Math 143 Midterm 2 Review

Topics on Midterm 2

1. Multiplying, dividing, differentiating, integrating series.
2. Euler's formula: $e^{it} = \cos t + i \sin t$.
3. Parametric equations (plotting, derivatives, arclength)
4. Polar equations (plotting, derivatives, arclength, polar rectangles, area)
5. $\mathbb{R}^3$ (distance, midpoints, basic plots including spheres and cylinders)
6. Vectors in $\mathbb{R}^3$ (length, unit vectors, dot product, cross product)

Sample questions

1. Find the first 4 terms in the series for $\sin x \cos(2x)$ at $x = 0$.

2. Approximate $\int_0^1 xe^{-x^4}\,dx$ to within $1/100$ of the true answer.

3. Plot, find the arclength, and find the area enclosed by the polar curve $r = \theta^2$ for $\theta \in [0, 2\pi]$.

4. Graph the parametric equations $\begin{cases} x = 2 + 3\sin t \\ y = 1 + 2\cos t \end{cases}$ for $t \in [0, 3\pi/2]$.

5. Find two vectors of length 2 which are perpendicular to both $\langle 2, 2, 3\rangle$ and $\langle -1, 0, 2\rangle$.

6. Consider the curve in the plane $\begin{cases} x = \cos t + t\sin t \\ y = \sin t - t\cos t \end{cases}$ where $t \in [0, 2\pi]$.

a. Find the (x, y) coordinates of all vertical and horizontal tangents.

b. Find the values of t for which this curve is concave down.

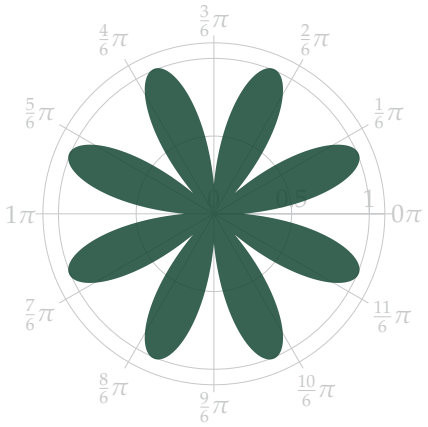
c. Find the arclength of the curve.

7. Consider the curve given parametrically by $\begin{cases} x = 2e^t - t \\ y = e^t - 3 \end{cases}$ for $t \in \mathbb{R}$. Find the parametric equations for the line tangent to the curve at $t = 1$.

8. Fix a vector $\mathbf{v} \in \mathbb{R}^3$. Which unit vector $\mathbf{w}$ maximizes the dot product $\mathbf{w} \cdot \mathbf{v}$?

9. If $a, b \in \mathbb{R}$, we let $\mathrm{Re}(a+ib) = a$ and $\mathrm{Im}(a+ib) = b$ denote the real and imaginary parts of the complex number $a+bi$. Plot the parametric equation $\begin{cases} x = \mathrm{Re}(3e^{2it}), \\ y = \mathrm{Im}(3e^{2it}) \end{cases}$ for $t \in [0, \pi/4]$.

10. Find the area enclosed by the polar curve $r = \sin(4\theta)$.



11. Find the arclength of the curve given by the parametric equations $\begin{cases} x = \cos(2t), \\ y = 2t - \sin(2t) \end{cases}$ for $t \in [0, \pi]$.

12. Find the arclength of the curve described by the parametric equations $\begin{cases} x = 3 + e^{-2t} \\ y = 2 - e^{-2t} \end{cases}$ for $t \in [0, 1]$.