# Math 143 Midterm 1 Review

## Topics on Midterm 1

1. Taylor polynomials of degree $n$ for $f(x)$ centered at $x = a$, namely $y = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$.
2. The error when $f(a)$ is approximated using the degree $n$ Taylor polynomial for $f(x)$ centered at $x = 0$ is less than $M/(n+1)!|a|^{n+1}$ where $|f^{(n+1)}(x)| \leq M$ for $-a \leq x \leq a$.
3. Taylor series for $f(x)$ centered at $x = a$, namely $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$.
4. The top 3 Taylor series:
   (a) The geometric series: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $-1 < x < 1$
   (b) The exponential function: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all $x$.
   (c) The sine function: $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$ for all $x$.
5. Ways to test for the convergence of infinite series:
   (a) Recognize a known series (a geometric series, a $p$-series)
   (b) Compare the series to a larger convergent series
   (c) Compare the series to a smaller divergent series
   (d) Compare the series to an improper integral of the form $\int_1^{\infty} f(x)\,dx$
   (e) Use the limit comparison test
   (f) Use the Ratio test
   (g) Use the Alternating series test
6. The interval and radius of convergence for functions of the form $\sum_{n=0}^{\infty} a_n x^n$.

## Sample questions

**1.** Do these series converge? If so, why?

a. $\sum_{n=2}^{\infty} \frac{4n^2-2}{3n^2+2}$

b. $\sum_{n=2}^{\infty} \frac{n}{n^2+1}$

c. $\sum_{n=2}^{\infty}(-1)^n \frac{n}{n^2+1}$

d. $\sum_{n=2}^{\infty} \frac{\ln n}{2^{n^2}}$

e. $\sum_{n=2}^{\infty} \frac{(1+n)^3}{(1+\sqrt{n})^4 \ln n}$

**2.** Find the interval and radius of convergence for these series

a. $\sum_{n=0}^{\infty} n^{n/2} x^n$

b. $\sum_{n=0}^{\infty} (x+4)^n / n^4$

c. $\sum_{n=0}^{\infty} (x-1)^{2n}$

**3.** Find the Taylor series for $1/(1-2x)$ centered at $x=0$.

**4.** Find the degree 5 Taylor polynomial for $x+\sqrt{x}$ centered at $x=1$.

**5.** Let $f(x)=\sqrt{5+2x}$.

a. Find the degree 2 Taylor polynomial for $f(x)$.

b. Find a bound on the error when approximating $f(1)$ by taking $x=1$ in part a.

**6.** Find the exact values of these series

a. $\sum_{n=2}^{\infty}(-1)^n \frac{2^n}{3^{n-1}}$

b. $\sum_{n=2}^{\infty}(-1)^n \frac{2^n}{(n-1)!}$