# Homework 1

CIS-623 STRUCTURED PROGRAMMING & FORMAL METHODS

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## Question 1:

Consider the following logic:
Syntax: A wff can be defined using the following BNF rules.
• Φ ::= p | (Φ + Φ) | (¬ Φ)
• p is propositional symbol representing a proposition from the set of propositions {p1, p2, p3, …}

Semantics:
• A proposition can map to "true" or "false"
• "¬" is interpreted as "negation" (¬true = false; ¬false = true)
• "+" is interpreted as "exclusive or"
o true + true = false
o false + false = false
o true + false = true
o false + true = true

Which strings are well-formed formulas (wff's) in this logic?

1. $p_1$ is a well-formed formula according to the BNF rules.
2. $(p_1 + (\neg p_2))$ is a wff.
3. $+p_1$ is not a wff because the exclusive or operator is defined as a binary operator in the BNF rules. The proposition on the left-hand side is missing.
4. $(\neg\Phi)$ is a wff.
5. $p_1 + p_2$ is not a wff because there is a missing set of parentheses.
6. $\neg(p_1)$ is not a wff. The negation symbol should be inside the parentheses.
7. $\neg(p_1$ is not a wff. There is a missing closed parenthesis.
8. $\left(p_1 + (p_2 + p_3)\right)$ is a wff.

## Question 2:

Let us assume that unary operator ¬ has higher precedence than + operator, and + is a left associative operator. What are the fully parenthesized versions of the following wffs?

1. $\left((\neg p_1) + (\neg p_2)\right)$
2. $\left(\left((\neg(\neg p_1)) + (\neg p_2)\right) + p_3\right)$
3. $\left(\left((\neg p_1) + p_2\right) + (\neg p_3)\right)$

## Question 3:

Axioms are the formulas that are regarded to be true in the logic. Can you give three axioms in this logic? (using schemas - generic versions of formulas)

The following three axioms are formulas that are valid (always true).

1. $\left((\neg p_1) + p_1\right)$

2. $\left(\left((\neg p_1) + p_1\right) + (p_1 + p_1)\right)$
3. $\left(\left(\left((\neg p_1) + p_1\right) + (p_1 + p_1)\right) + (p_1 + p_1)\right)$

## Question 4:

Can you give some inference rules in this logic? (using schemas generic versions of formulas)

Rule 1: $(\Phi + \Psi), \Phi \vdash (\neg\Psi)$

Rule 2: $(\Phi + \Psi), \Psi \vdash (\neg\Phi)$

Rule 3: $(\Phi + \Psi), \neg\Phi \vdash \Psi$

Rule 4: $\Phi, \neg\Psi \vdash (\Phi + \Psi)$

Rule 5: $\neg\Phi, \Psi \vdash (\Phi + \Psi)$

Rule 6: $(\Phi + \Psi), \neg\Psi \vdash \Phi$

Rule 7: $\neg\neg\Phi \vdash \Phi$

## Question 5:

Can you give a proof p3 from p1, p1+p2, p2+p3 using the inference rules from question 4?

$p_1, (p_1 + p_2), (p_2 + p_3) \vdash p_3$

1. $p_1$ (premise)
2. $(p_1 + p_2)$ (premise)
3. $(p_2 + p_3)$ (premise)
4. $(\neg p_2)$ $[Rule\ 1], 1, 2$
5. $p_3$ $[Rule\ 3], 3, 4$

## Question 6:

Is $\neg p_2$ a logical consequence of $p_1$ and $p_1 + p_2$?

$p_1, (p_1 + p_2) \vDash \neg p_2$ ?

| Model Number | $p_1$ | $p_2$ | $p_1 + p_2$ |
|---|---|---|---|
| 1 | F | F | F |
| 2 | F | T | T |
| 3 | T | F | T |
| 4 | T | T | F |

From the above truth table, only Model 3 satisfies the given premises that $p_1$ and $p_1 + p_2$ are both true. In this model, $p_2$ is false. Therefore, $\neg p_2$ is true.

$\neg p_2$ is a logical consequence of $p_1$ and $p_1 + p_2$.

$p_1, (p_1 + p_2) \vDash \neg p_2$