Linear Analysis II Set 10

1. Find the projection matrix P for the span of the vectors $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$. Warning: to use

$$P = \frac{1}{\mathbf{u}_1^\top \mathbf{u}_1}\mathbf{u}_1\mathbf{u}_1^\top + \cdots + \frac{1}{\mathbf{u}_k^\top \mathbf{u}_k}\mathbf{u}_k\mathbf{u}_k^\top$$

the vectors $\mathbf{u}_1, \ldots, \mathbf{u}_k$ must be pairwise orthogonal.

2. Use the projection matrix P to find the vector $\mathbf{w}$ in the span of $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ closest to $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

3. Let P be the projection matrix onto the subspace S of $\mathbb{R}^n$ and let $\mathbf{x}, \mathbf{y}$ be any other vectors in $\mathbb{R}^n$. Explain why $(P\mathbf{x}) \cdot \mathbf{y} = \mathbf{x} \cdot (P\mathbf{y})$.

4. Let P be the projection matrix onto the span of $\mathbf{u}_1, \ldots, \mathbf{u}_k$. Let I be the identity matrix and define Q to be the matrix $I - P$. Show that these properties hold for Q:

a. $Q^\top = Q$

b. $Q^2 = Q$

c. $PQ = QP$

5. Find the function $f(x) = mx + b$ that best fits the data $(0,0), (-1,1), (1,2)$. Solve the problem two ways; both with and without using the normal equation.

6. Use the normal equation to find the function $f(x) = a2^x + b2^{-x}$ that best fits the data $(0,0), (-1,1), (1,2)$.