Linear Algebra Midterm 2 Review Questions

1. Give an example of a rank 3 matrix A such that a basis for the nullspace of A is the single vector $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

2. Diagonalize $\begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & -1 \\ -1 & 0 & 0 \end{bmatrix}$ if possible. (That is, find P and a diagonal D for which $A = PDP^{-1}$.)

3. Let S be the subspace of polynomials $p(x)$ of degree 4 or less with $p'(0) = 0$. Find a basis and the dimension of S.

4. Let S be the set of 2×2 matrices A with $A + A^\top = I$. Either show S is a subspace or show that S is not a subspace of $M_{2,2}$.

5. **T**rue or **F**alse:

_______ **a.** The nullspace and the column space of a matrix can be equal.

_______ **b.** The number of linearly independent rows of a matrix is always equal to the number of linearly independent columns.

_______ **c.** If A is diagonalizable, then the determinant of A is the product of the eigenvalues of A.

6. Why did we only discuss eigenvalues and eigenvectors for square matrices?

7. What is the dimension of the span of the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ -1 \end{bmatrix}$?

8. Suppose that λ is an eigenvalue of A with eigenvector $\mathbf{x}$ and suppose that A^{-1} exists. Explain why λ^{-1} is an eigenvalue of A^{-1}.

9. Give an example of a matrix A with eigenvalue/eigenvector pairs 2, $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and -1, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

10. Let S be the set of all $n \times n$ matrices with rank 0 or 1. Is S a subspace?