

Math 344 Midterm 2 Sample

Midterm 2 topics

Topics on Midterm 2: Frobenius differential equations, Legendre polynomials, Orthogonality, Gram-Schmidt Projection matrices, Normal equation, inner products on $PS[a, b]$.

Exercise Set 11 is due next Tuesday after the exam, but please consider completing exercises 1, 2, and 4 on Set 11 as practice.

Sample problems

1. Find the projection matrix P for the projection onto the span of $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and use it to find the vector in the span of these two vectors closest to $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

2. Which function of the form $ax + bx^2$ in $PS[0, 1]$ is closest to the function 1? Of course the answer may be left as a sum or quotient of fractions.

3. Verify that $2\|f\|^2 + 2\|g\|^2 = \|f + g\|^2 + \|f - g\|^2$ for any functions f, g in $PS[a, b]$.

4. Describe how the Gram-Schmidt procedure can be used to find the dimension of $\text{span}\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$.

5. Find the first two nonzero terms in the two series solutions of $2x^2y'' + xy' - (1 + x)y = 0$.

6. Find the line $f(x) = ax + b$ that best fits $\{(0, 0), (1, 1), (1, 2), (2, 1)\}$.

7. Let P the projection matrix onto the span of some vectors in \mathbb{R}^n . Show that $(I + P/2)^{-1} = I - P/3$.

8. Suppose the Dirac delta function $\delta(x)$ is written as a sum of Legendre polynomials.

$$\delta(x) = a_0p_0(x) + a_1p_1(x) + a_2p_2(x) + \dots$$

Here $p_k(x)$ is the k th Legendre polynomial. What is a_3 ?