1. Let A be an $n \times n$ matrix and λ a constant. Is $S = \{\mathbf{x} \in \mathbb{R}^n : A\mathbf{x} = \lambda\mathbf{x}\}$ a subspace of $\mathbb{R}^n$? Why?

2. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $S = \{X \in M_{2,2} : AX = XA\}$. Show that S is a subspace. Find a basis for S.

3. Let $\mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ be nonzero vectors in $\mathbb{R}^n$. Perform the matrix multiplications $\mathbf{u}\mathbf{v}^\top$ and $\mathbf{u}^\top \mathbf{v}$ and then give the rank of each of these matrices.

4. Let A be an $m \times n$ matrix and let $B = \begin{bmatrix} \mathbf{b}_1 \cdots \mathbf{b}_p \end{bmatrix}$ be an $n \times p$ matrix. By now we know that

1. The vector $A\mathbf{b}_1$ is a linear combination of the columns of A.

2. Matrix multiplication is defined by $AB = \begin{bmatrix} A\mathbf{b}_1 \cdots A\mathbf{b}_p \end{bmatrix}$.

3. The rank of A is the number of linearly independent columns of A.

Why do these three facts imply that $\text{rank}(AB) \le \text{rank}(A)$?