

Math 143 Midterm 1 Review

Topics on Midterm 1

1. Taylor polynomials of degree n for $f(x)$ centered at $x = a$, namely $y = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x - a)^k$.
2. The error when $f(a)$ is approximated using the degree n Taylor polynomial for $f(x)$ centered at $x = 0$ is less than $M/(n+1)!|a|^{n+1}$ where $|f^{(n+1)}(x)| \leq M$ for $-a \leq x \leq a$.
3. Taylor series for $f(x)$ centered at $x = a$, namely $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$.
4. The top 3 Taylor series:
 - (a) The geometric series: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $-1 < x < 1$
 - (b) The exponential function: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all x .
 - (c) The sine function: $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ for all x .
5. Ways to test for the convergence of infinite series:
 - (a) Recognize a known series (a geometric series, a p -series)
 - (b) Compare the series to a larger convergent series
 - (c) Compare the series to a smaller divergent series
 - (d) Compare the series to an improper integral of the form $\int_1^{\infty} f(x) dx$
 - (e) Use the limit comparison test
 - (f) Use the Ratio test
 - (g) Use the Alternating series test
6. The interval and radius of convergence for functions of the form $\sum_{n=0}^{\infty} a_n x^n$.

Sample questions

1. Do these series converge? If so, why?

a. $\sum_{n=2}^{\infty} \frac{4n^2 - 2}{3n^2 + 2}$

b. $\sum_{n=2}^{\infty} \frac{n}{n^2 + 1}$

- c. $\sum_{n=2}^{\infty} (-1)^n \frac{n}{n^2 + 1}$
- d. $\sum_{n=2}^{\infty} \frac{\ln n}{2^{n^2}}$
- e. $\sum_{n=2}^{\infty} \frac{(1+n)^3}{(1+\sqrt{n})^4 \ln n}$
- f. $\sum_{n=0}^{\infty} \frac{n^2 + 2^n}{n^4 + 2^n}$

2. Find the interval and radius of convergence for these series

- a. $\frac{1}{1} + \frac{1}{1 \cdot 6}x + \frac{1}{1 \cdot 6 \cdot 11}x^2 + \frac{1}{1 \cdot 6 \cdot 11 \cdot 16}x^3 + \dots$
- b. $\sum_{n=1}^{\infty} \frac{\ln(n+2)}{(2n)^2} x^n$
- c. $\sum_{n=0}^{\infty} (x+4)^n / n^4$
- d. $\sum_{n=0}^{\infty} (x-1)^{2n}$

3. Find the Taylor series for $1/(1-2x)$ centered at $x = 0$.

4. Find the degree 5 Taylor polynomial for $x + \sqrt{x}$ centered at $x = 1$.

5. Find the exact values of these series

- a. $\sum_{n=2}^{\infty} (-1)^n \frac{2^n}{3^{n-1}}$
- b. $\sum_{n=2}^{\infty} (-1)^n \frac{2^n}{(n-1)!}$

6. Let $f(x) = \frac{4}{5}(1+x)^{5/2}$.

- a. Find the degree 2 Taylor polynomial for $f(x)$ at $x = 0$.
- b. Find a bound on the error when approximating $f(-1/2)$ by taking $x = -1/2$ in part a.