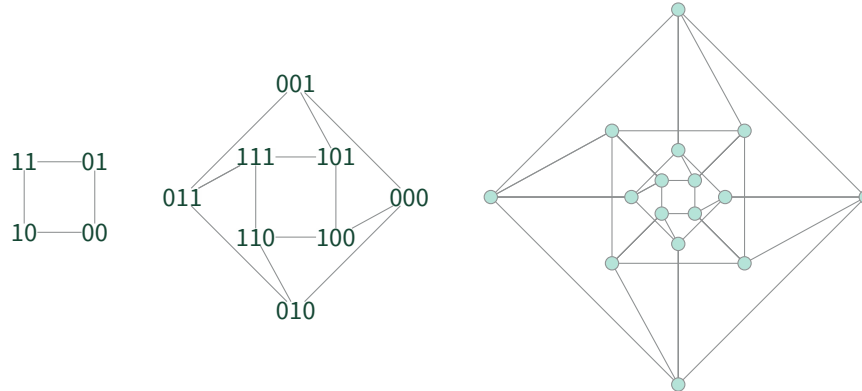
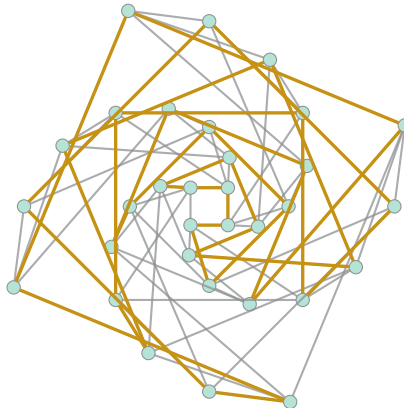


Graph Theory Set 6

27. The **cube graph** Q_n has vertices the bit strings of length n (these are sequences of 0's and 1's, such as 10110110) with an edge between two bit strings if and only if the strings differ in exactly one position. Below are Q_2 and Q_3 and an unlabeled Q_4 :



- Show that Q_n has 2^n vertices and $n2^{n-1}$ edges.
- Show that Q_n is bipartite.
- Use Menger's theorems to show that $\varepsilon(Q_n) = \kappa(Q_n) = n$.
- Show that Q_n is Hamiltonian by induction on n . One Hamiltonian cycle is highlighted on an unlabeled Q_5 graph below:



- Suppose G is critical (see Exercise 12). Show that $\kappa(G) \neq 1$.
- Show that deleting an edge from G reduces $\kappa(G)$ by at most 1.
- Show that if every vertex in G has an even degree, then $\varepsilon(G)$ is even.
- Suppose $\kappa(G) \geq 3$ and u, v, w are vertices in G . Show that G has a cycle containing u and v but not w .