

F1 We can use Mathematica to evaluate our series w/ concrete examples

• Ex: The Catalan Numbers

Series $\left[\frac{1 - \sqrt{1-4x}}{2x}, \{x, 0, 10\} \right] \leftarrow$ Mathematica command for the first 10 terms in the series

• Ex: The Bell Numbers

$B[x, y] := e^y (e^x - 1)^y \leftarrow$ define the function

Series $[B[x, y], \{x, 0, 6\}] //$ Simplify \leftarrow first 6 terms of series

How many set of partitions of n contain exactly 3 sets? What is the generating function?

$(D[B[x, y], \{y, 3\}] /. \{y \rightarrow 0\}) / 3!$

\uparrow derivative

Table $[n! \text{ SeriesCoefficient}[\frac{1}{6} (-1 + e^x)^3, \{x, 0, n\}], \{n, 0, 10\}]$

\leftarrow Mathematica commands to extract the 3rd coefficient of each series

What is the average number of sets in a set partition of 100?

If we take the derivative with respect to y and then plug in $y=1$, we get the generating function for the total number of sets in all set partitions of n .

$D[B[x, y], y] /. \{y \rightarrow 1\}$

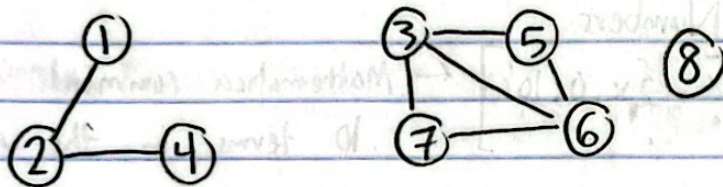
Table $[n! \text{ SeriesCoefficient}[e^{-1+e^x} (-1+e^x), \{x, 0, n\}], \{n, 0, 10\}]$

100! SeriesCoefficient $[e^{-1+e^x} (-1+e^x), \{x, 0, 100\}]$

Def: An exponential structure is an object created by assigning some structure to the sets in a set partition of n .

Ex: I can create a labeled graph by taking each set in a set partition and drawing a connected graph w/ vertices labeled the # in the set.

$\{\{1, 2, 4\}, \{3, 5, 6, 7\}, \{8\}\}$



Ex: A permutation of n is a bijection $\sigma: \{1, 2, \dots, n\} \rightarrow \{1, 2, \dots, n\}$.

We can write permutations in cycle notation:

$(1 \ \sigma(1) \ \sigma(\sigma(1)) \ \sigma^3(1) \dots) (2 \ \sigma(2) \ \sigma^2(2) \dots) (3 \dots)$
 ^ least # not in 1st parens.

Ex: $\sigma(1) = 3$ $(1 \ 3 \ 2 \ 4 \ 5)$

$\sigma(2) = 4$

$\sigma(3) = 5$

$\sigma(4) = 1$

$\sigma(5) = 2$

Ex: $\sigma(1) = 2$ $(1 \ 2)(3 \ 5)(4)$

$\sigma(2) = 1$

$\sigma(3) = 5$

$\sigma(4) = 4$

$\sigma(5) = 3$

Permutations in cycle notation can be created by forming a cycle with each set in a set partition.

ie. $\{\{1, 2, 4\}, \{3, 5, 6, 7\}, \{8\}\}$

$(1 \ 4 \ 2)(3 \ 5 \ 7 \ 6)(8)$