Math 143 Set 8

1. Give a rough estimate the value of these complex numbers by hand by rotating in the complex plane:

a. $e^{i\pi/3}4$

b. $(-1)e^{i5\pi/4}2$

2. Use the equation $\frac{1}{e^{i\theta}} = e^{-i\theta}$ to explain why division by $e^{i\theta}$ corresponds to rotating clockwise in the complex plane. Use this to explain why $\frac{1}{i} = -i$.

3. Use Euler's formula to explain why $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$.

4. Use Euler's formula to expand both sides of the equation $e^{(\alpha+\beta)i} = e^{\alpha i}e^{\beta i}$ in terms of sines and cosines. Then use the result to prove the trig identities

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta,$$
$$\sin(\alpha+\beta) = \cos\alpha\sin\beta + \cos\beta\sin\alpha.$$

5. Plot these parametric curves (with starting and ending points and an arrow indicating direction):

a. $\begin{cases} x = \sqrt{t}+1 \\ y = t+1 \end{cases}$ for $t \in [0,1]$.

b. $\begin{cases} x = 3t-5, \\ y = 2t+1 \end{cases}$ for $t \in (-\infty,\infty)$

c. $\begin{cases} x = t^2-2, \\ y = 5-2t \end{cases}$ for $t \in [-3,4]$

d. $\begin{cases} x = t^2, \\ y = t^3 \end{cases}$ for $t \in [-1,1]$

e. $\begin{cases} x = 2\cos(3t), \\ y = 3\sin(3t) \end{cases}$ for $t \in [-\pi/2, 3\pi/2]$

f. $\begin{cases} x = \ln t, \\ y = \sqrt{t} \end{cases}$ for $t \in [1,\infty)$