STA240 Final Project

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Introduction

In 2024 alone, the restaurant industry was predicted to reach $1 trillion in sales; however, with rising costs and increased competition, owners must find innovative ways to keep up profit (National Restaurant Association, 2024). With increased technology, many restaurants are turning to data to inform business decisions. Therefore, in this project, we aimed to use statistical modeling to guide our recommendation on the optimal number of chefs and dining tables that maximize profit and consumer satisfaction. To provide a comprehensive analysis, we modeled the behavior of arriving customers and evaluated key performance metrics such as waiting times, downtime, and profits in 3 scenarios.

Performance Metrics

To evaluate restaurant operation efficiency and consumer satisfaction, we measured and analyzed the following performance metrics:

1. Daily profit: the amount the restaurant earns in a day, defined by total revenue minus cost of tables and chefs, with a 50% revenue punishment for customers who waited over 30 minutes as they tend to tip less after bad consumer experience. A good profit is essential for the restaurant's financial health and sustainability.
2. Restaurant downtime: the proportion of time where less than 50% tables are occupied. Although we aim to maximize the efficiency of operation, downtime is necessary for cleaning, maintenance, and rest to ensure the quality of service.
3. Long wait: the proportion of customers who waited more than 30 minutes. Overall, customer satisfaction improves when there are fewer people having long waits.
4. Queue length: number of people waiting for a table. The first impression of customers about the restaurant experience. A long line may discourage or encourage customers from coming.

Mathematical Model

In the following models, we chose to model arrival times using a Poisson distribution and service times using an exponential distribution. For arrival times, which are independent of each other, a Poisson distribution represents a constant average rate of customers arriving throughout the day. To model service times, we use an exponential distribution because it has the memoryless property, where the probability of a future event is independent of the time that has already passed.

- W_k: Time between the $k-1^{\text{th}}$ arrival and the k^{th} arrival.
- T_k: Arrival time of the k^{th} customer. $T_k = T_{k-1} + W_k$
- S_k: total service time (ordering, cooking, and eating) once the customer arrives.

Scenario 1 Description:

We started with a simple configuration of one dining table and one chef, operating from 10am - 10pm. Customer arrival will be modeled as a Poisson process with a rate of 5 per hour. Arrival time is modeled by $W_k \sim Exp\ (\lambda = 5)$. Service time is modeled by $S_k \sim Exp\ (\lambda = 6)$.

First, to model arrival time and service time, we utilized two functions respectively. The arrival time function simulates the arrival time of customers over the specified parameters of time (12 hours of operation) and the given rate (customers per minute). Within the function, we generate the interarrival times (W_k) using an exponential distribution, utilize a for loop to calculate the cumulative sum of these time gaps (T_k) to obtain the arrival times. The service times are generated using an exponential distribution (S_k) based on the parameters of service rate.

Second, to model waiting times, we used a loop for each arriving customer. If the previous customer leaves before the next arrival, service begins immediately; otherwise, it starts when the previous customer leaves. Each customer's service end time is the service start time plus the service duration, and waiting time is the service start time minus the arrival time.

Scenario 1 Results and Analysis

Given its simplicity, we used scenario 1 to gain a general understanding of the distribution of waiting times and conducted a stress test to assess the reliability of our waiting time simulation.

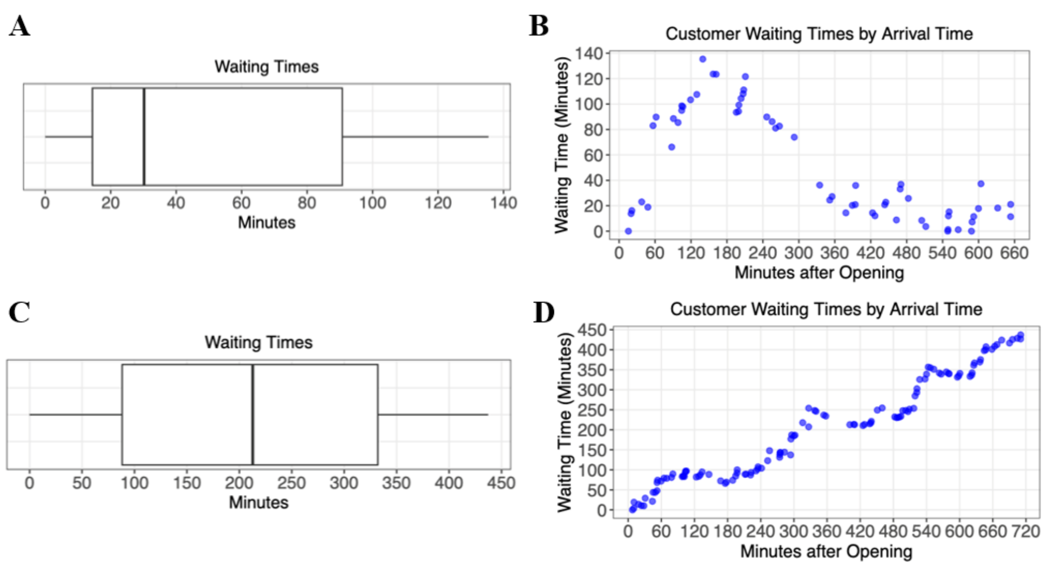

Figure 1. Waiting time analysis with 1 chef and 1 table. Waiting times when (A) arrival rate $\lambda = 5$ and (C) $\lambda = 10$ under stress test. Waiting time through one day when (B) arrival rate $\lambda = 5$ and (D) $\lambda = 10$.

With customer arrival rate $\lambda = 5$ and service rate $\lambda = 6$, the median waiting time is 30 minutes. Overall, the waiting times boxplot shows a positive skew (Fig. 1A). The variable waiting times throughout the day (Fig. 1B) are a result of random periods of more frequent arrivals. We then conducted a stress test by doubling the customer arrival rate to $\lambda = 10$ while keeping the service rate the same. As expected, the median waiting time increased to 213 minutes with maximum waiting time of 437 minutes (Fig. 1C). Since the arrival time is greater than the service rate, the waiting time accumulates and climbs to its maximum at the end of the day (Fig. 1D). Overall, the stress test results suggest that our waiting time simulation is reliable and consistent with real-world expectations.

Scenario 2 Description:

We then investigated the scenario with 5 dining tables and variable numbers of chefs, L, with a service rate of λ = 3L. The restaurant operates from 10am - 10pm, and customer arrivals will be modeled as a Poisson process with a rate of 30 per hour. Service time is modeled by $S_k \sim Exp\ (\lambda = 3L)$. For the profit calculation, we assumed that chefs are paid $40/hour, each customer brings in $50 of revenue, and the restaurant has a daily fixed cost of $5000. Finally, any customer that was in queue for more than 30 minutes contributes half the revenue ($25) of a customer that was more promptly seated.

With the increased complexity of having multiple tables, we used a function to simulate individual days of the restaurant's operation by iterating through every minute. Given inputs of arrival times for all customers and service times for each, the function keeps track of arriving customers and the status of each table in the restaurant. It outputs the queue length at each minute, duration of each person's wait in the queue, and the number of tables occupied by customers at each minute, with which we can calculate summary statistics (see Fig. 2). Starting with 1 chef and increasing to 5 chefs, 200 trials of the simulation were run for each staffing level. The results calculated for each day were summarized using a mean and standard deviation.

Scenario 2 Results and Analysis:

The goal of scenario 2 is to gain an understanding of customer's queue times and down times, and to identify the optimal number of chefs given 5 tables to minimize wait times and maximize profit.

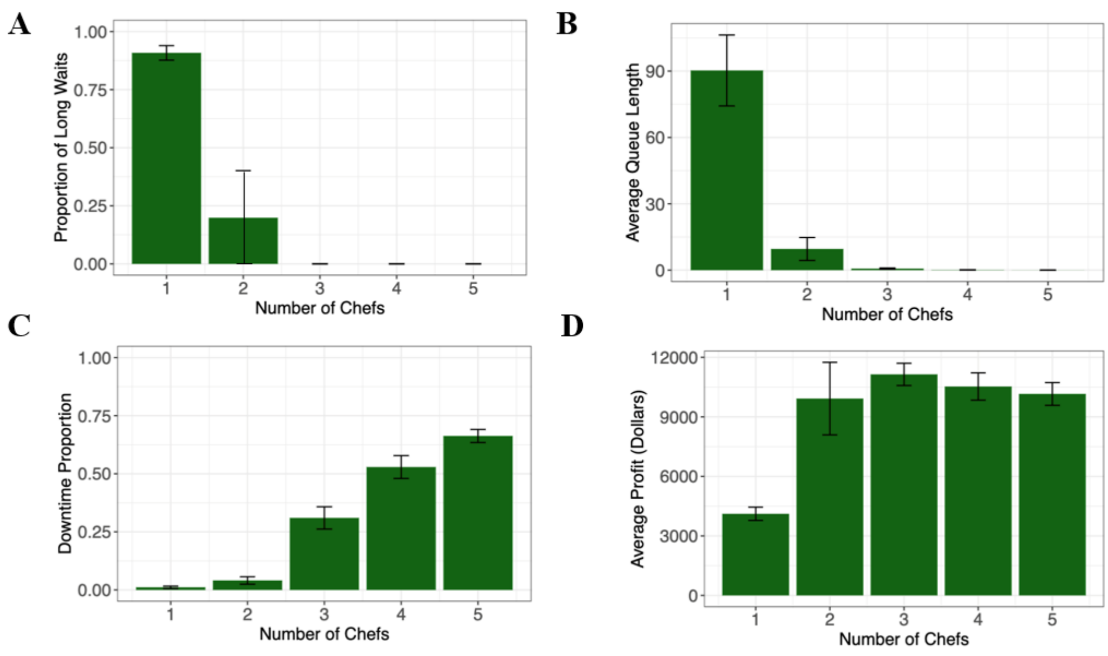

Figure 2. Analysis of Scenario 2 (variable chef, 5 tables). Customers arrive with rate λ = 30 per hour. Bar graphs show (A) the proportion of long waits, (B) average queue length, (C) proportion of downtime, and (D) average daily profit with different numbers of chefs. Data presented in mean ± standard deviation.

Because service rate is modeled according to $Exp\ (\lambda = 3L)$, when there is only 1 chef, 91% of customers need to queue for more than 30 minutes. With two chefs, the average proportion decreases by over 50%; however, there is still significant variability among the observations. With 3 or more chefs, almost no customers wait longer than 30 minutes (Fig. 2A). As expected, average queue length follows similar trends as the number of chefs increases (Fig. 2B). The proportion of downtime is correlated with the number of chefs, with the most downtime occurring with 5 chefs, where average downtime proportion reaches 0.6. In contrast to 4 workers, where there is an even balance between work and rest, having 5 chefs may suggest lost productivity. These trends are reflected in average profit across the number of chefs, where having 3 chefs maximizes average profit (Fig. 2D).

Overall, from these analyses, we conclude that in this scenario, having 3 chefs would be the most ideal scenario. Not only would the restaurant be maximizing profitability and productivity, but queue lengths and wait times would be at a minimum.

Scenario 3 Description

We introduce a more realistic third scenario, with updated assumptions about the arrival and service times. The restaurant still operates from 10am to 10pm; however, from 12pm to 2pm and 6pm to 8pm, 60 customers arrive every hour on average, while during other times, only 6 customers arrive each hour on average. These variable arrival rates correspond to lunch and dinner peak times. Customers' service times are now modeled with $S_k \sim Exp\ (\lambda = \ln\ (L + 1)$, where L is the number of chefs, so that additional chefs beyond the first two have a greater impact on service times. Additionally, to account for time spent at the restaurant which cannot be reduced by additional staffing, each customer will sit for a minimum of 45 minutes (the previously simulated service time is added to this minimum). To account for this change in the profit calculation, the cost for adding an additional table is set at $80 per table.

We first modeled a new distribution of arrival time that is closer to the real-world situation with increased customer arrival rates during lunch and dinner. The realistic arrival time function specified the parameter of peak time periods, and peak/baseline arrival rate. Using these updates models for arrival and service times, we used the same iterating function as in scenario 2 to simulate numerous days for a variety of configurations. Chefs varied from 1 up to 5, and tables varied from 1 up to a maximum of 101 in increments of 10. For each of these 55 scenarios, 100 trials of the simulation were run. The results were aggregated in a similar fashion to scenario 2.

Scenario 3 Results and Analysis

With the implementation of peaks in customer arrivals, a large majority of each day's customers are arriving for either lunch or dinner (Fig. 3A). This makes it much more difficult to have optimal utilization of the tables and chefs in the restaurant. Even for the configurations which yield top 5 average profits (Fig. 3C), the restaurant's tables are less than half-occupied ("Downtime") for 50% of the time each day (Fig. 3E). When we try to optimize for minimum downtime in the restaurant, a substantial queue develops during peak times, leading to profit loss. For the highest profit configuration, only 4.1% of customers needed to wait for more than 30 minutes, and within the top 5, the highest of these proportions was for a configuration of 2 chefs and 61 tables, where about 11.3% of customers had long waits (Fig. 3F).

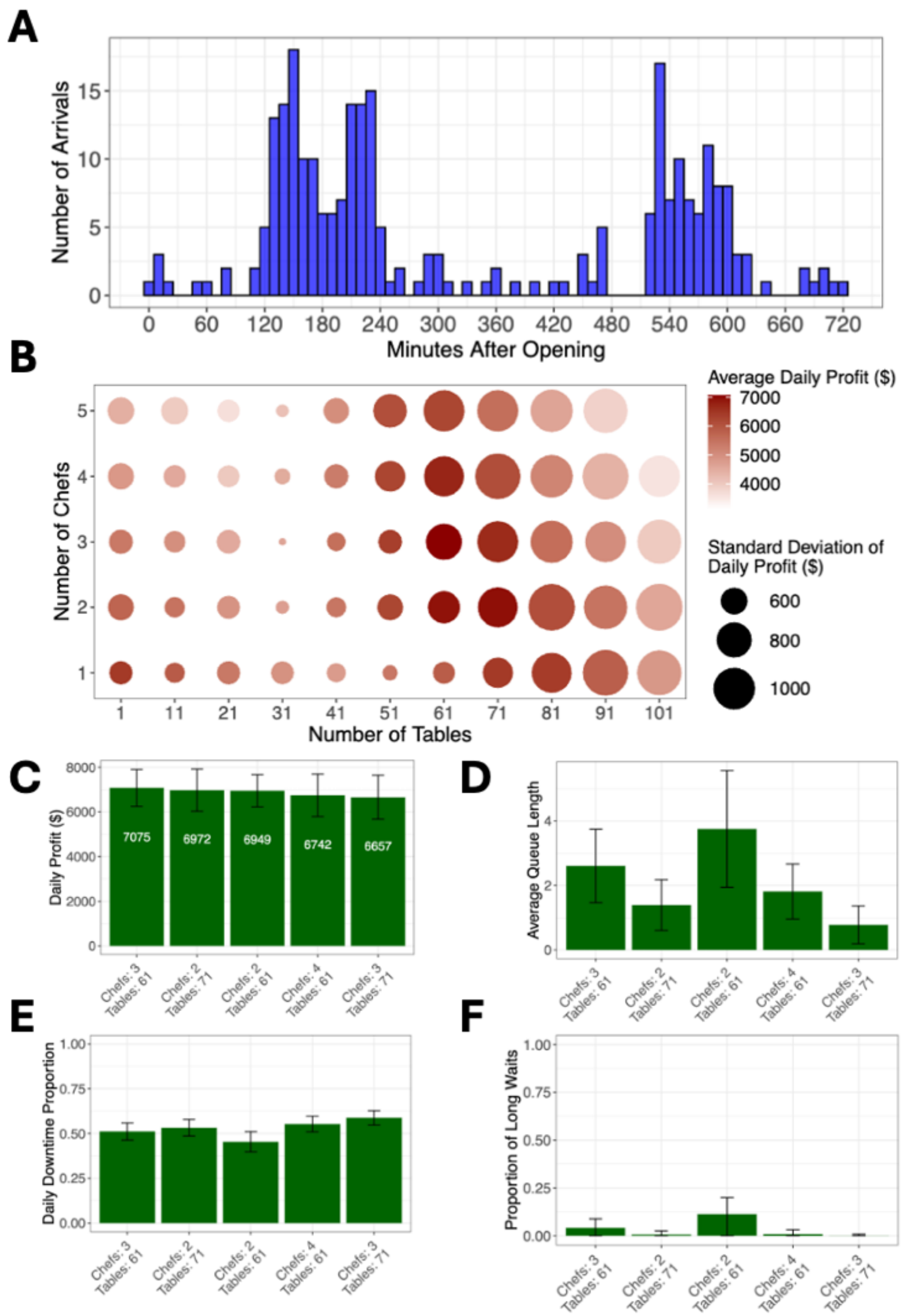

Figure 3. Analysis of scenario 3 (variable chef, variable tables). (A) Distribution of customer arrivals during a typical day. (B) Profit with different number of tables (x axis) and chefs (y axis). Dot color indicates mean profit and size represents standard deviation. The top five profitable combination of table were selected for visualization of (C) daily profit, (D) queue length, (E) proportion of downtime, and (F) proportion of long waits. Data presented in mean ± standard deviation.

Unsurprisingly, that configuration also had the longest average queue length (Fig. 3D). Overall, the average queue length across these simulated days was highly variable. Finally, when we examine profits of the restaurant configurations outside of the top 5, we find an interesting pattern of mean profits and standard deviations (Fig. 3B). Average profits are higher as it gets closer to the configuration of 3 chefs and 61 tables. Profits are lower until the number of tables shrinks to 21, which is when smaller numbers of tables are better again. When the restaurant has more than 51 tables, the daily profit is highly variable. Finally, under our current assumptions, the restaurant is always profitable: even the worst configuration has an average daily profit of $3136.50.

Final Recommendation

From scenario 3, which most reflects real-life restaurant conditions, we can make several recommendations by comparing the top 5 chef-table combinations (Fig. 3).

For a restaurant that would like to prioritize customer satisfaction, a good option would be to have 3 chefs and 71 tables. In this scenario, average wait time is low, as it has the shortest queue lengths. Furthermore, the standard deviation of queue length is smallest, which means that customers can be confident that the wait will not be too long, even on a busy day. One drawback is that among the top 5 choices, this option generates the lowest profit while experiencing a slightly higher average downtime. However, because the average downtime is still close to 50%, there is a balance between overworking staff and inefficient utilization. Overall, the benefits to a restaurant that prioritizes customer satisfaction may extend beyond wait times: if people enjoy their experience at a restaurant, they may recommend the place to others, which in the long term may increase arrival rates.

Alternatively, to maximize profitability, the best option would be to have 3 chefs and 61 tables. This configuration generates the highest average profit with medium downtime, at the cost of longer queues. Despite this, the proportion of long waits is still relatively low. Additionally, having a longer queue may not be inherently negative: if there are queues outside of a restaurant, people passing by may be drawn in by their curiosity, increasing the number of arrivals. Nevertheless, in scenarios like these, it would still be important to examine waiting times to ensure continual customer satisfaction.

Future Work and Limitations

One limitation to this project is that our analyses assume that there is only one customer at each table when realistically, restaurants have a variety of different sized tables. Another limitation is that when calculating profit, the only costs considered are the amount added for additional tables and chefs. In the real world, restaurants must consider other fixed and variable costs such as rent, utilities, costs of supplies and food, etc. Furthermore, our analysis doesn't consider what happens to customers who experience an especially long wait time. Although our simulation penalizes long wait times with decreased profit, it fails to consider the scenario in which people may leave or give up waiting. Lastly, we did not track whether the chefs are working on another task when we started our service time. It is likely in our simulation that the chef worked on multiple services at the same time, which is unrealistic.

Further research can examine these areas that went unaddressed. Specifically, future work can explore how a restaurant may optimize operations when there are different sized tables, when there are other fixed and variable costs, when customers leave the queue, and when other distributions, such as the normal distribution for spending, are considered for consumer behavior.

Citation: https://restaurant.org/research-and-media/research/research-reports/state-of-the-industry/#:~:text=Get%20the%20Report,were%20not%20profitable%20last%20year.