Math 344 Quiz 1

Names: _____

1. $\mathcal{L}^{-1}\left[\dfrac{s}{(s+a)^2+b^2}+e^{-as}\arctan\dfrac{b}{s}\right]$ where a, b are constants.

2. Solve the differential equation $y''(t)+4y(t)=\delta(t-2)$ if $y(0)=1$ and $y'(0)=0$.

3. Use the known series

$$\frac{1}{1-t} = \sum_{n=0}^{\infty} t^n, \qquad \sinh t = \sum_{n=0}^{\infty} \frac{t^{2n+1}}{(2n+1)!}, \qquad \operatorname{arctanh} t = \sum_{n=0}^{\infty} \frac{t^{2n+1}}{(2n+1)}$$

to find $\mathcal{L}\left[\sinh(at)\right]$ and $\mathcal{L}\left[\dfrac{\sinh(at)}{t}\right]$.

4. Solve the system $\begin{cases} x'(t) = -y(t) + \delta(t-1) \\ y'(t) = x(t) \end{cases}$ with the conditions $x(0) = y(0) = 1$.