Misner, Thorne and Wheeler's Gravitation Problems

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Chapter 1

Chapter 2

Chapter 3

The Electromagnetic Field

1. **Derive equations:**

$$\| F^{\alpha}_{\beta} \| = \begin{Vmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{Vmatrix}$$

and

$$\| F_{\alpha\beta} \| = \begin{Vmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{Vmatrix}$$

for the components of Faraday by comparing:

$$dp^{\alpha}/d\tau = eF^{\alpha}_{\beta}u^{\beta}$$

with

$$\frac{d\mathbf{p}}{d\tau} = \frac{1}{\sqrt{1-\mathbf{v}^2}}\frac{d\mathbf{p}}{dt} = \frac{e}{\sqrt{1-\mathbf{v}^2}}(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = e(u^0\mathbf{E} + \mathbf{u}\times\mathbf{B})$$

$$\frac{dp^0}{d\tau} = \frac{1}{\sqrt{1-\mathbf{v}^2}}\frac{dE}{dt} = \frac{1}{\sqrt{1-\mathbf{v}^2}}e\mathbf{E}\cdot\mathbf{v} = e\mathbf{E}\cdot\mathbf{u}$$

and by using definition:

$$F_{\alpha\beta} = \eta_{\alpha\gamma}F^{\gamma}_{\beta}$$

1 represents the four following equations:

$$\frac{dp^0}{d\tau} = e[F^0_0u^0 + F^0_1u^1 + F^0_2u^2 + F^0_3u^3]$$

$$\frac{dp^1}{d\tau} = e[F^1_0u^0 + F^1_1u^1 + F^1_2u^2 + F^1_3u^3]$$

$$\frac{dp^2}{d\tau} = e[F^2_0 u^0 + F^2_1 u^1 + F^2_2 u^2 + F^2_3 u^3]$$

$$\frac{dp^3}{d\tau} = e[F^3_0 u^0 + F^3_1 u^1 + F^3_2 u^2 + F^3_3 u^3]$$

using:

$$\frac{dp^0}{d\tau} = \frac{1}{\sqrt{1-\mathbf{v}^2}} \frac{dE}{dt} = \frac{1}{\sqrt{1-\mathbf{v}^2}} e\mathbf{E}$$