Homework 5

CIS-623 STRUCTURED PROGRAMMING & FORMAL METHODS

PROF. MEHMET KAYA

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Anthony Redamonti
SYRACUSE UNIVERSITY

Show that the following Hoare triples are valid.

Question 1:

$[y \geq 0]$ Precondition

$0 = 0x$

$a = 0;$

$[y \geq 0] \rightarrow 0 = 0x =$ Valid

$0 = ax;$

$z = 0;$

$z = ax;$

while $(a != y)\{$

$(a != y) \wedge (z = ax)$

$z + x = (a+1)x$

$z = z + x;$

$z = (a+1)x$

$a = a + 1;$

$z = ax;$

$\}$

$(a != y) \wedge (z = ax) \rightarrow z + x = (a+1)x$

$(a != y) \wedge (z = ax) \rightarrow (z = ax)$

Valid

$[\neg(a != y) \wedge (z = ax)]$

$[z = x \cdot y]$

$\neg(a != y) \wedge (z = ax) \rightarrow (z = xy)$

$\hookrightarrow a = y$

valid

y	a	x	z
3	0	2	0
3	1	2	2
3	2	2	4
3	3	2	6

Question 2:

$[y = y_0 \ \& \ y \geq 0]$ Precondition

$\rightarrow 0 = (y_0 - y)x$

$z = 0$

$z = (y_0 - y)x$

While $(y != 0)$ {

$(y != 0) \wedge (z = (y_0 - y)x)$

$\rightarrow z + x = (y_0 - (y-1))x$

$z = z + x;$

$\rightarrow z = (y_0 - (y-1))x$

$y = y - 1;$

$z = (y_0 - y)x$

}

$[\neg(y != 0) \wedge (z = (y_0 - y)x)]$

$[z = x \cdot y_0]$ Post condition

$[y = y_0 \ \& \ y \geq 0] \rightarrow 0 = (y_0 - y)x$

Valid

$(y != 0) \wedge (z = (y_0 - y)x) \rightarrow z + x = (y_0 - (y-1))x$

$(y != 0) \wedge (z = (y_0 - y)x) \rightarrow z = (y_0 - y)x$

Valid

$\neg(y != 0) \wedge (z = (y_0 - y)x) \rightarrow z = x\, y_0$

valid

y_0	y	x	z
3	3	2	0
3	2	2	2
3	1	2	4
3	0	2	6

$z = (y_0 - y)x$

Question 3:

$[T]$ $\qquad\qquad$ $[T] \rightarrow T$ = Valid

$(x > y) \rightarrow \phi_1 \wedge \neg(x > y) \rightarrow \phi_2 = T$

if $(x > y)$

$(W > X) \rightarrow W = max(X, Y, W) \wedge \neg(W > X) \rightarrow X = max(X, Y, W) = \phi_1$

$Z = X;$

$(W > Z) \rightarrow W = max(X, Y, W) \wedge \neg(W > Z) \rightarrow Z = max(X, Y, W)$

else

$(W > Y) \rightarrow W = max(X, Y, W) \wedge \neg(W > Y) \rightarrow Y = max(X, Y, W) = \phi_2$

$Z = Y;$

$(W > Z) \rightarrow W = max(X, Y, W) \wedge \neg(W > Z) \rightarrow Z = max(X, Y, W)$

if $(W > Z)$

$W = max(X, Y, W)$

$Z = W;$

$Z = max(X, Y, W)$

$[Z = max(X, Y, W)]$