

Math 244 Quiz 5

Names: _____

1. Let $A = \begin{bmatrix} 0 & -4 \\ 1 & 0 \end{bmatrix}$.

a. Find e^{At} and write it without using i . Without doing more calculations, what is e^{-At} ?

b. Let $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$. Solve the linear system $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ by writing it as $\frac{d}{dt} \left(e^{-At} \mathbf{x} \right) = e^{-At} \mathbf{f}$.

2. Let $A = \begin{bmatrix} 0 & 1 & -3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. Find e^{At} and use it to solve $\mathbf{x}' = A\mathbf{x}$ with the initial condition $\mathbf{x}(0) = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$.

Hint: Calculate $e^{At} = I + At + A^2t^2/2! + A^3t^3/3! + \dots$ without diagonalizing A first.

3 (Bonus! Only try when all other exercises are completed.) Solve $\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \mathbf{x}$ with the initial condition $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ using the matrix exponential.