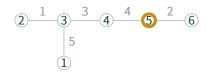
Graph Theory Set 4

- **16.** This exercise gives another proof of Cayley's theorem. Let t_n be the number of labeled trees on n vertices. Let A be the set of objects which can be created this way:
 - 1. Select a labeled tree with *n* vertices.
 - 2. Mark one vertex.
- 3. Label the n-1 edges with $1, \ldots, n-1$ such that each edge has a different label. For example, one possible element in A when n=6 is:



a. By following these instructions, how many elements are in A? (The answer should involve t_n since we are pretending that we do not know that $t_n = n^{n-2}$.)

There is another way to create elements in A:

- 1. Start with an empty graph with vertices $1, \ldots, n$. Set i = 1. Mark every vertex.
- 2. Select any vertex, say v.
- 3. Select a marked vertex, say w.
- 4. Remove the mark on w and draw an edge with label i between v and w.
- 5. If there are at least two marked vertices, increment *i* by 1 and go back to step 2.. If not, stop.
- **b.** By following the 5 above steps, how many elements are in A? Why does this prove that $t_n = n^{n-2}$?
- **17.** Suppose G has two spanning trees T_1 and T_2 . Let e be any edge in T_1 . Show that there is an edge f in T_2 such that the graph (T e) + f (remove e from T_1 and include f) is also a spanning tree.
- **18.** Let $\tau(G)$ be the number of spanning trees for G and let e be an edge in G. Show that $\tau(G) = \tau(G e) + \tau(G/e)$.
- **19.** It has been proved that if G is planar, then it can be drawn in the plane with straight line segments as edges. Exhibit such planar drawings for $K_5 e$ and $K_{3,3} e$.
- **20.** Remove and contract edges in the following graph to find $K_{3,3}$, showing that it is not planar.

