Task 2. Musical Chairs

Available marks: 17

The Progcomp house band consists of saxophonists (S), violinists (V) and drummers (D). There are *N* musicians of each kind. At any time exactly *N* musicians are playing, and are seated in a single row of chairs. At regular intervals one of the chairs (we'll call it the *musical chair*) is nominated and the musician seated there is replaced by another playing a different instrument.

A musician who plays instrument *A* can replace a musician who plays instrument *B*, only if the following rule applies.

	S	V	V	D
✓	S	**D**	V	D
✓	**V**	D	V	D
✗	V	D	**S**	D

> All musicians seated to the left* of the musical chair play an instrument that is neither *A* nor *B*.
> * *left* is from the audience's viewpoint.

For example, if the musician in the third chair from the left is a saxophonist and is replaced by a violinist, the first two chairs must be occupied by drummers. Two legal transitions are shown above, followed by an illegal one (the V being replaced is not the leftmost one).

The band has a signature tune that starts off entirely with violins and ends entirely with saxophones. Using the notation S, V and D for each instrument type, write a program that displays the shortest sequence of legal moves that produces this outcome for any *N*. Each step in the sequence lists the instruments from (our) left to right on one line, separated by a space. The near-trivial solution for *N*=2 would be shown as

```
V V
D V
D S
S S
```

If you're interested, the band recently went on a holiday to Hanoi, where they discovered an elegant recursive solution to this problem at the famous “Triple Towers” tourist destination.

Assessment

The judges will ask you to run your solution for one or two values of *N*, no greater than 10 as the solution has 2^N values.

If you need help with this task and are prepared to pay for it, a detailed **hint** is available for a modest fee of 5 marks, deducted in advance.

References

This variation is derived from a version using colours by Fernando Rodriguez Villegas, from the Department of Mathematics at the University of Texas, Austin, also discussed at cut-the-knot.org . Chair image by Anna Kvach.