Graph Theory Midterm 1 Review

Definitions: complete graph K_n, r-coloring, cycle graph C_n, path graph P_n, adjacent, bipartite, chromatic number $\chi(G)$, chromatic polynomial $P_G(x)$, complement, complete bipartite graph $K_{m,n}$, component, connected, cycle, degree, degree sequence, edge, graph, greedy coloring, incident, independent sets, isomorphic graphs, line graph, path, proper r-coloring, self-complementary, spanning tree, subgraph, unlabeled graph, vertex.

Theorems:

- K_n has $\binom{n}{2}$ edges.

- There are $2^{\binom{n}{2}}$ graphs on n vertices.

- The sum of the degree sequence is twice the number of edges.

- If G has n vertices and more than $\binom{n-1}{2}$ edges, then G is connected.

- If G and G^c are connected, then G has a P_4 subgraph.

- If p is prime, then $r^p - r$ is divisible by p for integer r.

- If G is not K_n or C_{2n+1}, then $\chi(G)$ is not more than the largest degree in G.

- G is bipartite if and only if G does not have an odd cycle.

- $P_G(x) = P_{G-e}(x) - P_{G/e}(x)$.

- $P_G(C_n) = (x-1)^n + (-1)^n(x-1)$.

- $P_G(K_n) = x(x-1)\cdots(x-n+1)$.

- $P_G(x) = x^{(\text{number of vertices})} - (\text{number of edges})x^{(\text{number of vertices})-1} + \cdots \pm ax^{(\text{number of components})}$.

- Let T be a graph with n vertices. The following statements are equivalent:

  a. T is a tree.

  b. T is connected and has no cycles (of length ≥ 3).

  c. T is connected and has $n-1$ edges.

  d. there is a unique path of distinct vertices between every pair of vertices u and v in T.

  e. $P_T(x) = x(x-1)^{n-1}$.

- There are n^{n-2} trees.

- There are $n^{m-1}m^{n-1}$ spanning trees for $K_{m,n}$.