

Linear Analysis II Exercise Set 13

1. Show that the Fourier transform of the second derivative $F[f''(t)]$ is equal to $-\omega^2 F[f(t)]$.
2.
 - a. Show that the Fourier Transform of $e^{-|t|}$ is $\frac{1}{\pi(\omega^2 + 1)}$.
 - b. Using the inverse Fourier Transform (listed as the “Fourier relations” on line 2 on our table of Fourier transforms) and part a., find the Fourier transform of $\frac{1}{t^2 + 1}$.
3. The one dimensional wave equation is the partial differential equation $u_{tt}(t, x) = k^2 u_{xx}(t, x)$ where k is a real number and $u(t, x)$ is a function of time t and one spacial dimension x . The wave equation models the displacement of a vibrating string at time t and location x .
By taking partial derivatives and plugging in the function

$$u(t, x) = \frac{f(x + kt) + f(x - kt)}{2} + \frac{1}{2k} \int_{x-kt}^{x+kt} g(s) ds$$

into the partial differential equation, show that the above function is a solution to the wave equation

$$\begin{cases} u_{tt}(t, x) = k^2 u_{xx}(t, x) \\ u(0, x) = f(x), u_t(0, x) = g(x). \end{cases}$$