Midterm Exam 2

CIS-675 DESIGN AND ANALYSIS OF ALGORITHMS

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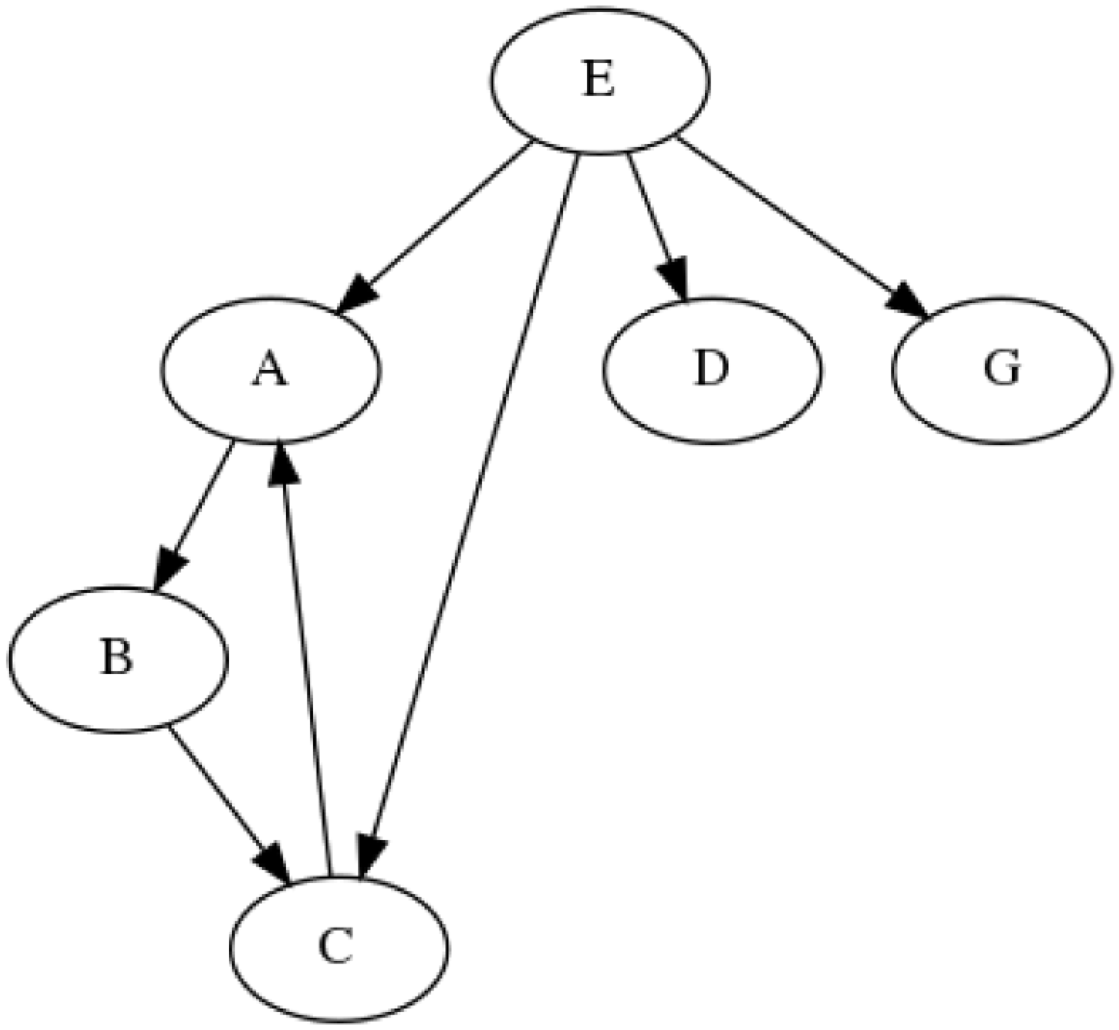
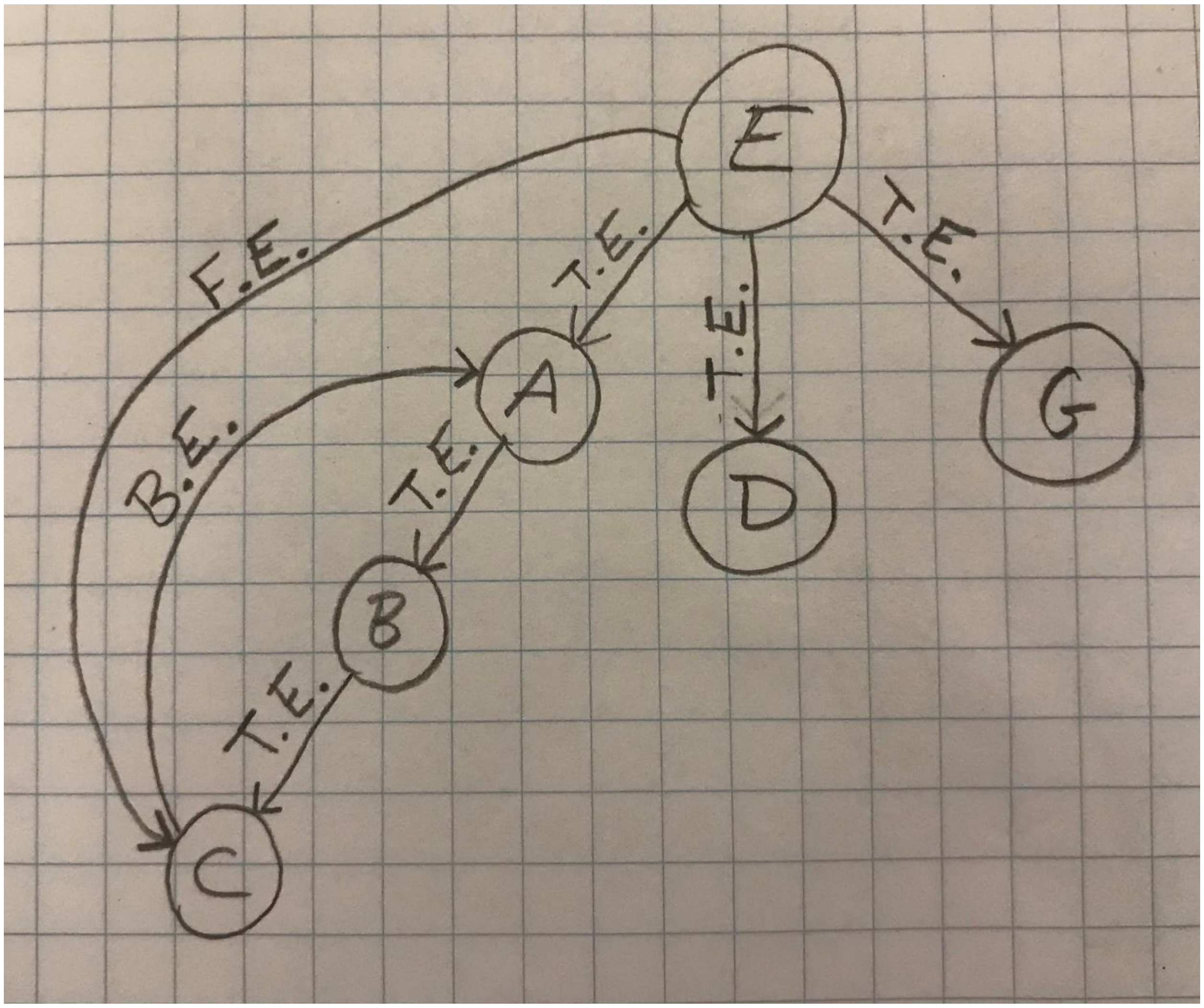
Questions 1, 2, 4, and 6 have been answered.

Question 1:

Worth 25pts
Find the strongly connected components in the following directed graph using the algo rithm from class. For full credit show the DFS tree on the graph including post/pre-view numbers, forward, back & cross edges.



The graph’s DFS tree is below with tree edges, cross edges, back edges, and forward edges labeled T.E., C.E., B.E. and F.E. respectively.



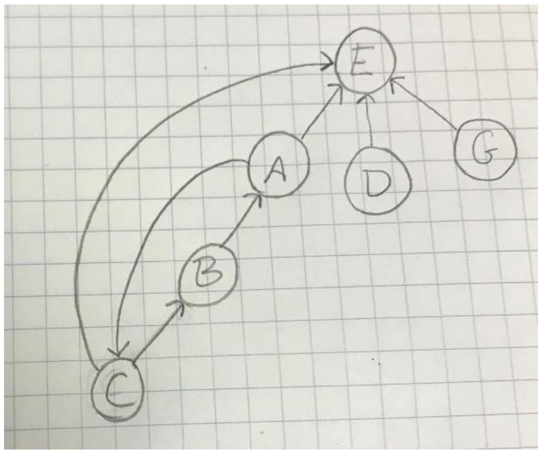
Vertex	Pre-order	Post-order
E	1	12
A	2	7
B	3	6
C	4	5
D	8	9
G	10	11

Run the algorithm to find all the strongly connected components and provide an ordering of the nodes by the G^R post-visit numbers.

The ordering of the post-visit numbers yields the following order on the stack.

Stack
E
G
D
A
B
C

Now create G^R, where G^R is a directed graph with all edge directions reversed.



Pop the elements off the stack and perform DFS on them in G^R.

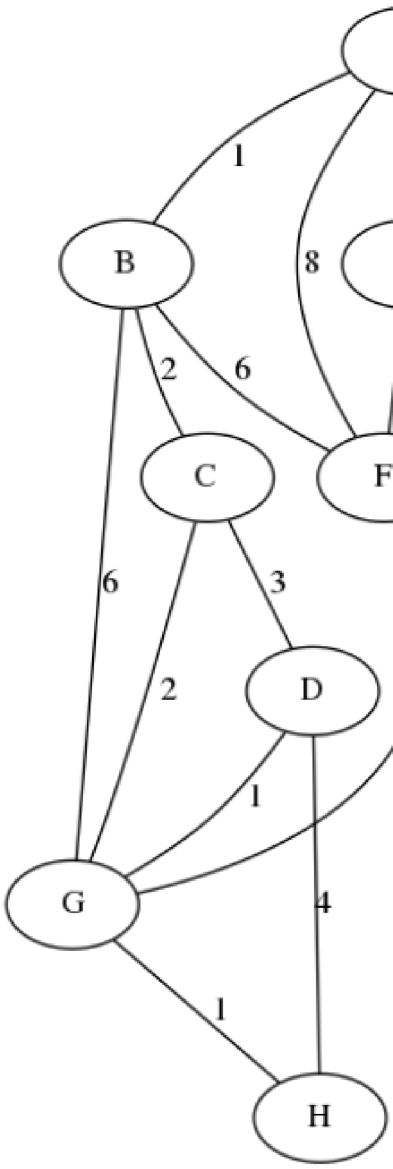
Vertex	Pre-order	Post-order
E	1	2
G	3	4
D	5	6
A	7	12
C	8	11
B	9	10

It is clear from the results of the algorithm that the following are the four strongly connected components in the graph:

- E
- G
- D
- {A,B,C}

Question 2:

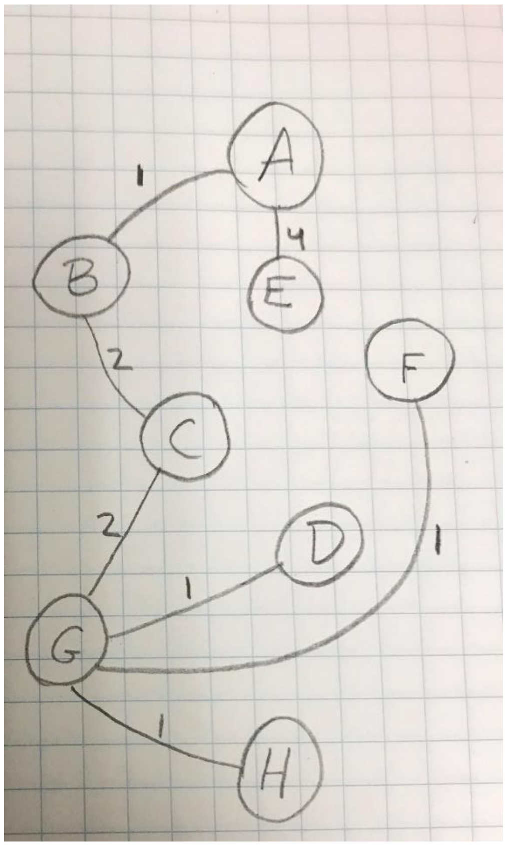
Question is worth 25 points
Run Prim's algorithm; whenever there is a choice of nodes, always use alphabetic ordering (e.g. start from node A). Draw a table showing the intermediate values of the cost.



Initially the cost values of all vertices are set to infinity except for the starting vertex A, which is set to 0. Below is the table showing the intermediate values of the cost per iteration.

Iteration	A	B	C	G	D	F	H	E
0	0	Infinity	Infinity	Infinity	Infinity	Infinity	Infinity	Infinity
1	0	1	Infinity	Infinity	Infinity	Infinity	Infinity	Infinity
2	0	1	2	Infinity	Infinity	Infinity	Infinity	Infinity
3	0	1	2	2	Infinity	Infinity	Infinity	Infinity
4	0	1	2	2	1	Infinity	Infinity	Infinity
5	0	1	2	2	1	1	Infinity	Infinity
6	0	1	2	2	1	1	1	Infinity
7	0	1	2	2	1	1	1	4

The minimum spanning tree is below. The total cost of the tree is 12.



Question 4:

Question is worth 25 pts
You are given N activities with their star and finish times. Devise a greedy algorithm that determines the maximum number of activities that can be performed by a single person, assuming that a person can only work on a single activity at a time. For simplicity you can assume that start and finish times are integers.

Consider the following example.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12
Start Times	1	3	2	6	6	8	4	4	9	10	2	7
Finish Times	6	5	7	8	9	9	7	7	12	14	6	10

Using the greedy approach to find the maximum number of activities that a single person can accomplish, the job with the lowest finish time is selected from the array. In the example above, A2 is chosen because it has the lowest finish time.

The algorithm will then swap A2 with the last element in the array and shrink its search size to exclude A2. The number of activities is now 1.

	A1	A12	A3	A4	A5	A6	A7	A8	A9	A10	A11	A2
Start Times	1	7	2	6	6	8	4	4	9	10	2	3
Finish Times	6	10	7	8	9	9	7	7	12	14	6	5

The algorithm will continue to search the array for activities with start times that are greater than or equal to the last activity chosen <u>and with the smallest finish times</u>. In the example, an eligible next activity has to have a starting time greater than or equal to 5 (A2's finish time). Therefore, the following are all eligible next activities: A4, A5, A6, A9, A10, and A12. The algorithm will choose A4 as the next activity because it has the lowest finish time among all the eligible next activities.

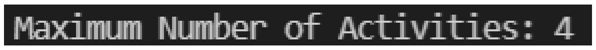
The algorithm is greedy because it selects the most optimal choice from the given search area for each iteration. The final resulting array is below. The value 4 is returned as the maximum number of activities that can be performed by a single person.

	A1	A12	A3	A11	A5	A10	A7	A8	A9	A6	A4	A2
Start Times	1	7	2	2	6	10	4	4	9	8	6	3
Finish Times	6	10	7	6	9	14	7	7	12	9	8	5

The algorithm described above was implemented in C below.

```c
#include <stdio.h>
#include <stdlib.h>

#define N 12 // number of activities

void swap_elements(int** array, int index1, int index2){
   int* array1= *array;
   int temp = array1[index1];
   array1[index1] = array1[index2];
   array1[index2] = temp;
}

//--------------------------- GREEDY ALGORITHM -------------------------------//
// Return the maximum number of activities able to be performed by a single perso
n.
int greedy_algorithm(int* start_times, int* finish_times){
   int i;
   int index_min = 0; // index at lowest finish time
   int count = 1;
   int** starttimes1 = &start_times;
   int** finishtimes1 = &finish_times;

   // find the activity with the lowest finish time
   for(i = 1; i < N; i++){
      if(finish_times[i] < finish_times[index_min]){
         index_min = i;
      }
   }

   // swap it with the last activity in the array
   swap_elements(finishtimes1, index_min, N-1);
   swap_elements(starttimes1, index_min, N-1);

   index_min = -1;
   int activity_found = 0;

   for(i = 0; i < N-count; i++){
      // check for eligibility
      if(finish_times[N-count] <= start_times[i]){
         activity_found = 1;
         if(index_min == -1){
            index_min = i;
         }
         // update optimal finish time
         if(finish_times[i] < finish_times[index_min]){
            index_min = i;
         }
      }

      // end of this cycle
      if(i == N-count-1){
         if(activity_found == 0){
            return(count);
         }

         // increment count and swap
         count++;
         swap_elements(finishtimes1, index_min, N-count);
         swap_elements(starttimes1, index_min, N-count);

         // reset values for next cycle
         i = -1;
         activity_found = 0;
         index_min = -1;
      }
   }
   return(count);
}

int main(){
   int start_times[N] =  {1, 3, 2, 6, 6, 8, 4, 4, 9,  10, 2, 7};
   int finish_times[N] = {6, 5, 7, 8, 9, 9, 7, 7, 12, 14, 6, 10};
   int max_number_activities = greedy_algorithm(start_times, finish_times);
   printf("Maximum Number of Activities: %d\n", max_number_activities);
}
```

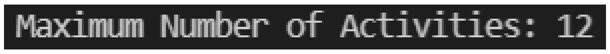
The output of the program is below. The result matches what is expected: {A2, A4, A6, A9} = 4 activities.



```
Maximum Number of Activities: 4
```

Other inputs were tested:

	A1	A12	A3	A11	A5	A10	A7	A8	A9	A6	A4	A2
Start Times	1	3	5	7	9	11	13	15	17	19	21	23
Finish Times	2	4	6	8	10	12	14	16	18	20	22	24

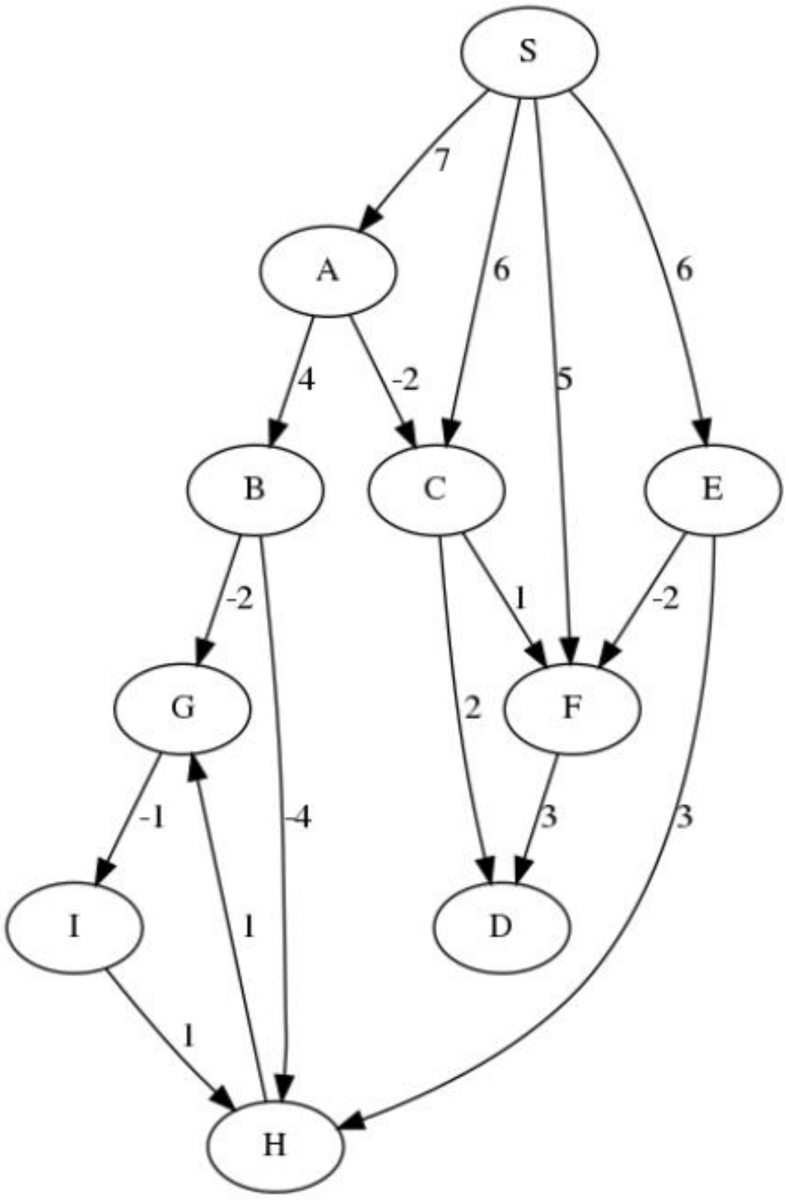


```
Maximum Number of Activities: 12
```

Question 6:

Use the Bellman-Ford algorithm starting at node A on the below graph

- Draw a table showing the intermediate distance values of all the nodes at each iteration of the algorithm
- Show the final shortest-path tree



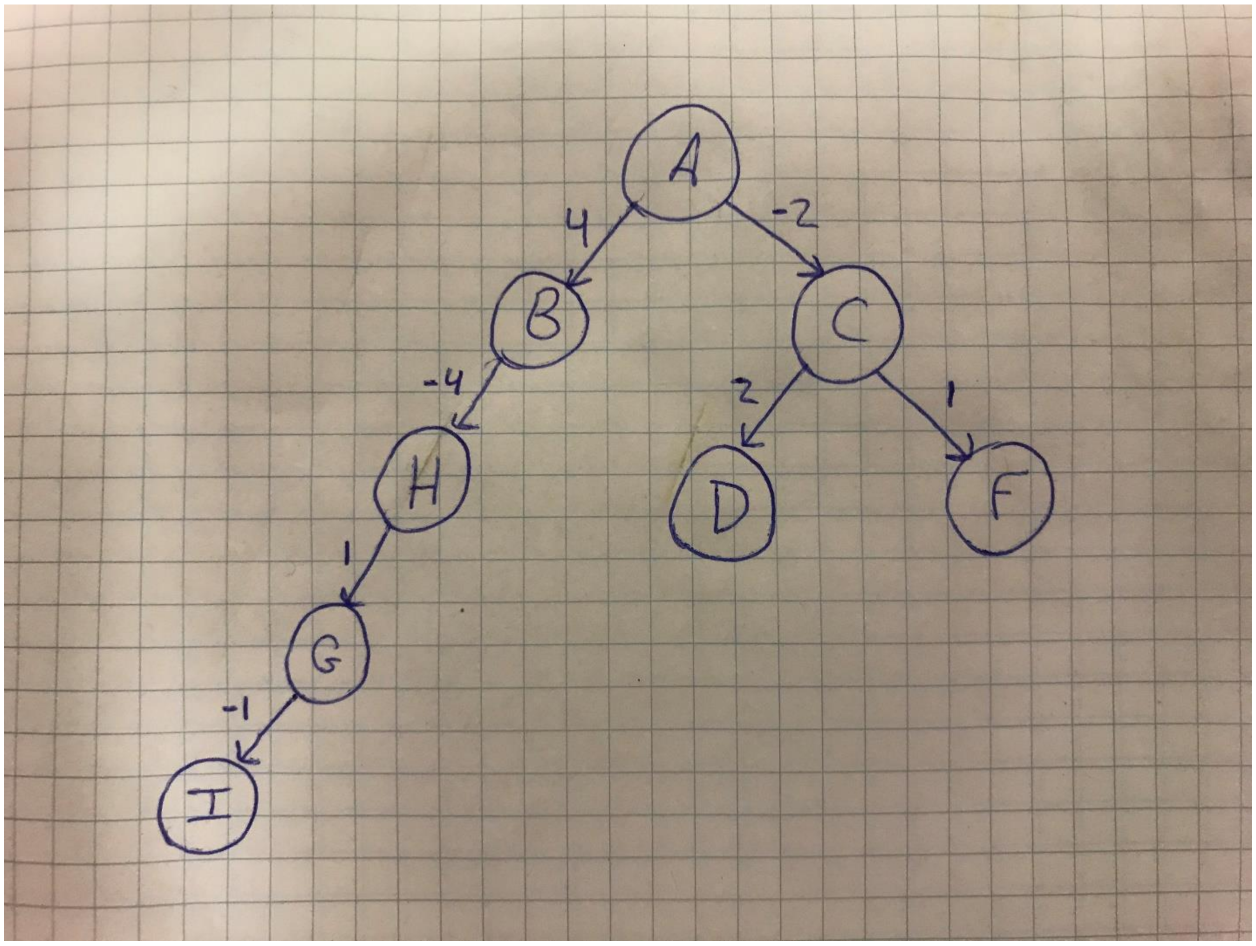
Starting at node A on the graph.

A) The table showing the intermediate distance values of the nodes at each iteration is below.

The algorithm will run for 3 iterations because there is no improvement in the distances between vertices from the second iteration to the third. Before the first iteration, the starting vertex A is initialized with distance 0, and all other vertices are initialized with distance infinity.

Iteration	A	B	C	F	G	H	I	D
1	0	4	-2	-1	1	0	1	0
2	0	4	-2	-1	1	0	0	0
3	0	4	-2	-1	1	0	0	0

B) The final shortest-path tree is below.


A
4
-2
B
C
-4
2
1
H
D
F
1
G
-1
I