Math 241 Midterm 1 Review

Topics

1. Level curves
2. Limits (including switching into polar and showing limits do not exist)
3. Partial derivatives and the chain rule (using the tree-like diagrams)
4. Directional derivatives and the gradient vector:
   (a) If $\mathbf{u}$ is a unit vector, $D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u}$.
   (b) The maximum possible directional derivative is $|\nabla f|$, found when $\mathbf{u} = \nabla f / |\nabla f|$
   (c) ∇f is perpendicular to level curves
5. Finding tangent planes and normal lines
6. Finding absolute maximums and minimums for $f(x, y)$ over a region R
7. The second derivative test to identify local maximums and minimums
8. Constrained optimization and Lagrange multipliers
9. Double integrals over a region R, namely $\iint_R f(x, y)\, dA$

Sample questions

1. Find the maximum rate of change of $x^2y + \sqrt{y}$ at the point $(1, 1)$. In which direction does this occur? Find a direction in which the directional derivative at $(1, 1)$ is 0.

2. Evaluate $\displaystyle\int_0^1 \int_{2y}^2 \cos(x^2)\, dx\, dy$.

3. Evaluate $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2}$ or show this limit does not exist.

4. Suppose $ze^{xyz} = 0$ where z is an unknown function of x and y. Write z_x in terms of x, y, and z.

5. Find the directional derivative of $f(x, y) = 3x^2 + 2xy$ in the direction of $\langle 2, 3\rangle$ at $(1, 1)$. What does this calculation mean?

6. Find and classify all local minimums and maximums for $x^3 - 6xy + 8y^3$.

7. Find the maximum and minimum for the function $(x - 1)^2 + (2y - 1)^2$ on the triangle with corners $(0, 0)$, $(2, 0)$, and $(0, 2)$.

8. Let S be the set of points which satisfy $z - e^{x+xy} = 0$ and let p be the point $(1, -1, 1)$. Find the plane tangent to S at p and find the line normal to S at p.

9. Let $z = f(x, y)$ with $x = \cos(st)$ and $y = s - t$. Find $\dfrac{\partial^2 z}{\partial t^2}$ in terms of $z_{xx}, z_{xy}, z_{yy}, s$ and t.

10. Find the point on the sphere $x^2 + y^2 + z^2 = 1$ which maximizes $x + 2y + z$.