Math 141 Sample Final questions

1. Draw the graph of a function f with all of the following properties

a. $\lim_{x\to -3^-} f(x) = 1$ and $\lim_{x\to -3^+} f(x) = 2$.

b. f is continuous at -2 but not differentiable at -2.

c. $\lim_{x\to 0^+} f(x)$ does not exist.

d. $f''(1) = 0$

e. $f'(x) > 0$ for all $x \in (2, 4)$.

2. Find the equation of the line tangent to the graph of $x^2 + \sin(2x)$ at $x = \pi/8$.

3. Use complete sentences to answer the following:

a. What does "$\lim_{x\to c} f(x)$" mean?

b. What does it mean for f to be continuous at c?

c. State the definition of the derivative of f at a.

d. State the fundamental theorem of calculus.

e. State the mean value theorem.

4. Evaluate $\displaystyle\int \left(\frac{\sqrt{x} + 2x^2 + 1}{x^3} + \sin x \right) dx$.

5. Suppose y is a function of x satisfying $y^2 - xy = 3$. Write y'' in terms of x and y.

6. A ball dropped from a height of 32 feet has a height of $f(t) = 32 - 16t^2$ feet after t seconds. By taking limits (do not use the power rule for derivatives), find the velocity of the ball when it hits the ground.

7. Find the rectangle with maximum area which lies in the first quadrant underneath the graph of $\sqrt{1-x}$.

8. You are 1 mile away from a rocket shooting straight up. The angle created between the rocket, you, and the launchpad is changing at $\frac{1}{5}$ radians per second. How fast is the rocket moving when it is 1 mile high?

9. Let $f(x) = \int_0^x t/(1+t^2)\, dt$. Find where $f(x) = 0$, where f is increasing, where f is concave up, and graph $f(x)$.