Linear Analysis II Set 12

1. Find the Fourier series for the unit step function $u_0(t)$ on $[-L, L]$. Write the solution in the form

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right)$$

or write the solution in the complex form

$$\sum_{n=-\infty}^{\infty} c_n e^{-in\pi x/L}.$$

2. Find the Fourier series for the Dirac delta function $\delta(x)$ on $[-L, L]$.

3. a. Find the Fourier series for the function x^2 on $[-L, L]$.

b. After taking $x = \pi$ and $L = \pi$ in the Fourier series in the previous part of this exercise, we have

$$\pi^2 = \frac{\pi^3}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{4}{n^2} \cos(n\pi).$$

Use this expression to find a formula for $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$.

4. Use the Pythagorean theorem to show that on $PS[-L, L]$ we have

$$\left\| \frac{a_0}{2} + \sum_{n=1}^{M} a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right) \right\|^2 = 2L \left(\frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{M} (a_n^2 + b_n^2) \right).$$