

Review

Highlights

$$\textcircled{1} [n]_q = \sum_{\sigma \in S_n} q^{\text{inv}(\sigma)} = \sum_{\sigma \in S_n} q^{\text{maj}(\sigma)}$$

$$\textcircled{2} \begin{bmatrix} n \\ k \end{bmatrix}_q = \sum_{r \in R(0^k, 1^{n-k})} q^{\text{inv}(r)} = \sum_{r \in R(0^k, 1^{n-k})} q^{\text{maj}(r)} = \sum_{\substack{\lambda \text{ fit in} \\ k \times (n-k) \text{ box}}} q^{|\lambda|} = (\# \text{ } k\text{-dim subspaces in } \mathbb{F}_q^n)$$

↳ Now we can understand $\begin{bmatrix} n+1 \\ k \end{bmatrix}_q = \sum_{j=0}^k q^{n-j} \begin{bmatrix} n-j \\ k-j \end{bmatrix}_q$ * prove identity with q -binom coeff, show rep same thing or make bij etc.

$\textcircled{3}$ Integer Partitions

a) G.F. $\prod_i \frac{1}{1-z^i}$, $\prod_i (1+z^i)$, etc. \rightarrow prove identities

b) prove identities bijectively \rightarrow direct construction
 \rightarrow Remmel's machine * like 1a on hw

$\textcircled{4}$ Euler's Pentagonal Number Theorem

$$\prod_i (1-z^i) = \sum_{k \in \mathbb{Z}} (-1)^k z^{k(3k-1)/2}$$

* make sure you understand, nice sign rev. in v.o., gives recursion (proof & result)

$\textcircled{5}$ Jacobi's Triple Product

$$(1+x) \prod_i (1-z^i)(1+xz^i)(1+x^{-1}z^i) = \sum_{k \in \mathbb{Z}} x^k z^{\frac{k(k-1)}{2}}$$

↳ slicing along staircase

$\textcircled{6}$ Symmetric Functions

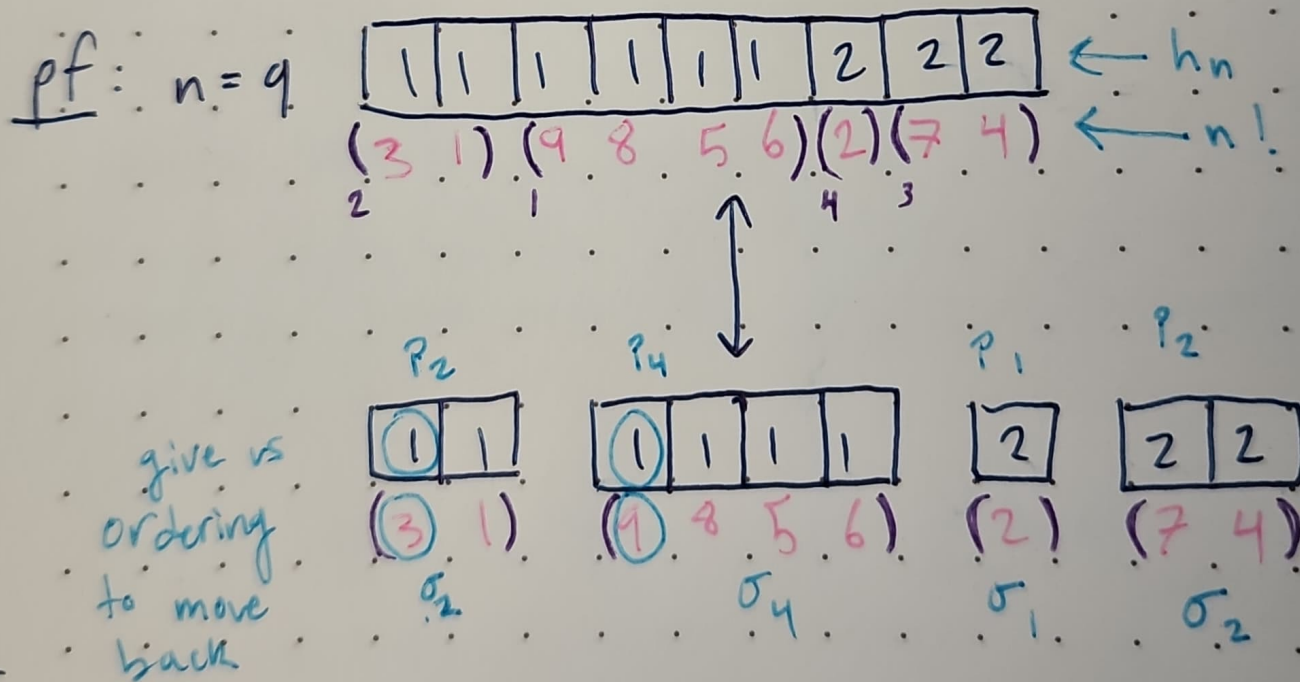
↳ m_λ, e_n, h_n, p_n

↳ Newton's identities

$$= p_{\lambda_1} p_{\lambda_2} \dots$$

Thm: $\sum_{\lambda \vdash n} |C_{\lambda}| p_{\lambda}$

$\leftarrow \{\sigma \in S_n \text{ w/ cycle type } \lambda\} = C_{\lambda}$



Thm: $n! e_n = \sum_{\lambda \vdash n} (-1)^{n-l(\lambda)} |C_{\lambda}| p_{\lambda}$