

Math 248 Sample Midterm 1

Midterm topics include: sets and set builder notation, Cartesian products, subsets and power sets, complements, indexed sets, statements, logical connectives, conditionals and biconditionals, truth tables and logical equivalence, quantifiers, negation, direct proofs of theorems and proof by cases.

The following problems are similar to those found on the midterm. They are just for practice and will not be collected.

1. Let P, Q and R be statements. Are the statements $(P \wedge Q) \implies R$ and $(P \wedge (\sim R)) \implies Q$ logically equivalent?

2. Prove that if $x \in \mathbb{R}$, then $x^2 + 4 > |2x - 1|$.

3. Let p_n be the n^{th} smallest prime. For each $n \in \mathbb{N}$, define $A_n = \{a \in \mathbb{N} : a \geq 2 \text{ and } p_n \text{ does not divide } a\}$.

What is the minimum element of $\bigcap_{n \in \{1,2,3,4\}} A_n$? Find and describe $\bigcap_{n \in \mathbb{N}} A_n$.

4. Negate the following statements:

a. $\forall x \in A, (\sim P(x)) \wedge (Q(x) \iff P(x))$.

b. If the Lakers win their next game or their last game and the Clippers do not win their last game, then the Lakers will make the playoffs.

5. Prove that if a and b are odd, then $8 \mid a^2 - b^2$.

6. Prove that if $a, b \in \mathbb{Z}$, then $\gcd(a, b) = \gcd(|a|, |b|)$.

7. Give an example of a subset A of \mathbb{Z}^3 such that $|A \times A| = |\mathcal{P}(A)|$.