# Math 344 Midterm 1 Solutions

**1.** Find $\mathcal{L}^{-1}\left[\dfrac{2s}{s^2+2s+4}+\dfrac{e^{-2s}}{(s+1)}+\dfrac{1}{s^{3/2}}+\dfrac{\mathcal{L}[f(t)]}{s^2}\right]$. (Write the last term as a convolution.)

**Solution.** $2e^{-t}\cos\sqrt{3}t-\dfrac{2}{\sqrt{3}}e^{-t}\sin\sqrt{3}t+u_2(t)e^{-(t-2)}+\dfrac{2}{\sqrt{\pi}}\sqrt{t}+f(t)*t.$

**2.** Solve $y''+y=f(t)$ where $f(t)=\begin{cases}1 & \text{if } 0\le t<1,\\ 0 & \text{if } 1\le t\end{cases}$ with $y(0)=0, y'(0)=1$.

**Solution.** $y(t)=1-\cos t+\sin t-u_1(t)\,(1-\cos(t-1))$.

**3.** Solve the system $\begin{cases}x'=y,\\ y'=-x+\delta(t-1),\end{cases}$ where $x(0)=0$ and $y(0)=0$.

**Solution.** $x(t)=u_1(t)\sin(t-1), y(t)=u_1(t)\cos(t-1)$.

**4.** Solve $x^2y''-xy'+4y=0$.

**Solution.** $y(x)=C_1x\cos\left(\sqrt{3}\ln x\right)+C_2x\sin\left(\sqrt{3}\ln x\right)$.

**5.** Find the two series solutions to $(1-x^2)y''-xy=0$ up to the $x^5$ term.

**Solution.** $y(x)=a_0\left(1+\dfrac{1}{3\cdot 2}x^3+\dfrac{1}{5\cdot 4}x^5+\cdots\right)+a_1\left(x+\dfrac{1}{4\cdot 3}x^4+\cdots\right)$.