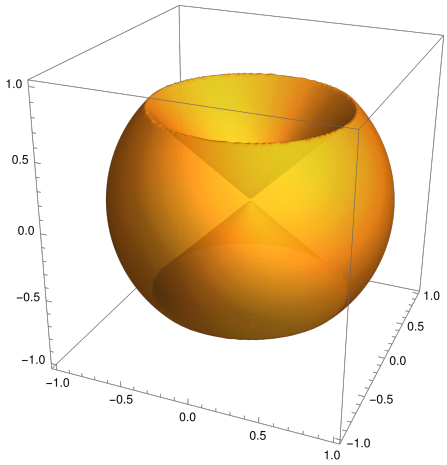
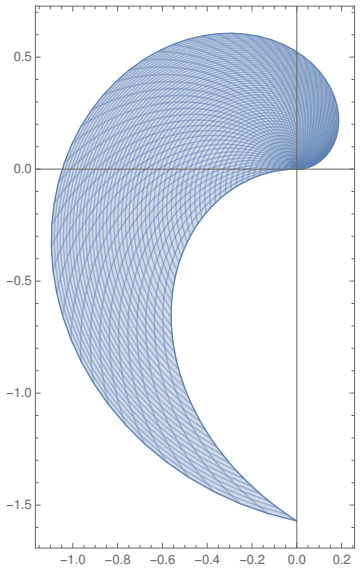
Math 241 Midterm 2

Name: _____

1. Find the volume of the region in $\mathbb{R}^3$ containing those points inside the sphere $x^2 + y^2 + z^2 = 1$ and outside of the double cone $z^2 = x^2 + y^2$.



2. Find the area of the region $\begin{cases} x = u\cos(u+v) \\ y = u\sin(u+v) \end{cases}$ for $v \in [0, \pi], u \in [0, v/2]$:



3. Find the point on the surface $x^2 + y + z^2 = 1$ that minimizes the magnitude of the curl of $\langle xz, -xz, x \rangle$.

4. Evaluate $\int_{-1}^{0} \int_{0}^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2} \, dy \, dx.$

5. Show that $\mathbf{F} = \langle xy^2 + yz, x^2y + xz, xy \rangle$ is conservative and find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is a curve in $\mathbb{R}^3$ with start at $(0, 0, 0)$ and end at $(2, 2, -1)$.