

Homework 8

CIS-623 STRUCTURED PROGRAMMING & FORMAL METHODS

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3/20/2022

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Question 1:

M1:

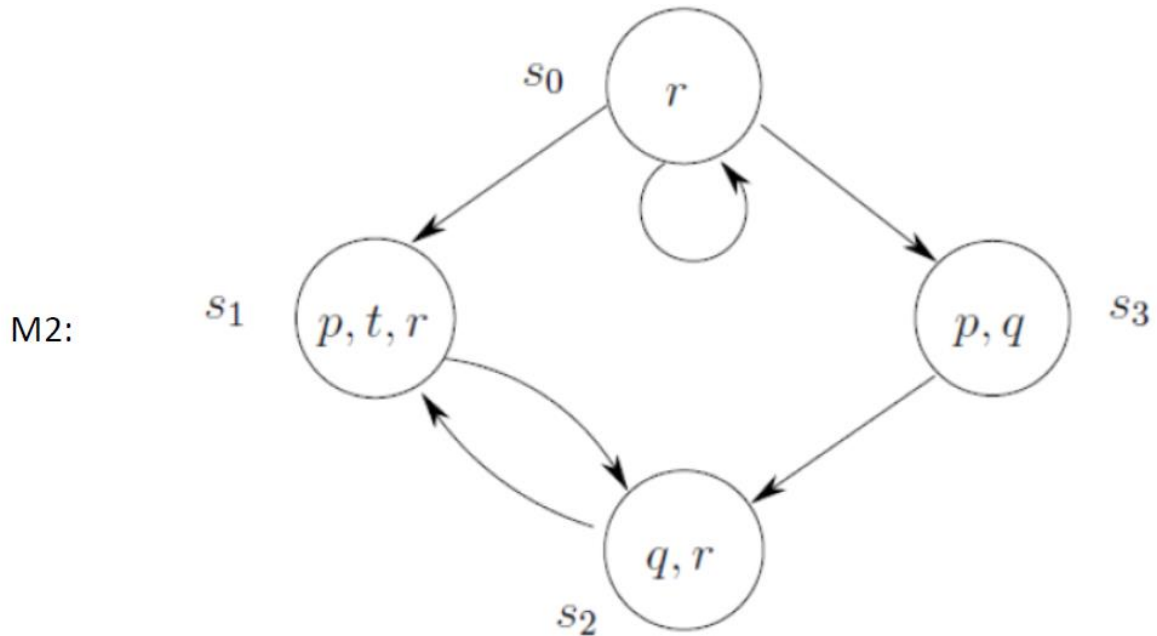


Does the model M1 and s1 satisfy the following formulas?

- a. $AG\ AF\ p$
 - b. $AG\ EF\ p$
- a. Not satisfied. All possible paths starting from s1 are s1's infinite loop and s1→s2 then s2's infinite loop. G means that every point along these two paths the following must be true: $AF(p)$. S1's infinite loop represents a path where $F(p)$ is not true. Therefore, the answer is "not satisfied".
 - b. Satisfied. All possible paths starting from s1 are s1's infinite loop and s1→s2 then s2's infinite loop. G means that every point along these two paths the following must be true: $EF(p)$. E means that there must exist a path from the starting point where something must be true. $F(p)$ means that there is a future path where p is true. S1 is connected to s2 and therefore always has the option of transitioning to s2, where p is true. The second path obviously satisfies the condition: $EF(p)$. Therefore, the answer is "satisfied."

Question 2:

Does the model M2 and s0 satisfy the following formulas?



- $\neg EG\ r$
- $AF\ q$
- $AG\ AF\ q$

- False. There is an infinite path where $G(r)$ is true.
- False. There is an infinite path where $F(q)$ is false.
- False. There is an infinite path where $AF(q)$ is false (s_0 recursive loop).

Question 3:

Prove or construct counterexamples for the following CTL formulas

a. $EG(p \ \& \ q) \rightarrow (EG \ p \ \& \ EG \ q)$

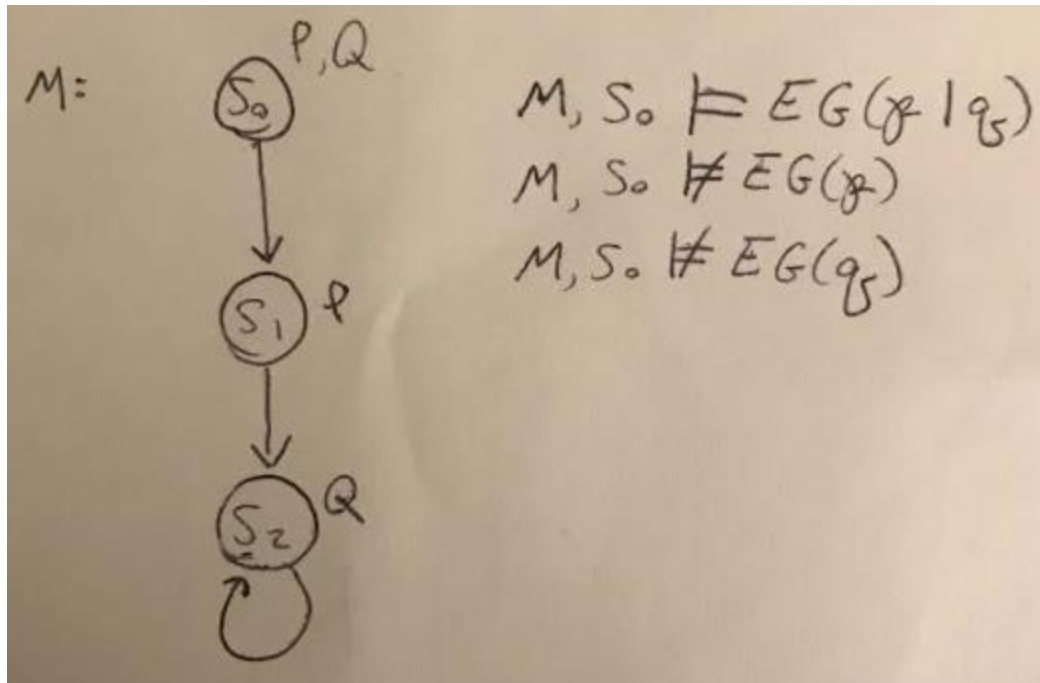
b. $EG(p \mid q) \rightarrow (EG \ p \mid EG \ q)$

a. True. See proof below.

$$EG(p \ \& \ q) \vdash (EG(p) \ \& \ EG(q))$$

1. $EG(p \ \& \ q)$ premise
2. $EG(p)$ $\wedge e1$
3. $EG(q)$ $\wedge e1$
4. $EG(p) \wedge EG(q)$ $\wedge x2,3$

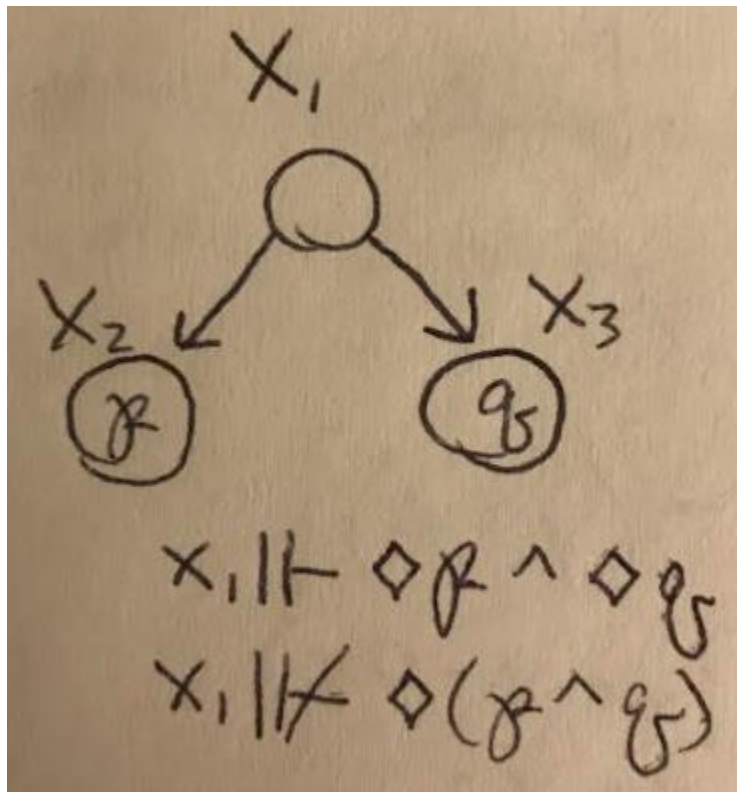
b. False. See counterexample below.



Question 4:

Give a model and a world in which only one of the following two formulas is true while the other is false.

$$\Diamond(p \wedge q) \text{ and } \Diamond p \wedge \Diamond q$$



Question 5:

Find natural deduction proofs for the following sequent over the basic modal logic K.

$$\Diamond(p \rightarrow q) \vdash \Box p \rightarrow \Diamond q$$

$$\begin{array}{l}
 \underline{\Diamond(p \rightarrow q) \vdash \Box p \rightarrow \Diamond q} \\
 1. \Diamond(p \rightarrow q) \text{ premise} \\
 \boxed{
 \begin{array}{l}
 2. \Box p \text{ assumption} \\
 3. p \quad \Box e 2 \\
 4. \Diamond \neg \Box \neg q \quad \rightarrow e 1, 3
 \end{array}
 } \\
 5. \Box p \rightarrow \Diamond q \rightarrow i 2-4
 \end{array}$$