

# Linear Analysis II Set 2

**1.** Use the known Laplace transforms of  $t^n$ ,  $e^{at}$ ,  $\cos(at)$ , and  $\sin(at)$  given in the videos or the table of Laplace transforms on our web site to find

$$\mathcal{L}^{-1} \left[ \frac{1}{s^5} + \frac{2s+1}{s^2+16} + \frac{6+4s^2}{s(s^2+2)} + \frac{a}{s^2-a^2} \right].$$

Hint: Use partial fractions on the last two terms.

**2.** Let  $y$  and  $x$  be functions of  $t$ . Use Laplace transforms to solve the following:

- a.  $y' - y = e^{2t}$  with  $y(0) = 3$ .
- b.  $y'' - 4y = 0$  with  $y(0) = 1, y'(0) = 2$ .
- c.  $y'' - y' = -t$  with  $y(0) = 0, y'(0) = 1$ .
- d.  $\begin{cases} x' = 9y \\ y' = -x \end{cases}$  with  $x(0) = 0, y(0) = 1$ .
- e.  $\begin{cases} x' = -4x - 2y \\ y' = x - y, \end{cases}$  with  $x(0) = 0, y(0) = 1$ .
- f.  $\begin{cases} x' = x + y + 1 \\ y' = x + y - 1, \end{cases}$  with  $x(0) = 1, y(0) = 0$ .

**3.** Verify the following identities involving the Laplace transform:

- a.  $\mathcal{L} \left[ \int_0^t f(x) dx \right] = \frac{1}{s} \mathcal{L}[f(t)]$ .
- b.  $\frac{d}{ds} \mathcal{L}[f(t)] = -\mathcal{L}[tf(t)]$ .