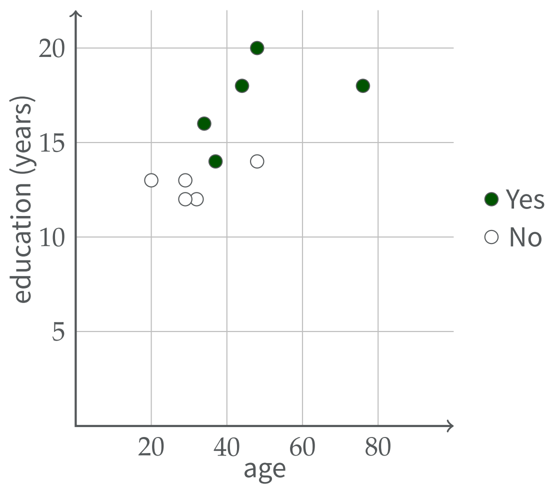
Linear Analysis II Exercise Set 10

1. If (x, y, b) is a row in the table below, let $g(x,y) = \begin{cases} 1 & \text{if } b = \text{Yes,} \\ -1 & \text{if } b = \text{No.} \end{cases}$

Find a function of the form $f(x,y) = ax + by + c$ which best approximates $g(x,y)$. What does this model predict for a 24 year old with 16 years of education?

	US census data	
age	*education (years)*	*income* $\geq$ *\$75k*
34	16	Yes
29	13	No
48	20	Yes
37	14	Yes
48	14	No
32	12	No
76	18	Yes
44	16	Yes
20	13	No
29	12	No



2. Find the projection matrix P for the span of the vectors $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$. Warning: to use

$$P = \frac{1}{\mathbf{u}_1^\top \mathbf{u}_1} \mathbf{u}_1 \mathbf{u}_1^\top + \cdots + \frac{1}{\mathbf{u}_k^\top \mathbf{u}_k} \mathbf{u}_k \mathbf{u}_k^\top$$

the vectors $\mathbf{u}_1, \ldots, \mathbf{u}_k$ must be pairwise orthogonal.

3. Use the projection matrix P to find the vector $\mathbf{w}$ in the span of $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ closest to $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

4. Let P be the projection matrix onto the subspace S of $\mathbb{R}^n$ and let $\mathbf{x}, \mathbf{y}$ be any other vectors in $\mathbb{R}^n$. Explain why $(P\mathbf{x}) \cdot \mathbf{y} = \mathbf{x} \cdot (P\mathbf{y})$.

5. Let P be the projection matrix onto the span of $\mathbf{u}_1, \ldots, \mathbf{u}_k$. Let I be the identity matrix and define Q to be the matrix $I - P$. Show that these properties hold for Q:

a. $Q^\top = Q$

b. $Q^2 = Q$

c. $PQ = QP$