Homework 3

CIS-623 STRUCTURED PROGRAMMING & FORMAL METHODS

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Question 1:

Are the following formulas valid, satisfiable, or unsatisfiable?

- $P(a) \rightarrow \exists x P(x)$
- $\exists x P(x) \rightarrow P(a)$
- $\exists x P(x) \vee \exists x! P(x) \rightarrow \forall x P(x)$

P(a) -> ExP(x) is valid based on the inductive Ex rule.

ExP(x) -> P(a) is satisfiable.

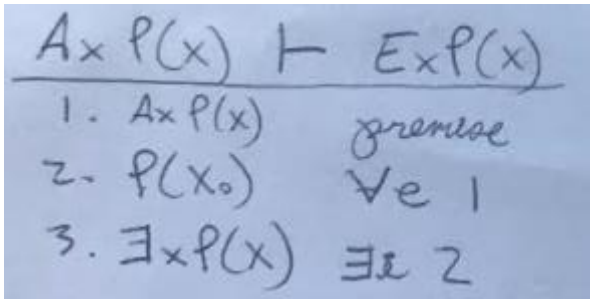
ExP(x) V Ex!P(x) -> AxP(x). The right side of the disjunction is always false. The left side is satisfiable. Therefore, the formula is satisfiable.

Question 2:

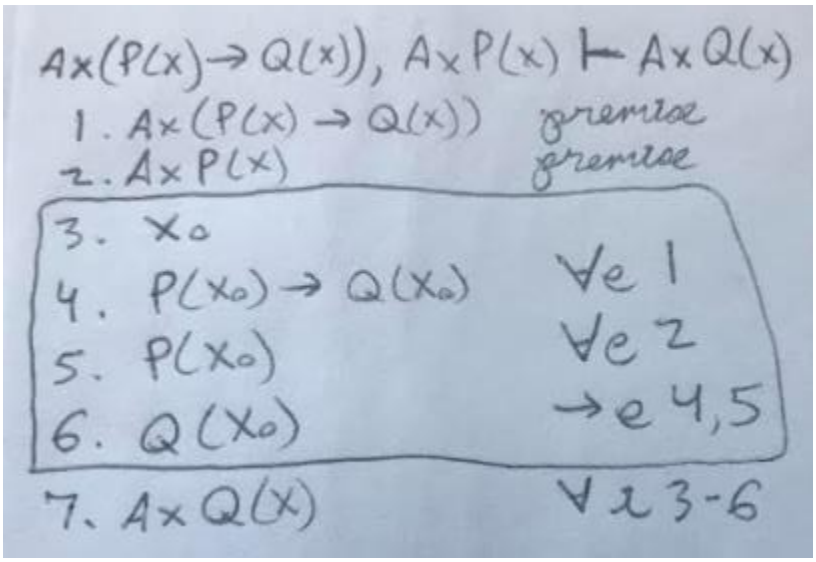
For each of the following sequences, give a formal proof if entailment holds.

- $\forall x P(X) \vdash \exists x P(x)$
- $\forall x(P(x) \rightarrow Q(x)), \forall x P(x) \vdash \forall x Q(x)$
- $\forall x(P(x) \rightarrow Q(x)), \exists x P(x) \vdash \exists x Q(x)$
- $!\exists x P(x) \vdash \forall x !P(X)$
- $!\forall x P(x) \vdash \exists x !P(x)$

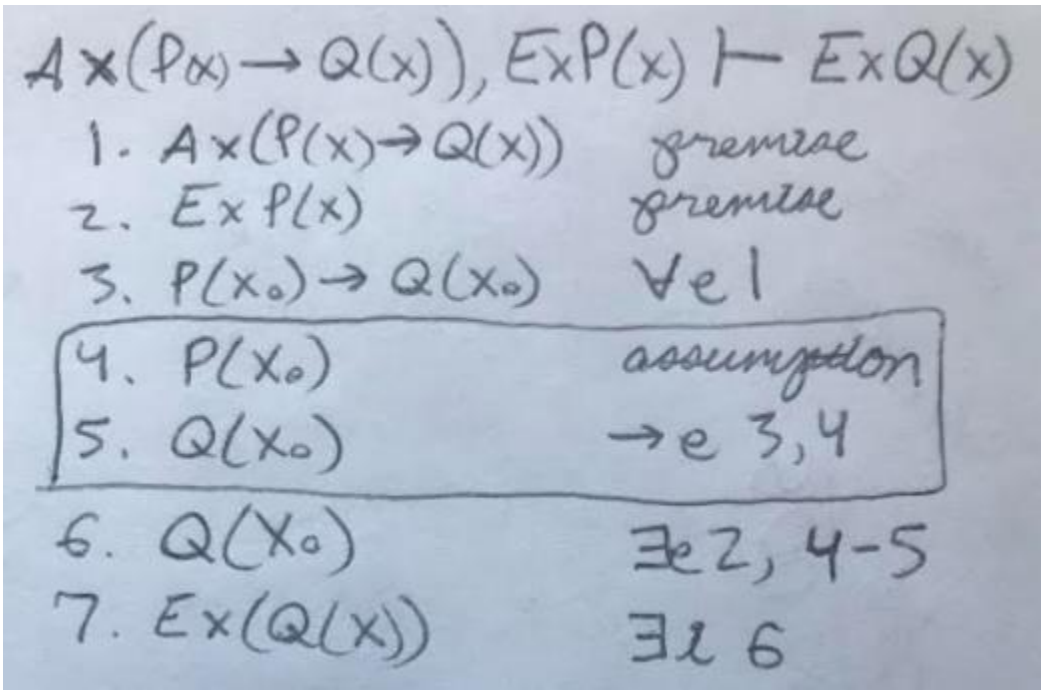
1. See proof in picture below.



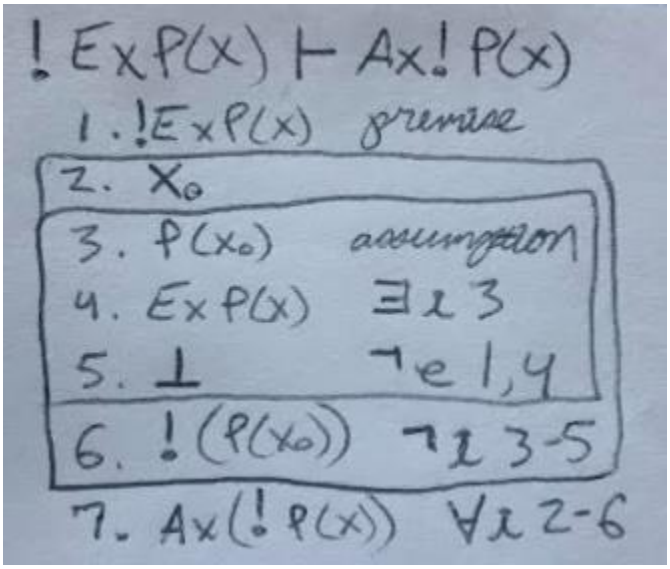
2. See proof in picture below.



3. See proof in picture below.



4. See proof in picture below.



5. See proof in picture below.