Math 344 Midterm 1 Practice

Topics on Midterm 1 include: All topics on Laplace transforms (taking basic Laplace transform and inverse Laplace transforms, Shifting theorems, Dirac delta functions and convolutions), Cauchy Euler differential equations, and series solutions to differential equations.

These are practice questions that may be similar to the ones on the first midterm exam.

1. Find a function $f(t)$ such that $f(t) = t - 4\int_0^t xf(t-x)\,dx$.

2. Solve $y'' + 2y' + 5y = \delta(t-3)$ if $y(0) = 0$ and $y'(0) = 1$. What is a physical interpretation for this differential equation?

3. Solve $x^2y'' - xy' + 13y = 0$.

4. Solve $y'' + y = f(t)$ where $f(t)$ is -1 on $[0,1]$, t on $[1,2]$, and 0 elsewhere.

5. Find the two series solutions to $(1-x^2)y'' - xy = 0$ up to the x^5 term.

6. Solve $tf(t) = \int_0^t f(x)f(t-x)\,dx$.

7. Find $\mathcal{L}^{-1}\left[1 + \frac{s}{(s-a)^2+b^2} + e^{-2s}\arctan\left(\frac{\pi}{s}\right)\right]$.

8. If $\mathcal{L}[f(t)] = F(s)$, write $\mathcal{L}[f(at)]$ in terms of F.

9. Solve $x^2y'' + xy' = 0$.

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)]$	
$f(t)$	$\int_0^\infty f(t)e^{-st}\,dt$	Definition of Laplace transform
t^n	$\frac{n!}{s^{n+1}}$	Valid for $n = 0, 1, 2, \ldots$
t^r	$\frac{r}{s}\mathcal{L}[t^{r-1}]$	Valid for $r > 0$
$t^{-1/2}$	$\sqrt{\frac{\pi}{s}}$	
e^{at}	$\frac{1}{s-a}$	
$\cos at$	$\frac{s}{s^2+a^2}$	
$\sin at$	$\frac{a}{s^2+a^2}$	
$\frac{\sin at}{t}$	$\arctan\left(\frac{a}{s}\right)$	
$\frac{e^{at}-1}{t}$	$\ln\left(\frac{s}{s-a}\right)$	
$f'(t)$	$s\mathcal{L}[f(t)] - f(0)$	First derivative in t
$f''(t)$	$s^2\mathcal{L}[f(t)] - sf(0) - f'(0)$	Second derivative in t
$e^{at}f(t)$	$F(s-a)$ where $F(s) = \mathcal{L}[f(t)]$	Shifting Theorem 1
$u_a(t)f(t-a)$	$e^{-as}\mathcal{L}[f(t)]$	Shifting Theorem 2
$\delta(t-a)$	e^{-as}	Dirac delta function
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}\mathcal{L}[f(t)]$	Derivatives in s
$f(t) * g(t)$	$\mathcal{L}[f(t)]\mathcal{L}[g(t)]$	The Convolution Theorem