CIS 623 – Homework 1 **due Wednesday, January 19**

A Logic Similar to Propositional Logic

Consider the following logic:

Syntax: A wff can be defined using following BNF rules.

- Φ ::= p | (Φ + Φ) | (¬ Φ)
- p is propositional symbol representing a proposition from the set of propositions {p1, p2, p3, …}

Semantics:

- A proposition can map to “true” or “false”
- “¬” is interpreted as “negation” (¬true = false; ¬false = true)
- “+” is interpreted as “exclusive or”
    - true + true = false
    - false + false = false
    - true + false = true
    - false + true = true

Questions-1: Which strings are wffs in this logic?

1. p1

2. (p1+(¬p2))

3. +p1

4. (¬Φ)

5. p1+p2

6. ¬ (p1)

7. (¬ p1

8. (p1+(p2+p3))

Questions-2: Let us assume that unary operator ¬ has higher precedence than + operator, and + is a left associative operator. What are the fully parenthesized versions of the following wffs?

1. ¬ p1 + ¬ p2

2. ¬ ¬ p1 + ¬ p2 + p3

3. ¬ p1 + p2 + ¬ p3

Questions-3: Axioms are the formulas that are regarded to be true in the logic. Can you give three axioms in this logic? (using schemas - generic versions of formulas)

Questions-4: Can you give some inference rules in this logic? (using schemas generic versions of formulas)

Rule 1	Rule 2	Rule 3
$\dfrac{\begin{matrix}\Phi + \Psi \\ \Phi\end{matrix}}{?}$	$\dfrac{\begin{matrix}\Phi + \Psi \\ \Psi\end{matrix}}{?}$	$\dfrac{\begin{matrix}\Phi + \Psi \\ \neg\,\Phi\end{matrix}}{?}$
Rule 4	**Rule 5**	**Rule 6**
$\dfrac{\begin{matrix}\Phi \\ \neg\,\Psi\end{matrix}}{?}$	$\dfrac{\begin{matrix}\neg\,\Phi \\ \Psi\end{matrix}}{?}$	$\dfrac{\begin{matrix}\Phi + \Psi \\ \neg\,\Psi\end{matrix}}{?}$
Rule 7		
$\dfrac{\neg\neg\,\Phi}{?}$		

Questions-5: Can you give a proof p3 from p1, p1+p2, p2+p3 using the inference rules from question 4?

p1, p1+p2, p2+p3 $\vdash$ p3

Questions-6: Is $\neg$ p2 a logical consequence of p1 and p1+p2?

p1, p1+p2 $\models \neg$ p2 ?