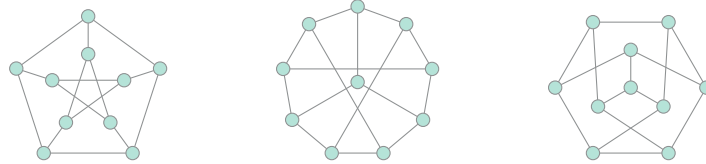



Graph Theory Set 1

1. Prove that a graph with two or more vertices has two vertices of the same degree.
2. Prove that either a graph or its complement is connected.
3. Label the vertices in each of the three unlabeled graphs below to show they are all the same. The graph in this exercise is the **Petersen graph**.



4. The **line graph** of G , denoted $L(G)$, is the graph with vertices the edges of G and edges defined such that two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G are incident.
 - a. Draw the line graph for K_4 .
 - b. Show that $(L(K_5))^c$ is isomorphic to the Petersen graph in Exercise 3.
 - c. Prove that the line graph of a connected graph is connected.
 - d. The **claw graph** is . Prove that if G has a subgraph isomorphic to the claw graph, then $G \neq L(H)$ for any graph H .
5. A graph G is **self-complementary** if G and G^c are isomorphic.
 - a. Find the only self-complementary unlabeled graph with 4 vertices.
 - b. How many edges does a self-complementary graph with n vertices have?
 - c. Why must $n = 4k$ or $n = 4k+1$ for some integer k in order for a self-complementary graph on n vertices to exist?