Math 142 Sample Midterm 2

Topics

1. Know everything (graphs, limits, derivatives, integrals, …) about
   - $\arcsin x$
   - $\arctan x$
2. Integration techniques.
   - Just know $\int 1/x\,dx$, $\int 1/(1+x^2)\,dx$, $\int 1/\sqrt{1-x^2}\,dx$, $\int e^x\,dx$, $\int x^k\,dx$, $\int \sin x\,dx$, $\int \cos x\,dx$.
   - u-substitution
   - Integration by parts
   - Integration of powers of $\sin x$ and $\cos x$
   - Integration of rational functions
   - Integration involving $\sqrt{a^2 \pm x^2}$
3. Numerical integration
4. L'Hôpital's rule
5. Improper integrals
6. Arclength

Sample Questions

The blue book exercises are a good measure of difficulty for exam questions. A good way to prepare for the exam is to redo these exercises with small changes (change the constants, change the functions, etc.).

The following questions have appeared on previous Math 142 exams.

1. Integrate $\int \sin^{\text{red}-1} x \cos^{\text{blue}-1} x\,dx$ where red and blue are randomly chosen from $\{1, \ldots, 6\}$.

2. Integrate $\int x \dfrac{e^{x^2} - e^{-x^2}}{2}\,dx$.

3. Integrate $\int_0^\infty x^2 e^{-x}\,dx$.

4. Evaluate $\lim\limits_{x\to\infty} \left(1 + \dfrac{a}{x}\right)^{bx}$ where a, b are real numbers.

5. Approximate the arclength of $\sin x$ on $[0, \pi]$.

6. Integrate $\int_0^1 \dfrac{x^3}{\sqrt{2-x^2}}\,dx$.

7. Integrate $\displaystyle\int \frac{x}{16+x^4}\,dx.$

8. Evaluate $\displaystyle\int_0^1 \frac{3x}{(x-2)(x+1)}\,dx.$

9. Integrate $\displaystyle\int \frac{x^2+2x+1}{x(x^2+1)}\,dx.$

10. Integrate $\displaystyle\int_0^\infty (x+2)^2 e^{-2x}\,dx.$

11. Integrate $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx.$

12. Integrate $\displaystyle\int \frac{1}{a^2-x^2}\,dx$ where a is a positive number.

13. Integrate $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx$ where a is a positive number.

14. Integrate $\displaystyle\int \frac{1}{x^2+a^2}\,dx$ where a is a positive number.

15. Integrate $\displaystyle\int_0^{\ln(3)/2} \frac{e^x}{e^{2x}+1}\,dx.$

16. Integrate $\displaystyle\int_1^{\sqrt{3}} \frac{2x^2+1}{x^2(x^2+1)}\,dx.$