Math 248 Midterm 1 Review

Midterm 1 topics include: sets and set builder notation, Cartesian products, subsets and power sets, complements, indexed unions and intersections, statements, logical connectives, conditionals and bi-conditionals, truth tables and logical equivalence, quantifiers, negation, direct proofs of theorems and proof by cases.

The following problems are practice for the midterm:

1. Let P, Q and R be statements. Are the statements $(P \wedge Q) \implies R$ and $(P \wedge (\sim R)) \implies Q$ logically equivalent?

2. Prove that if $x \in \mathbb{R}$, then $x^2 + 4 > |2x - 1|$.

3. Let p_n be the n^{th} smallest prime. For each $n \in \mathbb{N}$, define $A_n = \{a \in \mathbb{N} : a \geq 2 \text{ and } p_n \text{ does not divide } a\}$.

What is the minimum element of $\bigcap_{n \in \{1,2,3,4\}} A_n$? Find and describe $\bigcap_{n \in \mathbb{N}} A_n$.

4. Negate the following statements:

a. $\forall x \in A, (\sim P(x)) \wedge (Q(x) \iff P(x))$.

b. If the Lakers win their next game or their last game and the Clippers do not win their last game, then the Lakers will make the playoffs.

5. Prove that if a and b are odd, then $8 \mid a^2 - b^2$.

6. Define the greatest common divisor of $a, b \in \mathbb{Z}$, denoted $\gcd a, b$, to be the largest integer c such that $c \mid a$ and $c \mid b$. Prove that if $a, b \in \mathbb{Z}$, then $\gcd(a, b) = \gcd(|a|, |b|)$.

7. Give an example of a subset A of $\mathbb{Z}^3$ such that $|A \times A| = |\mathcal{P}(A)|$.