

Task 1. FoxTrot Maths

Available marks: 12

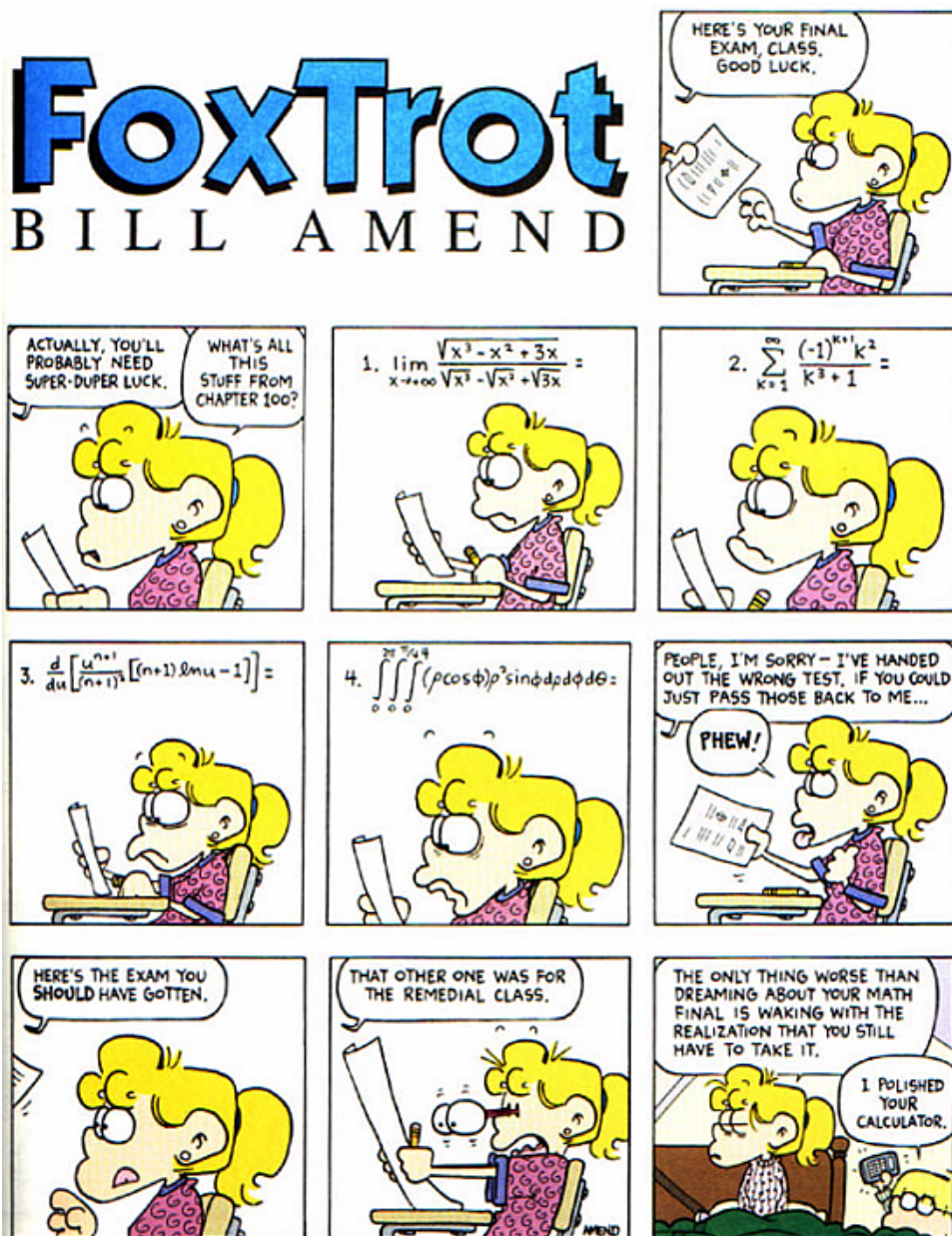
The comic strip shown here was first published on 2 June 1996. The artist, Bill Amend, posed a number of interesting mathematical problems.

Problem 1 (7 marks)

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^3 - x^2 + 3x}}{\sqrt{x^3} - \sqrt{x^2} + \sqrt{3x}}$$

The limit in the first problem is exactly 1, but for any finite, real value for x the formula calculates a slightly larger value. Using bisection* or a similar strategy, determine for what value of x (to 12 significant figures) the formula has a value as close as possible to $100/99 = 1.01010101\ldots$ (it's less than a million).

In case your floating point processor and sqrt library function are feeling a bit grumpy today, if your answer differs from ours you must display the formula result to 15 significant figures.



Ref: Amend, B (1998). *Camp FoxTrot*. Republished from Wolfram Mathworld, <http://mathworld.wolfram.com/Limit.html>

Problem 2 (5 marks)

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k^3 + 1}$$

The series in the second problem converges extremely slowly. Write a program that approximates an answer to the series, using a sufficient number of terms that successive evaluations agree to 6 decimal places.

Again, display the last two sums to demonstrate the convergence.

* Bisection maintains two values either side of an answer, evaluates the midpoint and moves one of the end-points to the middle accordingly.