Def An integer partition of n is weakly decreasing list of integers that of non-negative sum to n.

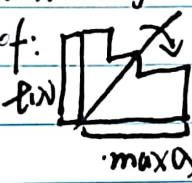
Let $\ell(\lambda)$ = # positive integers in λ $\quad \lambda \vdash n$ (λ is an integer partition of n) \vlash

$\max(\lambda)$ = max integer of λ $\quad |\lambda| = n$

Ex: $n=5$

$\lambda \vdash n$	alt notation	Young/Ferres diagram	$\ell(\lambda)$	$\max(\lambda)$
(5)	5		1	5
$(4,1)$	14		2	4
$(3,2)$	23		2	3
$(3,1,1)$	$1^2 3$		3	3
$(2,2,1)$	$1\,2^2$		3	2
$(2,1,1,1)$	$1^3 2$		4	2
$(1,1,1,1,1)$	1^5		5	1

Thm. # integer $\lambda \vdash n$ with $\ell(\lambda)=k$ = # integer partition $\lambda \vdash n$ with $\max(\lambda)=k$

proof: 

Def: The reflection of λ about the diagonal is a conjugate partition. often ~~d~~denoted with λ'.

Thm $\begin{bmatrix} n \\ k \end{bmatrix}_q = \sum_{\substack{\text{integer partition } \lambda \\ \text{with Young diagrams} \\ \text{fills in } k\times(n-k) \\ \text{rectangle}}} q^{|\lambda|}$

Ex $k=2$ $\quad n=4$

 q^4  q^2

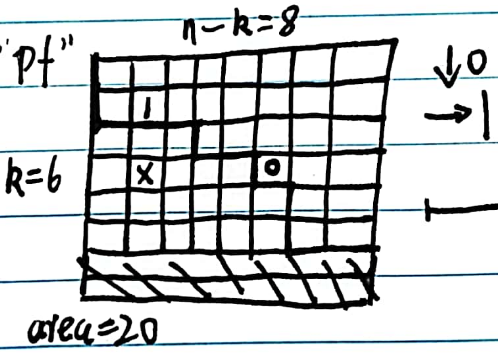
 q^3  q^1

 q^2  q^0

1100 $\quad$ 0110

1010 $\quad$ 0101 $\nearrow$ The range of respective Young diagrams

1001 $\quad$ 0011

"pf" 

$\longmapsto$ 00111011010011

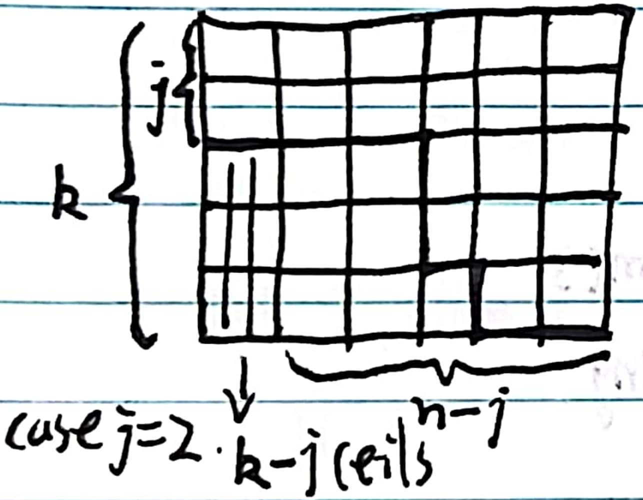
↓

A element in $R(0^k, 1^{n-k})$.

each $(1,0) \longmapsto \square$ in diagram

$$\begin{bmatrix} n+1 \\ k \end{bmatrix} = \sum_{j=0}^{k} q^{k-j} \begin{bmatrix} n-j \\ k-j \end{bmatrix}_q$$

Pf: Sent integer partitions that fill in a $k \times (n+1-k)$ box by $\ell(\lambda)$



λ fills in $k \times (n+1-k)$

j is pick height to column 1

$k-j$ is # ceils in column 1

$\begin{bmatrix} n-j \\ k-j \end{bmatrix}$ is rest part