Math 248 Group Quiz 1

Names: _____

1. A subset $A \subseteq \mathbb{R}^n$ for some $n \in \mathbb{N}$ is **convex** if $\{\lambda \mathbf{x} + (1-\lambda)\mathbf{y} : \lambda \in [0,1]\} \subseteq A$ for all $\mathbf{x}, \mathbf{y} \in A$.

a. Give an example of a convex subset of $\mathbb{R}^2$. (Explain why your example is convex.)

b. Give an example of a subset of $\mathbb{R}^2$ that is not convex. (Explain why your example is not convex.)

c. Give a more exotic example of a convex subset of $\mathbb{R}^3$. (Explain!)

2. The **convex hull** of $A \subseteq \mathbb{R}^n$ is $\bigcap_{\substack{B \supseteq A \\ B \text{ is convex}}} B$. Sketch the convex hull of $\{(x^2, x) : x \in [0,1]\}$ in $\mathbb{R}^2$.

3. Let A and B be statements. Define the **Sheffer stroke** of A and B, denoted $A \uparrow B$, to be the statement that is False if and only if both A and B are True.

a. Write a truth table containing columns for $A \uparrow B$, $(A \uparrow B) \uparrow A$, and $(A \implies B) \uparrow (B \wedge (A \uparrow A))$.

b. Find logically equivalent expressions for $\sim A$, $A \wedge B$, and $A \vee B$ that only use the logical operation of the Sheffer stroke.