Math 206 Group Quiz 2

Names: _____

1. A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is *one-to-one* if $f(\mathbf{x}) = f(\mathbf{y})$ implies that $\mathbf{x} = \mathbf{y}$ for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$.

a. Give an example of a matrix A such that $f(\mathbf{x}) = A\mathbf{x}$ is a one-to-one function from $\mathbb{R}^2$ to $\mathbb{R}^3$.

b. Find a matrix A such that $f(\mathbf{x}) = A\mathbf{x}$ is not a one-to-one function from $\mathbb{R}^2$ to $\mathbb{R}^3$.

c. Why is it impossible to give an example of a one-to-one function from $\mathbb{R}^2$ to $\mathbb{R}^1$?

2. A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is *onto* if for every $\mathbf{y} \in \mathbb{R}^m$ there is a $\mathbf{x} \in \mathbb{R}^n$ with $f(\mathbf{x}) = \mathbf{y}$.

a. Give an example of a matrix A such that $f(\mathbf{x}) = A\mathbf{x}$ is an onto function from $\mathbb{R}^3$ to $\mathbb{R}^2$.

b. Find a matrix A such that $f(\mathbf{x}) = A\mathbf{x}$ is not an onto function from $\mathbb{R}^3$ to $\mathbb{R}^2$.

c. Why is it impossible to give an example of a onto function from $\mathbb{R}^1$ to $\mathbb{R}^2$?