Math 344 Quiz 3

Names: _____

1. L_k is the polynomial solution to $\dfrac{d}{dx}\left[xe^{-x}y'\right] = -ke^{-x}y$ that satisfies $L_k(0) = 1$. Find L_2.

2. Show that if $k \neq m$, then $\displaystyle\int_0^\infty e^{-x}L_kL_m\,dx = 0$.

3. Given $\int_0^\infty e^{-x} L_k^2 \, dx = 1$, find a formula for the constant a_i which makes the approximation

$$f(x) \approx a_0 L_0 + \cdots + a_i L_i + \cdots + a_n L_n$$

as accurate as possible.

4 (Bonus! Only try when done with other problems). Let $p_k(x)$ be the kth Legendre polynomial and let $q(x)$ be a polynomial of degree less than k. Explain why $\int_{-1}^{1} q(x) p_k(x) \, dx = 0$.