Chapter 6 - The Fundamental Law of Active Management

Section A: Alpha, Active Risk and Information Ratio

Every portfolio has a benchmark against which the portfolio's performance would be evaluated. For example, the S&P500 index is usually the benchmark for a long-only U.S. domestic large cap equity portfolio; cash (LIBOR rates) could be the benchmark for a market neutral alternative portfolio. A portfolio's **alpha,** or called active return**,** is often defined as the return difference between the portfolio and its benchmark:

$$\alpha_P = r_P - r_B.$$

While the alpha definition is equivalent to the performance difference within the whole holding period, we usually annualize this metric and get the average magnitude of the portfolio's outperformance/underperformance on an annual basis.

Example A1 Assume a portfolio's monthly returns are $\{r_P(t)\}_{t=1}^T$, and its benchmark's monthly returns are $\{r_P(t)\}_{t=1}^T$. The portfolio's monthly alpha α_m can be calculated as follows:

$$\alpha_m = [\prod_{t=1}^T \frac{1+r_P(t)}{1+r_B(t)}]^{1/T} - 1\,,$$

and the portfolio's annualized alpha α_y has the form: $\alpha_y = (1+\alpha_m)^{12} - 1$.

Active risk, also called tracking error, measures the volatility of return differences between the portfolio and its benchmark. The allowed range of active risk is usually specified in advance (ex ante) by the investment's mandate, and portfolio managers take the active risk specification as a key constraint input in portfolio design and construction. The realized active risk would be constantly monitored in risk management as portfolio goes alive.

Example A2 The monthly active risk v_m of the portfolio in *Example A1 equals* the standard deviation of the monthly return differences $\{r_P(t) - r_B(t)\}_{t=1}^T$, and its annualized active risk is $v_y = \sqrt{12}\,v_m$.

A portfolio's **information ratio**, denoted as IR, is defined as the ratio of (annualized) alpha to (annualized) active risk. Informatio ratio is a benchmark-aware performance metric which measures the portfolio's active-risk-adjusted active performance. For the portfolio in the above example,

$$IR = \frac{\alpha_y}{v_y}.$$

Note: it's very important to use annualized alpha and annualized active risk to calculate information ratio under the commonly-accepted convention. Just like Sharpe ratio, monthly information ratio is $\frac{1}{\sqrt{12}}$ *of its annualized counterpart. For clear communications, when we use*

Sharpe ratio or information ratio, we implicitly refer to their annualized measures.

Caculations of alpha, active risk and information ratio given a portfolio's realized returns provide ex post active performance measures, i.e., measures for what has already happened. We use these ex-post measures to evaluate:

- whether the manager has outperformed the benchmark;
- whether the manager has taken too much risk, i.e., when the active risk exceeded the pre-specified limit; or the manager took little risk, which opens the question on the fee level of the portfolio;
- whether the manger has achieved success in managing the portfolio in the peer group.

The book *Active Portfolio Managment* by Grinold and Kahn offers a table showing the approximate distribution of empirically observed (before-fee) information ratios.

Table A: Approximate Empirical Information Ratio Distirbution

Percentile	Information Ratio
90	1.0
75	0.5
50	0.0
25	-0.5
10	-1.0

As you can see above, ***a top-quartile manager has an information ratio of 0.5.***

Section B: The Fundamental Law of Active Management

In the previous section, we introduced the basics for calculating ex-post information ratio. The fundamental law of active management provides a concise formula for a manager to make a forecast on the approximate (ex-ante) information ratio of his to-be-constructed portfolio.

The fundamental law of active management is in the following formula:

$$IR = IC \cdot \sqrt{BR},$$

where

- IR is the portfolio's (or the strategy's) information ratio.
- IC is the manager's **information coefficient**, which measures the manager's skill. It's calculated as the correlation between the manager's forecasts and actual outcomes.
- BR is the strategy's **breadth**, which is defined as the number of independent active forecasts the manager makes in a year.

The law breaks down the information ratio to two parts: the skill and the breadth. Note: the formula is approximately true because it has certain other assumptions involved.

The immediate insight from the fundamental law of active management is: **it's important to play often (high BR) and to play well (high IC).**

(The other take-away is play often when not play well is the worst investment practice)

Example B1 In a gambling game, for each play with an initial bet of \$1, the casino wins \$1 with 52% chance and loses \$1 with 48% chance.

a. What's the expected return for the casino on each play ?

$$\frac{\$1}{\$1} \times 52\% + \frac{-\$1}{\$1} \times 48\% = 4\%$$

b. What's the standard deviation of the return?

$$\sqrt{(100\% - 4\%)^2 \times 52\% + (-100\% - 4\%)^2 \times 48\%} = 0.9992$$

c. The information ratio of each play for the casino is: $\frac{4\%}{0.9992} = 0.04$.

d. What's an approximate information coeffcient for the given probability distribution of 52% winning and 48% losing?

Imagine there exists a game with binary outomes of 1 and -1, the casino can correctly forecast it with 52% of time. Say $x(t)$ is the casino's forecast at time t and it turns out the real outcome is $y(t)$. Let's assume x and y both have mean 0 and standard deviation of 1. By definition of informatio coefficient, it's the correlation between the two time series. If the casino makes N bets, then:

$$IC = \frac{Cov(x, y)}{\sigma_x \sigma_y} = Cov(x, y) = \frac{1}{N} \sum_{t=1}^{N} x(t) \cdot y(t) = \frac{1}{N} [N_1 - (N - N_1)] = \frac{2N_1}{N} - 1,$$

where N_1 is the number of correct bets out of total Nbets. On the right hand side of the above formula, we know $\frac{N_1}{N} \approx 52\%$ when N is large enough, which gives us $IC \approx 0.04$. (IC is approximately equal to 2 times hit ratio minus 1)

e. From d we know the value of the equivalent IC for the casino in the gambling game. Let's apply the law, since there is only one play, $BR = 1$. Therefore, $IR = IC \times \sqrt{BR} \approx 0.04 \times \sqrt{1} = 0.04$.

Example B2 Suppose a manager follows 400 stocks and takes a position on these on average of once per year. The manager's information coefficient is 0.03. (what's the implied hit ratio?) From the law, this manager would achieve information ratio of $0.6 = 0.03 \times \sqrt{400}$.

Additivity

The fundamental law is additive in the squared information ratios. Suppose there are two classes of stocks. For Class 1, you have breadth BR_1 and a skill level of IC_1; for Class 2, your breadth is BR_2 with a skill level of IC_2. Assuming optimal implementaion of the alphas across the two classes (it's a necessary condition for the fundamental law to hold), the information ratio for the aggregate will be:

$$IR^2 = IC_1^2 \times BR_1 + IC_2^2 \times BR_2.$$

Example B3 Given information in *Example B2*, suppose the same manager in addition to make stock selection calls, he makes a quarterly forecast on Japanese currency Yen, for which the information coeffcient is 0.1. Then we can calculate the manager's information for trading Yen is $0.2 = 0.1 \times \sqrt{4}$. Combined with his stock calls, we can apply the additivity property of the fundamental law and calculate the manager's overall information ratio will be the squared root of the sum of the squared information ratios for stock calls and Yen calls, i.e., $\sqrt{0.6^2 + 0.2^2} \approx 0.63$.

Example B4 Assume we are based in U.S., and we are managing an international portfolio and we invest in the four contries: the United States, Japan, Germany, and the United Kingdom. What's the portfolio's approximate (ex-ante) information ratio given the following assumptions:

1. In each year, there are three currency bets availabe to us; we revise our currency positions each quarter. Our IC for currency bets is only 0.04.
2. We actively make forecasts on a quarterly basis for the largest 50 stocks in each market. Our IC for stock selection in the four markets are 0.02,0.01,0.01,0.01 respectively.

Assumptions for the Fundamental Law

The derivation of the fundamental law is by solving an utility optimization problem where the distribution of ex-ante alpha and active risk are conditional on signals. It assumes all signal added-values are of the equal magnitude. We want to emphasize the three key assumptions underpinning the law: $IR = IC \cdot \sqrt{BR}$.

Assumption 1*: The manager has an accurate enough measure of his or her own skills and exploits information in an optimal way*

This is the most crucial assumption. They need a precise idea of what they know and more importantly what they don't know. Moreover, they need to know how to turn their ideas into portfolios and gain the benefits of their insights.

Assumption 2*: All forecasts have the same level of skill*

If not the case, we need to apply the additivity property of the law and re-calculate the overall information ratio.

Assumption 3*: All forecasts should be independent to each other*

A forecast should not be based on the same source of information which has already been utilized in a separate forecast. For example, suppose our first forecast is on Emerging Market currency performance and our second forecast is on Emerging Market equity performance. The two forecasts are highly correlated given the observation that EM currency performance always accounts for a large portion of EM equity performance. In this case, the two forecasts are definitely not independent to each other. An example of independent forecasts would be quarterly adjustment of EM currency views because each view is based on a separate set of information (a new set of info for a new quarter).

In situation of stock selection when you make forecasts on a firm-by-firm basis, it's possible you may like firms in a particular industry, or firms with a specific factor exposure. In this case, the number of bets you are makings doesn't equal the number of stocks you did research on, instead, you may only make a limited number of factor bets.

Also each forecast should be based on new signal/info. For example, if you evaluate Japanaese Yen only in the beginning of each year, however, you rebalance the yen position to a target four times a year (quarterly basis). Then you only make 1 forecast each year for Yen (and execute the forecast 4 times a year).

When two forcasts are dependent and are believed to embed the same level of skill of IC. We know the two sources of information (signals) for the two forecasts are dependent, i.e., part of the second source's information will reinforce what we have observed from the first source, and the remaining part is new and incremental information. As one can imagine, the greater the dependence between the two information sources, the lower the value of the incremental information. If ρ measures the correlation between the two information sources, then the skill level of the combined sources Ii.e. we think the two forecasts as a combined one forecast) has the value:

$$\widetilde{IC} = IC \cdot \sqrt{\frac{2}{1+\rho}}.$$

When there is no correlation between the two sources ($\rho = 0$), then the combined forecast is equivalent to two independent forecasts with the same skill of IC. As ρ increases toward 1, the value of the second source diminishes. (Note: $\rho \geq 0$ because the two signals are all objectively existent, and they should not conflict with each other). Diversification among distinct signals adds value by increasing IC.

Summary

We have shown how the information ratio of an acive manager can be explained by two components: the skill (IC) of the investment manager and the breadth (BR) of the strategy. The fundamental law is a guideline, not an operational tool. Indeed, the law has shown its ability to make reasonable prediction on a portfolio's information ratio. Clear message: you must play often and play well to win at the investment management game. It takes only a modest amount of skill to win as long as that skill is deployed frequently and across a large number of different bets.