Vectors and $A\mathbf{x} = \mathbf{b}$.

1. Determine if $\mathbf{b}$ is a linear combination of the columns of A when:

a. $A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$.

b. $A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$.

2. List 4 vectors in the span of $\mathbf{v}_1, \mathbf{v}_2$ in the cases below. For each example, show the weights on $\mathbf{v}_1$ and $\mathbf{v}_2$ used to generate the example vectors.

a. $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$.

b. $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -6 \\ -2 \\ 4 \end{bmatrix}$.

3. True or false:

a. Another notation for $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 2 \end{bmatrix}$.

b. An example of a linear combination of vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is $\frac{1}{2}\mathbf{v}_1$.

c. Asking whether the linear system corresponding to the augmented matrix $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{bmatrix}$ has a solution is equivalent to asking if $\mathbf{b}$ is in the span of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

d. The coefficients $c_1, \ldots, c_n$ in a linear combination $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n$ cannot be all 0.

4. Write the system as a matrix equation $A\mathbf{x} = \mathbf{b}$:

a.

$$\begin{aligned} 5x_1 + x_2 - 3x_3 &= -2 \\ 7x_2 + x_3 &= 0 \end{aligned}$$

b.

$$\begin{aligned} 4x_1 - x_2 &= 9 \\ 7x_1 + x_2 &= 0 \\ 7x_1 + 3x_2 &= 1 \end{aligned}$$

5. Given the following examples of A and $\mathbf{b}$, solve $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$. Write the solutions as a vector.

a. $A = \begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & 2 \\ -3 & -7 & 6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -2 \\ 4 \\ 12 \end{bmatrix}$.

b. $A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -4 & 2 \\ 5 & 2 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$.

6. Let $\mathbf{u} = \begin{bmatrix} 4 \\ -1 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 5 & -1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$. Is $\mathbf{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A? Why?

7. Can every vector in $\mathbb{R}^4$ be written as a linear combination of the columns in $\begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$? Do these columns span $\mathbb{R}^4$?

8. True or false:

a. A vector $\mathbf{b}$ is a linear combination of the columns of a matrix A if and only if the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution.

b. Any linear combination of vectors can always be written as $A\mathbf{x}$ for some matrix A and vector $\mathbf{x}$.