Math 241 Final Review

Exam times

- The exam for the 9am section is on Wednesday June 14 at 8:10am until 10am in our classroom.
- The exam for the 10am section is on Friday June 16 at 10:10am until 12pm in our classroom.

Topics

The final exam is cumulative and will cover the topics found on Midterm 1 and Midterm 2 in addition to the topics below. See the previous exam reviews for those topics!

1. Parameterizing surfaces and surface integrals.
2. Calculating flux of a vector field through a curve or a surface.
3. Divergence and the Divergence theorem
4. Stokes' theorem

Sample questions

1. Show that Green's theorem is a special case of Stokes' theorem.

2. Verify the divergence theorem for $\mathbf{F} = \langle 2x^2y, -y^2, +4xz^2\rangle$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$.

3. Let $\mathbf{F} = \langle P, Q\rangle$ and $\mathbf{G} = \langle Q, -P\rangle$ be vector fields in $\mathbb{R}^2$. Explain why

$$\int_C \mathbf{F}\cdot\mathbf{T}\,ds = \int_C \mathbf{G}\cdot\mathbf{N}\,ds$$

where $\mathbf{T}$ and $\mathbf{N}$ are the unit normal and tangent vectors along C.

4. Let S be the surface $z + x^2 + y^2 = 2$ above the x, y plane. Evaluate $\displaystyle\iint_S \frac{1}{\sqrt{4x^2 + 4y^2 + 1}}\,d\sigma$.

5. Verify Stokes' theorem for $\mathbf{F} = \langle 2x - y, -yz^2, -y^2z\rangle$ where S is the upper have of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

6. Use the methods in this course to find the surface area of a sphere of radius r.

7. Find the center of mass for the solid in the first octant bounded by the plane $z = 2 - 3x - y$.

8. Find the point on the sphere $x^2 + y^2 + z^2 = 1$ which maximizes $4x + 4y + 2z$.

9. Find the volume of the portion of the cylinder $x^2 + y^2 = 4$ which lies above the x, y and below the plane $x + y + z = 4$.

10. How much work is done by the vector field $\mathbf{F} = \langle x - y, y \rangle$ in moving an object along the graph of $y = \sqrt{x}$ on the interval from 0 to 1?

11. Maximize the function $f(x, y) = 4xy - x^2 - y^2$ for values of x, y such that $x^2 + y^2 \leq 1$.

12. Find the directional derivative of $\sin(x^2 - 2y)$ in the direction of $\langle 1, -1 \rangle$ at the point $(0, 0)$.

13. Find the volume of the region containing those points inside the sphere $x^2 + y^2 + z^2 = 1$ but not inside the double cone $z^2 = x^2 + y^2$.

14. Find the maximum value of $\ln x + 2 \ln y$ if x and y are positive and must satisfy $x^2 + y^2 = 1$.

15. Find the equation of the plane tangent to the surface $y^2/(1 + x^2 y)$ at the point $(1, 1, 1/2)$.

16. Find a point on the surface $x^2 - 2y^2 + z^2 - 1$ such that the tangent plane at that point is parallel to the plane $x + y - 2z = 4$.

17. Suppose $xze^{xyz} = y$ where z is an unknown function of x and y. Write z_x in terms of x, y, and z.

18. Find the volume of the solid below the graph of $z = 3 - 8x^2 - y^2$ and above the graph of $z = x^2 + 8y^2$.

19. Suppose $e^{xyz} + xz = 0$ where z is an unknown function of x and y. Write z_x in terms of x, y, and z.

20. Find the absolute maximum of the function $f(x, y) = 5xy - x^2 - y^2$ if $x^2 + y^2 \leq 1$.