

# Math 141 Sample Midterm 1 Questions

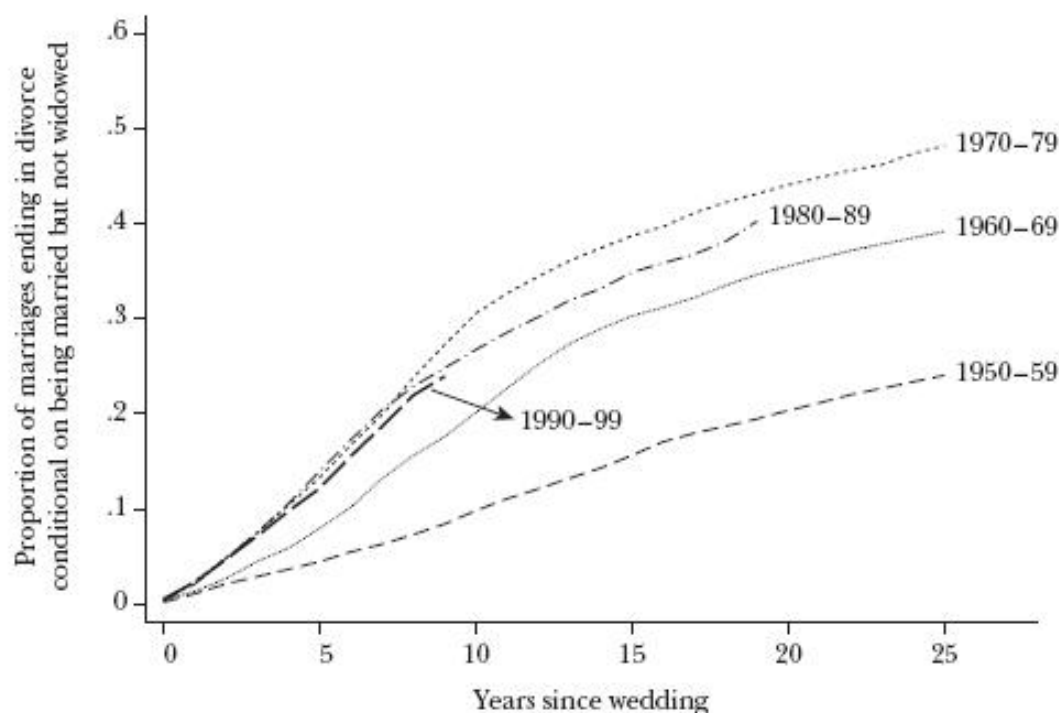
Topics on the midterm include all topics discussed in class up to and including Tuesday, October 14th.

1. Let  $f(t)$  be the percentage of the world population who can access the internet in year  $t$ .

- Sketch the graph of  $f(t)$  (just from what you think the graph should look like).
- Sketch the graph of  $f'(t)$ .
- Estimate  $f'(2000)$ . What is the meaning of  $f'(2000)$ ?
- When do you think  $f'(t)$  is increasing? When is  $f'(t)$  decreasing?

2. Consider the following graph:

**First Marriages Ending in Divorce, by Year of Marriage**



Source: Retrospective marital histories recorded in the 2001 Survey of Income and Program Participation.

Let

$f(t)$  = the % of marriages ending in divorce after  $t$  years of marriage for couples married between 1950-59,  
 $g(t)$  = the % of marriages ending in divorce after  $t$  years of marriage for couples married between 1970-79.

- What is the average value of  $f(t)$  on  $[0, 25]$ ? What does this number mean?
- Draw the graph of  $g'(t)$ .

- c. Which year has a maximum value of  $g'(t)$ ? What does this number mean?
- d. Is it reasonable to say that  $f'(t) = g'(t)$  for  $t$  in  $(15, 25)$ ? Why or why not?

**3.** Using complete sentences, give the following mathematical definitions or state the following theorems:

- a. What does " $\lim_{x \rightarrow 4} f(x) = 6$ " mean?
- b. What does " $\lim_{x \rightarrow 2^-} f(x) = \infty$ " mean?
- c. What does it mean for a function  $f$  to be continuous at 3?
- d. State the intermediate value theorem.

**4.** Find the equation of the line tangent to the graph of  $\sin(1/x^2)$  at  $x = 1/\sqrt{\pi}$ .

**5.** A function  $f$  satisfies  $\frac{x^3}{\sin x} < f(x) < 1 - \cos(2x)$  for  $x$  in  $(-1, 0)$  and  $(0, 1)$ . Find  $\lim_{x \rightarrow 0} f(x)$ .

**6.** Give all numbers for which the function  $\frac{\sin(x)}{1 + \cos(x)}$  is not continuous. Then find the derivative of this function.

**7.** Carefully explain why there must be a solution to  $\cos x = \sqrt{x}$  for some  $x$  satisfying  $0 < x < 1$ .

**8.** Find  $\frac{d}{dx} \left( \frac{1}{1+x^2} - 2 \cos(x/2) + \sin(x \cos x) + \sqrt{1-x+x^2+4} \right)$

**9.** Find  $\lim_{x \rightarrow \sqrt{5}} \frac{x^2 + 2x + 1}{-x^2 + 5}$ .

**10.** Find the line tangent to the graph of  $x^2 = y^3 - 2y + 2$  at the point  $(\sqrt{6}, 2)$ .

**11.** Maximize  $\cos(2x) + \sin(2x)^2$  on the interval  $[0, \pi/2]$ .

**12.** Maximize  $x + 1/x$  on  $[2, 3]$ .

**13.** Simplify  $\frac{d}{dx} \left( \frac{1}{x} + \frac{x^2}{x^2+1} + \cos(\sin x) + x \sin x + 5 \right)$ .

**14.** A pebble is thrown into a calm pond, causing circular ripples. When the area of the disturbed water is  $1 \text{ m}^2$ , the radius of the largest ripple is moving at a rate of  $3 \text{ m/s}$ . How fast is the area of disturbed water increasing then?