Math 241 Quiz 1

Names: _____

1. Either find the following limits or show they do not exist:

a. $\lim_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\ln\left(x^2+y^2\right)}$

b. $\lim_{(x,y)\to(0,0)} \dfrac{x^2\sin(x/y)}{x^2+y^2}$

c. $\lim_{(x,y)\to(0,0)} \dfrac{x\left(x^2+y^2\right)}{y^2+\left(x^2+y^2\right)^2}$

2. If z is a function of x and y, $x = re^{\vartheta}$, and $y = re^{-\vartheta}$, write $\dfrac{\partial^2 z}{\partial r \partial \vartheta}$ in terms of $r, \vartheta, z_x, z_y, z_{xx}, z_{yy}$, and z_{xy}.

3. Let $f(x, y) = y^2 e^{x^2 - 2xy}$. Find the x, y coordinates where $\nabla f = \mathbf{0}$. What does $\nabla f = \mathbf{0}$ mean geometrically?