x^8 + 5x^4 - 4$$

5. Graph these functions, identifying any zeros, vertical asymptotes, other asymptotes:

a.
$$\frac{(x-1)^2(x+1)}{x^2+1}$$

b.
$$\frac{x^2 - x - 1}{x}$$

c.
$$x^2 + 1 - \frac{4 - x}{3 - x}$$

d.
$$\frac{x^2 + 4x + 4}{x^2 - 4}$$

e.
$$e^{-3x+1}$$

f.
$$1 - \ln(5 + x)$$

g.
$$4^{-x}$$

h.
$$\log_{10}(-x)$$

6. Solve these equations or inequalities:

a.
$$1 + x + \frac{1}{x} \le 0$$

b.
$$\frac{(x-1)^2(x+1)^3}{x(x-4)} \le 0$$

$$c. \ \frac{2-x}{x} > 3$$

d.
$$x^5 + 3x^3 - 5x \le 0$$

e.
$$ln(x+1) - ln(x-1) = 3$$

f.
$$e^{4x} + 2e^{2x} = -1$$
.

7. How much money would need to be initially invested at a rate of 12% continuously compounded interest as to earn 1 million dollars in 40 years?

8. Newton's law of cooling says that the temperature of a cooling object in a room with surrounding temperature T is $f(t) = T + e^{at+b}$ for some constants a and b. A hot cup of water at 200° is placed in a 40° freezer, after one minute it is 100°. What is the temperature after 5 minutes?

9. Find the inverse function to

a.
$$e^{-2x+1}$$

b.
$$\frac{1}{1 - e^x}$$

c.
$$2^{x} - 1$$