Math 143 Midterm 2 Review

Topics on Midterm 2

1. Multiplying, dividing, differentiating, integrating series.
2. Euler's formula: $e^{it} = \cos t + i \sin t$.
3. Parametric equations (plotting, derivatives, arclength)
4. Polar equations (plotting, derivatives, arclength, polar rectangles, area)
5. $\mathbb{R}^3$ (distance, midpoints, basic plots including spheres and cylinders)
6. Vectors in $\mathbb{R}^3$ (length, unit vectors, dot product, cross product)
7. Lines and planes in $\mathbb{R}^3$

These identities will be given on the midterm: $\cos^2 t = (1 + \cos 2t)/2$, $\sin^2 t = (1 - \cos 2t)/2$.

Sample questions

1. Plot, find the arclength, and find the area enclosed by the polar curve $r = \theta^2$ for $\theta \in [0, 2\pi]$.

2. Find the arclength of the curve described by the parametric equations $\begin{cases} x = 3 + e^{-2t} \\ y = 2 - e^{-2t} \end{cases}$ for $t \in [0, 1]$.

3. Graph the parametric equations $\begin{cases} x = 2 + 3\sin t \\ y = 1 + 2\cos t \end{cases}$ for $t \in [0, 3\pi/2]$.

4. Find two vectors of length 2 which are orthogonal to $\langle 2, 2, 3 \rangle$ and $\langle -1, 0, 2 \rangle$.

5. Find the equation of the plane which passes through the origin and is perpendicular to both $x + y + z = 3$ and $x + 2y + 3z = 3$.

6. Find the angle between the planes $x + y + z = 1$ and $x + 2y - z = 2$.

7. Consider the curve in the plane $\begin{cases} x = \cos t + t \sin t \\ y = \sin t - t \cos t \end{cases}$ where $t \in [0, 2\pi]$.

a. Find the (x, y) coordinates of all vertical and horizontal tangents.

b. Find the values of t for which this curve is concave down.

c. Find the arclength of the curve.

8. Consider the curve given parametrically by $\begin{cases} x = 2e^t - t \\ y = e^t - 3 \end{cases}$ for $t \in \mathbb{R}$. Find the parametric equations for the line tangent to the curve at $t = 1$.

9. Fix a vector $\mathbf{v} \in \mathbb{R}^3$. Which unit vector $\mathbf{w}$ maximizes the dot product $\mathbf{w} \cdot \mathbf{v}$?

10. If $a, b \in \mathbb{R}$, we let $\mathrm{Re}(a + ib) = a$ and $\mathrm{Im}(a + ib) = b$ denote the real and imaginary parts of the complex number $a + bi$. Plot the parametric equation $\begin{cases} x = \mathrm{Re}(3e^{2it}), \\ y = \mathrm{Im}(3e^{2it}) \end{cases}$ for $t \in [0, \pi/4]$.

11. Find the area enclosed by the polar curve $r = \sin(4\theta)$.