Graph Theory Set 2

6. The **Ramsey number** $R(\ell, m)$ is the least n such that every graph G with n vertices either has a K_ℓ subgraph or a K_m^c subgraph. The numbers $R(\ell, m)$ are only known for very small values of ℓ, m.

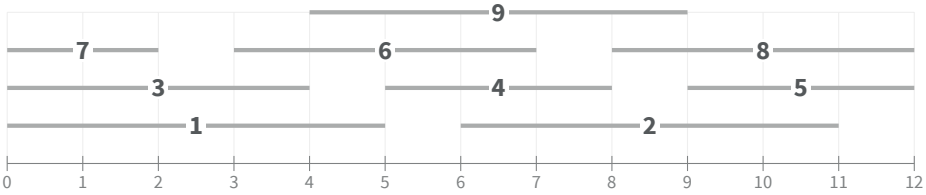
- **a.** Show that $5 < R(3,3)$ by giving an example of a graph with 5 vertices that does not have a K_3 subgraph and does not have a K_3^c subgraph.
- **b.** Show that $R(3,3) \leq 6$ by showing that every graph with 6 vertices either has a K_3 subgraph or a K_3^c subgraph.
- **c.** Show that $8 < R(3,4)$.

7. An "X" in the table below indicates a pair of animals that do not peacefully coexist:

	wolf	ostritch	cobra	tiger	rhino	baby
wolf		X		X	X	X
ostritch	X				X	X
cobra						X
tiger	X					X
rhino	X	X				X
baby	X	X	X	X	X	

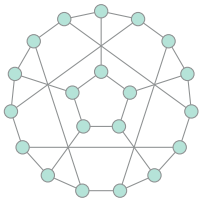
Find the minimum number of cages needed to safely separate these animals by creating a graph based on the above table and then finding its chromatic number.

8. There are 9 jobs that need to be done in 12 hours, each within a different time window as indicated below.



This diagram indicates, for example, that job 1 occupies the time window between hour 0 and hour 5. If exactly one worker is needed for each job, find the minimum number of workers needed to complete all jobs by creating a graph based on the above diagram and then finding its chromatic number.

9. Find the chromatic number of the **flower snark**:



10. Show that if $\chi(G) \geq 6$, then G has two odd cycles that do not share a vertex.