Math 143 Set 9

1. Find the line tangent to the curves at the indicated point:

a. $\begin{cases} x = 6\sin t \\ y = t^2 + t \end{cases}$ at the point found when $t = 1$.

b. $\begin{cases} x = \cos t + \cos(2t) \\ y = \sin t + \sin(2t) \end{cases}$ at the point $(-1, 1)$.

c. $\begin{cases} x = 2t - \pi \sin t \\ y = 2 - \pi \cos t \end{cases}$ at $t = \pi/2$.

2. Find the first and second derivatives of these parametric curves. For which values of t is the parametric equation concave up?

a. $\begin{cases} x = 2\sin t \\ y = 3\cos t \end{cases}$

b. $\begin{cases} x = t^3 - 12t \\ y = t^2 - 1 \end{cases}$

3. Find the exact length of the curve:

a. $\begin{cases} x = 1 + 3t^2, \\ y = 4 + 2t^3 \end{cases}$ for $t \in [0, 1]$

b. $\begin{cases} x = e^t + e^{-t}, \\ y = 5 - 2t \end{cases}$ for $t \in [0, 3]$

c. $\begin{cases} x = e^t \cos t, \\ y = e^t \sin t \end{cases}$ for $t \in [0, \pi]$

4. Consider the parametric equations

$$\begin{cases} x = \displaystyle\int_0^t \frac{\cos u}{1 + u^2}\, du, \\ y = \displaystyle\int_0^t \frac{\sin u}{1 + u^2}\, du \end{cases}$$

for $t \in [0, \infty)$. What is the first positive value of t for which this curve has a vertical tangent line? What is the length of the curve from $(0, 0)$ to this value?

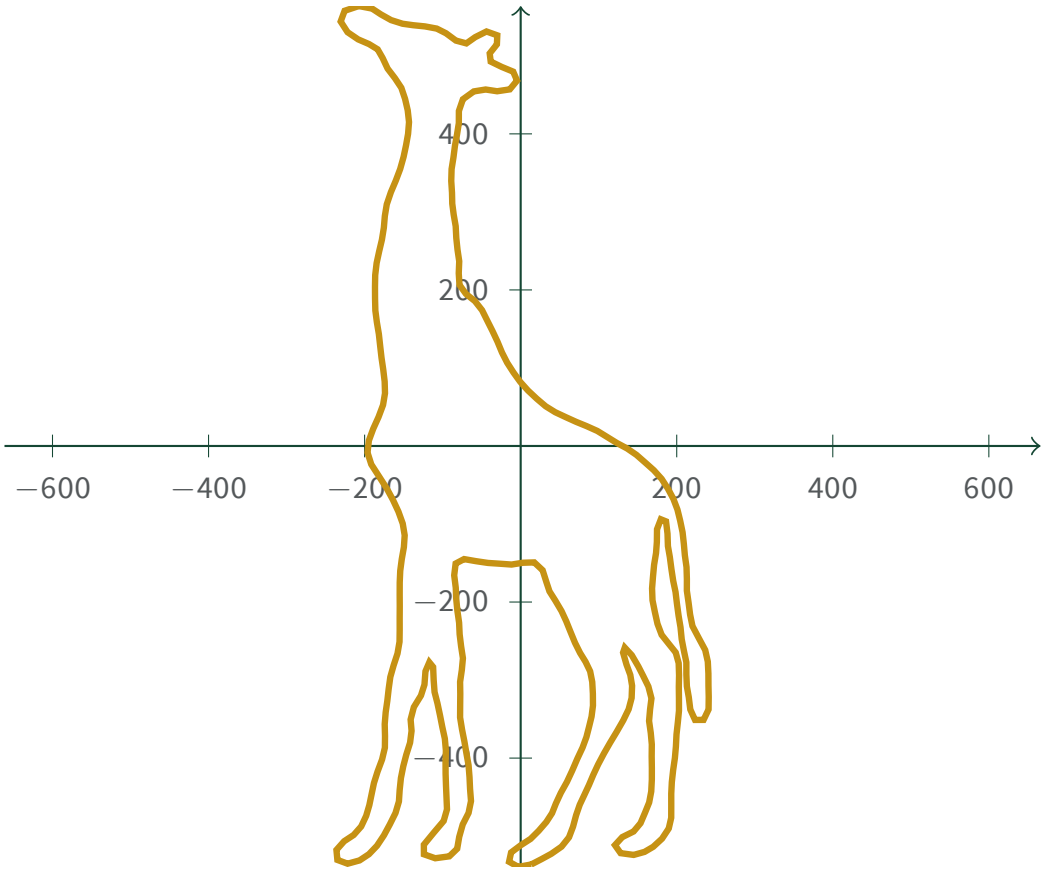
5 (Bonus! This question is optional.). The parametric equations for the g-raph (on the next page) are

$$
\begin{aligned}
x = {} & \frac{1367}{7}\sin(\frac{23}{9}-t) - \frac{389}{13}\sin(6t+\frac{1}{4}) - \frac{23}{8}\sin(20t+\frac{2}{3}) - \frac{11}{5}\sin(26t+\frac{16}{13}) - \frac{7}{5}\sin(36t+\frac{2}{3}) - \frac{8}{25}\sin(37t+\frac{1}{3}) - \frac{10}{9}\sin(38t+\frac{2}{9}) \\
& - \frac{9}{7}\sin(44t+\frac{11}{8}) - \frac{2}{7}\sin(53t+\frac{4}{5}) - \frac{1}{4}\sin(60t+\frac{7}{11}) - \frac{4}{9}\sin(62t+\frac{1}{3}) - \frac{2}{3}\sin(64t+\frac{4}{9}) - \frac{2}{9}\sin(75t+\frac{33}{34}) - \frac{3}{8}\sin(86t+1) \\
& - \frac{1}{7}\sin(90t+\frac{5}{12}) - \frac{1}{24}\sin(97t+\frac{1}{9}) - \frac{1}{7}\sin(98t+\frac{5}{9}) - \frac{1}{11}\sin(102t+\frac{1}{11}) - \frac{1}{5}\sin(114t+\frac{4}{11}) - \frac{1}{48}\sin(116t+\frac{15}{16}) - \frac{1}{7}\sin(118t+\frac{4}{5}) \\
& + \frac{892}{27}\sin(\frac{23}{5}-2t) + \frac{259}{12}\sin(\frac{2}{7}-3t) + \frac{34}{3}\sin(\frac{37}{16}-4t) + \frac{205}{14}\sin(\frac{25}{6}-5t) + \frac{31}{4}\sin(\frac{29}{8}-7t) + \frac{187}{9}\sin(\frac{77}{17}-8t) + \frac{1}{3}\sin(\frac{11}{10}-9t) \\
& + \frac{127}{6}\sin(\frac{37}{8}-10t) + \frac{17}{4}\sin(\frac{25}{9}-11t) + \frac{62}{13}\sin(\frac{37}{9}-12t) + \frac{60}{7}\sin(\frac{22}{13}-13t) + \frac{19}{3}\sin(\frac{171}{43}-14t) + \frac{43}{13}\sin(\frac{17}{11}-15t) + \frac{69}{11}\sin(\frac{30}{7}-16t) \\
& + \frac{10}{3}\sin(\frac{47}{10}-17t) + \frac{33}{8}\sin(\frac{67}{15}-18t) + \frac{11}{3}\sin(\frac{17}{8}-19t) + \frac{21}{8}\sin(\frac{37}{9}-21t) + \frac{23}{9}\sin(\frac{16}{9}-22t) + \frac{37}{10}\sin(\frac{9}{4}-23t) + \frac{3}{4}\sin(\frac{112}{37}-24t) \\
& + \frac{25}{13}\sin(\frac{43}{17}-25t) + \frac{11}{10}\sin(\frac{31}{7}-27t) + \frac{13}{3}\sin(\frac{51}{11}-28t) + \frac{27}{11}\sin(\frac{33}{13}-29t) + \frac{28}{13}\sin(\frac{25}{12}-30t) + \frac{11}{8}\sin(\frac{26}{7}-31t) + \frac{23}{10}\sin(\frac{43}{13}-32t) \\
& + \frac{24}{23}\sin(\frac{83}{21}-33t) + \frac{17}{12}\sin(4-34t) + \frac{9}{11}\sin(\frac{33}{8}-35t) + \frac{17}{10}\sin(\frac{23}{7}-39t) + \frac{7}{6}\sin(\frac{3}{2}-40t) + \frac{16}{11}\sin(\frac{20}{9}-41t) + \frac{7}{8}\sin(\frac{41}{9}-42t) \\
& + \frac{17}{11}\sin(\frac{43}{16}-43t) + \frac{3}{4}\sin(\frac{19}{9}-45t) + \frac{7}{12}\sin(\frac{17}{5}-46t) + \frac{16}{15}\sin(\frac{13}{9}-47t) + \frac{3}{7}\sin(\frac{15}{4}-48t) + \frac{29}{28}\sin(\frac{73}{18}-49t) + \frac{7}{6}\sin(\frac{38}{13}-50t) \\
& + \frac{1}{16}\sin(\frac{15}{4}-51t) + \frac{1}{6}\sin(\frac{8}{11}-52t) + \frac{3}{7}\sin(\frac{2}{11}-54t) + \frac{2}{3}\sin(\frac{27}{7}-55t) + \frac{5}{8}\sin(\frac{11}{9}-56t) + \frac{8}{11}\sin(\frac{24}{11}-57t) + \frac{31}{32}\sin(\frac{7}{12}-58t) \\
& + \frac{20}{19}\sin(\frac{7}{2}-59t) + \frac{3}{5}\sin(\frac{12}{5}-61t) + \frac{3}{10}\sin(\frac{5}{2}-63t) + \frac{1}{9}\sin(\frac{20}{7}-65t) + \frac{3}{7}\sin(\frac{29}{9}-66t) + \frac{3}{4}\sin(\frac{57}{23}-67t) + \frac{2}{5}\sin(\frac{24}{25}-68t) \\
& + \frac{1}{3}\sin(\frac{1}{3}-69t) + \frac{4}{13}\sin(\frac{59}{14}-70t) + \frac{2}{13}\sin(\frac{3}{5}-71t) + \frac{1}{8}\sin(\frac{17}{16}-72t) + \frac{1}{3}\sin(\frac{4}{13}-73t) + \frac{1}{5}\sin(\frac{9}{8}-74t) + \frac{4}{11}\sin(\frac{28}{17}-76t) \\
& + \frac{5}{9}\sin(\frac{52}{15}-77t) + \frac{3}{10}\sin(\frac{33}{14}-78t) + \frac{1}{25}\sin(\frac{4}{3}-79t) + \frac{7}{11}\sin(\frac{5}{14}-80t) + \frac{1}{9}\sin(\frac{32}{9}-81t) + \frac{2}{9}\sin(\frac{13}{11}-82t) + \frac{3}{10}\sin(\frac{32}{11}-83t) \\
& + \frac{1}{4}\sin(\frac{19}{13}-84t) + \frac{2}{11}\sin(\frac{33}{16}-85t) + \frac{1}{7}\sin(\frac{220}{73}-87t) + \frac{1}{9}\sin(\frac{25}{6}-88t) + \frac{1}{4}\sin(\frac{7}{8}-89t) + \frac{1}{3}\sin(\frac{11}{10}-91t) + \frac{4}{13}\sin(\frac{19}{11}-92t) \\
& + \frac{1}{35}\sin(\frac{9}{8}-93t) + \frac{1}{10}\sin(\frac{27}{10}-94t) + \frac{1}{5}\sin(\frac{47}{19}-95t) + \frac{1}{12}\sin(\frac{29}{13}-96t) + \frac{1}{49}\sin(\frac{11}{16}-99t) + \frac{1}{7}\sin(\frac{3}{2}-100t) + \frac{1}{23}\sin(\frac{57}{29}-101t) \\
& + \frac{1}{11}\sin(\frac{21}{5}-103t) + \frac{1}{12}\sin(\frac{1}{3}-104t) + \frac{1}{8}\sin(\frac{11}{6}-105t) + \frac{1}{5}\sin(\frac{1}{3}-106t) + \frac{1}{15}\sin(\frac{1}{81}-107t) + \frac{1}{7}\sin(\frac{12}{13}-108t) + \frac{1}{18}\sin(\frac{8}{5}-109t) \\
& + \frac{2}{11}\sin(\frac{2}{9}-110t) + \frac{1}{5}\sin(\frac{43}{11}-111t) + \frac{1}{11}\sin(\frac{22}{5}-112t) + \frac{2}{9}\sin(3-113t) + \frac{1}{8}\sin(\frac{13}{10}-115t) + \frac{1}{33}\sin(\frac{28}{9}-117t) + \frac{1}{11}\sin(\frac{10}{11}-119t) \\
& + \frac{1}{7}\sin(\frac{21}{8}-120t) - \frac{26}{5}
\end{aligned}
$$

and

$$
\begin{aligned}
y = {} & \frac{1846}{5}\sin(\frac{67}{15}-t) - \frac{482}{37}\sin(10t) - \frac{1325}{12}\sin(3t+\frac{9}{7}) - \frac{97}{15}\sin(5t+\frac{20}{13}) - \frac{33}{10}\sin(23t+\frac{10}{7}) - \frac{34}{9}\sin(27t+\frac{5}{6}) - \frac{11}{4}\sin(28t+\frac{4}{11}) \\
& - \frac{8}{7}\sin(31t+\frac{17}{12}) - \frac{17}{7}\sin(32t+\frac{4}{11}) - \frac{3}{5}\sin(36t+\frac{1}{5}) - \frac{8}{9}\sin(41t+\frac{31}{21}) - \frac{3}{5}\sin(43t+\frac{12}{11}) - \frac{3}{8}\sin(49t+\frac{7}{6}) - \frac{3}{10}\sin(57t+\frac{20}{13}) \\
& - \frac{5}{7}\sin(59t+\frac{11}{7}) - \frac{2}{5}\sin(66t+\frac{1}{4}) - \frac{1}{3}\sin(70t+\frac{15}{11}) - \frac{1}{5}\sin(71t+\frac{1}{9}) - \frac{1}{3}\sin(75t+\frac{7}{9}) - \frac{1}{5}\sin(77t+\frac{10}{7}) - \frac{1}{5}\sin(79t+\frac{1}{33}) \\
& - \frac{2}{11}\sin(94t+\frac{8}{17}) - \frac{1}{21}\sin(104t+\frac{12}{13}) - \frac{1}{7}\sin(105t+\frac{1}{48}) - \frac{1}{10}\sin(106t+\frac{34}{33}) - \frac{1}{10}\sin(109t+\frac{8}{7}) + \frac{3281}{15}\sin(\frac{7}{6}-2t) + \frac{127}{6}\sin(\frac{1}{12}-4t) \\
& + \frac{1106}{27}\sin(\frac{63}{16}-6t) + \frac{752}{9}\sin(\frac{63}{16}-7t) + \frac{369}{13}\sin(\frac{111}{28}-8t) + \frac{531}{19}\sin(\frac{3}{8}-9t) + \frac{172}{9}\sin(\frac{65}{14}-11t) + \frac{31}{12}\sin(\frac{47}{10}-12t) + \frac{193}{17}\sin(\frac{7}{11}-13t) \\
& + \frac{63}{10}\sin(\frac{37}{36}-14t) + \frac{48}{11}\sin(\frac{29}{7}-15t) + \frac{30}{7}\sin(\frac{209}{52}-16t) + \frac{25}{8}\sin(\frac{51}{11}-17t) + \frac{12}{7}\sin(\frac{67}{34}-18t) + \frac{29}{11}\sin(\frac{16}{7}-19t) + \frac{9}{10}\sin(\frac{19}{9}-20t) \\
& + \frac{1}{2}\sin(\frac{16}{5}-21t) + \frac{28}{11}\sin(\frac{18}{5}-22t) + \frac{37}{36}\sin(\frac{14}{3}-24t) + \frac{3}{2}\sin(\frac{25}{7}-25t) + \frac{2}{3}\sin(\frac{17}{7}-26t) + \frac{9}{11}\sin(\frac{49}{16}-29t) + \frac{53}{21}\sin(\frac{43}{22}-30t) \\
& + \frac{18}{17}\sin(\frac{38}{11}-33t) + \frac{8}{9}\sin(\frac{13}{6}-34t) + \frac{7}{6}\sin(\frac{29}{11}-35t) + \frac{10}{19}\sin(\frac{29}{19}-37t) + \frac{1}{3}\sin(\frac{25}{6}-38t) + \frac{5}{9}\sin(\frac{51}{11}-39t) + \frac{7}{11}\sin(\frac{31}{7}-40t) \\
& + \frac{3}{4}\sin(\frac{35}{8}-42t) + \frac{5}{8}\sin(\frac{21}{16}-44t) + \frac{4}{11}\sin(\frac{85}{19}-45t) + \frac{2}{5}\sin(\frac{47}{14}-46t) + \frac{1}{4}\sin(\frac{3}{7}-47t) + \frac{10}{7}\sin(\frac{9}{8}-48t) + \frac{3}{5}\sin(\frac{29}{7}-50t) \\
& + \frac{11}{15}\sin(\frac{38}{9}-51t) + \frac{2}{3}\sin(\frac{21}{16}-52t) + \frac{5}{14}\sin(\frac{18}{7}-53t) + \frac{7}{10}\sin(\frac{53}{18}-54t) + \frac{7}{12}\sin(\frac{41}{10}-55t) + \frac{1}{2}\sin(\frac{19}{11}-56t) + \frac{12}{23}\sin(\frac{6}{7}-58t) \\
& + \frac{4}{11}\sin(\frac{1}{6}-60t) + \frac{8}{13}\sin(\frac{47}{11}-61t) + \frac{4}{7}\sin(\frac{1}{14}-62t) + \frac{5}{11}\sin(\frac{18}{5}-63t) + \frac{5}{14}\sin(2-64t) + \frac{3}{4}\sin(\frac{25}{9}-65t) + \frac{2}{13}\sin(\frac{17}{7}-67t) \\
& + \frac{1}{5}\sin(\frac{43}{16}-68t) + \frac{4}{9}\sin(\frac{25}{11}-69t) + \frac{3}{7}\sin(\frac{23}{7}-72t) + \frac{1}{7}\sin(\frac{81}{23}-73t) + \frac{2}{5}\sin(\frac{45}{13}-74t) + \frac{1}{10}\sin(\frac{24}{11}-76t) + \frac{2}{11}\sin(\frac{20}{9}-78t) \\
& + \frac{1}{18}\sin(\frac{7}{6}-80t) + \frac{1}{8}\sin(\frac{45}{11}-81t) + \frac{4}{11}\sin(\frac{33}{16}-82t) + \frac{1}{9}\sin(\frac{20}{11}-83t) + \frac{1}{6}\sin(\frac{11}{3}-84t) + \frac{2}{7}\sin(\frac{45}{11}-85t) + \frac{1}{11}\sin(\frac{18}{11}-86t) \\
& + \frac{1}{8}\sin(\frac{3}{2}-87t) + \frac{3}{13}\sin(\frac{49}{15}-88t) + \frac{2}{9}\sin(\frac{31}{8}-89t) + \frac{1}{13}\sin(\frac{24}{7}-90t) + \frac{1}{21}\sin(\frac{3}{7}-91t) + \frac{1}{6}\sin(\frac{25}{13}-92t) + \frac{1}{6}\sin(\frac{68}{15}-93t) \\
& + \frac{1}{67}\sin(\frac{15}{16}-95t) + \frac{1}{5}\sin(\frac{1}{2}-96t) + \frac{1}{12}\sin(\frac{11}{3}-98t) + \frac{5}{16}\sin(\frac{13}{5}-99t) + \frac{1}{7}\sin(2-100t) + \frac{1}{9}\sin(\frac{41}{20}-101t) + \frac{1}{14}\sin(\frac{18}{5}-102t) \\
& + \frac{1}{6}\sin(\frac{40}{17}-103t) + \frac{1}{25}\sin(\frac{26}{11}-107t) + \frac{1}{5}\sin(\frac{15}{4}-108t) + \frac{1}{20}\sin(\frac{8}{5}-110t) + \frac{1}{15}\sin(\frac{13}{7}-111t) + \frac{1}{10}\sin(\frac{34}{15}-112t) + \frac{1}{9}\sin(\frac{2}{5}-113t) \\
& + \frac{2}{11}\sin(\frac{3}{5}-114t) + \frac{1}{13}\sin(\frac{3}{7}-115t) + \frac{1}{6}\sin(\frac{15}{7}-116t) + \frac{1}{13}\sin(\frac{20}{7}-117t) + \frac{1}{23}\sin(\frac{17}{16}-118t) + \frac{1}{6}\sin(\frac{45}{11}-119t) \\
& + \frac{1}{18}\sin(\frac{13}{4}-120t) - \frac{1721}{12}
\end{aligned}
$$

for $t \in [0, 2\pi]$. Find the arclength of the g-raph.


400
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