Linear Analysis II Exercise Set 13

1. a. Show that the Fourier Transform of $e^{-|t|}$ is $\dfrac{1}{\pi(\omega^2+1)}$.

b. Using the inverse Fourier Transform (listed as the "Fourier relations" on line 2 on our table of Fourier transforms) and part a., find the Fourier transform of $\dfrac{1}{t^2+1}$.

2. The one dimensional wave equation is the partial differential equation $u_{tt}(t,x) = k^2 u_{xx}(t,x)$ where k is a real number and $u(t,x)$ is a function of time t and one spacial dimension x. The wave equation models the displacement of a vibrating string at time t and location x.

By taking partial derivatives and plugging in the function

$$u(t,x) = \frac{f(x+kt)+f(x-kt)}{2} + \frac{1}{2k}\int_{x-kt}^{x+kt} g(s)\,ds$$

into the partial differential equation, show that the above function is a solution to the wave equation

$$\begin{cases} u_{tt}(t,x) = k^2 u_{xx}(t,x) \\ u(0,x) = f(x), u_t(0,x) = g(x). \end{cases}$$