ENM 503, Homework 1

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Problem 1

This problem can be solved by a combination of the multiplication and addition principle. For this problem, it is easier to find the total number of integers without repeating digits.

For...

- 1-digit integers: There are 9 total unique numbers
- 2-digit integers: There are 90 total numbers but 9 numbers (11,22,...,99) that contain repeating digits

$$90 - 9 = 81$$

- 3-digit integers: For integers 100 to 999, we can begin to use the multiplication principle. There are 9 possible non-zero digits, 9 possible digits excluding the digit in the first position, and 8 possible digits excluding the 2 previous chosen digits

$$9 * 9 * 8 = 648$$

- 4-digit integers: The same methodology can be used here as 3-digit integers.

$$9 * 9 * 8 * 7 = 4536$$

- 5-digit integers: Same methodology again

$$9 * 9 * 8 * 7 * 6 = 27216$$

Now to find the number of integers with at least one repeating digit, we take the total number of integers between 1 and 99,999 and subtract the sum of the numbers *without* repeating digits found from above

$$99999 - (9 + 81 + 648 + 4536 + 27216) = 67509$$

Problem 2

We can use the pigeonhole principal and multiplication principal to solve this problem. First we need to find the total number of boxes. A box can be described as {age,height,sex,weight}. For example one box can be {26,76 inches,male,210}. This can be done with the multiplication principle:

$$101 * 85 * 2 * 396 = 6,799,320 \ \ boxes$$

If we were to group all people in the US in these boxes, by the pigeonhole principle at least $\lceil 320mil/6799320 \rceil = 47$ people are guaranteed to have the same age, height, sex, and weight.

Problem 3

If there are n party-goers, and if we label each person in the party from 0 to $n-1$, the number of others that person k can know is in the range $[0, n-2]$.

In this case, we can say there are $n-1$ boxes representing number of people person k can know in a group. Every person must be assigned one of these $n-1$ boxes, but because there are n party-goers, one of the boxes must occupy more than one party-goer by the pigeonhole principle.

Problem 4

For license plates starting with a **letter**, we can find the number of plates by:

$$26 * 10 * 26 * 10 * 26 * 50 = 87880000$$

For license plates starting with a **number**, we do a similar calculation:

$$10 * 26 * 10 * 26 * 10 * 50 = 676000$$

The total number of valid license plate being $676000 + 87880000 = 8855600$

Problem 5

Given the prime factorization of the the number, if we take all of the *exponents*, add 1 to each, multiply the respective sums together, then we can find the number of divisors of 22,226,400.

$$(5+1) * (4+1) * (2+1) * (3+1) = 360 \ \ divisors$$