

Discrete Midterm 1

Name: _____

1. Find a generating function for the probability that a permutation of n does not have a cycle with a length that is a multiple of 3. (A fully correct response does not contain the symbol ' Σ '.)

2. Let a_n be the number of ways to tile a $2 \times n$ chessboard with dominoes of sizes 2×1 and 2×2 . For example, there are 11 such tilings when $n = 4$:



Explain why $a_0 = 1$, $a_1 = 1$, and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 2$ and then find a generating function for a_n .

3. What is the average value of the descent statistic $\text{des}(\sigma)$ over all $\sigma \in S_n$?

4. Suppose $f(x) = \frac{1}{1 - \sin x}$ has series expansion $\sum_{n=0}^{\infty} \frac{a_n}{n!} x^n$. Find an asymptotic formula for a_n .

5. Let L_n be the collection of ordered set partitions of size n and recall that

$$\sum_{n=0}^{\infty} \sum_{s \in L_n} y^{(\text{the number of sets in } s)} \frac{x^n}{n!} = \frac{1}{1 - y(e^x - 1)}.$$

Let a_n be the number of $s \in L_n$ such that (the number of sets in s) is odd. Find $\sum_{n=0}^{\infty} \frac{a_n}{n!} x^n$.