Math 248 Sample Final

Final topics are cumulative but emphasize functions, bijections, compositions, inverses, images and preimages, cardinality, and countable sets.

The following problems are similar to the final questions that involve the most recent topics. They are just for practice and will not be collected. There are also problems on the final that cover earlier material.

1. Is $\bigcup_{n\in\mathbb{N}} \mathbb{Q}^n$ countable? Why?

2. Prove that if $|A| = |B|$, then $|\mathcal{P}(A)| = |\mathcal{P}(B)|$.

3. Let A and B be sets. Find a counterexample to each of these false statements:

a. An injective function $f : A \to A$ is a surjection.

b. A surjective function $f : A \to A$ is an injection.

c. Let $f : A \to B$. If there exists a $g : B \to A$ such that $g \circ f = i_A$, then f is a bijection.

d. Let $f : A \to B$. If there exists a $g : B \to A$ such that $f \circ g = i_B$, then f is a bijection.

4. Let $f : A \to B$ be onto (a surjection) and let $Y \subseteq B$. Show that $f(f^{-1}(Y)) = Y$.