Linear Analysis II Set 3

1. Write these functions in terms of unit step functions and then find their Laplace transforms.

a. $f(t) = \begin{cases} 1 & \text{if } 0 \le t < 3, \\ -1 & \text{if } 3 < t. \end{cases}$

b. $g(t) = \begin{cases} t^3 & \text{if } 0 \le t < 1, \\ 0 & \text{if } 1 < t. \end{cases}$

c. $h(t) = \begin{cases} 0 & \text{if } t < 0, \\ 1 & \text{if } 0 \le t < 1, \\ -t & \text{if } 1 \le t < 2, \\ e^{-t} & \text{if } 2 \le t. \end{cases}$

2. Calculate $\mathcal{L}\left[e^{3t} \sin t + e^{-2t} \cos 3t + t^3 e^{3t} + \dfrac{e^{-2t}}{\sqrt{\pi t}} - u_{2\pi}(t) \sin 2t \right]$

3. Calculate $\mathcal{L}^{-1}\left[\dfrac{2s}{s^2 - 4s + 13} + \dfrac{6}{s^2 + 2s + 2} + \dfrac{s}{(s+2)^2 + 9} \right]$

4. Calculate $\mathcal{L}^{-1}\left[\dfrac{e^{-s}s}{s^2 + 1} + \dfrac{e^{-2s}}{s^2 + 1} + \dfrac{e^{-4s}}{s^2 - 4} \right]$

5. Solve $\begin{cases} x' = 2x - y \\ y' = x + 2y \end{cases}$ with $x(0) = 1, y(0) = 0$.