Math 248 Sample Midterm 2

Midterm topics include: Proof by contrapositive and contradiction, proving biconditionals and existence, proofs involving subsets and set equality, disproving statements, conjectures, induction, relations and equivalence relations.

The following problems are similar to those found on the midterm. They are just for practice and will not be collected.

1. Define a sequence g_n such that $g_1 = g_2 = 1$ and $g_{n+2} = g_{n+1} + 2g_n$ for $n \geq 1$. Prove or disprove: $g_{n+5} = 4g_{n+3} + g_n$ for all $n \geq 1$.

2. Let R be the relation on $\mathbb{Z} \times \mathbb{Z}$ such that $(a, b)R(c, d)$ provided $a - b = c - d$. Verify this is an equivalence relation and describe $[(1, 3)]$.

3. Prove that 3 divides a if and only if 3 divides a^2.

4. Prove that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$ for all $n \in \mathbb{N}$.

5. Let A, B and C be sets. Show that $A - (B - C) = (A - B) \cup (A \cap C)$.

6. Prove that there are not $a, b \in \mathbb{N}$ such that $a^2 - b^2 = 1$.