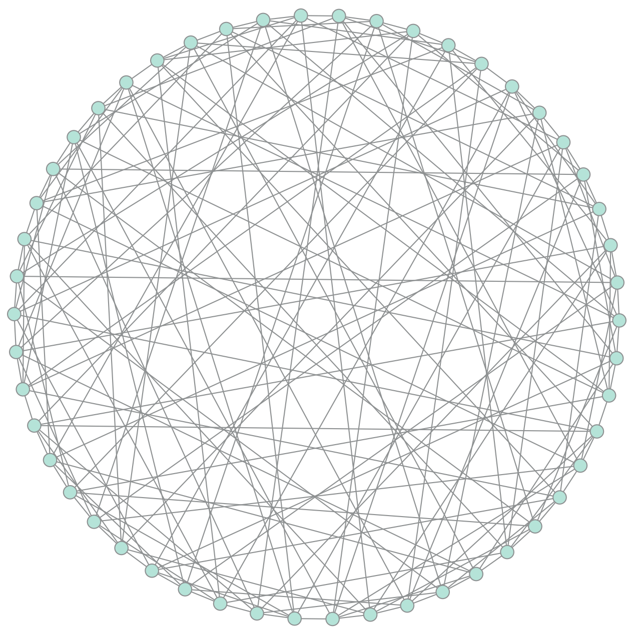
Graph Theory Set 9

43. Let G be a d regular graph. Show that $\lambda_1, \ldots, \lambda_n$ are the eigenvalues for the adjacency matrix $A(G)$ if and only if $d - \lambda_1, \ldots, d - \lambda_n$ are the eigenvalues for the Laplacian matrix $L(G)$.

44. Show that if $0, \mu_2, \ldots, \mu_n$ are the eigenvalues for the Laplacian $L(G)$ of a graph with n vertices, then $0, n - \mu_n, \ldots, n - \mu_2$ are the eigenvalues for the Laplacian $L(G^c)$.

45. The eigenvalues for the following 7-regular graph G are $\overbrace{-3, \ldots, -3}^{\text{21 times}}, \overbrace{2, \ldots, 2}^{\text{28 times}}, 7$:



a. How many triangles are in G?

b. Is G Eulerian?

c. Is G Hamiltonian?

d. What lower bound on the crossing number for G is given by Exercise 23?

e. What lower bound on the chromatic number $\chi(G)$ is given by Theorem 109?

f. What lower bound on the vertex connectivity $\kappa(G)$ is given by Theorem 121?

g. How many spanning trees does G have?

h. What is the diameter of G?

i. Show that G has a perfect matching.

j. Repeat all previous of this exercise but with G replaced with G^c.