Math 335 Midterm 3

Name: _____

1. State the definition of

a. the adjacency matrix for a graph G

b. a matching for G

c. a covering for G

d. a probability vector in $\mathbb{R}^n$

e. the Perron value for the adjacency matrix of a strongly connected network

2. The cube graph Q_n has vertices the bit strings of length n with an edge between two bit strings if and only if the strings differ in exactly one position. Show that Q_n is Hamiltonian.

3. Suppose G is color critical. Why is $\kappa(G) \geq 2$?

4. An $n \times n$ doubly stochastic matrix is a matrix of nonnegative real numbers such that each row and each column sums to 1. For example, one 3×3 doubly stochastic matrix is

$$\begin{pmatrix} 1/2 & 0 & 1/2 \\ 1/4 & 1/4 & 1/2 \\ 1/4 & 3/4 & 0 \end{pmatrix}$$

From an $n \times n$ doubly stochastic matrix M, create a bipartite graph G with independent sets the rows and columns of M and edges between row i and column j if the i, j entry of M is nonzero. Show that G has a perfect matching.

5. The eigenvalues for the adjacency matrix of a graph G are $\lambda_{\max}, \underbrace{-3, \ldots, -3}_{5 \text{ times}}, \underbrace{1, \ldots, 1}_{10 \text{ times}}$ for some $\lambda_{\max}$.

a. What is the Perron value $\lambda_{\max}$?

b. How many edges are in G? (Leave the answer as an unsimplified arithmetic expression.)

c. How many triangles are in G? (Leave the answer as an unsimplified arithmetic expression.)

d. Is G bipartite? Why?