A Relativist's Toolkit
Problems

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May 23, 2021

Chapter 1

Fundamentals

1. **The surface of a two-dimensional cone is embedded in three-dimensional flat space. The cone has an opening angle of 2α. Points on the cone which all have the same distance r from the apex define a circle, and ϕ is the angle that runs along the circle.**

   (a) **Write down the metric of the cone, in terms of the coordinates r and ϕ.**

   The opening angle of the cone relates the height and the raduis of the cone as:

   $$2\alpha = 2\tan^{-1}(\frac{r}{h})$$
   $$\frac{r}{h} = \tan\alpha$$

   (b) **Find the coordinate transformation $x(r, \phi_))$, $y(r, \phi)$ that brings the metric into the form $ds^2 = dx^2 + dy^2$. Do these coordinates cover the entire two-dimensional plane?**

   (c) **Prove that any vector parallel transported along a circle of constant r on the surface of the cone ends up rotated by an angle β after a complete trip. Express β in terms of α.**