Review

Highlights

① $[n]_q! = \sum_{\sigma \in S_n} q^{inv(\sigma)} = \sum_{\sigma \in S_n} q^{maj(\sigma)}$

② $\begin{bmatrix} n \\ k \end{bmatrix}_q = \sum_{r \in R(0^k, 1^{n-k})} q^{inv(r)} = \sum_{r \in R(0^k, 1^{n-k})} q^{maj(r)} = \sum_{\substack{\lambda \text{ fit in} \\ k \times (n-k) \text{ box}}} q^{|\lambda|} = (\# \text{ } k\text{-dim subspaces in } \mathbb{F}_q^n)$

↳ Now we can understand $\begin{bmatrix} n+1 \\ k \end{bmatrix}_q = \sum_{j=0}^{k} q^{k-j} \begin{bmatrix} n-j \\ k-j \end{bmatrix}$ ⋆ prove identity with q-binom coeff, show rep same thing or make bij etc.

③ Integer Partitions

ⓐ G.F. $\prod_i \frac{1}{1-z^i}$, $\prod (1+z^i)$, etc. → prove identities

ⓑ prove identities bijectively → direct construction
→ Remmel's machine ⋆ like 1a on hw

④ Euler's Pentagonal Number Theorem ⋆ make sure you understand, nice sign rev. in vo., gives recurssion (proof & result)

$\prod (1-z^i) = \sum_{k \in \mathbb{Z}} (-1)^k z^{k(3k-1)/2}$

⑤ Jacobi's Triple Product

$(1+x) \prod (1-z^i)(1+xz^i)(1+x^{-1}z^i) = \sum_{k \in \mathbb{Z}} x^k z^{\frac{k(k-1)}{2}}$

↳ slicing along staircase

⑥ Symmetric Functions

↳ m_λ, e_n, h_n, p_n

↳ Newton's identities

$= p_{\lambda_1} p_{\lambda_2} \cdots$

Thm: $\sum_{\lambda \vdash n} |C_\lambda| \, p_\lambda$

$\{\sigma \in S_n \text{ w/ cycle type } \lambda\} = C_\lambda$

pf: $n = 9$

$\boxed{1}\boxed{1}\boxed{1}\boxed{1}\boxed{1}\boxed{1}\boxed{2}\boxed{2}\boxed{2}$ ← h_n

$(3\ 1)(9\ 8\ 5\ 6)(2)(7\ 4)$ ← $n!$

2 1 4 3

$\updownarrow$

p_2	p_4	p_1	p_2
$\boxed{1}\boxed{1}$	$\boxed{1}\boxed{1}\boxed{1}\boxed{1}$	$\boxed{2}$	$\boxed{2}\boxed{2}$
$(3\ 1)$	$(9\ 8\ 5\ 6)$	(2)	$(7\ 4)$
σ_2	σ_4	σ_1	σ_2

give us ordering to move back

Thm: $n!\, e_n = \sum_{\lambda \vdash n} (-1)^{n - \ell(\lambda)} |C_\lambda| \, p_\lambda$