

## Math 248 Group Quiz 1

Names: \_\_\_\_\_

1. A subset  $A \subseteq \mathbb{R}^n$  for some  $n \in \mathbb{N}$  is **convex** if  $\{\lambda \mathbf{x} + (1 - \lambda)\mathbf{y} : \lambda \in [0, 1]\} \subseteq A$  for all  $\mathbf{x}, \mathbf{y} \in A$ .

a. Give an example of a convex subset of  $\mathbb{R}^2$ . (Explain why your example is convex.)

b. Give an example of a subset of  $\mathbb{R}^2$  that is not convex. (Explain why your example is not convex.)

c. Give a more exotic example of a convex subset of  $\mathbb{R}^3$ . (Explain!)

2. The **convex hull** of  $A \subseteq \mathbb{R}^n$  is defined as  $\bigcap_{B \supseteq A} B$ . Sketch the convex hull of  $\{(x^2, x) : x \in [0, 1]\}$  in  $\mathbb{R}^2$ .

3. Let  $A$  and  $B$  be statements. Define the **Sheffer stroke** of  $A$  and  $B$ , denoted  $A \uparrow B$ , to be the statement that is False if and only if both  $A$  and  $B$  are True.

a. Write a truth table containing columns for  $A \uparrow B$ ,  $(A \uparrow B) \uparrow A$ , and  $(A \implies B) \uparrow (B \wedge (A \uparrow A))$ .

b. Find logically equivalent expressions for  $\sim A$ ,  $A \wedge B$ , and  $A \vee B$  that only use the logical operation of the Sheffer stroke.