

# Homework 3

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CIS-623 STRUCTURED PROGRAMMING & FORMAL METHODS

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Question 1:

Are the following formulas valid, satisfiable, or unsatisfiable?

- $P(a) \rightarrow \text{Ex}P(x)$
- $\text{Ex}P(x) \rightarrow P(a)$
- $\text{Ex}P(x) \vee \text{Ex!}P(x) \rightarrow \text{Ax}P(x)$

$P(a) \rightarrow \text{Ex}P(x)$  is valid based on the inductive Ex rule.

$\text{Ex}P(x) \rightarrow P(a)$  is satisfiable.

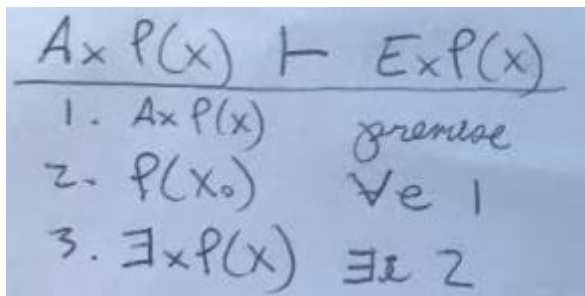
$\text{Ex}P(x) \vee \text{Ex!}P(x) \rightarrow \text{Ax}P(x)$ . The right side of the disjunction is always false. The left side is satisfiable. Therefore, the formula is satisfiable.

Question 2:

For each of the following sequences, give a formal proof if entailment holds.

- $\text{Ax}P(x) \vdash \text{Ex}P(x)$
- $\text{Ax}(P(x) \rightarrow Q(x)), \text{Ax}P(x) \vdash \text{Ax}Q(x)$
- $\text{Ax}(P(x) \rightarrow Q(x)), \text{Ex}P(x) \vdash \text{Ex}Q(x)$
- $\text{!Ex}P(x) \vdash \text{Ax!}P(x)$
- $\text{!Ax}P(x) \vdash \text{Ex!}P(x)$

1. See proof in picture below.



2. See proof in picture below.

$$\begin{array}{l}
 A \times (P(x) \rightarrow Q(x)), A \times P(x) \vdash A \times Q(x) \\
 1. A \times (P(x) \rightarrow Q(x)) \quad \text{premise} \\
 2. A \times P(x) \quad \text{premise} \\
 \boxed{
 \begin{array}{ll}
 3. x_0 & \\
 4. P(x_0) \rightarrow Q(x_0) & \forall e 1 \\
 5. P(x_0) & \forall e 2 \\
 6. Q(x_0) & \rightarrow e 4, 5
 \end{array}
 } \\
 7. A \times Q(x) & \forall i 3-6
 \end{array}$$

3. See proof in picture below.

$$\begin{array}{l}
 A \times (P(x) \rightarrow Q(x)), E \times P(x) \vdash E \times Q(x) \\
 1. A \times (P(x) \rightarrow Q(x)) \quad \text{premise} \\
 2. E \times P(x) \quad \text{premise} \\
 3. P(x_0) \rightarrow Q(x_0) \quad \forall e 1 \\
 \boxed{
 \begin{array}{ll}
 4. P(x_0) & \text{assumption} \\
 5. Q(x_0) & \rightarrow e 3, 4
 \end{array}
 } \\
 6. Q(x_0) & \exists e 2, 4-5 \\
 7. E \times (Q(x)) & \exists i 6
 \end{array}$$

4. See proof in picture below.

$$\begin{array}{l}
 !ExP(x) \vdash Ax!P(x) \\
 1. !ExP(x) \text{ premise} \\
 \boxed{
 \begin{array}{l}
 2. x_0 \\
 3. P(x_0) \text{ assumption} \\
 4. ExP(x) \exists x 3 \\
 5. \perp \neg e 1, 4 \\
 6. !(P(x_0)) \neg i 3-5
 \end{array}
 } \\
 7. Ax(!P(x)) \forall i 2-6
 \end{array}$$

5. See proof in picture below.

$$\begin{array}{l}
 !(AxP(x)) \vdash Ex(!P(x)) \\
 1. !(AxP(x)) \text{ premise} \\
 2. !(P(x_0)) \forall e 1 \\
 3. Ex(!P(x)) \exists i 2
 \end{array}$$