

Math 143 Quiz 5

Names: _____

1. Let \mathbf{a} and \mathbf{v} be fixed vectors. Parameterize the line $\mathbf{r}(t) = \mathbf{a} + t\mathbf{v}$ by arclength.

2. Suppose \mathbf{r} is a vector valued function such that $|\mathbf{r}| = 1$, meaning that $\mathbf{r} \cdot \mathbf{r} = 1$. Differentiate both sides of this identity to show that \mathbf{r} and \mathbf{r}' are orthogonal. Why does this mean that \mathbf{T} and \mathbf{N} are orthogonal?

3. Show that the curvature of the graph of $y = f(t)$ is given by $\kappa(t) = \frac{|f'(t)|}{(1 + f'(t)^2)^{3/2}}$.

4. Suppose \mathbf{r} is parameterized by arclength, meaning that $\kappa = |\mathbf{r}''|$. Show that if $\kappa = 0$, then \mathbf{r} is a line.

5 (**Bonus!**). Try on a separate page if done with all other problems: Explain why $\kappa(t) = |\mathbf{r}' \times \mathbf{r}''|/|\mathbf{r}'|^3$.