Math 143 Sample Final Exam

The final exam is cumulative but may emphasize the material not covered on previous exams. The exam is closed notes/resources but these identities will be given:

$$\cos^2 t = (1+\cos 2t)/2 \qquad \sin^2 t = (1-\cos 2t)/2 \qquad \sin^2 t + \cos^2 t = 1.$$

1. Approximate the value of $\displaystyle\sum_{n=4}^{\infty} \frac{(-1)^n}{\ln n}$ to within $1/100$ of the true value.

2. Let $f(x) = (1+x)^{1/2} + (1+x)^{3/2}$.

a. Find the degree 2 Taylor polynomial for $f(x)$ at $x = 0$.

b. Find a bound on the error when approximating $f(1/2)$ by taking $x = 1/2$ in part a.

3. Approximate $\displaystyle\int_0^1 \frac{\cos(2x)-1}{x^2}\,dx$ to within $1/100$ of the true answer.

4. Find the radius of convergence for these series:

a. $\displaystyle\sum_{n=1}^{\infty} x^n/(n-1)!$

b. $\displaystyle\sum_{n=1}^{\infty} 2(x+1)^n$

c. $\displaystyle\sum_{n=0}^{\infty} (-1)^n (x-2)^{2n+1}/(2n+1)!$

5. Find the degree 3 Taylor polynomial at $x = 0$ for $(1+2x)^{-1/2}$. Find the degree 3 Taylor polynomial at $x = 1$ for this same function.

6. Consider the curve given parametrically by $\begin{cases} x = \sin t - t\cos t \\ y = \cos t + t\sin t \end{cases}$ for $t \in \mathbb{R}$. Find all values of t which give vertical tangents and find the arclength of this curve on $[-2\pi, 4\pi]$.

7. Find an equation for the plane containing the point $(3, 0, -1)$ and the line common to the planes $x - y + z = 1$ and $-x - y + z = 1$.

8. Where is the curve in the plane described by $\langle t^2, t^3 \rangle$ concave up?

9. Parameterize $\langle e^t - 1, 2e^t + 2, e^t \rangle$ by arclength.

10. Find the circle which is tangent to and matches the curvature of the graph of $x^3/3 - x$ at $x = -1$.

11. Find $\mathbf{T}$, $\mathbf{N}$, $\mathbf{B}$, velocity, speed, curvature and acceleration for your favorite vector valued function.