Math 143 Final Sample Questions

1. Let $f(x) = (1+x)^{1/2} + (1+x)^{3/2}$.

a. Find the degree 2 Taylor polynomial for $f(x)$ at $x = 0$.

b. Find a bound on the error when approximating $f(1/2)$ by taking $x = 1/2$ in part a.

2. For which values of x do these series converge?

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n+1} x^n$.

b. $\displaystyle\frac{1-1}{1} + \frac{3-2}{1\cdot 4}x + \frac{3^2-3}{1\cdot 4\cdot 7}x^2 + \frac{3^3-4}{1\cdot 4\cdot 7\cdot 10}x^3 + \frac{3^4-5}{1\cdot 4\cdot 7\cdot 10\cdot 13}x^4 + \cdots$.

3. Approximate $\displaystyle\int_0^1 \frac{\cos(2x)-1}{x^2}\,dx$ to within $1/100$ of the true answer.

4. Find the series representations for $\sqrt{x}\sin(\sqrt{3x})$ and $(e^{-x^2}-1)/x$.

5. Find the first three terms in the series of $\tan x$ and $e^x \sin x$.

6. For which values of x do these sums converge? What functions are they equal to when they do converge?

a. $\displaystyle\sum_{n=0}^{\infty} (x-1)^n/n!$

b. $\displaystyle\sum_{n=1}^{\infty} x^n/(n-1)!$

c. $\displaystyle\sum_{n=1}^{\infty} 2x^n$

d. $\displaystyle\sum_{n=0}^{\infty} (-1)^n(3x-2)^{2n+1}/(2n+1)!$

e. $\displaystyle\sum_{n=0}^{\infty} \left(\sum_{k=1}^{\infty} x^k/k\right)^n /n!$

7. Do these series converge?

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2+\sqrt{n+1}}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{1-6^n}{1+2^n}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 2\sin n}{(n+1)^5}$

d. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\sqrt{n}}}$

8. Find the degree 3 Taylor polynomial at $x = 0$ for $(1+2x)^{-1/2}$. Find the degree 3 Taylor polynomial at $x = 1$ for this same function.

9. The degree 5 Taylor polynomial for $\cos x$ is $1 - x^2/2! + x^4/4!$. Find a bound on the error when using this to approximate $\cos 3$.

10. Consider the curve given parametrically by $\begin{cases} x = \sin t - t\cos t \\ y = \cos t + t \sin t \end{cases}$ for $t \in \mathbb{R}$. Find all values of t which give vertical tangents and find the arclength of this curve on $[-2\pi, 4\pi]$.

11. Consider the vector valued function

$$\mathbf{r}(t) = \langle 1, \sin t + \cos t, \sin t - \cos t \rangle .$$

Find the velocity, speed, acceleration, the unit tangent vector $\mathbf{T}$, the unit normal vector $\mathbf{N}$, the unit binormal vector $\mathbf{B}$, the arclength of the curve traced by $\mathbf{r}$ from 0 to π, the curvature κ, the line tangent to the curve at $t = \pi/4$, and the plane normal to the curve at $t = \pi/4$.

12. Find an equation for the plane containing the point $(3, 0, -1)$ and the line common to the planes $x - y + z = 1$ and $-x - y + z = 1$.

13. Parameterize $\left\langle \frac{t}{\sqrt{1+t^2}}, \arctan t, \frac{1}{\sqrt{1+t^2}} \right\rangle$ by arclength. (Recall: $\arctan t = \int \frac{1}{1+t^2}\, dt$.)

14. Find the area enclosed by the polar equation $r(\theta) = \sin(2\theta)$.

15. Where is the curve in the plane described by $\langle t^2, t^3 \rangle$ concave up?

16. Find the line tangent to the curve $\langle t\cos t, t^2, t \sin t \rangle$ at $(-\pi, \pi^2, 0)$.

17. Parameterize $\langle e^t - 1, 2e^t + 2, e^t \rangle$ by arclength.

18. Find the curvature of the ellipse $\mathbf{r}(t) = \langle a\cos t, b \sin t, 0 \rangle$.

19. Find the circle which is tangent to and matches the curvature of the graph of $x^3/3 - x$ at $x = -1$.