Math 344 Midterm 1

Name: _____

1. Find $\mathcal{L}^{-1}\left[\dfrac{2s}{s^2+2s+4}+\dfrac{e^{-2s}}{(s+1)}+\dfrac{1}{s^{3/2}}+\dfrac{\mathcal{L}[f(t)]}{s^2}\right]$. (Leave the last term as a convolution.)

2. Solve $y' + y = f(t)$ where $f(t) = \begin{cases} e^t & \text{if } 0 \leq t < 1, \\ 0 & \text{if } 1 \leq t \end{cases}$ with $y(0) = 1$.

3. Solve the system $\begin{cases} x' = -y + \delta(t-1), \\ y' = x \end{cases}$ where $x(0) = 0$ and $y(0) = 0$.

4. Solve $x^2y'' - 2xy' + 3y = 0$.

5. Find the two series solutions to $y'' + xy' + y = 0$ up to the x^4 term.

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)]$	
$f(t)$	$\int_0^\infty f(t)e^{-st}\,dt$	Definition of Laplace transform
t^n	$\frac{n!}{s^{n+1}}$	Valid for $n = 0, 1, 2, \ldots$
t^r	$\frac{r}{s}\mathcal{L}[t^{r-1}]$	Valid for $r > 0$
$t^{-1/2}$	$\sqrt{\frac{\pi}{s}}$	
e^{at}	$\frac{1}{s-a}$	
$\cos at$	$\frac{s}{s^2+a^2}$	
$\sin at$	$\frac{a}{s^2+a^2}$	
$\frac{\sin at}{t}$	$\arctan\left(\frac{a}{s}\right)$	
$\frac{e^{at}-1}{t}$	$\ln\left(\frac{s}{s-a}\right)$	
$f'(t)$	$s\mathcal{L}[f(t)] - f(0)$	First derivative in t
$f''(t)$	$s^2\mathcal{L}[f(t)] - sf(0) - f'(0)$	Second derivative in t
$e^{at}f(t)$	$F(s-a)$ where $F(s) = \mathcal{L}[f(t)]$	Shifting Theorem 1
$u_a(t)f(t-a)$	$e^{-as}\mathcal{L}[f(t)]$	Shifting Theorem 2
$\delta(t-a)$	e^{-as}	Dirac delta function
$t^n f(t)$	$(-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)]$	Derivatives in s
$f(t) * g(t)$	$\mathcal{L}[f(t)]\mathcal{L}[g(t)]$	The Convolution Theorem