

Math 142 Final Review

Topics

The final exam is cumulative! There may be a slight emphasis on the topics which have not appeared on a midterm. Those topics are

1. The center of mass of a flat, uniformly dense object in the shape of the graph of $f(x)$ on $[a, b]$ is $(\int_a^b x f(x) dx, \int_a^b f(x)^2 / 2 dx) / \int_a^b f(x) dx$.
2. Pappus' theorem: the volume of a solid formed by rotating an area around a line is the product of the area and the distance traveled by the center of mass.
3. The probability that a random variable with probability density function $f(x)$ is between a and b is $\int_a^b f(x) dx$. The exponential probability density function is $f(x) = ce^{-cx}$ for $x \geq 0$ where $1/c$ is the mean. The normal probability density function is $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$ where μ is the mean and σ is the standard deviation.
4. Separable differential equations.
5. Linear differential equations and solving with the integrating factor.

Sample questions

1. Find the y -coordinate of the center of mass of a flat object in the shape of $\sin x \sqrt{\cos x}$ on $[0, \pi/2]$.
2. The graph of $\sqrt{3xe^{-x^2}}$ on the interval $[0, 4]$ is spun around the x -axis to create a solid. Find its volume.
3. Integrate $\int x^2 \cos x dx$.
4. Evaluate $\lim_{x \rightarrow \infty} \frac{x^3}{e^{3x}}$.
5. Find the center of mass of a flat object with uniform density in the shape of the region below the graph of $\frac{2}{\sqrt{1-x^2}}$ on the interval $[0, 1/2]$. What is the volume of the object created when this region is spun around the y axis? What is the volume of the object created when this region is spun around the line $y = -x - 1$?
6. The mean of the probability density function $f(x)$ is equal to the x coordinate for the center of mass of the graph of $f(x)$. Use this to verify that the means for the exponential and the normal density functions are $1/c$ and μ , respectively.
7. Sketch the functions $|x - x^2|$ and $1 - x^2$ together on the axes provided below before finding the area of the region bounded by both curves.

8. For what values of x is the function xe^{5x} concave upward?

9. Integrate $\int \sin^2 x \cos^2 x \, dx$.

10. Either find the maximum value of the function $-x \ln x$ for $x \in (0, \infty)$ or show that this function does not have a maximum on this interval.

11. Evaluate $\int \frac{x-8}{x^2-x-2} \, dx$.

12. Integrate $\int_0^1 \frac{1}{\sqrt{2-x^2}} \, dx$.

13. Evaluate $\int \sin(3x) \sin(4x) \, dx$.

14. Integrate $\int x \frac{\sin(x^2)}{\cos(x^2)} \, dx$.

15. Find the center of mass of a flat object with uniform density in the shape of the region below the graph of $\sqrt{4-x^2}$ on the interval $[0, 2]$.

16. Solve $xy' + y = xy + 2xe^x$ where y is a function of x .

17. Solve $y' = x^2y^2 + y^2$ where y is a function of x .