Discrete Midterm

Name: ______________________________

1. Let X be an irreducible representation of G and let h be in the center of G (meaning that $hg = gh$ for all $g \in G$). Prove that $X(h) = \omega I$ where ω is a root of unity.

2. Let X and Y be representations of G. Show that there is a fixed matrix T such that

$$(X \otimes Y)(g) = T((Y \otimes X)(g))T^{-1}$$

for all $g \in G$.

3. a. Find the character table for S_3.

b. The group S_3 acts on itself by conjugation; that is, σ acts on τ by $\sigma\tau\sigma^{-1}$. Let Y be the matrix representation for this group action. How does Y break up into a direct sum of irreducibles?

4. The character table for a group G is

	C_1	C_2	C_3	C_4	C_5	C_6
$\chi^{(1)}$	1	1	1	1	1	1
$\chi^{(2)}$	7	-1	-1	1	0	0
$\chi^{(3)}$	8	0	0	-1	1	1
$\chi^{(4)}$	6	2	0	0	-1	-1
$\chi^{(5)}$	3	-1	1	0	α	$\overline{\alpha}$
$\chi^{(6)}$	3	-1	1	0	$\overline{\alpha}$	α

for some $\alpha \in \mathbb{C}$.

a. What is the size of the group? (There is no need to simplify the arithmetic!)

b. What is α?

c. Is G simple? Why or why not?