Math 143 Final Review

The final exam is on December 9. It begins at 10:10am and ends at 12pm.
Blue book exercises will be collected at the final exam, or should be turned in on Canvas.

Topics

The final exam is cumulative and will cover the topics found on Midterm 1 and Midterm 2 (see the previous exam reviews for a list of those topics) in addition to the topics below.

1. Lines and planes
2. Vector valued functions
3. Parameterizing by arclength.
4. $\mathbf{T}(t), \mathbf{N}(t), \mathbf{B}(t)$.
5. Curvature $\kappa(t) = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$.
6. Velocity, speed, and acceleration and $\mathbf{r}''(t) = \dfrac{d}{dt}\left(|\mathbf{r}'(t)|\right)\mathbf{T}(t) + |\mathbf{r}'(t)|^2\kappa(t)\mathbf{N}(t)$.

Sample questions

1. Let $f(x) = (1+x)^{1/2} + (1+x)^{3/2}$.

a. Find the degree 2 Taylor polynomial for $f(x)$ at $x = 0$.

b. Find a bound on the error when approximating $f(1/2)$ by taking $x = 1/2$ in part a.

2. For which values of x do these series converge?

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n+1} x^n$.

b. $\dfrac{1-1}{1} + \dfrac{3-2}{1\cdot 4}x + \dfrac{3^2-3}{1\cdot 4\cdot 7}x^2 + \dfrac{3^3-4}{1\cdot 4\cdot 7\cdot 10}x^3 + \dfrac{3^4-5}{1\cdot 4\cdot 7\cdot 10\cdot 13}x^4 + \cdots$.

3. Approximate $\displaystyle\int_0^1 \frac{\cos(2x)-1}{x^2}\,dx$ to within $1/100$ of the true answer.

4. Find the series representations for $\sqrt{x}\sin(\sqrt{3x})$ and $(e^{-x^2}-1)/x$.

5. Find the first three terms in the series of $\tan x$ and $e^x \sin x$.

6. For which values of x do these sums converge? What functions are they equal to when they do converge?

a. $\sum_{n=0}^{\infty}(x-1)^n/n!$

b. $\sum_{n=1}^{\infty}x^n/(n-1)!$

c. $\sum_{n=1}^{\infty}2x^n$

d. $\sum_{n=0}^{\infty}(-1)^n(3x-2)^{2n+1}/(2n+1)!$

e. $\sum_{n=0}^{\infty}\left(\sum_{k=1}^{\infty}x^k/k\right)^n/n!$

7. Do these series converge? Why?

a. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2+\sqrt{n+1}}$

b. $\sum_{n=1}^{\infty}\frac{1-6^n}{1+2^n}$

c. $\sum_{n=1}^{\infty}\frac{n^2+2\sin n}{(n+1)^5}$

d. $\sum_{n=2}^{\infty}\frac{1}{(\ln n)\sqrt{n}}$

8. Find the degree 3 Taylor polynomial at $x=0$ for $(1+2x)^{-1/2}$. Find the degree 3 Taylor polynomial at $x=1$ for this same function.

9. The degree 5 Taylor polynomial for $\cos x$ is $1-x^2/2!+x^4/4!$. Find a bound on the error when using this to approximate $\cos 3$.

10. Consider the curve given parametrically by $\begin{cases} x=\sin t - t\cos t \\ y=\cos t + t\sin t \end{cases}$ for $t\in\mathbb{R}$. Find all values of t which give vertical tangents and find the arclength of this curve on $[-2\pi,4\pi]$. Some of the calculations might be messy, such messy calculations will not appear on the exam.

11. Consider the vector valued function

$$\mathbf{r}(t)=\langle 1,\sin t+\cos t,\sin t-\cos t\rangle .$$

Describe how to find the velocity, speed, acceleration, the unit tangent vector $\mathbf{T}$, the unit normal vector $\mathbf{N}$, the unit binormal vector $\mathbf{B}$, the arclength of the curve traced by $\mathbf{r}$ from 0 to π, the curvature κ, the line tangent to the curve at $t=\pi/4$, and the plane normal to the curve at $t=\pi/4$. Some of the calculations might be messy, such messy calculations will not appear on the exam.

12. Parameterize $\left\langle \frac{t}{\sqrt{1+t^2}}, \arctan t, \frac{1}{\sqrt{1+t^2}} \right\rangle$ by arclength. (Recall: $\arctan t = \int \frac{1}{1+t^2}\, dt$.)

13. Find the area enclosed by the polar equation $r(\theta) = \sin(2\theta)$.

14. Where is the curve in the plane described by $\langle t^2, t^3 \rangle$ concave up?

15. Find the line tangent to the curve $\langle t\cos t, t^2, t\sin t \rangle$ at $(-\pi, \pi^2, 0)$.

16. Parameterize $\langle e^t - 1, 2e^t + 2, e^t \rangle$ by arclength.

17. Find the curvature of the ellipse $\mathbf{r}(t) = \langle a\cos t, b\sin t, 0 \rangle$.

18. Find the equation of the plane containing the line $\begin{cases} x = 3 + 2t \\ y = 3 - 2t \\ z = 3t \end{cases}$ and the point $(1, 3, 1)$.