1. Find the area of the largest rectangle which fits below the graph of $1/(1+x^2)$ and lies above the x-axis.
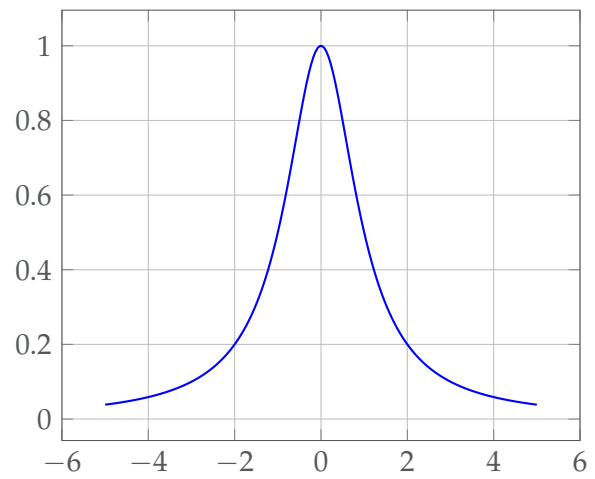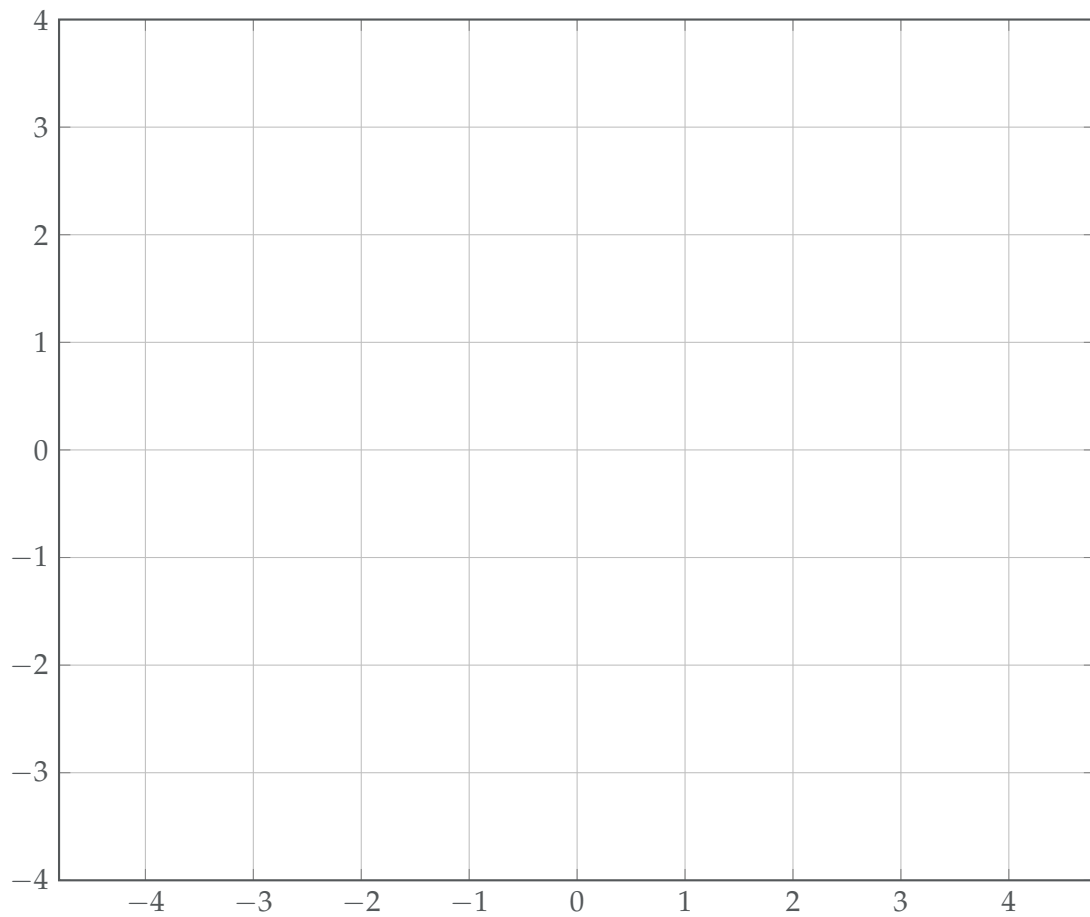
2. Let $f(x) = \dfrac{x+1}{\sqrt{x^2+1}}$:

   a. Find $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$,

   b. Find where $f(x)$ is increasing.

c. Find where $f(x)$ is concave up.

d. Sketch the graph of $f(x)$.

3. a. Carefully state the Mean Value Theorem, including all hypothesis (when it can be used).

b. The average body mass index of an American was 25 kg/m^2 in 1960 and 29 kg/m^2 in 2020. What does the Mean Value Theorem say here?

4. Find a function which satisfies $f''(x) = 2x - \cos x + 1$, $f'(0) = 2$, and $f(0) = 1$.