Chapter 1 Problems

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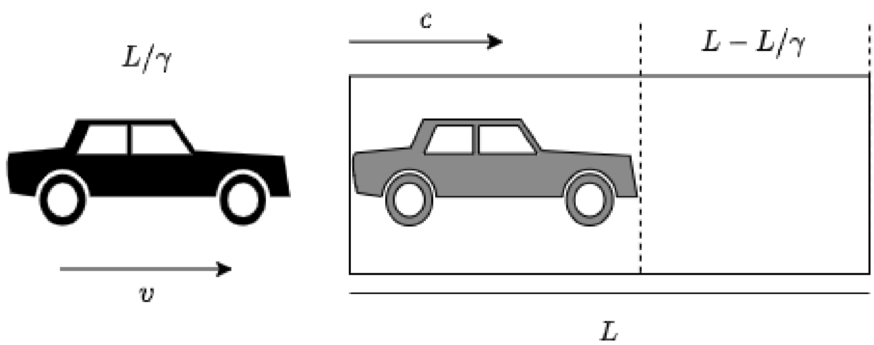

Figure 1: The car and garage paradox in the doorman's frame.

1. **Car and Garage Paradox. The lack of a notion of absolute simultaneity in special relativity leads to many supposed paradoxes. One of the most famous of these involves a car and a garage of equal proper length. The driver speeds toward the garage, and a doorman at the garage is instructed to slam the door shut as soon as the back end of the car enters the garage. According to the doorman, "the car Lorentz contracted and easily fitted into the garage when I slammed the door". According to the driver, "the garage Lorentz contracted and was too small for the car when I entered the garage." Draw a spacetime diagram showing the above events and explain what really happens. Is the doorman's statement correct? For definiteness, assume that the car crashes through the back wall of the garage without stopping or slowing down.**

   The doorman's statement is correct but the doorman and driver disagree on the order of events. In the doorman's frame, he sees the end of the car coincident with him and closes the door. In his frame, the car is Lorentz contracted so that it has not broken through the wall yet. To be a nice guy, he decides to send a message to the driver telling them to stop the car so that the car does not break through the wall.

   Let L be the proper length of the car and garage, v be the velocity of the car in the doorman's reference frame, and γ be the Lorentz factor.

   In the doorman's frame, he observes the car to be L/γ. When he closes the door, the car has not crashed through the end of the garage and the distance from the front of the car to the end of the garage is:

   $$\Delta L = L(1 - \frac{1}{\gamma})$$

   as shown in 1. Assume for simplicity that there is a detector at the end of the garage that will absorb the signal from the doorman. If the signal has been absorbed it is green and if not it is red. The driver simply observes the detector when he passes to see if he should stop or not.

When the doorman emits the light signal to tell the driver to stop the car, the light takes a time:

$$\Delta t_c = L/c$$

to reach the end of the garage. The car however takes:

$$\Delta t_v = \frac{\Delta L}{v} = \frac{L}{v}(1 - \frac{1}{\gamma})$$

If $\Delta t_c \leq \Delta t_v$ the car stops but if $\Delta t_c > \Delta t_v$ the car crashes through the end of the garage. Assume:

$$\begin{aligned}
\Delta t_c &> \Delta t_v \\
\frac{L}{c} &> \frac{L}{v}(1 - \frac{1}{\gamma}) \\
\frac{v}{c} &> (1 - \frac{1}{\gamma}) \\
\frac{v}{c} &> (1 - \sqrt{1 - \frac{v^2}{c^2}}) \\
\sqrt{1 - \frac{v^2}{c^2}} &> 1 - \frac{v}{c} \\
\sqrt{(1 - \frac{v}{c})(1 + \frac{v}{c})} &> 1 - \frac{v}{c} \\
\sqrt{1 + \frac{v}{c}} &> \sqrt{1 - \frac{v}{c}}
\end{aligned}$$

This is always true, so the car always crashes through the end of the garage before the doorman can tell the driver to stop. This resolves the apparent paradox.

However, it is clear that the order of events is not the same. As mentioned in the problem definition, the driver sees the garage Lorentz contracted so he crashes through the garage and then observes the door close. In the doorman's frame the order of events is inverted.

In hindsight, this is obvious from constructing a correct Minkowski diagram as shown in 2.

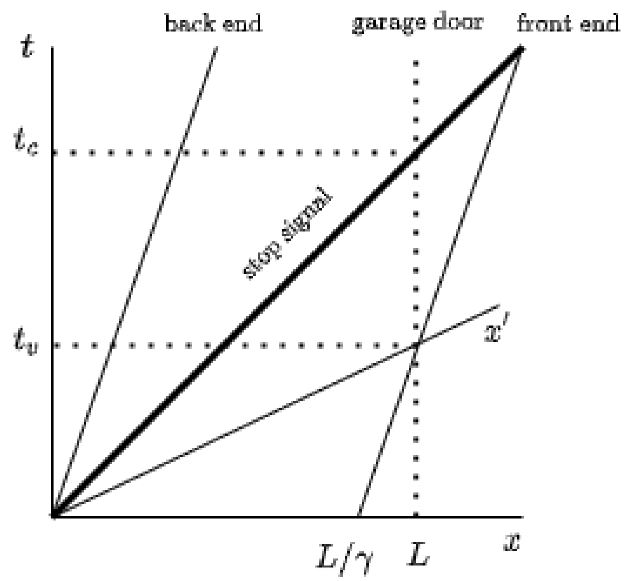

Figure 2: Minkowski diagram in the reference frame of the doorman