

1 Systems and Row Echelon Form

1. Find the general solutions to the systems with these augmented matrices by putting the matrix into Row Echelon Form:

a.
$$\begin{bmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

b.
$$\begin{bmatrix} 1 & -5 & 4 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

c.
$$\begin{bmatrix} 1 & -1 & 0 & 0 & 5 \\ 0 & 1 & -2 & 0 & 7 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

d.
$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$

e.
$$\begin{bmatrix} 1 & -3 & 0 & -5 \\ -3 & 7 & 0 & 9 \end{bmatrix}$$

f.
$$\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & 2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Solve the system of equations:

a.

$$\begin{aligned} x_2 + 5x_3 &= -4 \\ x_1 + 4x_2 + 3x_3 &= -2 \\ 2x_1 + 7x_2 + x_3 &= -2 \end{aligned}$$

b.

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= -3 \\ 2x_1 - 7x_2 + 3x_3 &= -2 \\ -2x_1 + x_2 + 7x_3 &= -1 \end{aligned}$$

c.

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ 2x_1 - 7x_2 &= -5 \end{aligned}$$

3. Determine if the following system is consistent. Do not completely solve the system.

$$\begin{aligned} x_1 - 6x_2 &= 5 \\ x_2 - 4x_3 + x_4 &= 0 \\ -x_1 + 6x_2 + x_3 + 5x_4 &= 3 \\ -x_2 + 5x_3 + 4x_4 &= 0 \end{aligned}$$

4. Do these three planes have at least one point in common? Why?

$$\begin{aligned} 2x_1 + 4x_2 + 4x_3 &= 4 \\ x_2 - 2x_3 &= -2 \\ 2x_1 + 3x_2 &= 0 \end{aligned}$$

5. True or false:

- a. Every elementary row operation is reversible.
- b. A 5×6 matrix has six rows.
- c. Elementary row operations on an augmented matrix never change the solution set of the associated linear system.

6. Give an example of an inconsistent system of two equations in three unknowns.