Linear Analysis II Set 2

1. Use the known Laplace transforms of t^n, e^{at}, $\cos(at)$, and $\sin(at)$ given in the videos or the table of Laplace transforms on our web site to find

$$\mathcal{L}^{-1}\left[\frac{1}{s^5}+\frac{2s+1}{s^2+16}+\frac{6+4s^2}{s(s^2+2)}+\frac{a}{s^2-a^2}\right].$$

Hint: Use partial fractions on the last two terms.

2. Let y and x be functions of t. Use Laplace transforms to solve the following:

a. $y'-y=e^{2t}$ with $y(0)=3$.

b. $y''-4y=0$ with $y(0)=1, y'(0)=2$.

c. $y''-y'=-t$ with $y(0)=0, y'(0)=1$.

d. $\begin{cases} x'=9y \\ y'=-x \end{cases}$ with $x(0)=0, y(0)=1$.

e. $\begin{cases} x'=-4x-2y \\ y'=x-y, \end{cases}$ with $x(0)=0, y(0)=1$.

f. $\begin{cases} x'=x+y+1 \\ y'=x+y-1, \end{cases}$ with $x(0)=1, y(0)=0$.

3. Verify the following identities involving the Laplace transform:

a. $\mathcal{L}\left[\int_0^t f(x)\,dx\right]=\frac{1}{s}\mathcal{L}[f(t)]$.

b. $\frac{d}{ds}\mathcal{L}[f(t)]=-\mathcal{L}[tf(t)]$.