Last time

$$\sum_{n=0}^{\infty} \frac{a_{2n}}{(2n)!} z^{2n} \qquad a_{2n} := \# \text{ of alternating permutations}$$

$$= \frac{1}{\cos(z)} = 1 + 0z^{1} + \frac{1}{2!}z^2 + \frac{0}{3!}z^3 + \frac{5}{4!}z^4 + \cdots$$

Consider,
$$\frac{1}{\cos\sqrt{z}} = 1 + \frac{1}{2!}z + \frac{5}{4!}z^2 + \cdots$$
$$= \sum_{n=0}^{\infty} \frac{a_{2n}}{(2n)!} z^n$$

The smallest singularity, i.e radius of convergence, is $R = \left(\frac{\pi}{2}\right)^2$

Recall, $\lim_{z \to R} (R-z)^{\alpha} f(z) = C$ where $C \neq 0$ $C < \infty$

then the coefficient of z^n in $f(z) \sim \frac{C n^{\alpha-1}}{R^{n+\alpha}\Gamma(\alpha)}$

here $\lim_{z \to (\frac{\pi}{2})^2} \frac{\left(\left(\frac{\pi}{2}\right)^2 - z\right)}{\cos\sqrt{z}} \overset{LH}{=} \pi$

$$\frac{a_{2n}}{2n!} \sim \frac{\pi n^{1-1}}{R^{n+1}\Gamma(1)} = \frac{\pi}{R^{n+1}} = \frac{\pi}{\frac{\pi^{2n+2}}{4^{n+1}}} = \frac{\pi 4^{n+1}}{(\pi^2)^{n+1}} = \frac{4^{n+1}}{\pi^{2n+1}}$$

with stirling's formula

$$a_{2n} \sim \sqrt{2\pi(2n)}\left(\frac{2n}{e}\right)^{2n} \cdot \frac{4^{n+1}}{\pi^{2n+1}}$$

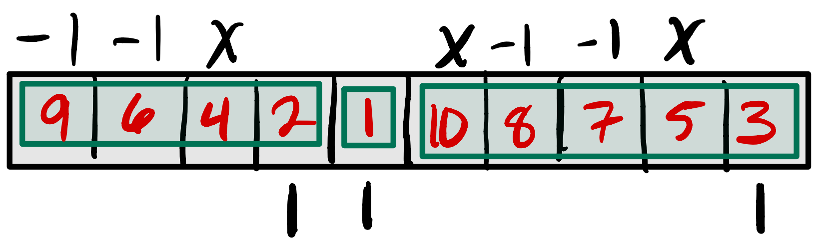
Let $R_{n-1,i}$ be the number of rearrangements of i copies of x's & $n-i-1$ copies of -1 s.t. no x's appear consecutively

Ex: $R_{4,2}$

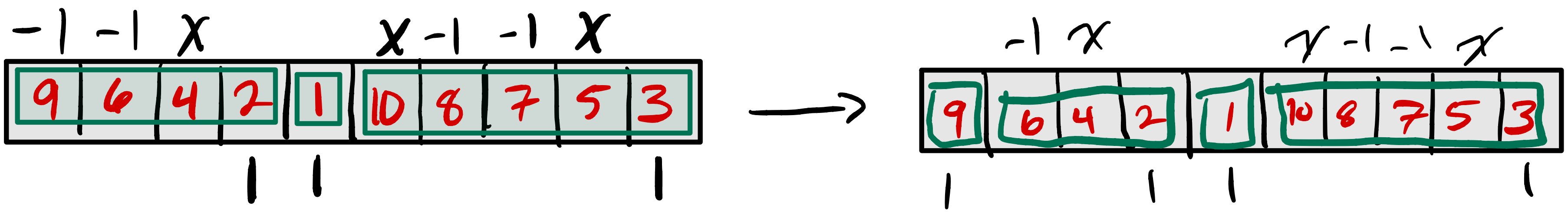
$x \; -1 \; -1 \; x$
$x \; -1 \; x \; -1$
$-1 \; x \; -1 \; x$

Define $\varphi(e_n) = \frac{(-1)^{n-1}}{n!} f(n)$ where $f(n) = \sum_{i=0}^{n-1} R_{n-1,i}\, x^i (-1)^{n-i-1}$

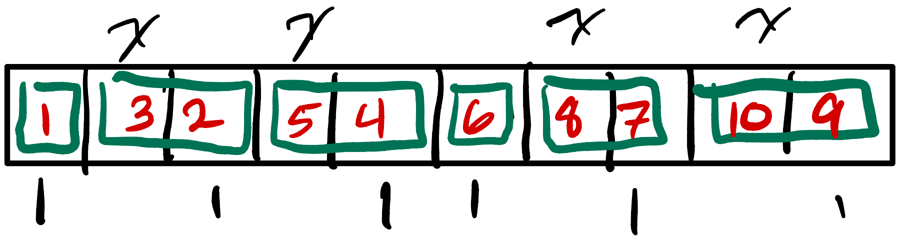
Then
$$n!\,\varphi(h_n) = n! \sum_{\lambda \vdash n} (-1)^{n-\ell(\lambda)} |B_{\lambda,(n)}|\, \varphi(e_\lambda)$$
$$= n! \sum_{\lambda \vdash n} (-1)^{n-\ell(\lambda)} |B_{\lambda,(n)}| \frac{(-1)^{\lambda_1-1}}{\lambda_1!} \cdot f(\lambda_1) \cdots$$
$$= \sum_{\lambda \vdash n} \binom{n}{\lambda_1, \lambda_2, \ldots} |B_{\lambda,(n)}| \left(\sum_{i=0}^{\lambda_1-1} R_{\lambda_1-1,i}\, x^i (-1)^{\lambda_1-i-1} \right) \cdots$$



1) Take a strip of length n ($n=10$ here)
2) Fill it with arbitrary length bricks
3) Write a decreasing sequence of $1,2,\ldots,n$ in each brick s.t. union is $1,2,\ldots,n$
4) Write a 1 in the last cell of each brick.
5) Write x or -1 in remaining cells s.t. no two x's appear next to one another.
6) Scan L to R looking for a -1 or consecutive bricks that can be combined.



Fixed points have no negative 1's & must increase between bricks & decrease between two bricks



So we have $\displaystyle\sum_{\sigma \in S_n} x^{des(\sigma)}$
σ never has consecutive descents

Apply φ to $\displaystyle\sum h_n z^n = \frac{1}{\sum_{n=0}^{\infty} (-1)^n e_n z^n}$ to find

$$\sum_{n=0}^{\infty} \frac{1}{n!} \sum_{\substack{\sigma \in S_n \\ \text{w/ no consecutive descents}}} x^{des(\sigma)} z^n = \frac{e^{z/2}}{\cos\left(\frac{z\sqrt{4x-1}}{2}\right) - \frac{1}{\sqrt{4x-1}} \sin\left(\frac{z\sqrt{4x-1}}{2}\right)}$$