pef A set partition of N 15 a set of nonempty disjoint sets WI Union { 1, n} Ex. N=5: $\left\{ \{1\}, \{2,3,4\}, \{5\} \right\}$ $\{\{\{1,2,5\},\{3,4\}\}\}=\{\{4,3\},\{1,5,2\}\}$ { {1,2,3,4,5}} Let bn = # of set points of n (Bell #s) our sequence: 1,1,2,5, ...

Thm bnti = $\sum_{k=0}^{n} \binom{n}{k}$ bn-k for nzo

Pf. Select the subset of $\{1,...,n\}$ in

Which to place "n+1"

Select a set partition w/leftover

Ex. n=5 {1,3,6}, {2,4}, {5}

numbers.

Let
$$B(x) = \sum_{N=0}^{\infty} \frac{b_N}{N!} x^N$$

= $1 + x + \frac{2}{2!} x^2 + \frac{5}{3!} x^3 + \cdots$

$$B'(X) = \sum_{N=0}^{\infty} N \frac{b_{n}}{N!} X^{N-1} = \sum_{N=1}^{\infty} \frac{b_{n}}{(N-1)!} X^{N-1} = \sum_{N=0}^{\infty} \frac{b_{n+1}}{N!} X^{n}$$

$$= \sum_{N=0}^{\infty} \sum_{k=0}^{N} \frac{n!}{k! (n-k)!} \frac{b_{n-k}}{n!} X^{n}$$

$$= \sum_{N=0}^{\infty} \sum_{k=0}^{n} \frac{N!}{k!(N-k)!} \frac{b_{N-k}}{k!} \chi^{n}$$

$$= \left(\sum_{N=0}^{\infty} \frac{\chi^{n}}{N!}\right) \left(\sum_{N=0}^{\infty} \frac{b_{N}}{k!} \chi^{n}\right) = e^{x} B(x)$$

$$= \left(\begin{array}{c} \sum_{N=0}^{\infty} \frac{X^{N}}{N!} \end{array} \right) \left(\begin{array}{c} \sum_{N=0}^{\infty} \frac{b_{N}}{N!} X^{N} \end{array} \right) = e^{X} B(X)$$

$$=\left(\begin{array}{c} \frac{x}{2} \frac{x^n}{n!} \end{array}\right) \left(\begin{array}{c} \frac{x}{2} \frac{b_n}{n!} x^n \end{array}\right) = e^x B(x)$$
Where is
$$B'(x) = e^x B(x)$$

Our de 15
$$\begin{cases} B'(x) = e^{x}B(x) \\ B(x) = 0 \end{cases}$$

$$B(x) = 0$$

$$\frac{1}{B}B' = e^{x}$$

$$\Rightarrow \int \frac{1}{B}dB = \int e^{x} dx$$

B = eex+C

The initial condition gives
$$B(x) = e^{-1}e^{e^{x}}$$

$$B(x) = e^{-1} \int_{N=0}^{\infty} \frac{(e^{x})^{n}}{n!} = \frac{1}{e} \int_{N=0}^{\infty} \frac{e^{nx}}{n!}$$

In
$$B = e^x + c$$
 $B = e^{e^x + c}$
The initial Condition give

 $= \frac{1}{e} \cdot \sum_{N=0}^{\infty} \frac{1}{N!} \sum_{k=0}^{\infty} \frac{(NX)^k}{k!}$

 $= \frac{1}{e} \sum_{k=0}^{\infty} \left(\sum_{k=0}^{\infty} \frac{N^k}{N!} \right) \frac{\chi^k}{k!}$

Conclusion: $D_k = \frac{1}{e} \sum_{k=1}^{\infty} \frac{N^k}{n!}$