Math 143 Quiz 3

Names: _____

1. Euler's amazing formula is $e^{it} = \cos t + i \sin t$. Use this to explain why this identity is true:

$$(\cos t + i \sin t)^n = \cos(nt) + i \sin(nt).$$

2. Use Euler's formula to expand both sides of the equation $e^{(a+b)i} = e^{ai} e^{bi}$ in terms of sines and cosines. Compare the real and imaginary components to explain why these two identities are true:

$$\cos(a + b) = \cos a \cos b - \sin a \sin b,$$
$$\sin(a + b) = \cos a \sin b + \cos b \sin a.$$

3. Find the equation of the line tangent to the curve $\begin{cases} x = \cos(2t) + t, \\ y = \sin(2t) + t, \end{cases}$ for $t \in \mathbb{R}$ at the point $(1 + \pi, \pi)$.
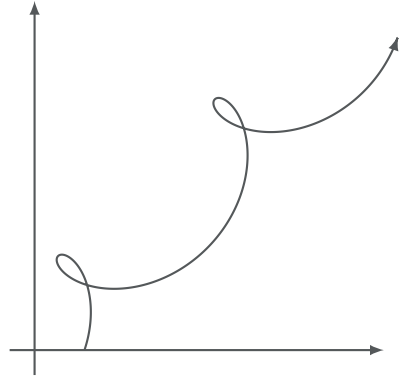
4. Find the values of t for which there are vertical tangent lines in the graph in question 3.