# Math 241 Midterm 1 Review

## Topics

1. Level curves
2. Limits (including switching into polar and showing limits do not exist)
3. Partial derivatives and the chain rule (using the tree-like diagrams)
4. Directional derivatives and the gradient vector:
   (a) If $\mathbf{u}$ is a unit vector, $D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u}$.
   (b) The maximum possible directional derivative is $|\nabla f|$, found when $\mathbf{u} = \nabla f / |\nabla f|$
   (c) $\nabla f$ is perpendicular to level curves
5. Finding tangent planes and normal lines
6. Finding absolute maximums and minimums for $f(x, y)$ over a region $R$
7. The second derivative test to identify local maximums and minimums
8. Constrained optimization and Lagrange multipliers

## Sample questions

**1.** Find the maximum rate of change of $x^2y + \sqrt{y}$ at the point $(1, 1)$. In which direction does this occur? Find a direction in which the directional derivative at $(1, 1)$ is $0$.

**2.** Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^2 + y^2}$ or show this limit does not exist.

**3.** Suppose $ze^{xyz} = 0$ where $z$ is an unknown function of $x$ and $y$. Write $z_x$ in terms of $x$, $y$, and $z$.

**4.** Find the directional derivative of $f(x, y) = 3x^2 + 2xy$ in the direction of $\langle 2, 3\rangle$ at $(1, 1)$. What does this calculation mean?

**5.** Find and classify all local minimums and maximums for $x^3 - 6xy + 8y^3$.

**6.** Find the maximum and minimum for the function $(x - 1)^2 + (2y - 1)^2$ on the triangle with corners $(0, 0)$, $(2, 0)$, and $(0, 2)$.

**7.** Let $S$ be the set of points which satisfy $z - e^{x+xy} = 0$ and let $p$ be the point $(1, -1, 1)$. Find the plane tangent to $S$ at $p$ and find the line normal to $S$ at $p$.

**8.** Let $z = f(x, y)$ with $x = \cos(st)$ and $y = s - t$. Find $\dfrac{\partial^2 z}{\partial t^2}$ in terms of $z_{xx}$, $z_{xy}$, $z_{yy}$, $s$ and $t$.

**9.** Find the point on the sphere $x^2 + y^2 + z^2 = 1$ which maximizes $x + 2y + z$.

**10.** Minimize $x^2 + 2xy + y$ subject to the constraint $x^2 + y^2 = 1$.