STA240 Final Project

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Scenario 1

Customer Arrival

Poisson process (rate = λ)

- T_k: Arrival time of the kth customer
- W_k: Time between the $k-1$th arrival and the kth arrival

$$W_k = T_k - T_{k-1}.$$

$W_k \sim Pois(\lambda)$

where $\lambda = 5$ customers per hour

Service Time

$S_k \sim Exp(\lambda)$

where $\lambda = 6$ customers per hour, so the average customer needs to wait 1/6 hours = 10 minutes.

Arrival Times

Open at 10am, close at 10pm, 5 customers arrive per hour on average

(Expressed in minutes after opening)

```
  [1]   7.740219   9.999120  10.609184  18.872636  24.020133  28.291100
  [7]  30.856584  43.897808  45.305376  49.139653  52.205852  52.291225
 [13]  53.193630  59.614829  64.818087  69.834038  78.455901  80.939994
 [19]  97.775911  99.660970  99.950087 101.768512 103.503894 103.962354
 [25] 105.383493 123.241789 127.660131 130.540695 133.787696 146.175237
 [31] 167.355173 175.789029 177.715756 189.252091 194.639219 197.065984
 [37] 197.402851 211.522654 213.494293 221.653137 222.570392 231.387538
 [43] 234.537606 235.205883 241.370710 252.514780 255.907988 274.251801
 [49] 274.489324 274.836889 275.364864 282.989124 294.067864 294.579267
 [55] 295.966617 299.893453 301.984833 315.940180 326.696706 326.780675
 [61] 337.774267 339.941354 353.951355 357.645249 401.234376 408.369909
 [67] 409.867689 425.006873 427.725484 437.115205 439.385691 440.060948
 [73] 451.028466 459.809412 482.520053 485.808313 488.697602 490.941677
 [79] 493.587513 496.969160 502.265192 506.555447 508.921832 516.911897
 [85] 519.271801 522.395077 522.760927 527.613383 537.272545 540.065662
 [91] 543.119006 547.080068 553.244399 563.228063 566.512064 575.943943
 [97] 579.953130 580.925464 595.961832 598.243164 600.437756 618.836567
[103] 621.836564 622.590031 625.575909 626.773011 636.658736 637.725852
[109] 644.539666 647.120297 647.830928 658.401464 663.362935 667.576699
[115] 676.071382 690.265893 695.575545 704.400624 710.291872 710.471580
```

Arrival Times Analysis

In this simulation, the number of customers that will be arriving within the operating hours is 120, with the first customer arriving 7 minutes after opening and the last customer arriving 10 minutes before closing

Serving Times

The average customer takes 1/6 hours, or 10 minutes to serve. So $= 6$

(The number of minutes taken by each customer after sitting down in the restaurant)

```
[1] 6
```

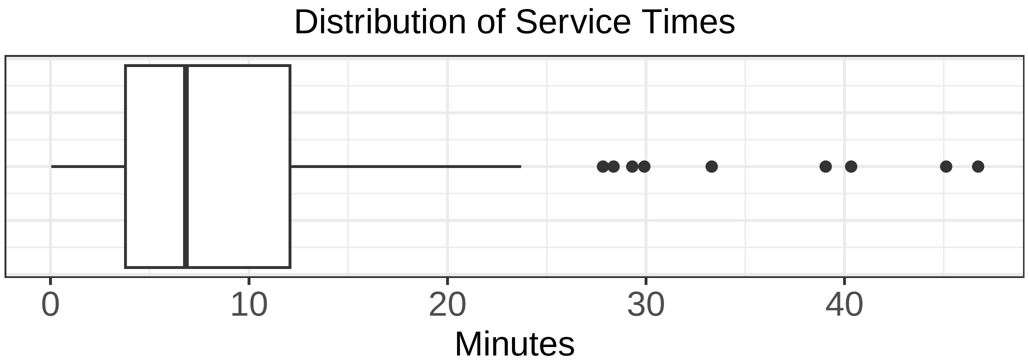
Time of the day with arrival time

```
  [1] "10:08" "10:10" "10:11" "10:19" "10:24" "10:28" "10:31" "10:44" "10:45"
 [10] "10:49" "10:52" "10:52" "10:53" "11:00" "11:05" "11:10" "11:18" "11:21"
 [19] "11:38" "11:40" "11:40" "11:42" "11:44" "11:44" "11:45" "12:03" "12:08"
 [28] "12:11" "12:14" "12:26" "12:47" "12:56" "12:58" "13:09" "13:15" "13:17"
 [37] "13:17" "13:32" "13:33" "13:42" "13:43" "13:51" "13:55" "13:55" "14:01"
 [46] "14:13" "14:16" "14:34" "14:34" "14:35" "14:35" "14:43" "14:54" "14:55"
 [55] "14:56" "15:00" "15:02" "15:16" "15:27" "15:27" "15:38" "15:40" "15:54"
 [64] "15:58" "16:41" "16:48" "16:50" "17:05" "17:08" "17:17" "17:19" "17:20"
 [73] "17:31" "17:40" "18:03" "18:06" "18:09" "18:11" "18:14" "18:17" "18:22"
 [82] "18:27" "18:29" "18:37" "18:39" "18:42" "18:43" "18:48" "18:57" "19:00"
 [91] "19:03" "19:07" "19:13" "19:23" "19:27" "19:36" "19:40" "19:41" "19:56"
[100] "19:58" "20:00" "20:19" "20:22" "20:23" "20:26" "20:27" "20:37" "20:38"
[109] "20:45" "20:47" "20:48" "20:58" "21:03" "21:08" "21:16" "21:30" "21:36"
[118] "21:44" "21:50" "21:50"
```

Waiting Times

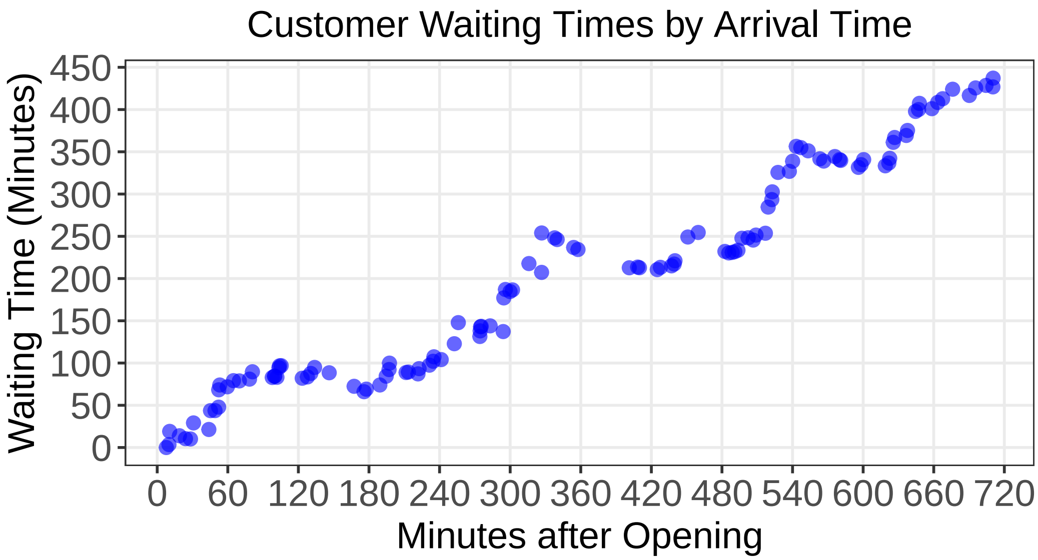
```
  customer arrival_time service_length service_start service_end waiting_time
1        1     7.740219       5.753559      7.740219    13.49378     0.000000
2        2     9.999120      16.216943     13.493779    29.71072     3.494659
3        3    10.609184       3.071809     29.710722    32.78253    19.101537
4        4    18.872636       1.719255     32.782531    34.50179    13.909895
5        5    24.020133       3.919356     34.501786    38.42114    10.481653
  time_of_day
1       10:08
2       10:10
3       10:11
4       10:19
5       10:24
```

Serving and Waiting Times Analysis



The average service time is 10 minutes, with the data skewed right, consistent with an exponential distribution. This indicates that service times tend to lower.

Waiting times tends to be slightly right-skewed and on average, the waiting time is 203 minutes.



Scenario 2

Assumptions:

1. 5 dining tables and L chefs with operating hours 10am - 10pm. We choose here that L = 2
2. each table only seats one customer
3. service time modeled by an exponential distribution with rate S = 3L, so that the more chefs there are, the faster the service times become
4. 10 customers arrive every hour

Restaurant Profits

Assumptions:

1. each customer spends \$50 per meal (customers who are still in the queue when the restaurant closes won't pay)
2. each chef earns a wage of \$40 per hour (paid for the entire duration of the restaurant's operating hours)
3. Each table cost \$80 per day (extra service cost, rent, etc.)

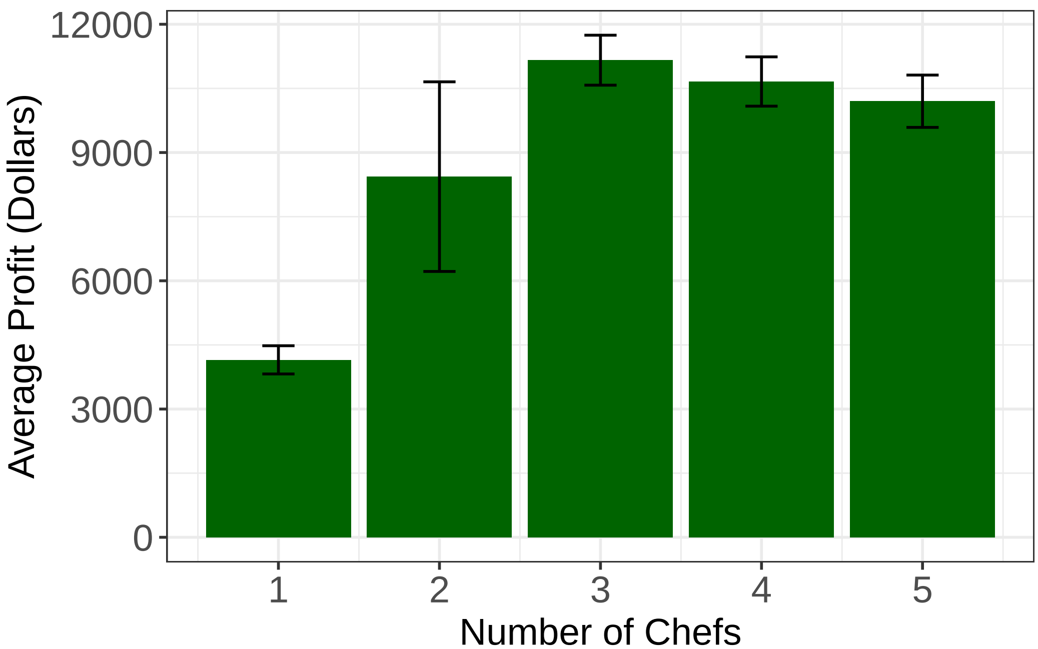
4. For customers who waited more than 30 minutes, they earn the restaurant half the amount of customers who didn't.

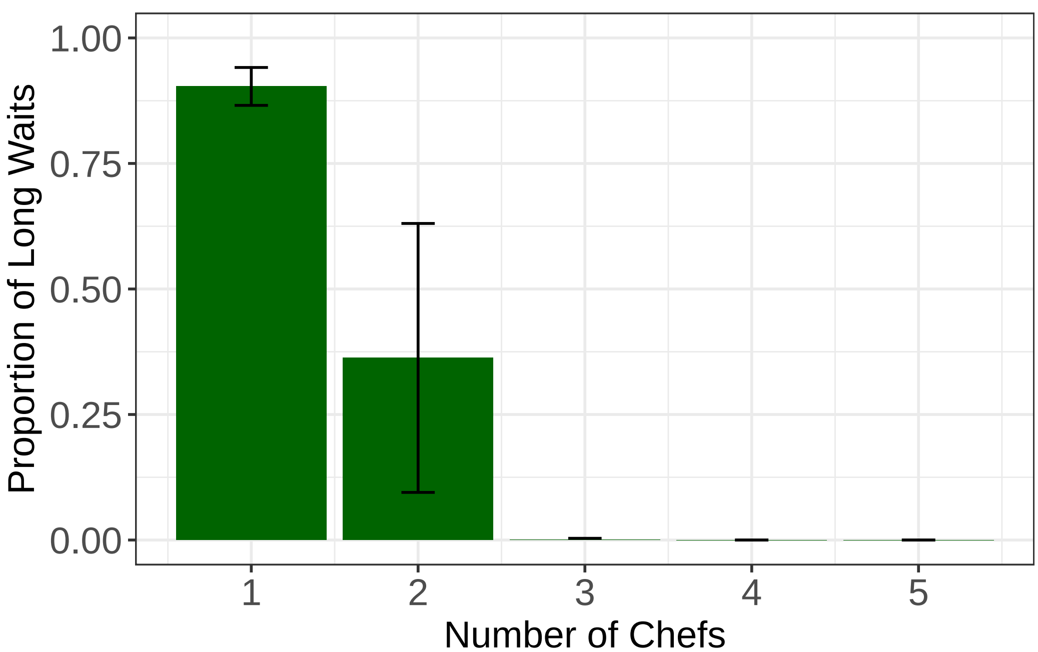
Maximizing Profits

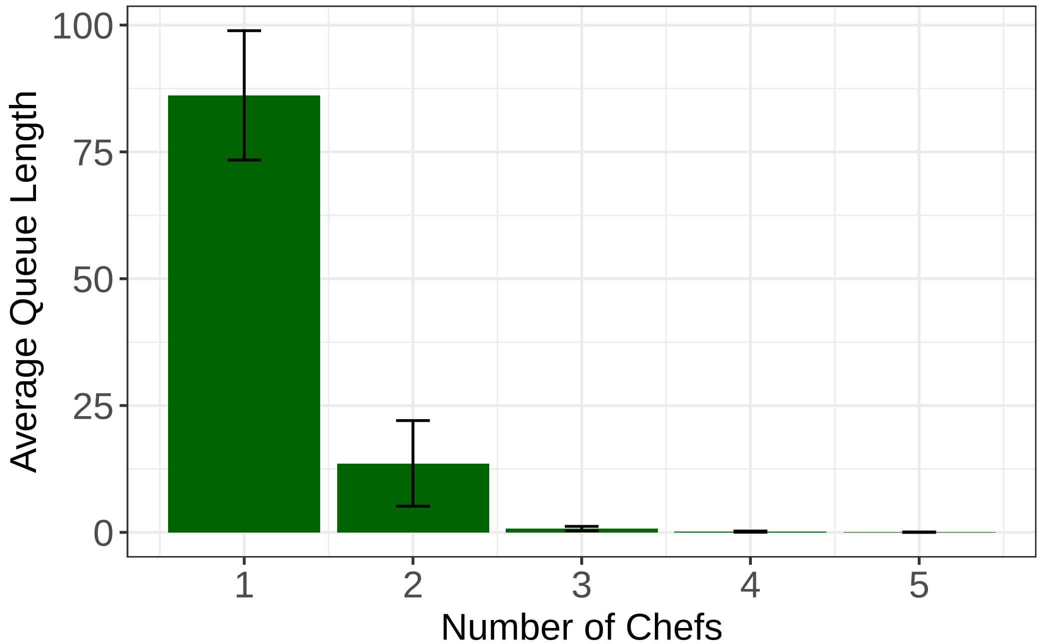
With 5 tables, 24 customers arriving per hour, and these dollar amounts, how many chefs should we hire? We will run our simulation 100 times with 1 to 5 chefs on staff, to see which will maximize the expected profit.

	total_customers	profit	num_chefs	num_tables	avg_waiting_time	long_waits
1	354	5965	2	5	59.81920904	231
2	357	10930	4	5	0.18207283	0
3	352	11160	3	5	1.40340909	0
4	358	10500	5	5	0.05865922	0
5	341	9650	5	5	0.02932551	0
6	352	10200	5	5	0.01420455	0
7	360	4340	2	5	48.98888889	308
8	347	4145	1	5	158.05187320	309

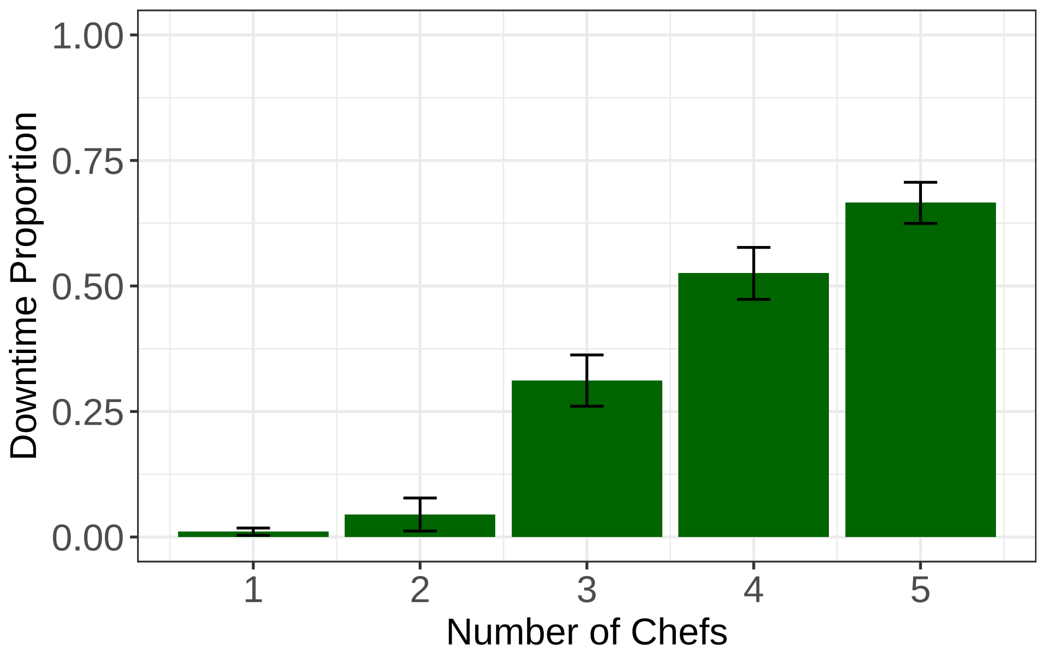
	avg_queue_length	max_queue_length	avg_tables_occupied	downtime_proportion
1	29.411111111	63	4.576389	0.044444444
2	0.090277778	3	2.597222	0.505555556
3	0.686111111	10	3.170833	0.344444444
4	0.029166667	2	2.055556	0.675000000
5	0.013888889	2	1.827778	0.715277778
6	0.006944444	1	1.854167	0.715277778
7	24.494444444	42	4.813889	0.008333333
8	76.172222222	153	4.912500	0.008333333


12000
9000
6000
3000
0
Average Profit (Dollars)
1
2
3
4
5
Number of Chefs


1.00
0.75
0.50
0.25
0.00
Proportion of Long Waits
1
2
3
4
5
Number of Chefs



100
75
50
25
0
Average Queue Length
1
2
3
4
5
Number of Chefs



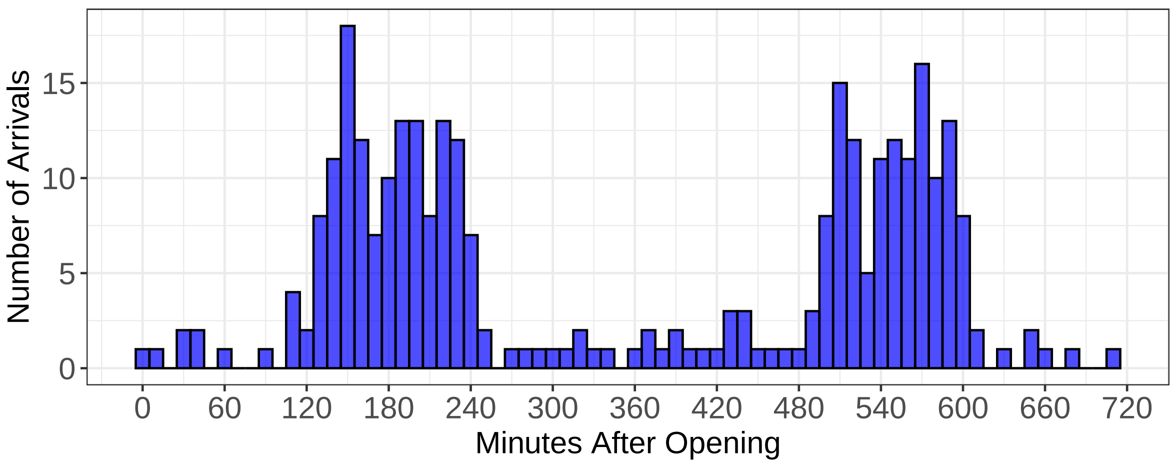
1.00
0.75
0.50
0.25
0.00
Downtime Proportion
1
2
3
4
5
Number of Chefs

Scenario 3

To make the simulation more realistic, we have a third scenario.

Assumptions: 1. Open at 10am, close at 10pm 2. From 12pm to 2pm and 6pm to 8pm, 60 customers arrive every hour. Otherwise, 6 arrive every hour. 3. Instead of simulating service times with Exp( ) where = 3 times the number of chefs, we do = ln(chefs + 1), so that additional chefs beyond 2 make more of an impact. 4. Each customer will sit for a minimum of 45 minutes. This flat value will be added to the simulated service time, and is unaffected by staffing. 5. In the profit calculation, there is a cost of adding additional tables (which are now variable), which is $40 per table. 6. Chefs still cost $40 per hour to hire, and each customer earns $50.

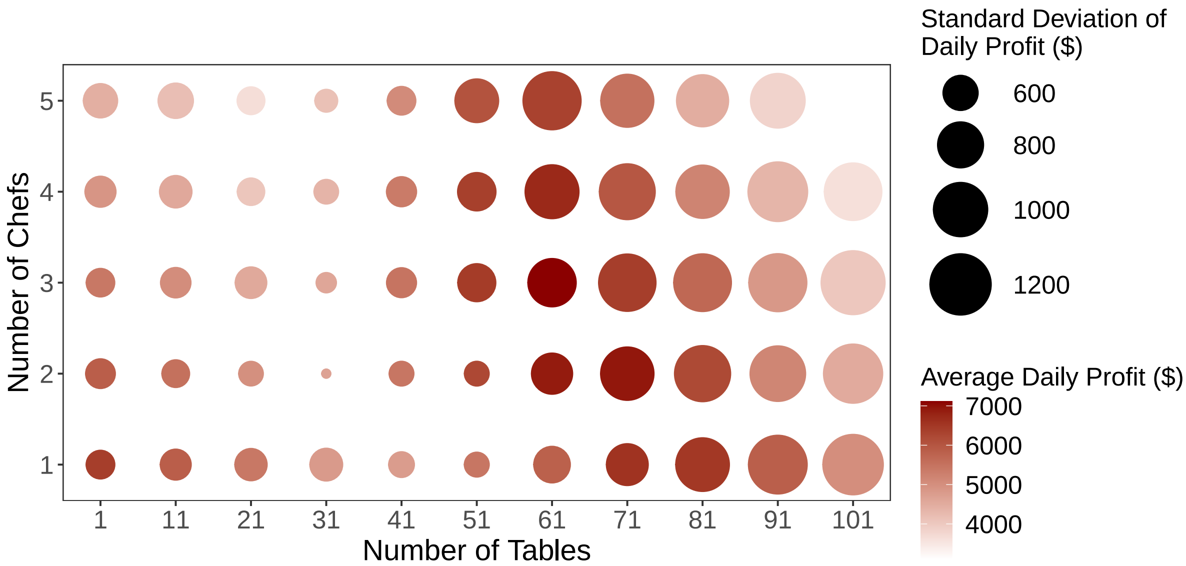
Arrival Times



Maximizing Profits

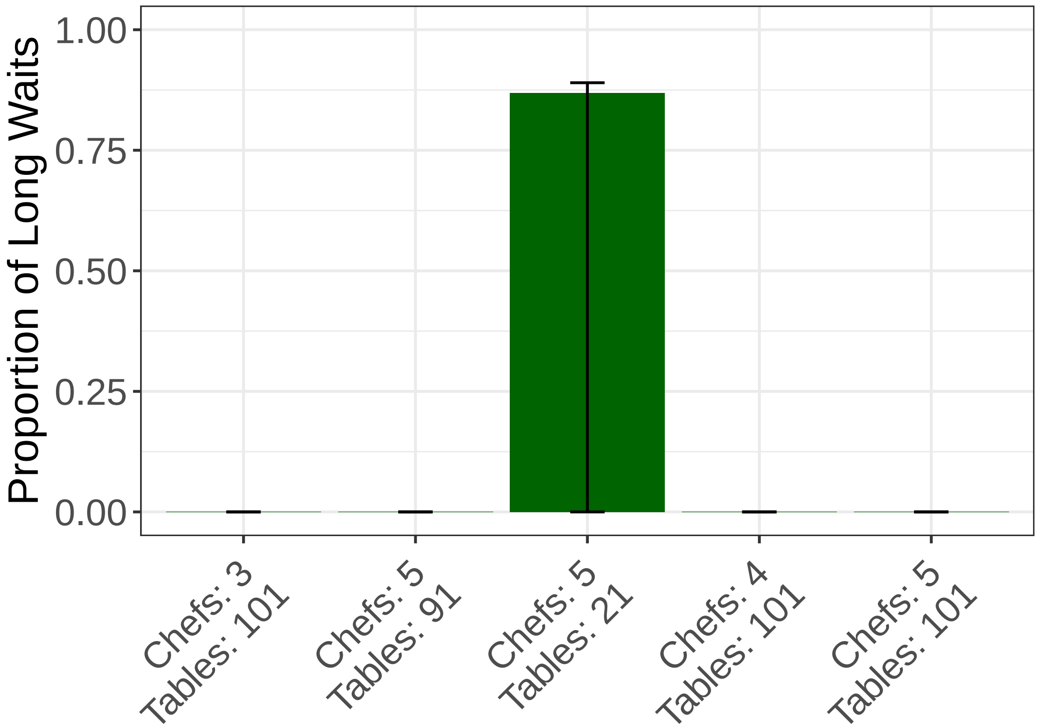
Under this scenario, how can we maximize profits?

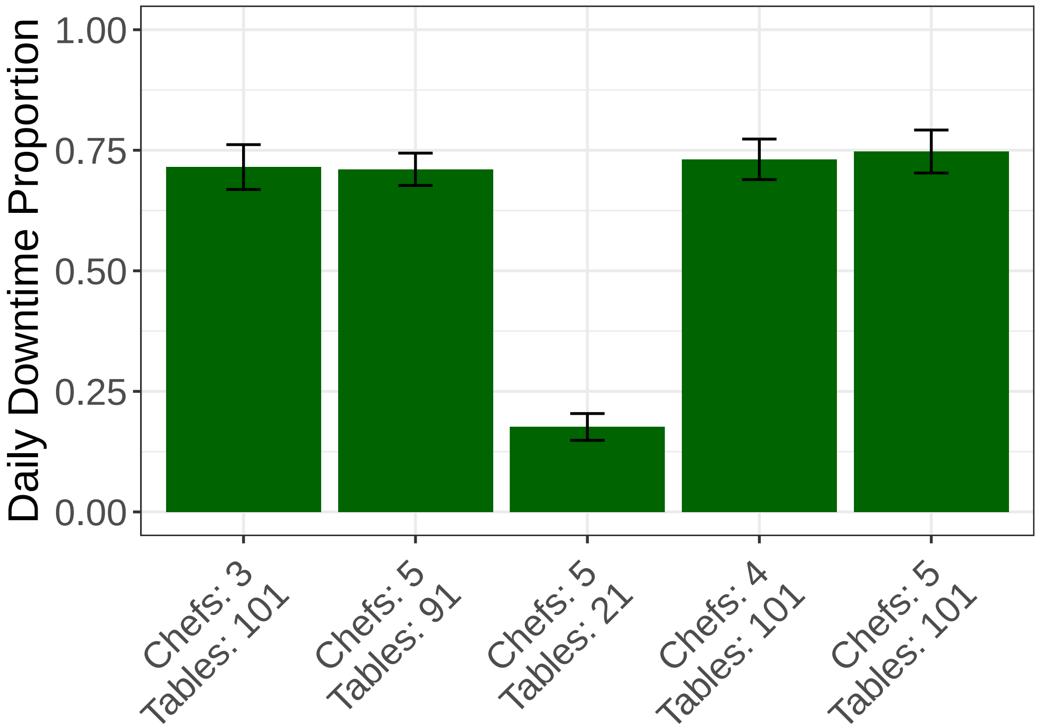
```
`summarise()` has grouped output by 'num_chefs'. You can override using the
`.groups` argument.
```

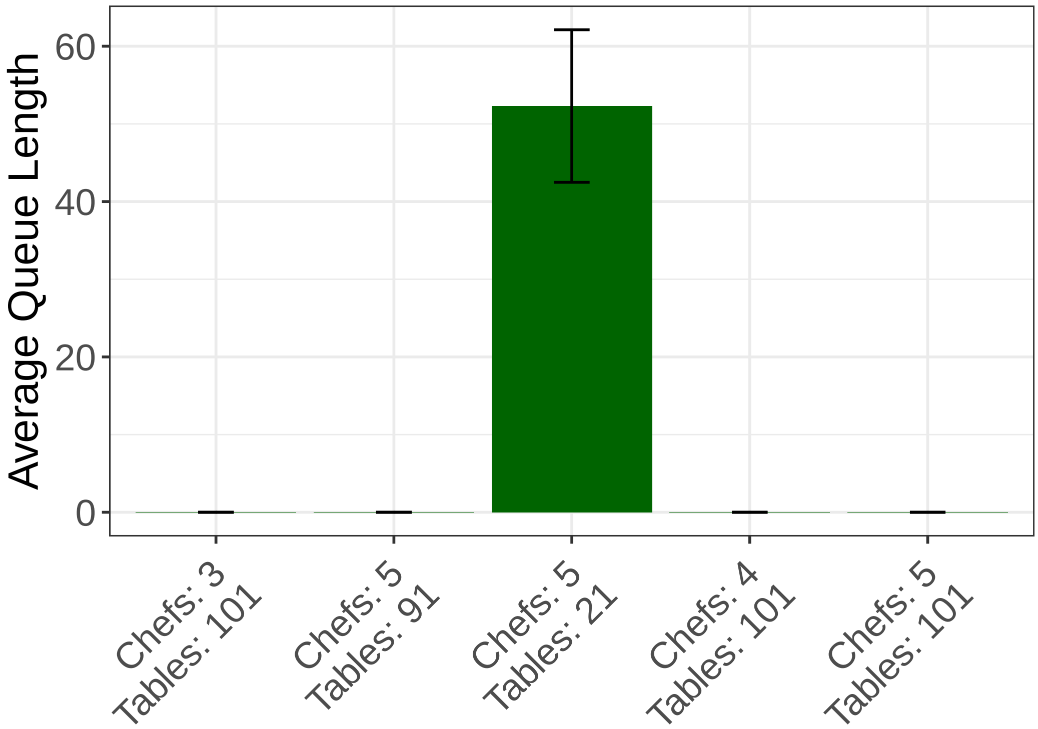



```
`summarise()` has grouped output by 'num_chefs'. You can override using the
`.groups` argument.
```

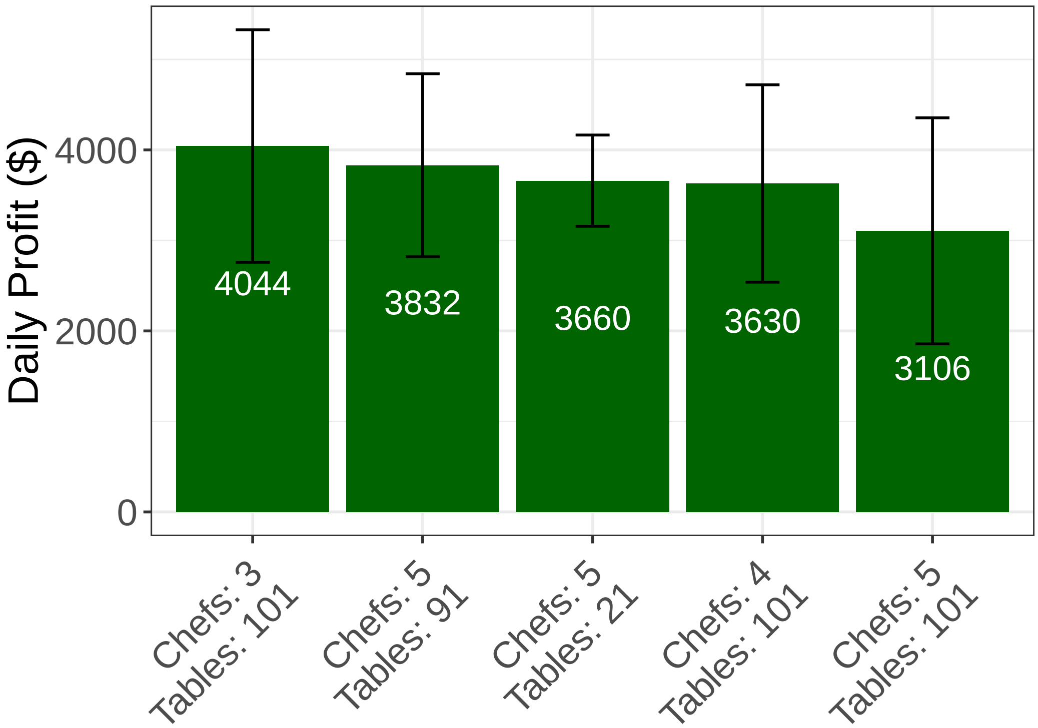
```
# A tibble: 5 x 19
  num_chefs num_tables mean_profit sd_profit mean_avg_waiting_time
      <dbl>      <dbl>       <dbl>     <dbl>                 <dbl>
1         5        101       3106      1249.                     0
2         4        101       3630.     1091.                     0
3         5         21       3660.      504.                   137.
4         5         91       3832.     1011.                     0
5         3        101       4044.     1285.                     0
# i 14 more variables: sd_avg_waiting_time <dbl>, mean_long_waits <dbl>,
#   sd_long_waits <dbl>, mean_avg_queue_length <dbl>,
#   sd_avg_queue_length <dbl>, mean_max_queue_length <dbl>,
#   sd_max_queue_length <dbl>, mean_avg_tables_occupied <dbl>,
#   sd_avg_tables_occupied <dbl>, mean_downtime_proportion <dbl>,
#   sd_downtime_propotion <dbl>, mean_long_waits_proportion <dbl>,
#   sd_long_waits_proportion <dbl>, description <chr>
```


Proportion of Long Waits
1.00
0.75
0.50
0.25
0.00
Chefs: 3
Tables: 101
Chefs: 5
Tables: 91
Chefs: 5
Tables: 21
Chefs: 4
Tables: 101
Chefs: 5
Tables: 101


Daily Downtime Proportion
1.00
0.75
0.50
0.25
0.00
Chefs: 3
Tables: 101
Chefs: 5
Tables: 91
Chefs: 5
Tables: 21
Chefs: 4
Tables: 101
Chefs: 5
Tables: 101



Average Queue Length
60
40
20
0
Chefs: 3
Tables: 101
Chefs: 5
Tables: 91
Chefs: 5
Tables: 21
Chefs: 4
Tables: 101
Chefs: 5
Tables: 101


Daily Profit ($)
0
2000
4000
4044
3832
3660
3630
3106
Chefs: 3
Tables: 101
Chefs: 5
Tables: 91
Chefs: 5
Tables: 21
Chefs: 4
Tables: 101
Chefs: 5
Tables: 101