Linear Analysis II Set 6

1. Solve the differential equations below using series. Write solutions in the form

$$y(x) = a_0 y_1(x) + a_1 y_2(x)$$

where $y_1(x)$ and $y_2(x)$ are series. It is acceptable to only write down the first three nonzero terms in the series for $y_1(x)$ and $y_2(x)$ followed by "$\cdots$". For example, instead of writing $y_1(x) = \sin x$, it is acceptable to write

$$y_1(x) = \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots.$$

a. $y'' - 2xy' - 2y = 0$

b. $y'' + xy' + 3y = 0$

c. $y'' - y' - xy = 0$

d. $(1 + x^2)y'' + 4xy' + 2y = 0$

e. $(x^2 - 1)y'' - 6xy' + 12y = 0$

2. Find one solution to $xy'' + y' + xy = 0$.