Friday Oct 27

Thm: (The # $\lambda \vdash n$ w/ distinct parts) = (The # of $\lambda \vdash n$ w/ only odd parts)

n=6	distinct	odd
	6	15
	5,1	1^6
	4,2	$1^3 3$
	3,2,1	3^2

Proof: $\sum_{n=0}^{\infty}$ (# $\lambda \vdash n$ w/ distinct parts) $z^n = (1+z)(1+z^2)(1+z^3)\ldots$

mult by 1 $= \frac{\cancel{(1-z)}(1+z)}{(1-z)} \frac{\cancel{(1-z^2)}(1+z^2)}{(1-\cancel{z^2})} \frac{(1-z^3)(1+z^3)}{(1-z^3)}\ldots$ even terms cancel

$(1+z^5+(z^5)^2+(z^5)^3+\ldots)$

$= \frac{1}{1-z}\cdot\frac{1}{1-z^3}\cdot\frac{1}{1-z^5}\cdots = \sum_{n=0}^{\infty}$ (# $\lambda \vdash n$ w/ only odd parts) z^n $\boxtimes$

Proof 2: let $\lambda \vdash n$ have distinct parts. $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_\ell)$

write each $\lambda_i = 2^{a_i}\theta_i$ where θ_i is odd.

n=6

$6 = 2^1\cdot 3 \to 3^2$

$6=5+1$
$=2^0\cdot 5+2^0\cdot 1$
$=(2^0)1+(2^0)5$

$5,1 \to 15$

$6=4+2=(2^2)1+(2^1)1$
$=(2^2+2^1)1$

$4,2 \to 1^6$

$6=3+2+1$
$=2^0\cdot 3+2^1\cdot 1+2^0\cdot 1$
$=(2^1+2^0)\cdot 1+(2^0)\cdot 3$

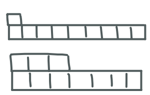
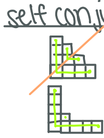
$3,2,1 \to 1^3 3$

Then $n = \lambda_1 + \ldots + \lambda_\ell$

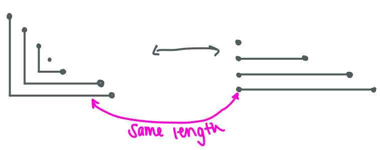
$= 2^{a_1}\theta_1 + \ldots + 2^{a_\ell}\theta_\ell$

$= \underbrace{(\text{sum of powers of } 2)}_{\text{how many 1's in a } \lambda' \text{ w/ odd parts}}\cdot 1 + \underbrace{(\text{sum of powers of } 2)}_{\text{how many 3's}}\cdot 3 + \ldots$

Thm: (# $\lambda \vdash n$ w/ odd distinct parts) = (# $\lambda \vdash n$ that are self conjugate)

n=10	odd & distinct	self conjugate
9,1		 (4,3,2,1)
7,3		(5,2,1,1,1)

Proof:



Thm: $\prod_{i=1}^{\infty} \frac{1}{1-xz^i} = \sum_{n=0}^{\infty} \frac{x^n z^n}{(1-z)\cdots(1-z^n)}$

Proof: $\prod_{i=1}^{\infty}\frac{1}{1-xz^i} = \underbrace{(1+xz+x^2z^2+x^3z^3+\ldots)}_{=\frac{1}{1-xz}}\underbrace{(1+xz^2+(xz^2)^2+(xz^2)^3+\ldots)}_{=\frac{1}{1-xz^2}}\underbrace{(1+(xz^3)+(xz^3)^2+(xz^3)^3+\ldots)}_{=\frac{1}{1-xz^3}}\ldots(1\cdots)$

$1^2 \quad 2^1 \quad 3^2 \quad 4^0 5^0\ldots$

$1^2 2^1 3^2 4^0 5^0 \sim x^5 z^{10}$
$(3,3,2,1,1)$
$3+3+2+1+1=10$

$= \sum_{\lambda} x^{\ell(\lambda)} z^{|\lambda|}$

$= \sum_{n=0}^{\infty} x^n \sum_{\lambda \text{ w/ } \ell(\lambda)=n} z^{|\lambda|}$

$= \sum_{n=0}^{\infty} \frac{x^n z^n}{(1-z)\cdots(1-z^n)}$